Nucleon properties from basis light front quantization

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  - ✓ Form Factors
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Basis Light-Front Quantization (BLFQ)

J. Vary *et. al.*, PRC 81 (2010)

BLFQ: approach for solving quantum field theory

- Nonperturbative:

  for systems with strong interaction

- First-principles:

  effective Hamiltonian as input/ direct access to wavefunction of bound states

- Light-front dynamics:

  spectrum and light-front Fock-state wavefunctions are obtained from

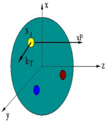
$$\boxed{H_{LF}|\psi\rangle = M^2|\psi\rangle}$$

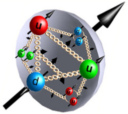
$$H_{LF} \equiv P_\mu P^\mu = P^+P^- - \mathbf{P}_\perp^2$$

$$P^\pm = P^0 \pm P^3$$

LF wavefunctions

Proton 3D imaging | **Proton spin**





FF | GPD | TMD...

General Procedure for BLFQ

✓ Derive/write the Light-Front Hamiltonian: $P^- = H_{\text{eff}}$

✓ Construct the basis state: $|\alpha\rangle$

- Calculate the Hamiltonian matrix elements:

$$H_{\text{eff}}^{\alpha'\alpha} = \langle\alpha'|H_{\text{eff}}|\alpha\rangle$$

$\langle\alpha'|$ & $|\alpha\rangle$ are the basis state of BLFQ, such as $|qqq\rangle$.

- Diagonalize H_{eff} and obtain its eigen spectrum

$$H_{\text{eff}}|\beta\rangle = H_{\text{eff}}^{\beta}|\beta\rangle$$

- $|\beta\rangle$ is the physical state and eigenstate of Hamiltonian. In case of proton $|\beta\rangle = |P_{\text{proton}}\rangle$.

✓ Evaluate observables:

$$\mathcal{O} = \langle\beta|\hat{\mathcal{O}}|\beta\rangle$$

Previous applications of BLFQ

QCD systems

- Heavy quarkonium: decay constant, elastic form factor, radii, radiative transitions, distribution amplitude, GPDs

—Y Li, G Chen, X Zhao, P Maris, J Vary, L Adhikari, M Li, A El-Hady (2016 - 2019)

- Light mesons: spectrum, decay constant, elastic form factor, radii, distribution amplitude, PDFs and scale evolution

—S Jia, J Vary, J Lan, CM, X. Zhao (2018 - 2019)

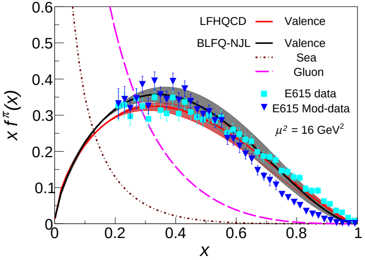
QED systems

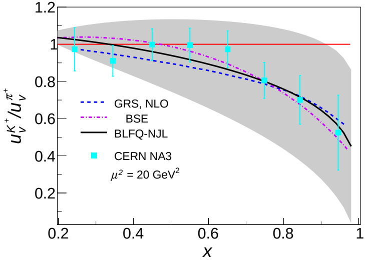
- Electron: anomalous magnetic moments
- positronium wave function, spectroscopy
- GPDs of the electron and positronium

—X. Zhao, P. Wiecki, Y. Li, H. Honkanen, D. Chakrabarti, P. Maris, J. P. Vary, S. J. Brodsky (2013 - 2018)

Example: light meson PDFs

Lan, CM, Jia, Zhao, Vary: to appear in PRL (2019)





Light front effective Hamiltonian, $H_{\rm eff}$: ($\mu^2_{0\pi} = 0.240 \pm 0.024\ \mathrm{GeV}^2$)

$$\underbrace{\frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x}}_{\text{LF Kinetic energy}} + \underbrace{\kappa^4 x(1-x)\vec{r}_\perp^{\,2}}_{\text{Transverse}} - \underbrace{\frac{\kappa^4}{(m_q + m_{\bar{q}})^2}\partial_x\big(x(1-x)\partial_x\big)}_{\text{Longitudinal}} + H^{\rm eff}_{\rm NJL},$$

- Light front wavefunctions ▶ eigenvectors of this Hamiltonian. PDFs evolution ▶ based on the NNLO DGLAP equations.
- The robustness of our results motivates the application of analogous effective Hamiltonians to the baryons.

