Probing Dihadron Fragmentation Functions and Twist-3 Parton Distribution Functions at CLAS12

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For the STAR Collaboration

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The Plan

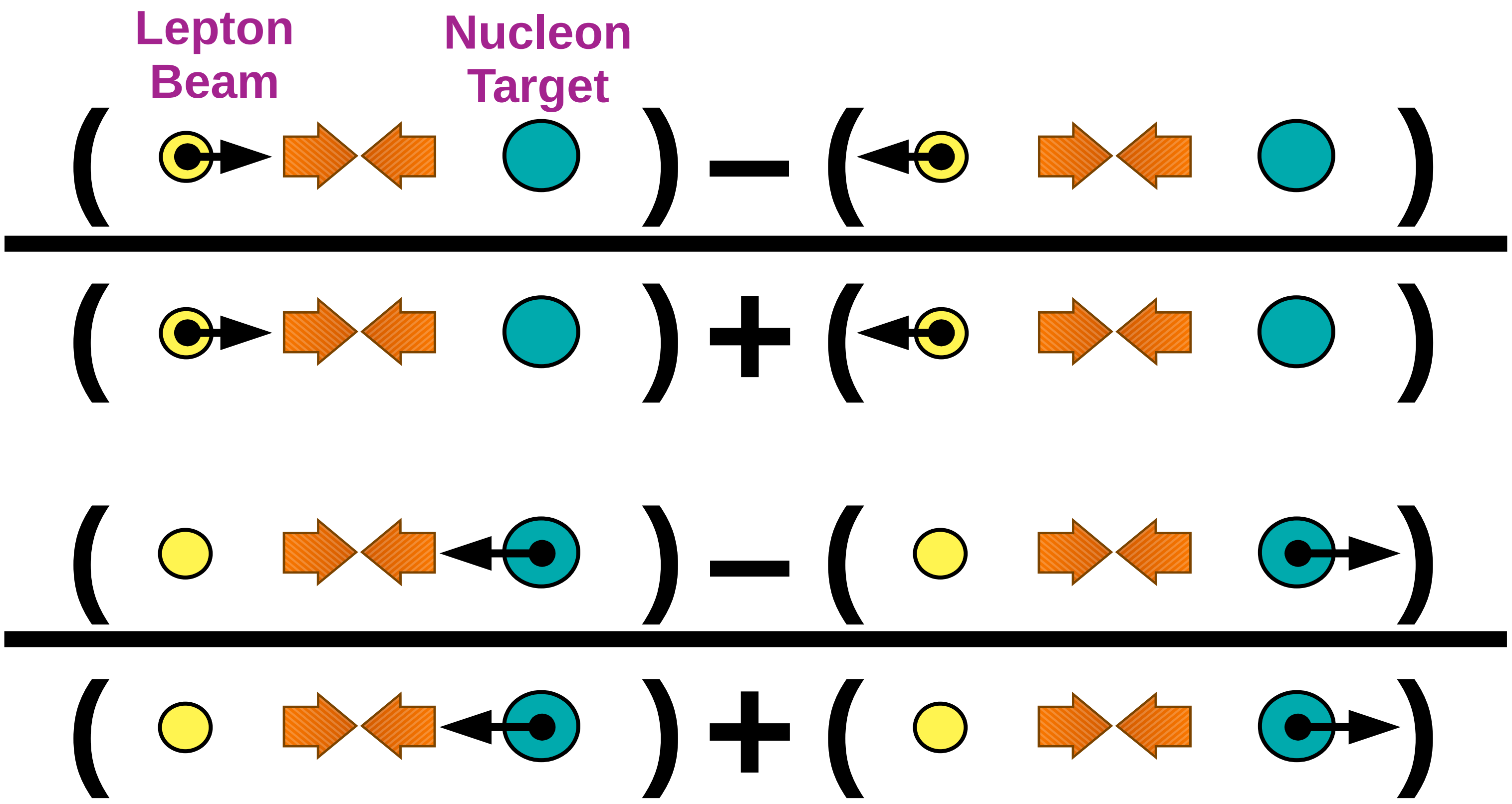
Measure **Longitudinal Spin Asymmetries** in **SIDIS Dihadron Production** to access **Dihadron Fragmentation Functions** and **Twist-3 Parton Distributions**

Beam Spin Asymmetry

$A_{LU} =$

Target Spin Asymmetry

$A_{UL} =$



Dihadron Production in SIDIS

$$\ell(l) + N(P) \rightarrow \ell(l') + h_1(P_1) + h_2(P_2) + X$$

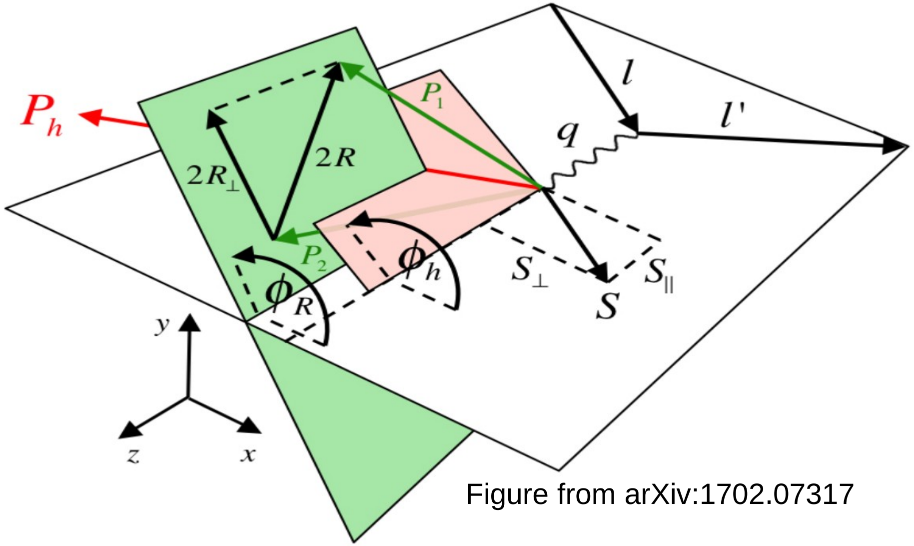


Figure from arXiv:1702.07317

Dihadron Spin asymmetries depend on momentum combinations P_h and R

Modulations in Φ_h and Φ_R are sensitive to different fragmentation and parton distributions

- Dihadron degrees of freedom:

$$P_h = P_1 + P_2$$

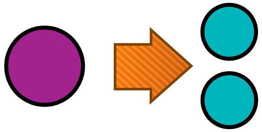
$$R = \frac{1}{2}(P_1 - P_2)$$

- Reaction plane is spanned by q and the incoming/outgoing lepton momenta
- Φ_h is the angle between the reaction plane and **plane spanned by P_h and q**
- Φ_R is the angle between the reaction plane and the **plane spanned by R and q**
- M_h denotes dihadron invariant mass

Dihadron Fragmentation Functions (DiFFs)

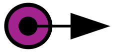
Unpolarized DiFF

D_1 =



Handedness / Helicity-Dependent DiFF

$G_1^{\perp}$ =



correlated with vector product of hadrons' transverse momenta

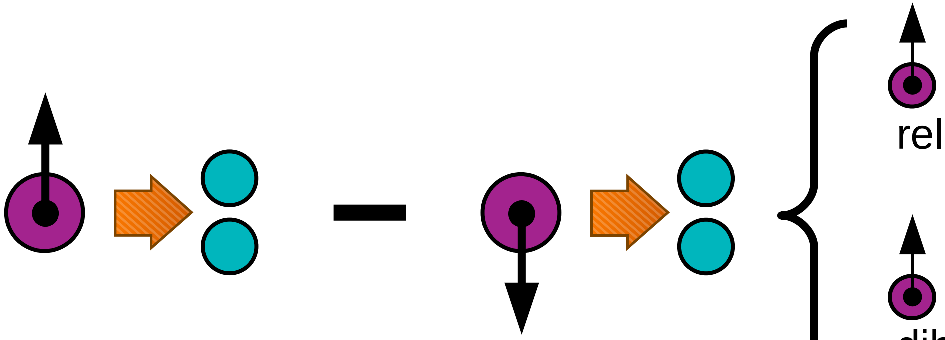
Interference Fragmentation Function

$H_1^{\sphericalangle}$

Collins-like DiFF

$H_1^{\perp}$

=



correlated with relative p_T of the pair

correlated with dihadron's total p_T

Dihadron Fragmentation Functions (DiFFs)

D_1 and $H_1^{\sphericalangle}$ – reasonably well-extracted

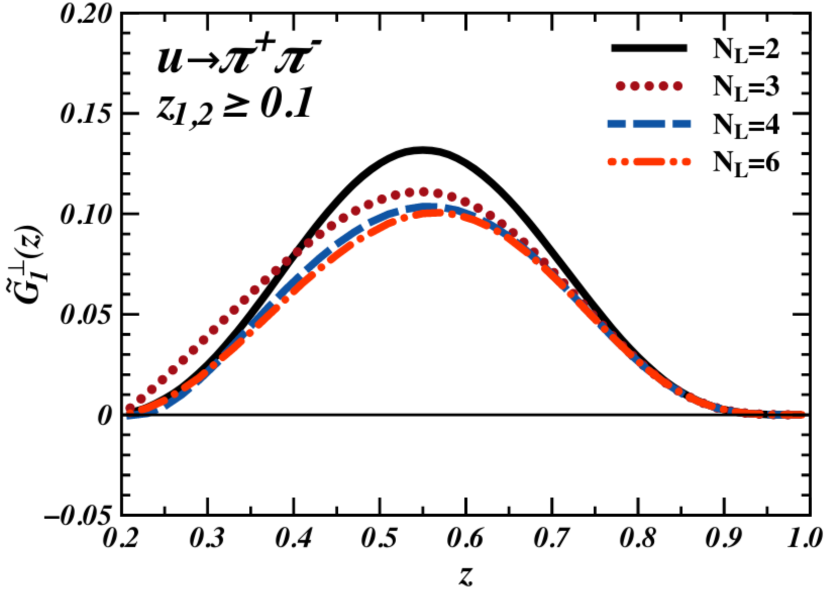
see backup slides, and, e.g.,
Phys.Rev. D85 (2012) 114023
arXiv:1202.0323

$H_1^{\perp}$ – dihadron analog of Collins Fragmentation Function

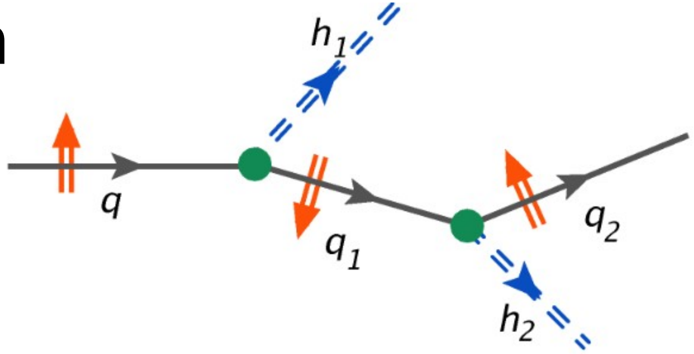
$G_1^{\perp}$ **– models exist, but not yet constrained by experimental data**

$G_1^{\perp}$ Predictions and Interpretation

$$\tilde{G}_1^{\perp}(z) \equiv \frac{1}{M_1 M_2} G_1^{\perp}(z)$$ (integrated over M_h)



Matevosyan, Kotzinian, and Thomas,
Phys.Rev. D96 (2017) no.7, 074010



- Calculations from the quark-jet hadronization framework show sizable $G_1^{\perp}$
- T-odd, chiral-even
  - → couples to chiral-even PDFs f_1 and g_1
- $G_1^{\perp}$ is sensitive to **spin-momentum correlations** in hadronization
- Recoiling quark q_1 in the hadronization chain can acquire **nonzero transverse polarization** via "wormgear" type splitting
- Net transverse polarization can transfer to an azimuthal correlation of the hadrons (Collins effect)