Effective Light-front Hamiltonian for nucleon

Light-Front Hamiltonian

$$P^{-}_{baryon} = H_{K.E.} + H_{trans} + H_{longi} + H_{OGE}$$

$$H_{K.E.} = \sum_i \frac{p_i^2 + m_q^2}{p_i^+} \qquad m_u = 0.6GeV, m_d = 0.57GeV$$

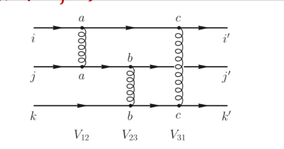
$$H_{trans} \sim \kappa_T^4 b^4 \zeta^2$$ -- Brodsky, Teramond arXiv: 1203.4025

$$H_{longi} = -\sum_{ij} \kappa_L^4 b^4 \partial_{x_i} \left(x_i x_j \partial_{x_j} \right)$$ ---Y Li, X Zhao , P Maris , J Vary PLB 758(2016)

$$H_{OGE} = -\frac{C_F 4\pi\alpha_s(Q^2)}{Q^2} \sum_{i,j(i<j)} \bar{u}_{s_i'}(k_i')\gamma^{\mu}u_{s_i}(k_i)\bar{u}_{s_j'}(k_j')\gamma_{\mu}u_{s_j}(k_j)$$

Infrared cut off : $m_g = 0.05\ GeV, C_F = -\frac{2}{3}$

$$|P_{baryon}\rangle = \boxed{|qqq\rangle} + |qqqg\rangle + |qqq\ q\bar{q}\rangle + \cdots\cdots$$



Although we truncate to the **leading Fock sector**, we can solve the **baryon system** with **multi-particle** (at least three particle). In future, we can include **higher Fock sector**, and study the **sea quark** and **gluon** distribution fcuntion.

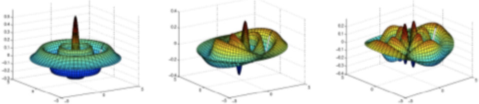
Basis construction

- Fock's space expansion:

$$|\,N\,\rangle_{\text{baryon}} = a|\,qqq\,\rangle + b|\,qqqg\,\rangle + c|\,qqqq\bar{q}\,\rangle + \cdots .$$

- For each Fock particle:
  - Transverse: 2D harmonic oscillator basis: $\Phi^{b}_{n,m}(\vec{p}_\perp)$
    labeled by radial (angular) quantum number n (m); scale parameter b

    e.g., n=4



m=0 m=1 m=2

  - Longitudinal: plane-wave basis, labeled by k
  - Helicity: labeled by λ

- For the first Fock sector:

$$|\,qqq\,\rangle = |\,\mathbf{n_{q_1}}, \mathbf{m_{q_1}}, \mathbf{k_{q_1}}, \lambda_{\mathbf{q_1}}\,\rangle \otimes |\,\mathbf{n_{q_2}}, \mathbf{m_{q_2}}, \mathbf{k_{q_2}}, \lambda_{\mathbf{q_2}}\,\rangle \otimes |\,\mathbf{n_{q_3}}, \mathbf{m_{q_3}}, \mathbf{k_{q_3}}, \lambda_{\mathbf{q_3}}\,\rangle .$$

Basis Truncation Scheme

- Symmetries of Hamiltonian:
  - Net fermion number: $\sum_i n_i^f = N^f$
  - Total angular momentum projection: $\sum_i (m_i + s_i) = J_z$
  - Longitudinal momentum: $\sum_i k_i = K$

- Further truncation:
  - Fock sector truncation
  - Discretization in longitudinal direction $k_i = \begin{cases} 1, 2, 3.... & \text{bosons} \\ 0.5, 1.5, 2.5 ... & \text{fermions} \end{cases}$
  - "N_{max}" truncation in transverse directions $\sum_i [2n_i + |m_i| + 1] \leq N_{max}$

  UV cutoff $\Lambda \sim b\sqrt{N_{\max}}$; IR cutoff $\lambda \sim b/\sqrt{N_{\max}}$

Flavor Form Factor in BLFQ

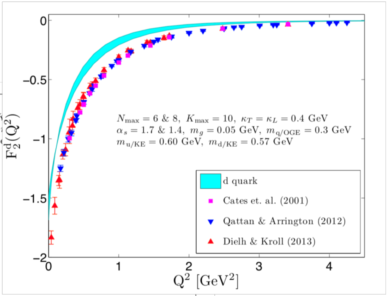
[Work in progress, C. Mondal, Siqi Xu, *et.al*]

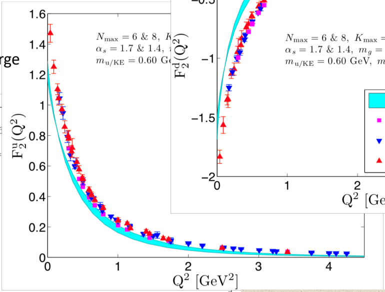
different quark masses
in kinetic and one gluon exchange term to
minimize the effect of higher Fock Sector

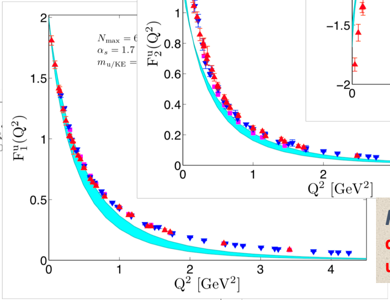
Dirac form factor

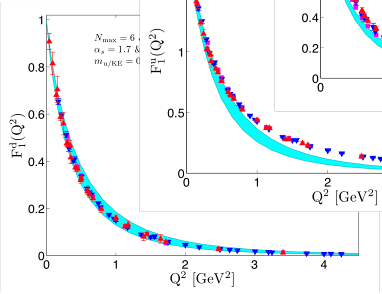
d quark: good agreement with data;

u quark: deviates from the data at large Q^2









Pauli Form factor

d quark: $F_2(0) = -1.73$ (Exp. : -2.03)

u quark: $F_2(0) = 1.27$ (Exp. : 1.67)

With ***increasing the basis size***,
our result ***approach to the experiment data***.

Nucleon Form Factor

[Work in progress, C. Mondal, Siqi Xu, *et.al*]

$$G_E(Q^2) = \sum_q e_q F_1^q(Q^2) - \frac{Q^2}{4M^2} \sum_q e_q F_2^q(Q^2)\,,$$
$$G_M(Q^2) = \sum_q e_q F_1^q(Q^2) + \sum_q e_q F_2^q(Q^2).$$

Charge form factor

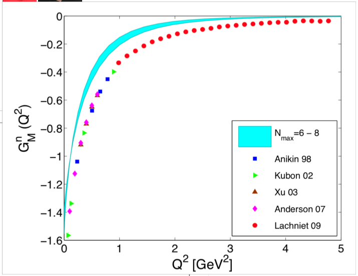
Neutron : agrees with experiment data

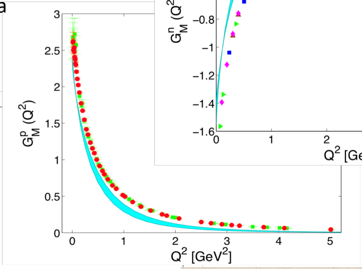
Proton : also agrees with the data

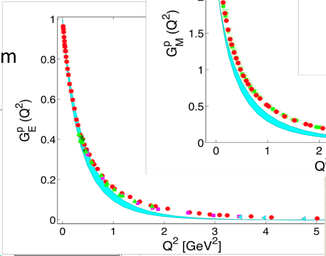
Magnetic form factor

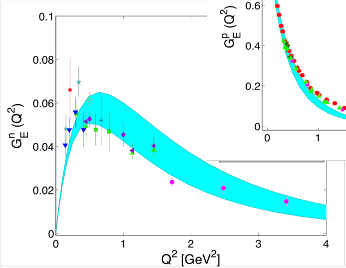
Proton : more or less agrees with data

Neutron : deviates from the data









Anomalous magnetic moment

Proton : $\mu_N = 2.42$ (Exp. : 2.79)

Neutron : $\mu_N = -1.57$ (Exp. : -1.91)