Accessing $G_1^\perp$ in Dihadrons from SIDIS

Accessible in the sin(Φ_h–Φ_R) modulation in longitudinal single spin asymmetries, weighted by $P_h^\perp$

$$\left\langle \frac{P_{h\perp}\sin(\phi_h-\phi_R)}{M_h}\right\rangle = \int d\sigma_{LU}\frac{P_{h\perp}\sin(\phi_h-\phi_R)}{M_h}$$

$$\langle 1\rangle = \int d\sigma_{UU}$$

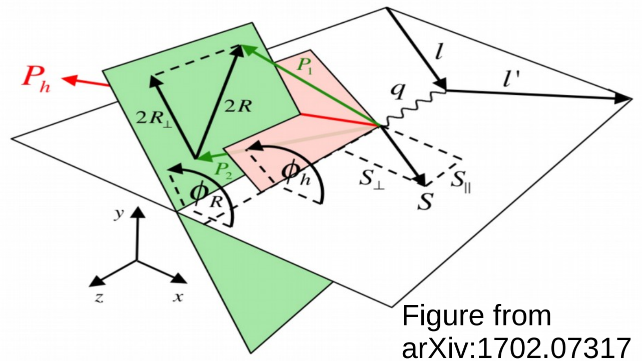


Figure from arXiv:1702.07317

Beam Spin Asymmetry → $G_1^\perp$ coupled to unpolarized PDF

$$A_{LU}^{\vec{\Rightarrow}}(x,y,z,M_h^2) = \frac{1}{M_h}\frac{\langle P_{h\perp}\sin(\varphi_h-\varphi_R)\rangle}{\langle 1\rangle} = \lambda_l\frac{C'(y)}{A'(y)}\frac{\sum_a e_a^2\,\boxed{f_1^a(x)}\,z\,\boxed{G_1^{\perp a}(z,M_h^2)}}{\sum_a e_a^2\;f_1^a(x)\;D_1^a(z,M_h^2)}$$

Target Spin Asymmetry → $G_1^\perp$ coupled to helicity PDF

$$A_{UL}^{\vec{\Rightarrow}}(x,z,M_h^2) = S_L\frac{\sum_a e_a^2\,\boxed{g_{1L}^a(x)}\,z\,\boxed{G_1^{\perp a}(z,M_h^2)}}{\sum_a e_a^2\;f_1^a(x)\;D_1^a(z,M_h^2)}$$

- Matevosyan, Kotzinian, and Thomas, Phys.Rev.Lett. 120 (2018) no.25, 252001
- Matevosyan, PoS DIS2018 (2018) 150

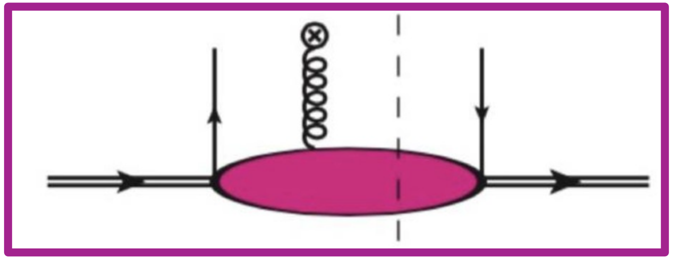
Twist-3 Parton Distribution Functions

Collinear PDFs
- **Twist-2**
- **Twist-3**

Quark Polarization

Nucleon Polarization	U	L	T
U	f_1		e
L		g_1	h_L
T		g_T	h_1

- Although twist-3 observables are suppressed by 1/Q, they are reasonably accessible at CLAS with $Q^2 \sim 1 - 6$ GeV^2
- Expressible in terms of multi-parton correlators (see backup slide)
- Fundamental to hadron mass generation and to Transverse Momentum Dependent PDFs (TMDs)
- $e(x)$ and $h_L(x)$ are T-even and chiral-odd



The $\sin(\Phi_R)$ modulation of A_{LU} and A_{UL} in dihadrons is sensitive to Twist-3 PDFs: $e(x)$ and $h_L(x)$

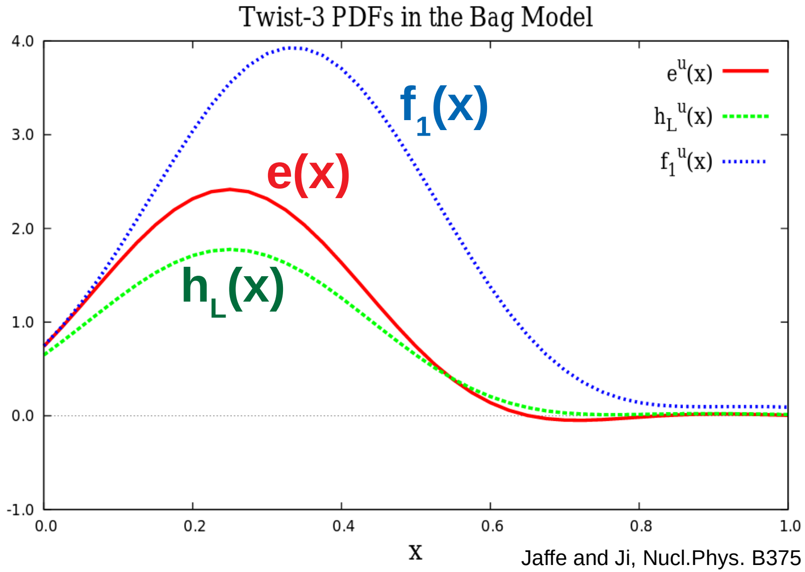
Twist-3 PDF Interpretations

e(x)

- Decomposable in terms of:
  - Unpolarized PDF $f_1(x)$ [twist-2]
  - Pure twist-3 part
- 1^{st} moment → pion-nucleon σ term, representing the contribution to the nucleon mass from the finite quark masses
- 2^{nd} moment → proportional to quark mass and number of valence quarks
- 3^{rd} moment → transverse polarization dependence of the transverse color-Lorentz force experienced by a struck quark, in an unpolarized nucleon

$h_L(x)$

- Decomposable in terms of:
  - Helicity PDF $g_1(x)$ [twist-2]
  - Wormgear TMD moment $h_{1L}^{\perp(1)}$ [twist-2]
  - Pure twist 3 part
- Related to the distribution of transversely polarized quarks in a longitudinally polarized nucleon



Jaffe and Ji, Nucl.Phys. B375 (1992) 527-560

Bacchetta and Radici, Phys.Rev. D69 (2004) 074026
Jaffe and Ji, Nucl.Phys. B375 (1992) 527-560
Efremov and Schweitzer, JHEP 0308 (2003) 006
Courtoy, arXiv:1405.7659
Burkardt, Phys.Rev. D88 (2013) 114502
Pereira, PoS (DIS2014) 231
Mulders, Tangerman, Nucl.Phys. B461 (1996) 197-237
Sirtl, PhD Thesis

Accessing Twist-3 PDFs via SIDIS Dihadrons

Beam Spin Asymmetry

$$F_{LU}^{\sin\phi_R} = -x\frac{|\boldsymbol{R}|\sin\theta}{Q}\left[\frac{M}{M_h}x\boxed{e^q(x)}\boxed{H_1^{\sphericalangle q}(z,\cos\theta,M_h)}+\frac{1}{z}\boxed{f_1^q(x)}\boxed{\widetilde{G}^{\sphericalangle q}(z,\cos\theta,M_h)}\right]$$

Target Spin Asymmetry

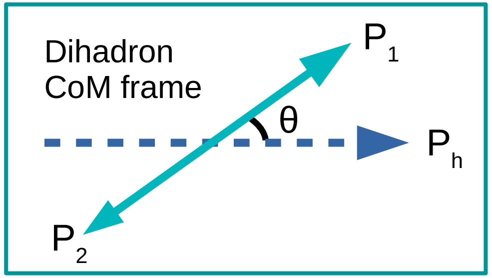
$$F_{UL}^{\sin\phi_R} = -x\frac{|\boldsymbol{R}|\sin\theta}{Q}\left[\frac{M}{M_h}x\boxed{h_L^q(x)}\boxed{H_1^{\sphericalangle q}(z,\cos\theta,M_h)}+\frac{1}{z}\boxed{g_1^q(x)}\boxed{\widetilde{G}^{\sphericalangle q}(z,\cos\theta,M_h)}\right]$$

Twist-3 PDFs

IFF

PDFs

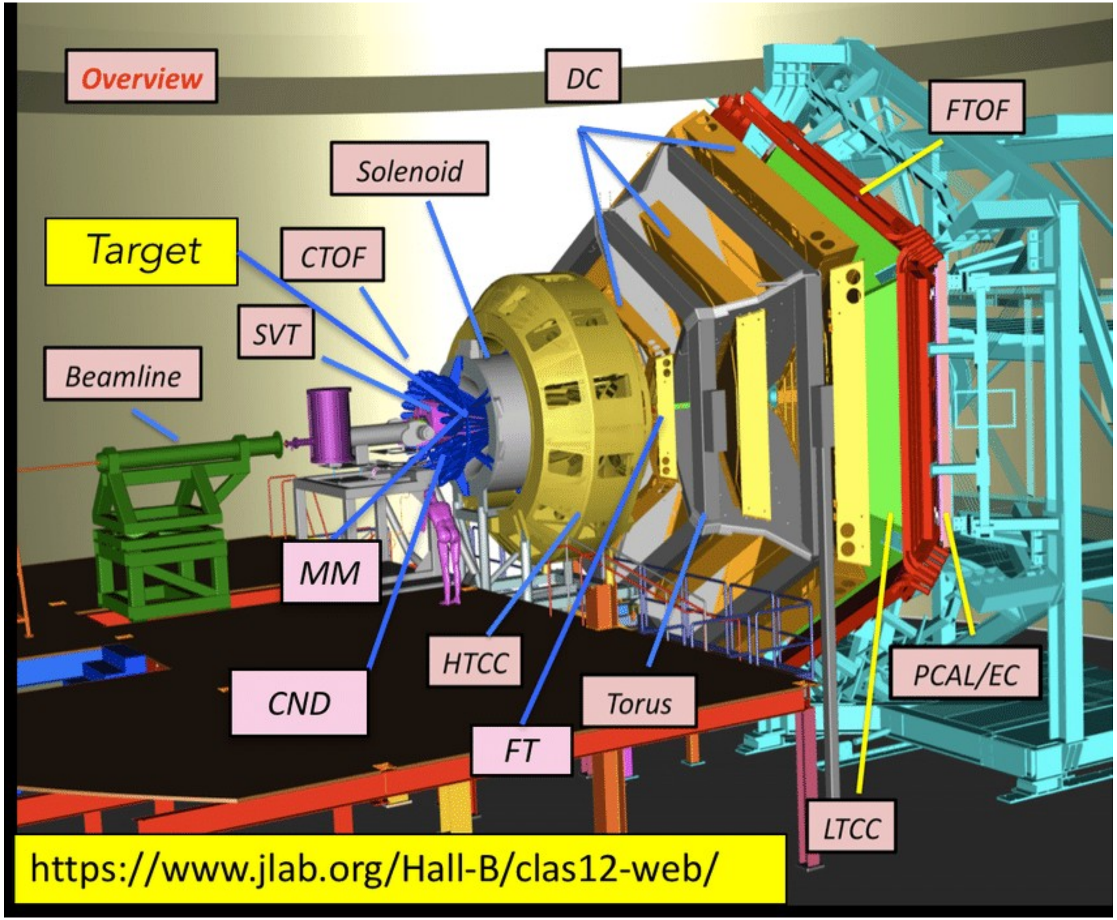
Twist-3 DiFFs (likely small, see backup slides)



Twist-3 PDFs are accessible in the sin(Φ_R) modulation in longitudinal single spin asymmetries

Pereira, PoS (DIS2014) 231
Bacchetta and Radici, Phys.Rev. D67 (2003) 094002
Bacchetta and Radici, Phys.Rev. D69 (2004) 074026