With ***increasing the basis size***, the magnetic form factor ***approach to the experiment data***

Electromagnetic radii

CM, X. Siqi, J. Lan $et.\ al.$ in preparation

$$\langle r_E^2 \rangle^N = -6 \frac{dG_E^N(Q^2)}{dQ^2}\Big|_{Q^2=0},$$

$$\langle r_M^2 \rangle^N = -\frac{6}{G_M^N(0)} \frac{dG_M^N(Q^2)}{dQ^2}\Big|_{Q^2=0}$$

The Sachs form factors are defined as

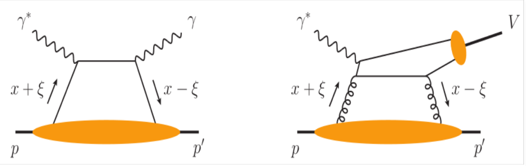
$$G_E^N(Q^2) = F_1^N(Q^2) - \frac{Q^2}{4M_N^2} F_2^N(Q^2),$$

$$G_M^N(Q^2) = F_1^N(Q^2) + F_2^N(Q^2).$$

Quantity	$\mathbf{N}_{\mathrm{max}} = \mathbf{6}$	$\mathbf{N}_{\mathrm{max}} = \mathbf{8}$	Data from PDG
r_E^p (fm)	0.966	0.877	0.877 ± 0.005
r_M^p (fm)	1.094	1.008	0.777 ± 0.016
$\langle r_E^2 \rangle^n$ (fm^2)	-0.1175	-0.1398	-0.1161 ± 0.0022
r_M^n (fm)	1.278	1.2045	$0.862^{+0.009}_{-0.008}$

Generalized Parton Distribution functions (GPDs)

➢ ***Deeply Virtual Compton Scattering (DVCS)/*** vector meson productions ***experiment:***



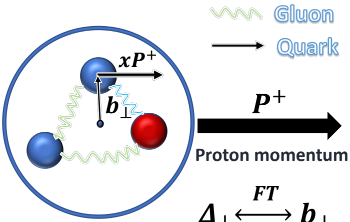
Encode the information about three dimensional spatial structure the spin and orbital angular momentum

➢ **GPDs** appear in **DVCS** processes.

- GPDs are functions of three variables :
  - ***Longitudinal momentum fraction*** $x = \frac{k^+}{P^+}$
  - ***Longitudinal momentum transfer -->***

    skewness $\xi = \frac{\Delta^+}{P^+} = 0$
  - ***Square of total mom transfer***

    $t = \Delta^2 = (P' - P)^2$



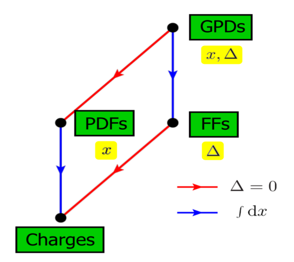
$$\Delta_\perp \xleftrightarrow{FT} b_\perp$$

where the $b_\perp$ is transverse position of parton

Generalized Parton Distribution Functions (GPD)

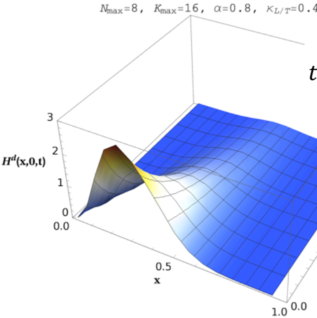
GPD encode the information about three dimensional spatial structure of nucleon, as well as the spin and orbital angular momentum of the constituents

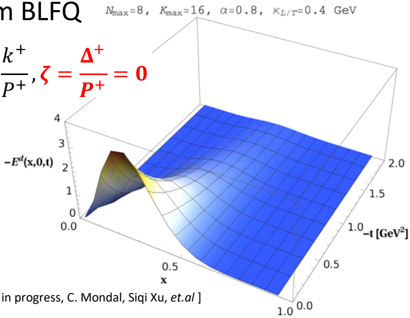
With **increasing momentum transfer (t)**, the **peaks of distributions** shift to **larger x**;
At **large x**, **BLFQ results** follow the **quark-diquark model**