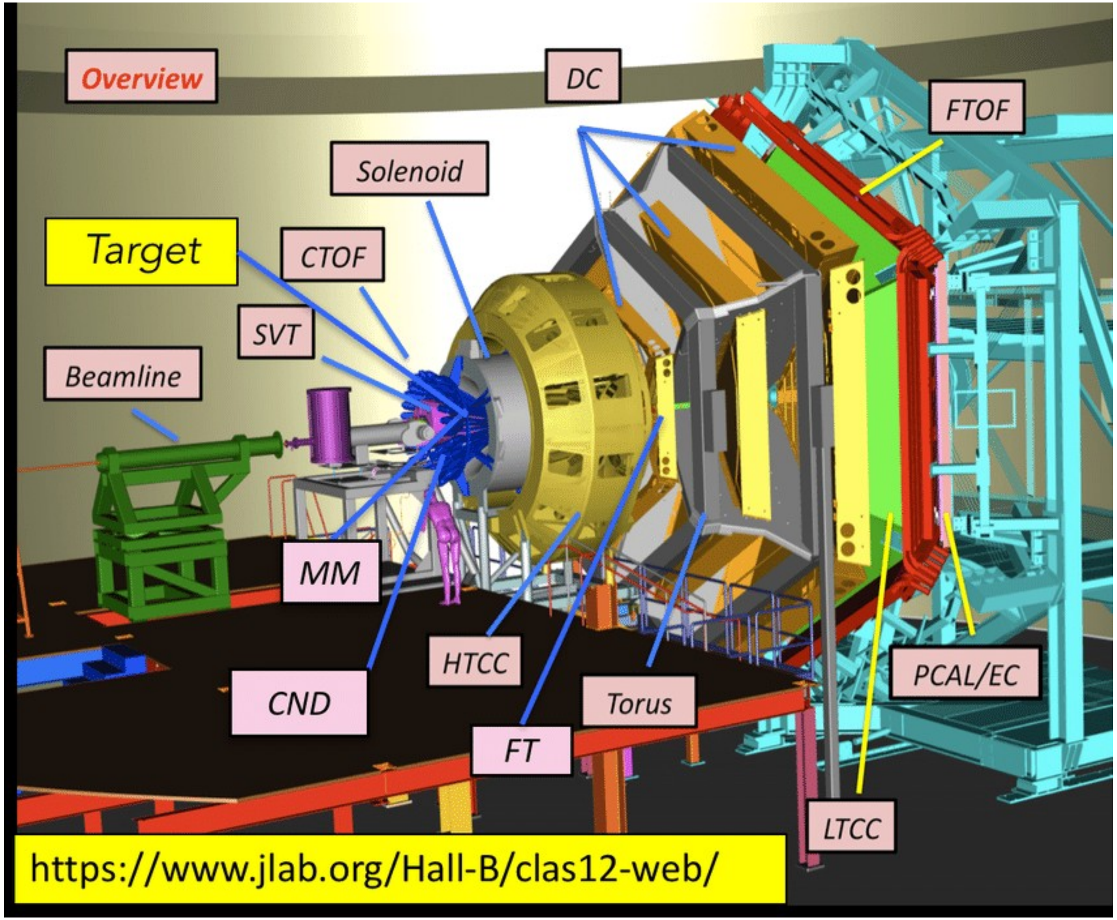
CLAS12 Experiment at Jefferson Lab



- Electron beam from CEBAF
  - ~85% Longitudinal Polarization
  - E = 10.6 GeV
- Fixed Nuclear Target
  - Liquid hydrogen H_2
  - Liquid deuterium D_2

  For flavor separation
  - Plans to run with polarized targets in the near future
    - Longitudinally polarized solid ammonia NH_3 and ND_3
    - Transversely polarized deuterium hydride HD (HD-Ice)

CLAS12 Experiment at Jefferson Lab



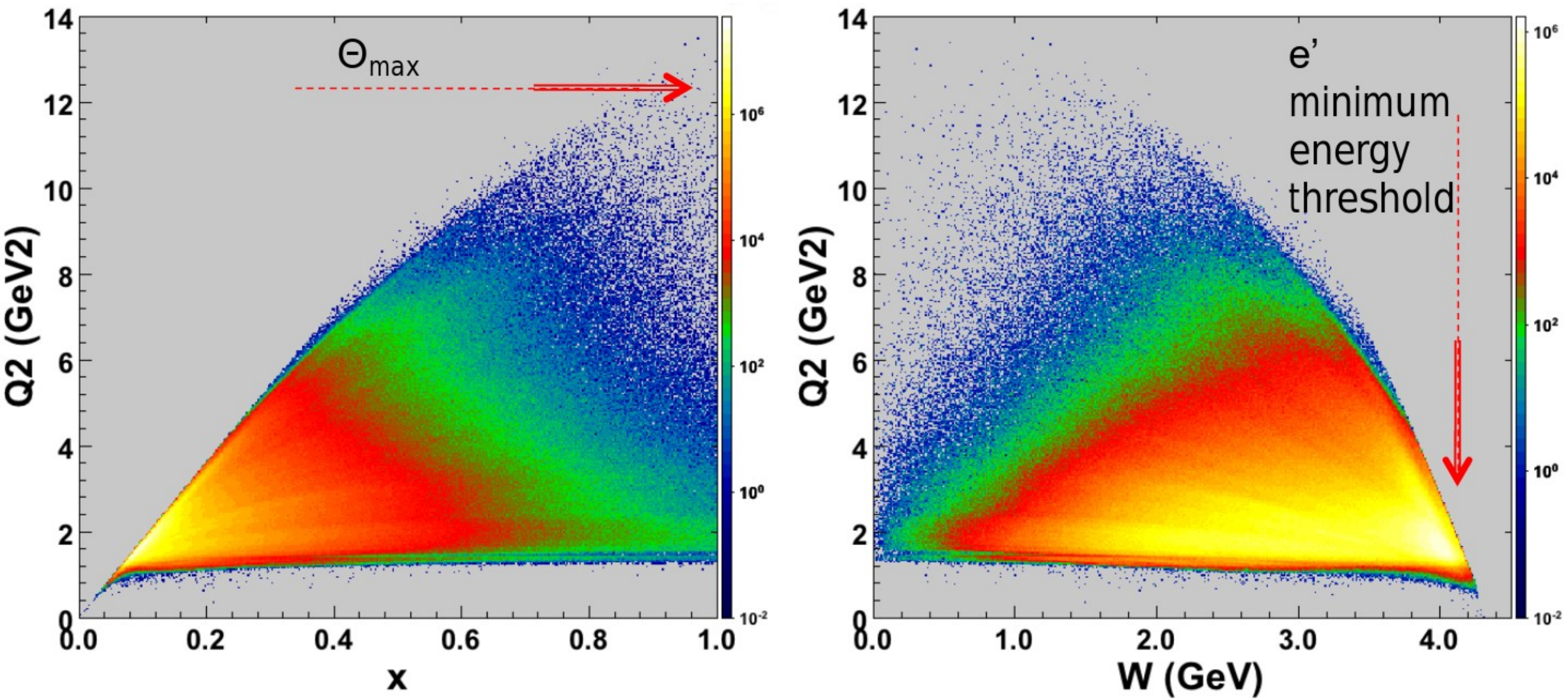
Central Detector:

- Solenoid Magnet
- Barrel Silicon Tracker (SVT)
- Micromegas Vertex Tracker (MM)
- Central Time of Flight (CTOF)
- Central Neutron Detector (CND)

Forward Detector

- Torus Magnet
- Drift Chamber (DC)
- Forward Time of Flight (FTOF)
- High-threshold Cherenkov Counter (HTCC)
- Low-threshold Cherenkov Counter (LTCC)
- Ring Imaging Cherenkov Detector (RICH)
- Preshower + Electromagnetic Calorimeter (PCAL/EC)
- Forward Tagger (FT)

Kinematic Reach at CLAS12



CLAS12 Data Acquisition Status

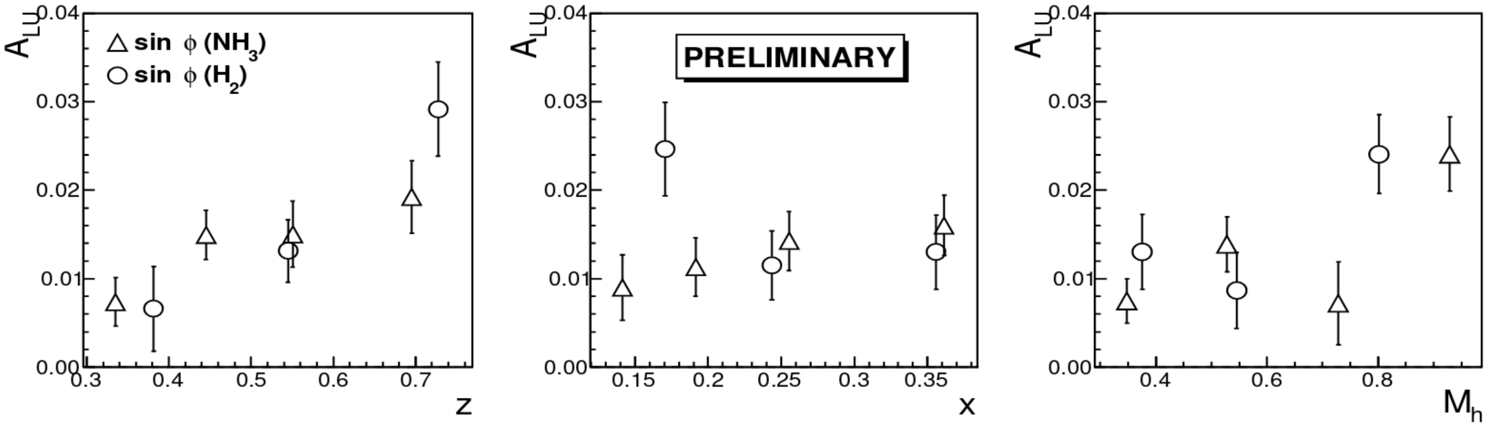
Run Group	Target	Period	Observable	Sensitivity
A	Liquid H_2 (Unpolarized)	Spring 2018 Autumn 2018 Spring 2019	A_{LU}	$e(x)$, $G_1^{\perp}$
B	Liquid D_2 (Unpolarized)	Spring 2019 Autumn 2019	A_{LU}	$e(x)$, $G_1^{\perp}$
C	Solid NH_3 Solid ND_3 Longitudinal Polarization	Possibly 2021	A_{UL} (and A_{LU}, A_{LL})	$h_L(x)$, $G_1^{\perp}$ (and $e(x)$, also twist-3 DiFFs*)

* Double-spin asymmetry A_{LL}, as well as the z and M_h-dependences of the ratio A_{LU}/A_{UL} are sensitive to twist-3 DiFFs, which are likely very small (see backup slides)

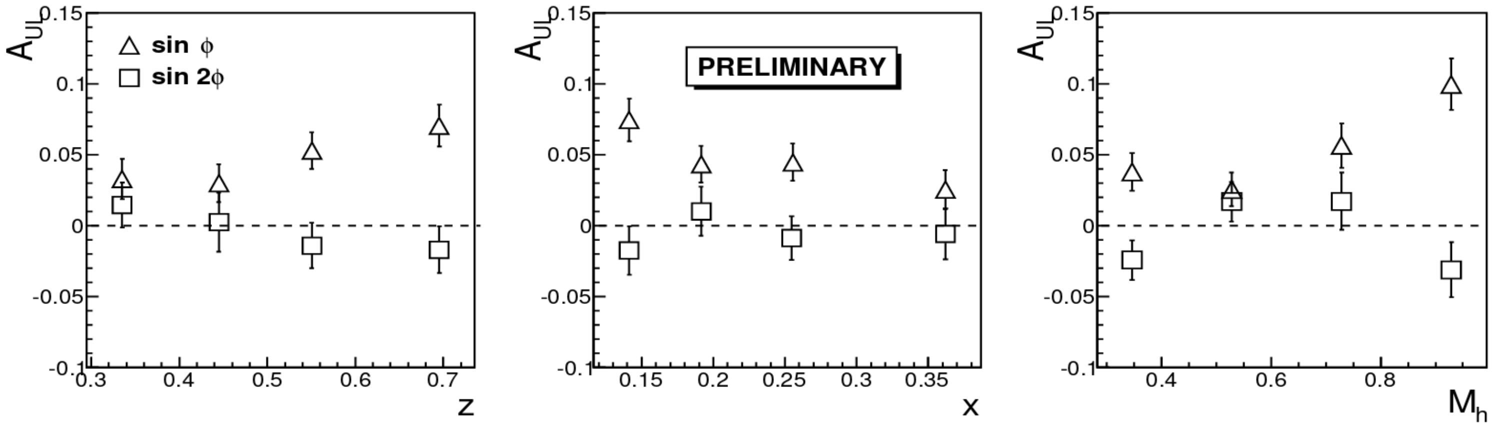
Dihadron Asymmetries from CLAS6

- 5.5 GeV electrons scattered on:
  - Longitudinally Polarized solid NH_3 (compared to BSAs with H_2 target)
- 85% beam polarization, 80% target polarization
- $1 < Q^2 < 6$ GeV^2
- $\sin\Phi_R$ modulation, sensitive to e(x)

Beam Spin Asymmetries A_{LU}



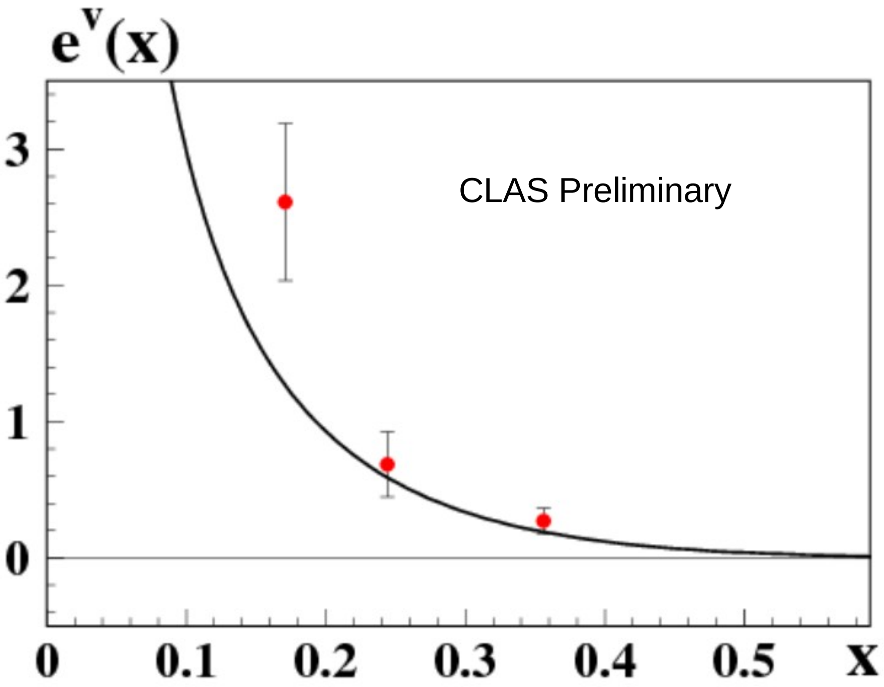
Target Spin Asymmetries A_{UL}



C. Dilks

Pereira, PoS(DIS2014)231

e(x) extraction



$$e^V(x) = \frac{4}{9}e^{u_V}(x) - \frac{1}{9}e^{d_V}(x)$$

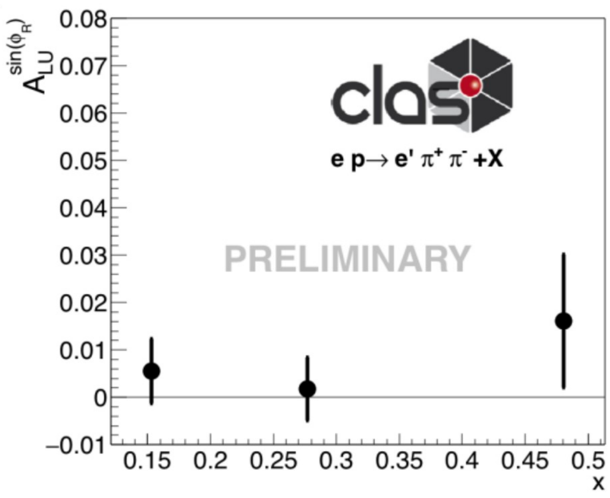
Extracted from CLAS6 preliminary A_{LU} and A_{UL}

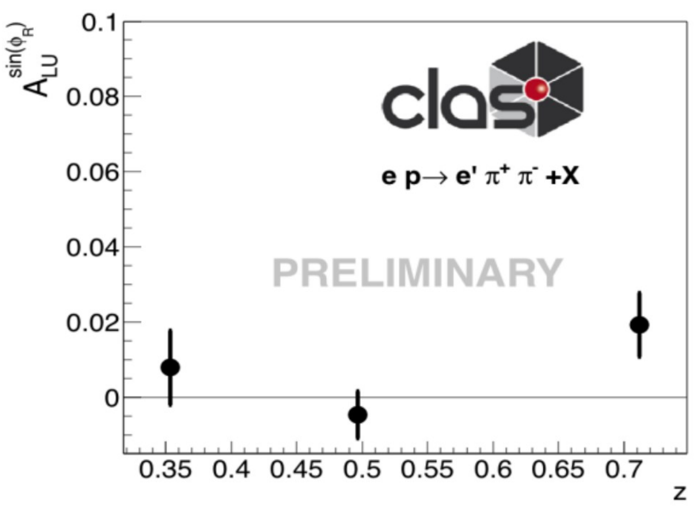
Black curve shows the comparison to the LFCQM model prediction

Extraction: Courtoy, arXiv:1405.7659
LFCQM Model: Lorcé , Pasquini, Schweitzer, JHEP 1501 (2015) 103
[figure from Pisano, Radici, Eur.Phys.J. A52 (2016) no.6, 155]

CLAS12 Beam Spin Asymmetries of Dihadrons

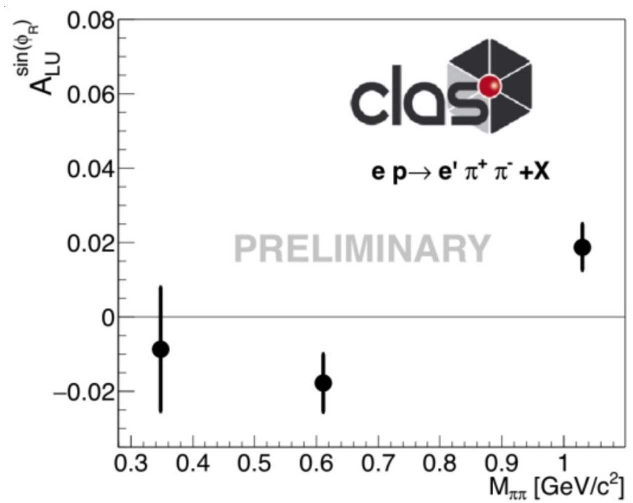
- ~10% of spring 2018 run (~3% of approved run time)
- ~86% beam polarization
- H_2 target
- $\sin\Phi_R$ modulation, sensitive to e(x)





Event Selection

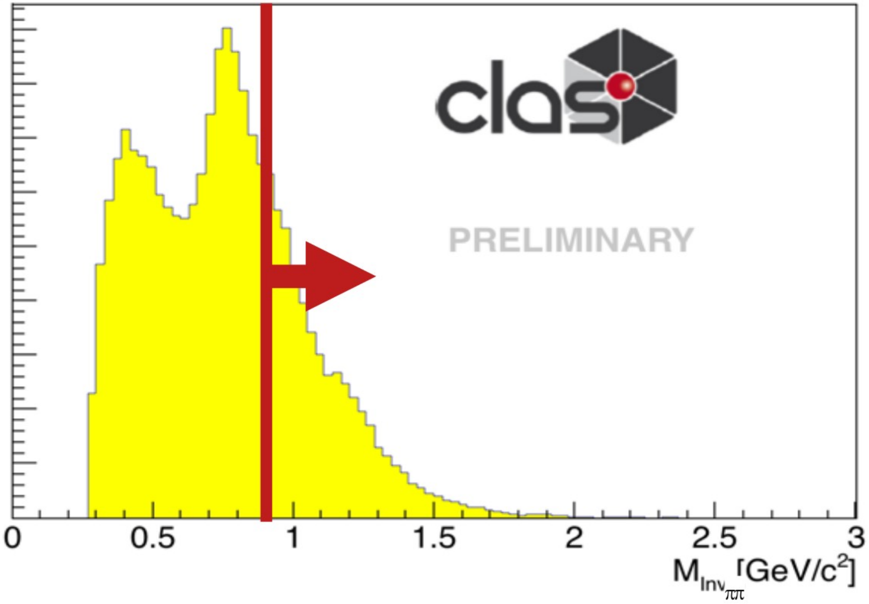
- $Q^2 > 1\ GeV^2$
- $W > 2\ GeV/c^2$
- $z_i > 0.1$
- $z_{pair} < 0.95$
- $M_{miss} > 1.05\ GeV/c^2$
- $x_F > 0$
- $y < 0.8$
- $p_\pi > 1\ GeV/c$

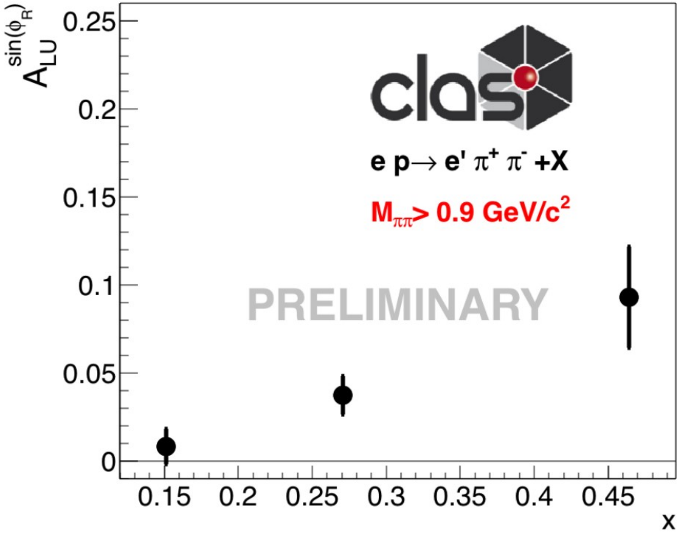


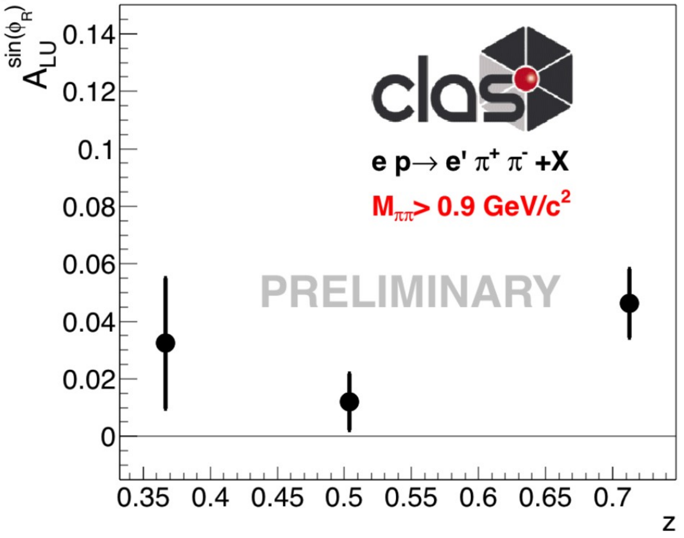
Vossen, DNP2018

CLAS12 Beam Spin Asymmetries of Dihadrons

Dihadron Invariant Mass Distribution







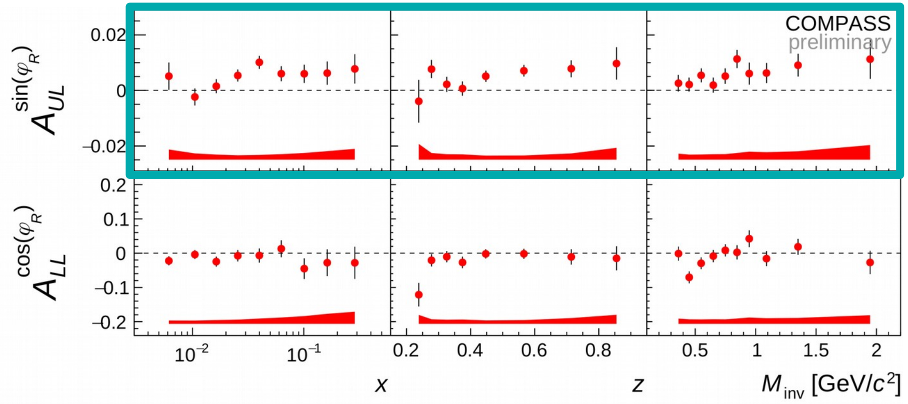
- Large M_h: dihadron fragmentation process can be derived perturbatively from single-hadron FFs
- M_h < hard scale (1 GeV/c^2 at CLAS12): nonperturbative DiFFs
- Large M_h asymmetries are enhanced and rise with x