From BLFQ

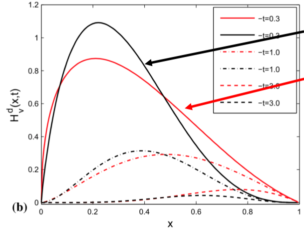
$$t = \Delta^2, x = \frac{k^+}{P^+}, \zeta = \frac{\Delta^+}{P^+} = 0$$





[Work in progress, C. Mondal, Siqi Xu, *et.al*]

Generalized Parton Distribution Functions



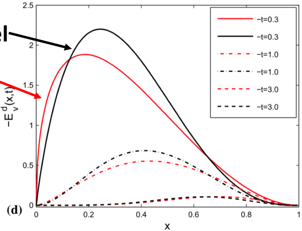
quark-diquark model

AdS/QCD

$$t = \Delta^2$$

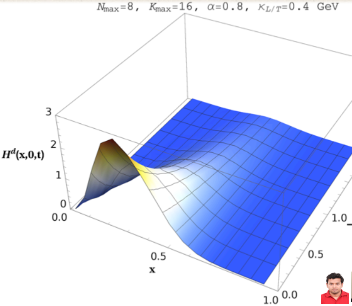
$$x = \frac{k^+}{P^+}, \boldsymbol{\zeta = \frac{\Delta^+}{P^+} = 0}$$

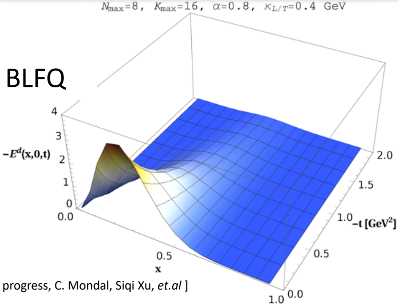
For d quark



With **increasing momentum transfer (t)**, the **peaks of distributions** shift to **larger x**;
At **Large x**, **BLFQ results** follow the **quark-diquark model**

From BLFQ





[Work in progress, C. Mondal, Siqi Xu, *et.al*]

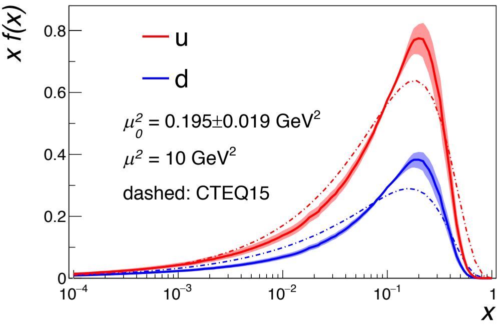
Parton distribution functions (PDF)

Parton distribution functions in BLFQ:

$$< x_q >= \int dx\, x f_1^q(x)$$

Average momentum is carried by the quark.

Here, we use the NNLO DGLAP equation to evolve the PDF. Qualitative behavior of PDF is more or less same with **the global fit CTEQ 15 PDF**.



U Quark

Low x region: agrees with global fit

High x region: deviates from the fit

D Quark

Low x region: deviates from the fit

High x region: more or less agrees with global fit

We need the contribution from ***higher Fock sector*** such as $|qqqg\rangle, |qqq\, q\bar{q}\rangle$ to obtain **better agreement**, and to study the ***sea quark*** and ***gluon*** distributions.

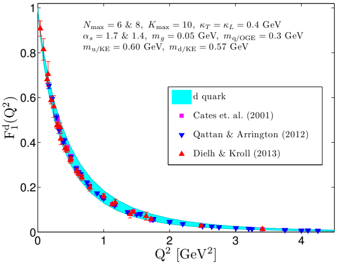
[Work in progress, C. Mondal, Siqi Xu, Jiangshan Lan, *et.al*]

Conclusions

- We have discussed the preliminary results of nucleon form factors, PDFs $\&$ GPDs in BLFQ approach.
- In the effective Hamiltonian, we have the kinetic energy $\&$ the confining potential in both the transverse and longitudinal direction and one gluon exchange with fixed coupling. Here, we consider only the leading Fock sector.
- BLFQ formalism provides promising results in order to understand the nucleon structure.

Outlook:

- Increase basis size
- Include the higher Fock component $|qqqg\rangle$.
- Investigate other nucleon properties..
- Investigate the structure of other baryons.



Thank You