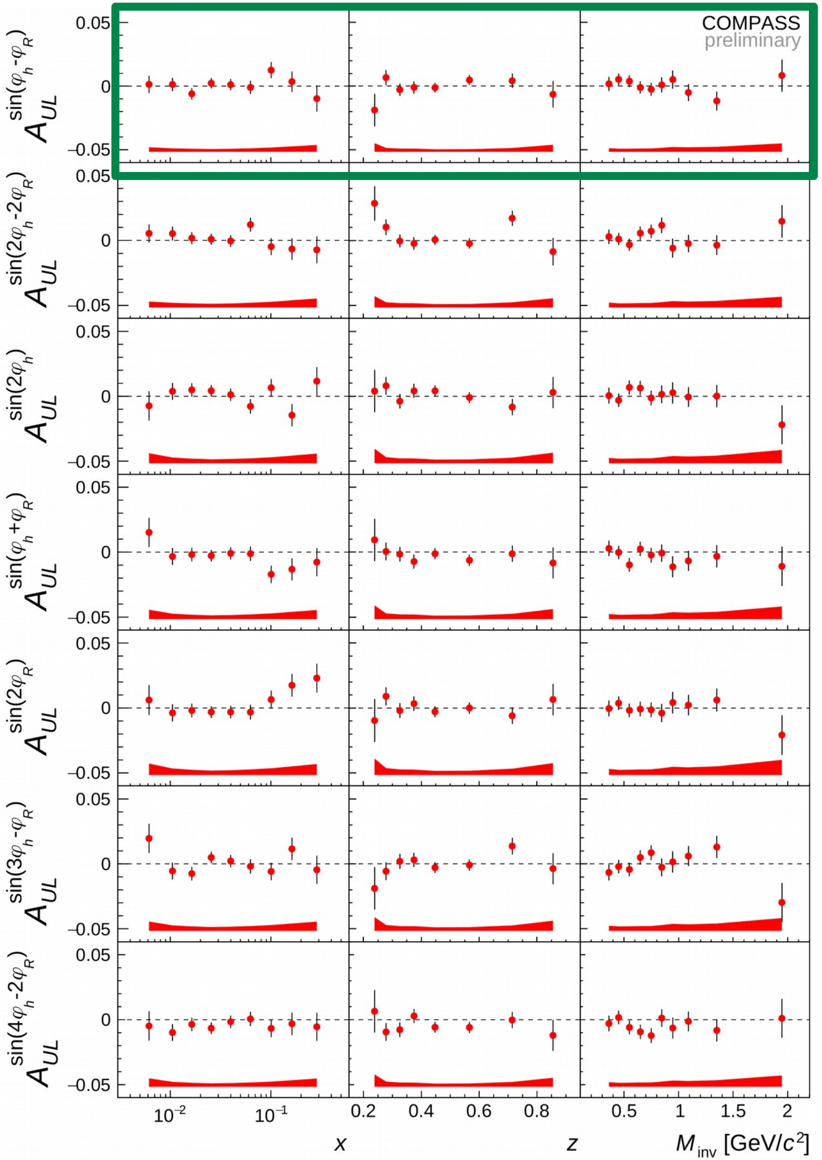
Vossen, DNP2018

COMPASS Target Spin Asym.

- μ^+ scattering on longitudinally polarized solid NH_3 target
- $Q^2 > 1$ GeV^2, $0.0025 < x < 0.7$
- $\sin(\Phi_R)$ modulation ~ 0.5%   [$h_L\, H_1^{<}$]
- $\sin(\Phi_h – \Phi_R)$ modulation ~0%   [$g_1\, G_1^{\perp}$]

  (but was not p_T-weighted...)





Sirtl, 22nd International Symposium on Spin Physics (SPIN 2016)
Urbana, IL, USA, September 25-30, 2016
arXiv:1702.07317

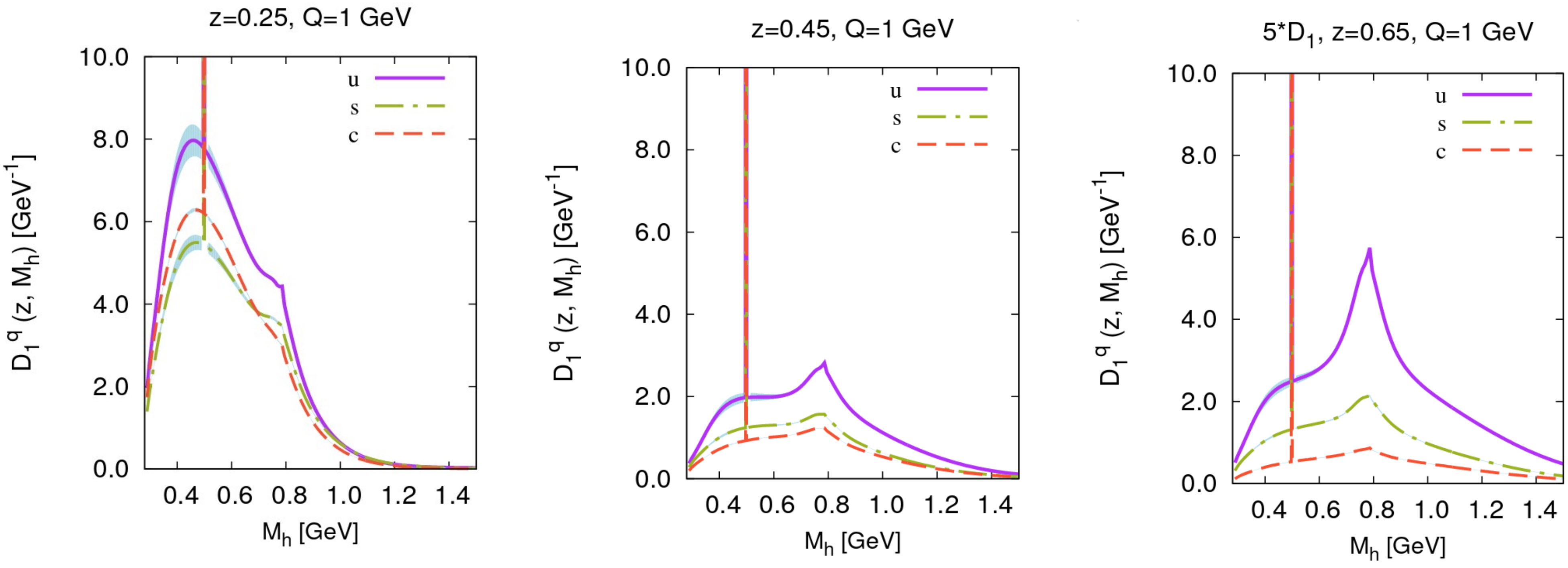
Summary and Outlook

New CLAS12 dihadron correlations data will help measure:

- $e(x)$ and $h_L(x)$ – twist-3 PDF
  - $\mathrm{Sin}(\Phi_R)$ modulations in A_{LU} and A_{UL}
  - Fundamental to TMDs
  - Moments are related to transverse force on struck quark
  - Ratio of A_{LU} / A_{UL} could help understand significance of the twist-3 DiFF contributions (see backup slides)
- $G_1^{\perp}$ – quark helicity dependent DiFF
  - $\mathrm{Sin}(\Phi_h – \Phi_R)$ modulations in A_{LU} and A_{UL}
  - The only DiFF not yet constrained by data
- Compare asymmetries for Hydrogen vs. Deuterium targets to study flavor dependence
- Polarized target data-taking will likely begin in the near future

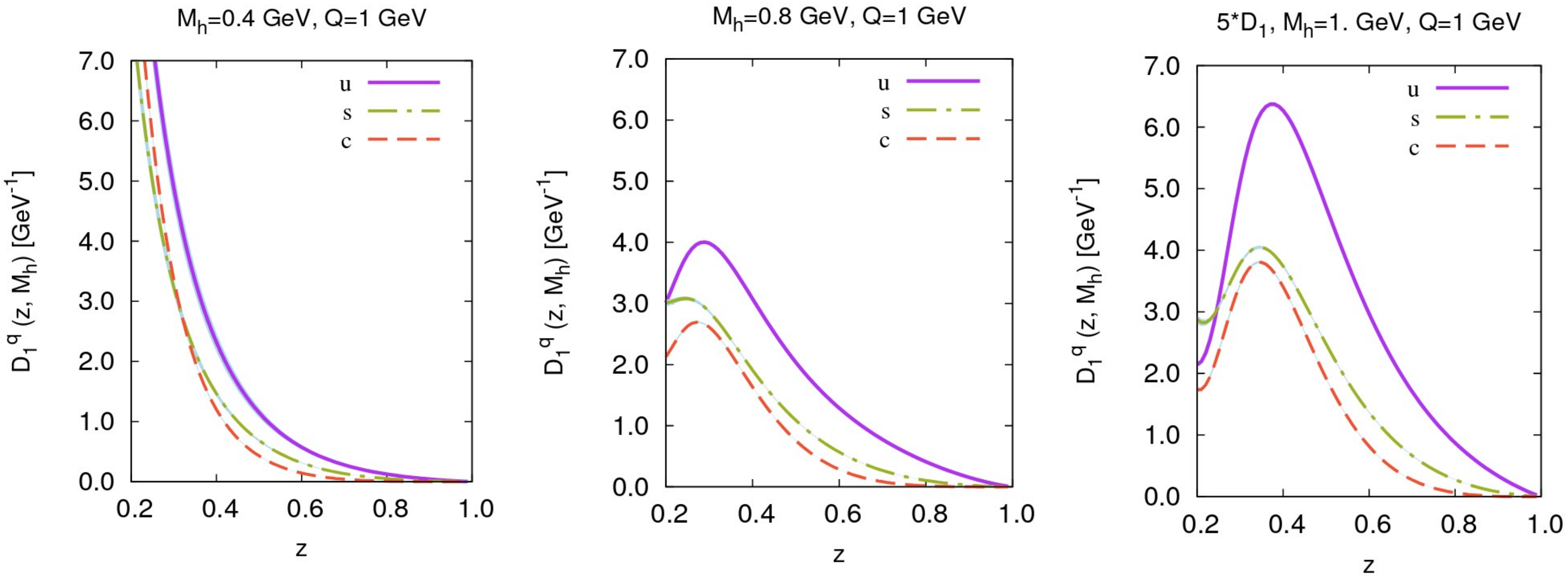
backup

Dihadron Fragmentation Function D_1



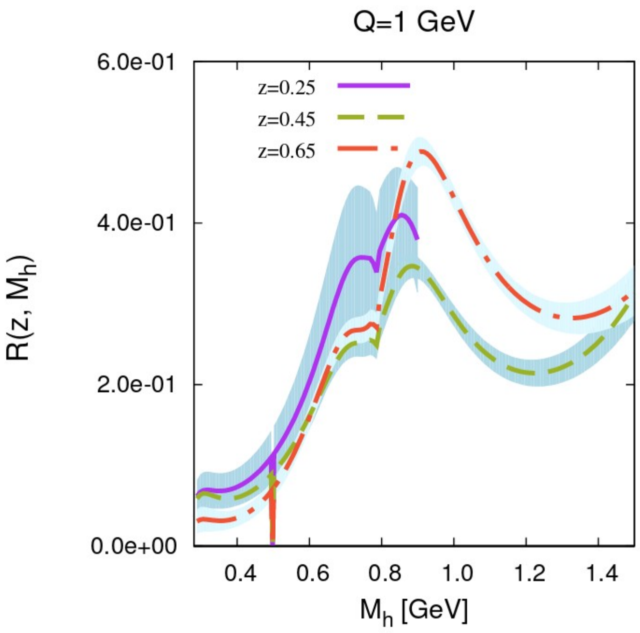
Phys.Rev. D85 (2012) 114023
arXiv:1202.0323

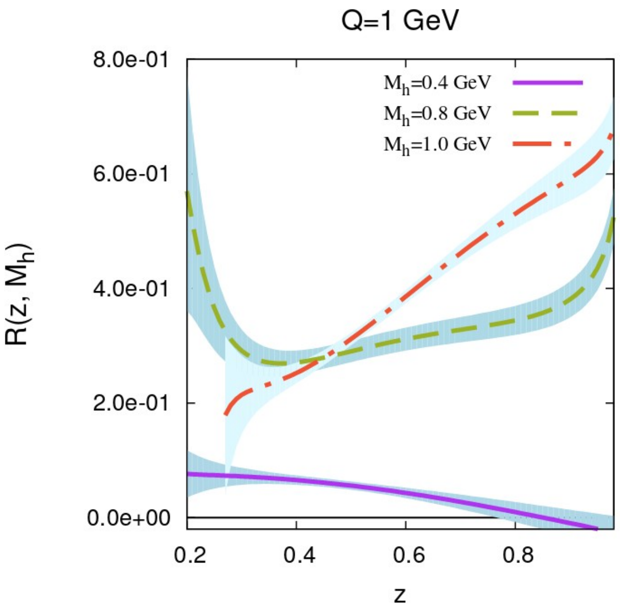
Dihadron Fragmentation Function D_1



Phys.Rev. D85 (2012) 114023
arXiv:1202.0323

Interference Fragmentation Function $H_1^{\sphericalangle}$





$$R(z, M_h) = \frac{|\boldsymbol{R}|}{M_h} \frac{H_{1,sp}^{\sphericalangle u}(z, M_h; Q_0^2)}{D_1^u(z, M_h; Q_0^2)}$$

Phys.Rev. D85 (2012) 114023
arXiv:1202.0323

SIDIS Dihadron Production Cross Section Terms

$$d^7\sigma_{OO} = \frac{\alpha^2}{2\pi Q^2 y} \sum_a e_a^2 \Big\{ A(y)\, f_1(x)\, D_1\big(z,\zeta,M_h^2\big)$$
$$- V(y)\, \cos\phi_R\, \frac{|\vec{R}_T|}{Q} \left[\frac{1}{z}\, f_1(x)\, \widetilde{D}^{\sphericalangle}\big(z,\zeta,M_h^2\big) + \frac{M}{M_h}\, x\, h(x)\, H_1^{\sphericalangle}\big(z,\zeta,M_h^2\big) \right] \Big\}, \qquad (44)$$

$$d^7\sigma_{OL} = \frac{\alpha^2}{2\pi Q^2 y}\, S_L \sum_a e_a^2\, V(y)\, \sin\phi_R\, \frac{|\vec{R}_T|}{Q} \left[\frac{M}{M_h}\, x\, h_L(x)\, H_1^{\sphericalangle}\big(z,\zeta,M_h^2\big) + \frac{1}{z}\, g_1(x)\, \widetilde{G}^{\sphericalangle}\big(z,\zeta,M_h^2\big) \right], \qquad (45)$$

$$d^7\sigma_{OT} = \frac{\alpha^2}{2\pi Q^2 y}\, |\vec{S}_\perp| \sum_a e_a^2 \Big\{ B(y)\, \sin(\phi_R+\phi_S)\, \frac{|\vec{R}_T|}{M_h}\, h_1(x)\, H_1^{\sphericalangle}\big(z,\zeta,M_h^2\big)$$
$$+ V(y)\, \sin\phi_S\, \frac{M_h}{Q} \left[h_1(x) \left(\frac{1}{z}\, \widetilde{H}\big(z,\zeta,M_h^2\big) + \frac{|\vec{R}_T|^2}{M_h^2}\, H_1^{\sphericalangle\, o\,(1)}\big(z,\zeta,M_h^2\big) \right) - \frac{M}{M_h}\, x\, f_T(x)\, D_1\big(z,\zeta,M_h^2\big) \right] \Big\}, \qquad (46)$$

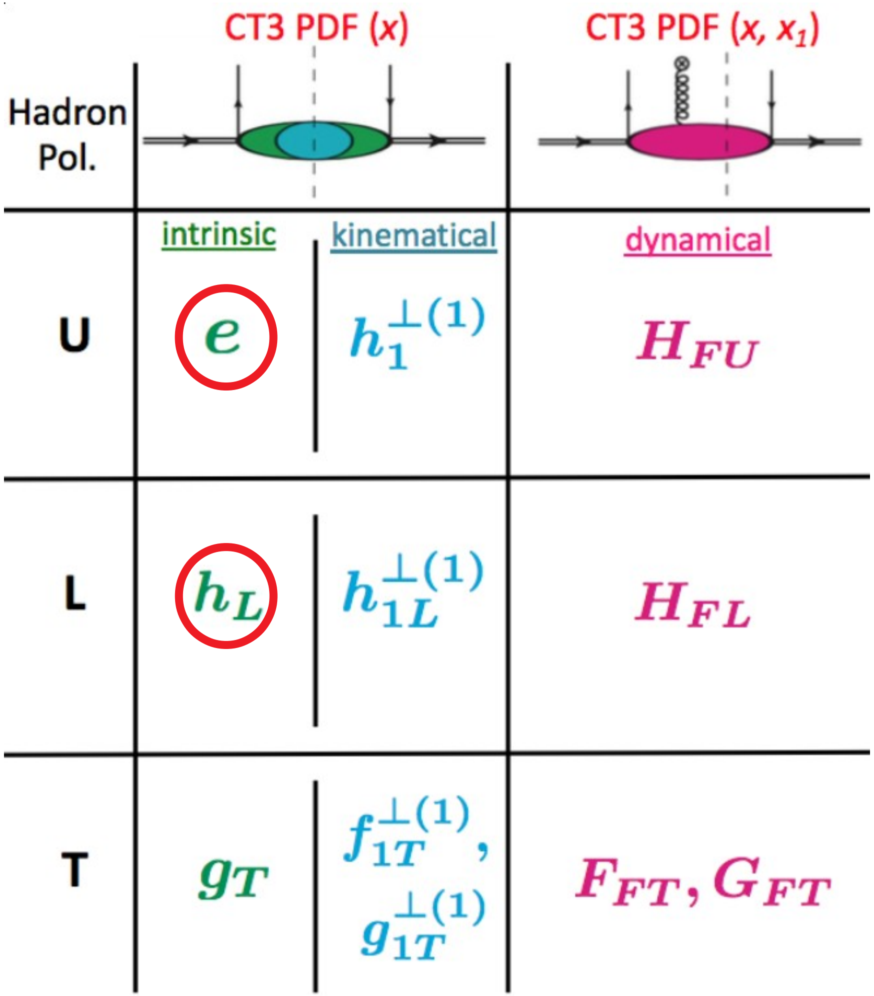
$$d^7\sigma_{LO} = \frac{\alpha^2}{2\pi Q^2 y}\, \lambda \sum_a e_a^2\, W(y)\, \sin\phi_R\, \frac{|\vec{R}_T|}{Q} \left[\frac{M}{M_h}\, x\, e(x)\, H_1^{\sphericalangle}\big(z,\zeta,M_h^2\big) + \frac{1}{z}\, f_1(x)\, \widetilde{G}^{\sphericalangle}\big(z,\zeta,M_h^2\big) \right], \qquad (47)$$

$$d^7\sigma_{LL} = \frac{\alpha^2}{2\pi Q^2 y}\, \lambda\, S_L \sum_a e_a^2 \Big\{ C(y)\, g_1(x)\, D_1\big(z,\zeta,M_h^2\big)$$
$$- W(y)\, \cos\phi_R\, \frac{|\vec{R}_T|}{Q} \left[\frac{1}{z}\, g_1(x)\, \widetilde{D}^{\sphericalangle}\big(z,\zeta,M_h^2\big) - \frac{M}{M_h}\, x\, e_L(x)\, H_1^{\sphericalangle}\big(z,\zeta,M_h^2\big) \right] \Big\}, \qquad (48)$$

$$d^7\sigma_{LT} = \frac{\alpha^2}{2\pi Q^2 y}\, \lambda\, |\vec{S}_\perp| \sum_a e_a^2\, W(y)\, \cos\phi_S\, \frac{M_h}{Q} \left[-\frac{M}{M_h}\, x\, g_T(x)\, D_1\big(z,\zeta,M_h^2\big) - \frac{1}{z}\, h_1(x)\, \widetilde{E}\big(z,\zeta,M_h^2\big) \right]. \qquad (49)$$

Bacchetta and Radici, Phys.Rev. D69 (2004) 074026

Twist-3 Parton Distribution Functions



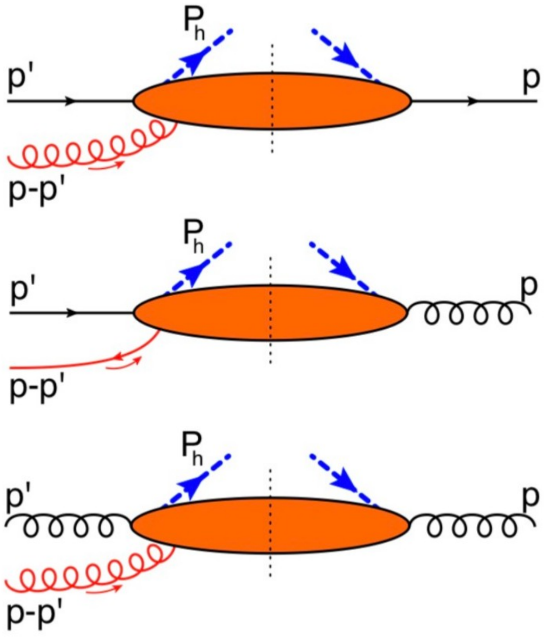
Hadron Pol.	CT3 PDF (x) intrinsic	kinematical	CT3 PDF (x, x_1) dynamical
U	e	$h_1^{\perp(1)}$	H_{FU}
L	h_L	$h_{1L}^{\perp(1)}$	H_{FL}
T	g_T	$f_{1T}^{\perp(1)}$, $g_{1T}^{\perp(1)}$	F_{FT}, G_{FT}

Image from Gamberg, FF2019 Workshop

Intrinsic: in terms of collinear quark-quark matrix elements; dependent on x
Kinematical: in terms of quark-quark matrix elements including quark transverse momentum dependence; dependent on x and k_T
Dynamical: matrix elements that involve 3 partons; dependent on x of one parton, x of the other parton, and k_T

Intrinsic and kinematical twist-3 distribution functions can be expressed solely in terms of dynamical twist-3 functions and twist-2 distributions; the same is true for twist-3 FFs

Kanazawa, Koike, Metz, Pitonyak, Schlegel, Phys.Rev. D93 (2016) no.5, 054024



Images from Pitonyak, FF2019 Workshop

Decomposition of Collinear Twist-3 PDFs

$$e(x) \equiv \int d^2\boldsymbol{p}_T\, e(x,\boldsymbol{p}_T^2) = \frac{m}{M}\frac{f_1(x)}{x} + \tilde{e}(x)$$

Unpolarized PDF — Pure twist-3 part

$$h_L(x) \equiv \int d^2\boldsymbol{p}_T\, h_L(x,\boldsymbol{p}_T^2) = -2\,\frac{h_{1L}^{\perp(1)}(x)}{x} + \frac{m}{M}\frac{g_1(x)}{x} + \tilde{h}_L(x)$$

Wormgear moment — Helicity PDF — Pure twist-3 part

$$g_T(x) \equiv \int d^2\boldsymbol{p}_T\, \left[g'_T(x,\boldsymbol{p}_T^2) + \frac{\boldsymbol{p}_T^2}{2M^2}\, g_T^\perp(x,\boldsymbol{p}_T^2)\right] = \frac{g_{1T}^{(1)}(x)}{x} + \frac{m}{M}\frac{h_1(x)}{x} + \tilde{g}_T(x)$$

Wormgear moment — Transversity PDF — Pure twist-3 part

Appendix C of
Mulders, Tangerman,
Nucl.Phys. B461 (1996) 197-237, Erratum: Nucl.Phys. B484 (1997) 538-540

Decomposition of e(x)

$$e^q(x) = e^q_{\text{sing}}(x) + e^q_{\text{tw3}}(x) + e^q_{\text{mass}}(x)$$

- $e_{\text{sing}}(x)$ – proportional to $\delta(x)$ and gives rise to the pion-nucleon sigma term
- $e_{\text{tw3}}(x)$ – quark-antiquark-gluon correlation function, the pure twist-3 interaction dependent contribution to e(x)
  - Intepretable as the interference between scattering from a coherent quark-gluon pair and from a single quark
  - x^2 moment is intepretable as the the transverse force experienced by a struck transversely polarized quark in an unpolarized nucleon
- $e_{\text{mass}}(x)$ – proportional to current quark mass and moments of $f_1^q(x)$

Efremov and Schweitzer, JHEP 0308 (2003) 006
Burkardt, Phys.Rev. D88 (2013) 114502

Flavor Separation Example for e(x)

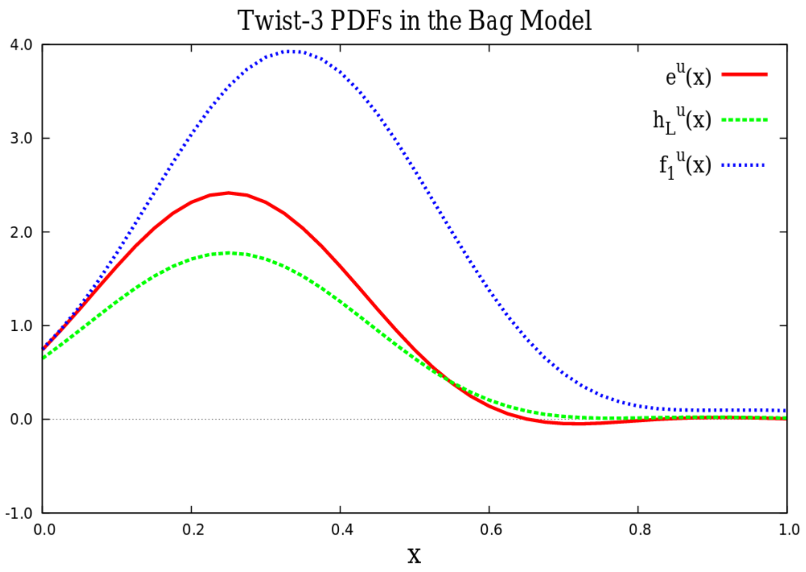
Flavor separation can be achieved with different targets:

Proton Target:
$$A_{LU,p}^{\sin\phi_R \sin\theta}(z, m_{\pi\pi}, x; Q, y) = -\frac{W(y)}{A(y)} \frac{M}{Q} \frac{|\boldsymbol{R}|}{m_{\pi\pi}} \frac{\boxed{\left(4\, xe^{u_V}(x) - xe^{d_V}(x)\right)}\, H_{1,sp}^{\sphericalangle,u}(z, m_{\pi\pi})}{\left(4\; f_1^{u_V}(x) + \; f_1^{d_V}(x)\right) D_1^u(z, m_{\pi\pi})}$$

Deuteron Target:
$$A_{LU,d}^{\sin\phi_R \sin\theta}(z, m_{\pi\pi}, x; Q, y) = -\frac{W(y)}{A(y)} \frac{M}{Q} \frac{|\boldsymbol{R}|}{m_{\pi\pi}} \frac{3\, \boxed{\left(xe^{u_V}(x) + xe^{d_V}(x)\right)}\, H_{1,sp}^{\sphericalangle,u}(z, m_{\pi\pi})}{5\, \left(f_1^{u_V}(x) + \; f_1^{d_V}(x)\right) D_1^u(z, m_{\pi\pi})}$$

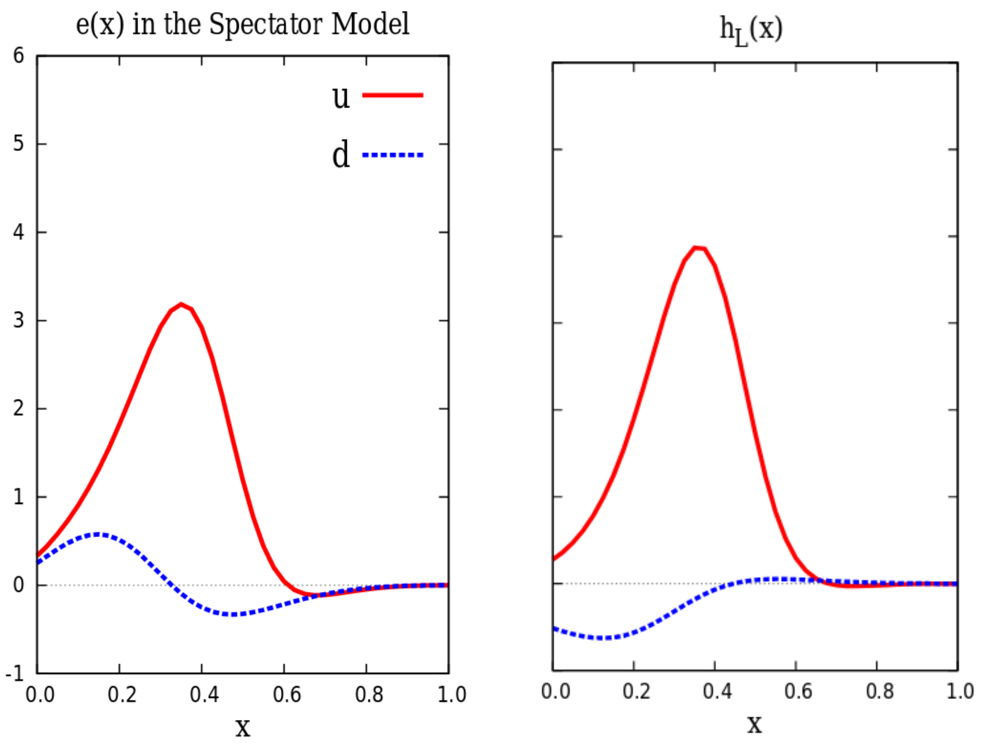
e(x) and h_L(x) Predictions

Bag Model



Jaffe and Ji, Nucl.Phys. B375 (1992) 527-560

Spectator Model

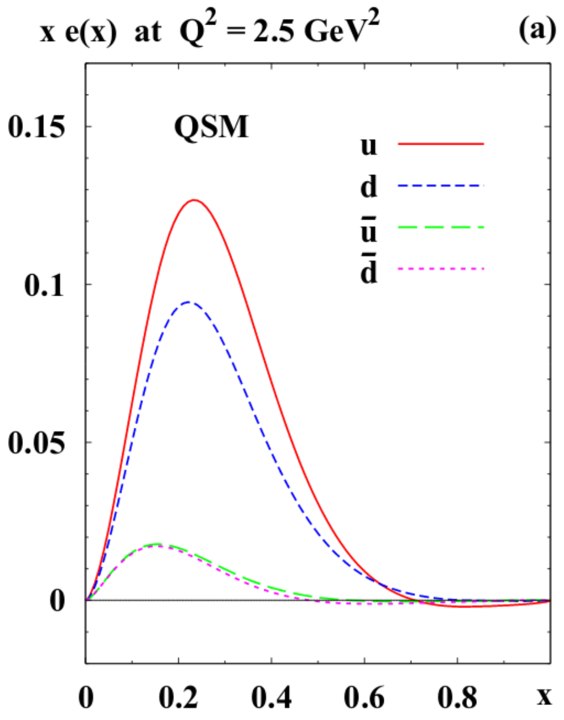


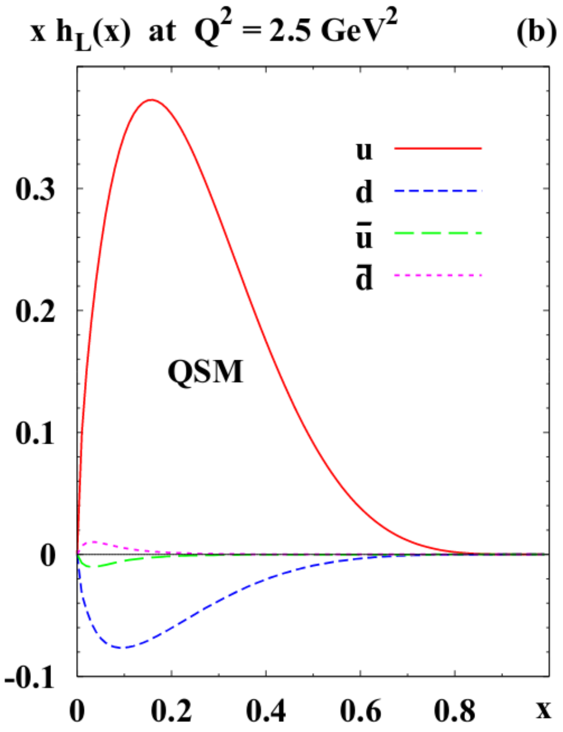
Jakob, Mulders, and Rodrigues, Nucl.Phys. A626 (1997) 937-965

Figures from JLab Proposal E12-06-112B/E12-09-008B

e(x) and h_L(x) Predictions

Chiral Quark Soliton Model



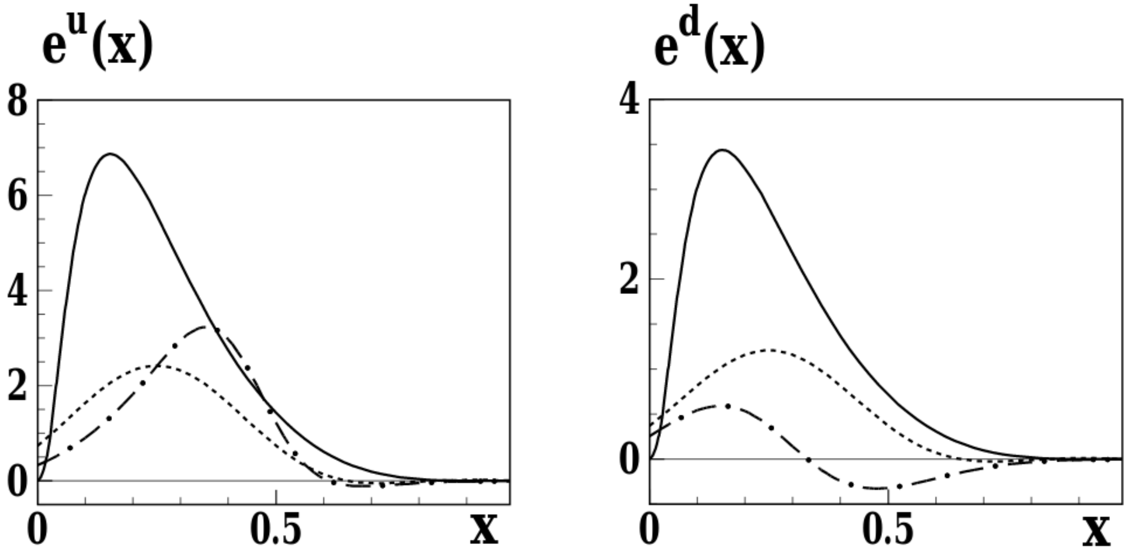


Cebulla et al., Acta Phys.Polon. B39 (2008) 609-640

Figures from JLab Proposal E12-06-112B/E12-09-008B

e(x) and h_L(x) Predictions

Light Front Constituent Quark Model



Lorcé , Pasquini, Schweitzer, JHEP 1501 (2015) 103

Solid: LFCQM model
Dot-Dashed: spectator model
Dashed: bag model

- Relatively larger magnitude partly due to mass effects

Twist-3 Dihadron Fragmentation Function

$$F_{LU}^{\sin\phi_R} = -\sum_q e_q^2\, x \frac{|R|\sin\theta}{Q}\left[\frac{M}{m_{hh}}\, x\, e^q(x)\, H_1^{\sphericalangle q}(z,\cos\theta,m_{hh}) + \frac{1}{z} f_1^q(x)\, \widetilde{G}^{\sphericalangle q}(z,\cos\theta,m_{hh})\right]$$

Extracting e(x) and h_L(x) involves

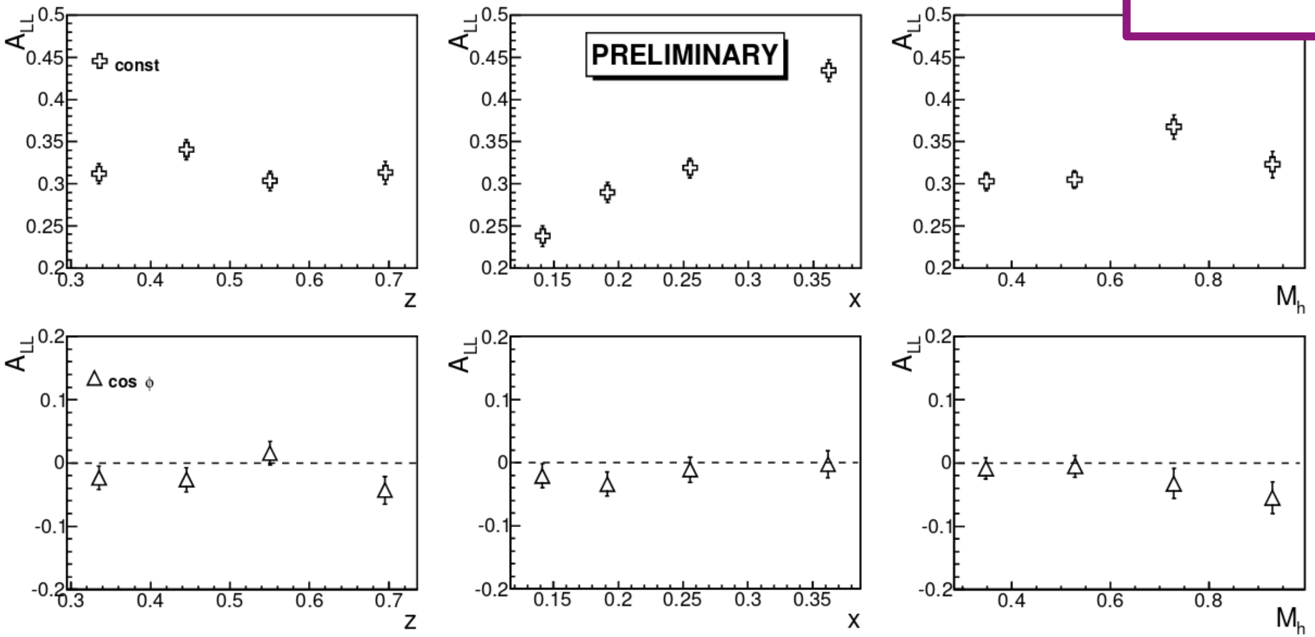
$\widetilde{G}^{\sphericalangle q}$ – a pure twist-3 DiFF

- ~0 under Wandzura-Wilzcek approximation
- What can we learn from data?

Double spin asymmetries can help:

$$F_{LL}^{const} = x g_1^q(x)\, D_1^q(z,\cos\theta,M_h),$$

$$F_{LL}^{\cos\phi_R} = -x\frac{|\boldsymbol{R}|\sin\theta}{Q}\frac{1}{z} g_1^q(x)\, \widetilde{D}^{\sphericalangle q}(z,\cos\theta,M_h)$$



$\widetilde{D}^{\sphericalangle q}$ is expected to be larger than $\widetilde{G}^{\sphericalangle q}$

$\widetilde{D}^{\sphericalangle q}$ is very small, since $\cos\Phi_R$ asymmetry is small, therefore $\widetilde{G}^{\sphericalangle q}$ is also likely very small

Pereira, PoS(DIS2014)231

Twist-3 Dihadron Fragmentation Function

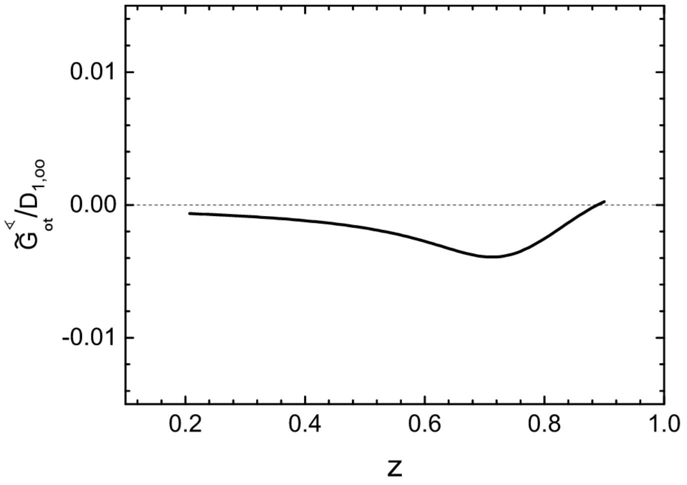
$$A_{\mathrm{SIDIS}}^{\mathrm{LU}}(x,z,M_h;Q) = -\frac{W(y)}{A(y)}\frac{M}{Q}\frac{|\boldsymbol{R}|}{M_h}\frac{\sum_q e_q^2\left[x\boxed{e^q(x)}\boxed{H_{1\,sp}^{\sphericalangle\,q}(z,M_h^2)}+\frac{M_h}{zM}\boxed{f_1^q(x)}\boxed{\tilde{G}_{sp}^{\sphericalangle\,q}(z,M_h^2)}\right]}{\sum_q e_q^2\,f_1^q(x)\,D_1^q(z,M_h^2)}$$

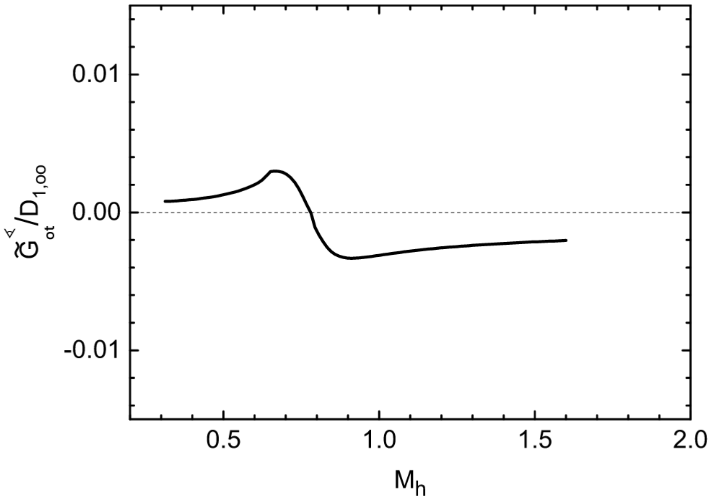
$$A_{\mathrm{SIDIS}}^{\mathrm{UL}}(x,z,M_h;Q) = -\frac{V(y)}{A(y)}\frac{M}{Q}\frac{|\boldsymbol{R}|}{M_h}\frac{\sum_q e_q^2\left[x\boxed{h_L^q(x)}\boxed{H_{1\,sp}^{\sphericalangle\,q}(z,M_h^2)}+\frac{M_h}{zM}\boxed{g_1^q(x)}\boxed{\tilde{G}_{sp}^{\sphericalangle\,q}(z,M_h^2)}\right]}{\sum_q e_q^2\,f_1^q(x)\,D_1^q(z,M_h^2)}$$

$\frac{A_{LU}}{A_{UL}}$ should not depend on (z, M_h) if $\tilde{G}^{\sphericalangle}$ is negligible

- Extraction of e(x) is more difficult if this is not the case
- Higher-precision data from CLAS12 will help address this

Twist-3 Dihadron Fragmentation Function





Spectator Model Calculation:

Leading order term of $\widetilde{G}^{\sphericalangle}$ is <0.5% of that of the leading-twist DiFF D_1

Partial Wave Expansions

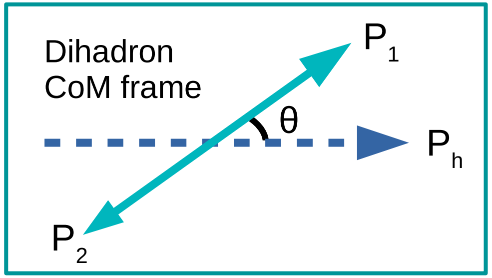
$$\widetilde{G}^{\sphericalangle}(z,\cos\theta,M_h^2) = \boxed{\widetilde{G}_{ot}^{\sphericalangle}(z,M_h^2)} + \widetilde{G}_{lt}^{\sphericalangle}(z,M_h^2)\cos\theta$$

$$D_1^a(z,\cos\theta,M_h^2) = \boxed{D_{1,oo}^a(z,M_h^2)} + D_{1,ol}^a(z,M_h^2)\cos\theta + D_{1,ll}^a(z,M_h^2)(3\cos^2\theta - 1)$$

W. Yang , X. Wang, Y. Yang, Z. Lu
Phys.Rev. D99 (2019) no.5, 054003

Partial Wave Expansion

ζ(cosθ) is linear → expansion in terms of Legendre polynomials of cosθ



$$\zeta = \frac{2R^-}{P_h^-} = \frac{1}{M_h}\left(\sqrt{M_1^2 + R^2} - \sqrt{M_2^2 + R^2 - 2|R|\cos\theta}\right)$$

$$z = \frac{P_h^-}{k^-}$$

Expansion of DiFFs:

$$\frac{2|\vec{R}|}{M_h} D_1\left(z, \zeta(\cos\theta), M_h^2\right) = \sum_n D_{1,n}(z, M_h^2)\, P_n(\cos\theta)$$

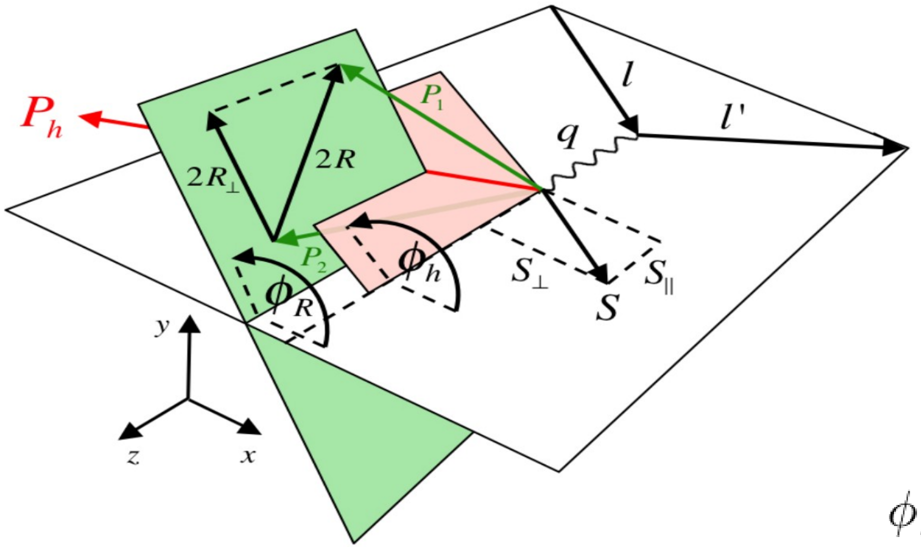
$$\frac{2|\vec{R}|}{M_h} H_1^{\sphericalangle}\left(z, \zeta(\cos\theta), M_h^2\right) = \sum_n H_{1,n}^{\sphericalangle}(z, M_h^2)\, P_n(\cos\theta)$$

$$D_{1,n}(z, M_h^2) = \int_{-1}^{1} d\cos\theta\; P_n(\cos\theta)\, \frac{2|\vec{R}|}{M_h} D_1\left(z, \zeta(\cos\theta), M_h^2\right)$$

$$H_{1,n}^{\sphericalangle}(z, M_h^2) = \int_{-1}^{1} d\cos\theta\; P_n(\cos\theta)\, \frac{2|\vec{R}|}{M_h} H_1^{\sphericalangle}\left(z, \zeta(\cos\theta), M_h^2\right)$$

Bacchetta and Radici, Phys.Rev. D67 (2003) 094002
Bacchetta and Radici, Phys.Rev. D69 (2004) 074026

SIDIS Dihadron Angles



$$\boldsymbol{R}_\perp = \frac{z_2 \boldsymbol{P}_{1\perp} - z_1 \boldsymbol{P}_{2\perp}}{z_1 + z_2}$$

$$\phi_h = \frac{(\boldsymbol{q} \times \boldsymbol{l}) \cdot \boldsymbol{P}_h}{|(\boldsymbol{q} \times \boldsymbol{l}) \cdot \boldsymbol{P}_h|} \arccos\left(\frac{(\boldsymbol{q} \times \boldsymbol{l}) \cdot (\boldsymbol{q} \times \boldsymbol{P}_h)}{|\boldsymbol{q} \times \boldsymbol{l}| \cdot |\boldsymbol{q} \times \boldsymbol{P}_h|}\right)$$

$$\phi_R = \frac{(\boldsymbol{q} \times \boldsymbol{l}) \cdot \boldsymbol{R}_\perp}{|(\boldsymbol{q} \times \boldsymbol{l}) \cdot \boldsymbol{R}_\perp|} \arccos\left(\frac{(\boldsymbol{q} \times \boldsymbol{l}) \cdot (\boldsymbol{q} \times \boldsymbol{R}_\perp)}{|\boldsymbol{q} \times \boldsymbol{l}| \cdot |\boldsymbol{q} \times \boldsymbol{R}_\perp|}\right)$$

arXiv:1702.07317