Nucleon properties from basis light front quantization

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- Numerical results
  - Form Factors
  - Parton distribution Functions (PDFs)
  - Generalized parton distributions (GPDs)

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Basis Light-Front Quantization (BLFQ)

- **BLFQ**: approach for solving quantum field theory
  - **Nonperturbative**: for systems with strong interaction
  - **First-principles**: effective Hamiltonian as input/direct access to wavefunction of bound states
  - **Light-front dynamics**: spectrum and light-front Fock-state wavefunctions are obtained from

\[
H_{LF} |\psi\rangle = M^2 |\psi\rangle
\]

\[
H_{LF} \equiv P_\mu P^\mu = P^+ P^- - P^2_\
\]

\[
P^\pm = P^0 \pm P^3
\]

LF wavefunctions
Proton 3D imaging
Proton spin
FF
GPD
TMD...
General Procedure for BLFQ

✓ Derive/write the Light-Front Hamiltonian: \( P^- = H_{\text{eff}} \)

✓ Construct the basis state: \( |\alpha\rangle \)

Calculate the Hamiltonian matrix elements:

\[
H_{\text{eff}}^{\alpha'\alpha} = \langle \alpha' | H_{\text{eff}} | \alpha \rangle
\]

\( \langle \alpha' | \) & \( | \alpha \rangle \) are the basis state of BLFQ, such as \( |qqq\rangle \).

Diagonalize \( H_{\text{eff}} \) and obtain its eigen spectrum

\[
H_{\text{eff}} |\beta\rangle = H_{\text{eff}}^{\beta} |\beta\rangle
\]

\( |\beta\rangle \) is the physical state and eigenstate of Hamiltonian. In case of proton \( |\beta\rangle = |P_{\text{proton}}\rangle \).

✓ Evaluate observables:

\[
\mathcal{O} = \langle \beta | \hat{O} | \beta \rangle
\]
Previous applications of BLFQ

**QCD systems**

- **Heavy quarkonium**: decay constant, elastic form factor, radii, radiative transitions, distribution amplitude, GPDs
  

- **Light mesons**: spectrum, decay constant, elastic form factor, radii, distribution amplitude, PDFs and scale evolution
  

**QED systems**

- **Electron**: anomalous magnetic moments

- **positronium** wave function, spectroscopy

- GPDs of the electron and positronium

Light front effective Hamiltonian, $H_{\text{eff}}$:

$$\tilde{k}_\perp^2 + m_q^2 \frac{x}{x} + \tilde{k}_\perp^2 + m_{\bar{q}}^2 \frac{1}{1-x} + \kappa^4 x(1-x)\tilde{r}_\perp^2 - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2} \partial_x \left(x(1-x)\partial_x\right) + H_{\text{NJL}}$$

- **Light front wavefunctions** ► eigenvectors of this Hamiltonian.
- **PDFs evolution** ► based on the NNLO DGLAP equations.
- The robustness of our results motivates the application of analogous effective Hamiltonians to the baryons.
Effective Light-front Hamiltonian for nucleon

Light-Front Hamiltonian

\[ P_{\text{baryon}}^- = H_{\text{K.E.}} + H_{\text{trans}} + H_{\text{longi}} + H_{\text{OGE}} \]

\[ H_{\text{K.E.}} = \sum_i \frac{p_i^2 + m_q^2}{p_i^+} \quad m_u = 0.6\text{GeV}, m_d = 0.57\text{GeV} \]

\[ H_{\text{trans}} \sim \kappa_T^4 b^4 \zeta^2 \quad \text{-- Brodsky, Teramond arXiv: 1203.4025} \]

\[ H_{\text{longi}} = -\sum_{ij} \kappa^4_L b^4 \partial x_i \left( x_i x_j \partial x_j \right) \quad \text{--- Y Li, X Zhao, P Maris, J Vary PLB 758(2016)} \]

\[ H_{\text{OGE}} = -\frac{C_F 4\pi \alpha_s(Q^2)}{Q^2} \sum_{i,j(i<j)} \bar{u}_{s'_i}(k'_i) \gamma^\mu u_{s_i}(k_i) \bar{u}_{s'_j}(k'_j) \gamma_\mu u_{s_j}(k_j) \]

Infrared cut off: \( m_g = 0.05 \text{ GeV}, C_F = -\frac{2}{3} \)

\[ |P_{\text{baryon}}^-\rangle = |qqq\rangle + |qqqg\rangle + |qqq \bar{q}\rangle + \ldots \]

Although we truncate to the leading Fock sector, we can solve the baryon system with multi-particle (at least three particle). In future, we can include higher Fock sector, and study the sea quark and gluon distribution function.
Basis construction

■ Fock’s space expansion:

\[ |N\rangle_{\text{baryon}} = a|qqq\rangle + b|qqqg\rangle + c|qqqq\bar{q}\rangle + \cdots \].

■ For each Fock particle:
  - **Transverse**: 2D harmonic oscillator basis: \( \Phi^b_{n,m}(\hat{p}_\perp) \)
    labeled by radial (angular) quantum number \( n \) (m); scale parameter \( b \)
    
    e.g., \( n=4 \)

  - **Longitudinal**: plane-wave basis, labeled by \( k \)
  - **Helicity**: labeled by \( \lambda \)

■ For the first Fock sector:

\[ |qqq\rangle = |n_{q1}, m_{q1}, k_{q1}, \lambda_{q1}\rangle \otimes |n_{q2}, m_{q2}, k_{q2}, \lambda_{q2}\rangle \otimes |n_{q3}, m_{q3}, k_{q3}, \lambda_{q3}\rangle. \]
Basis Truncation Scheme

• Symmetries of Hamiltonian:
  - Net fermion number:
    \[ \sum_i n_i^f = N_f \]
  - Total angular momentum projection:
    \[ \sum_i (m_i + s_i) = J_z \]
  - Longitudinal momentum:
    \[ \sum_i k_i = K \]

• Further truncation:
  - Fock sector truncation
  - Discretization in longitudinal direction
    \[ k_i = \begin{cases} 
      1, 2, 3, \ldots & \text{bosons} \\
      0.5, 1.5, 2.5 \ldots & \text{fermions} 
    \end{cases} \]
  - “N_max” truncation in transverse directions
    \[ \sum_i [2n_i + |m_i| + 1] \leq N_{\text{max}} \]
  UV cutoff \( \Lambda \sim b \sqrt{N_{\text{max}}} \); IR cutoff \( \lambda \sim b / \sqrt{N_{\text{max}}} \)
Flavor Form Factor in BLFQ

different quark masses
in kinetic and one gluon exchange term to minimize the effect of higher Fock Sector

Dirac form factor
d quark: good agreement with data;
u quark: deviates from the data at large $Q^2$

Pauli Form factor
d quark: $F_2(0) = -1.73$ (Exp. : $-2.03$)
u quark: $F_2(0) = 1.27$ (Exp. : 1.67)

With increasing the basis size, our result approach to the experiment data.
Nucleon Form Factor

\[ G_E(Q^2) = \sum q e_q F_1^q(Q^2) - \frac{Q^2}{4M^2} \sum q e_q F_2^q(Q^2), \]
\[ G_M(Q^2) = \sum q e_q F_1^q(Q^2) + \sum q e_q F_2^q(Q^2). \]

**Charge form factor**
- **Neutron**: agrees with experiment data
- **Proton**: also agrees with the data

**Magnetic form factor**
- **Proton**: more or less agrees with data
- **Neutron**: deviates from the data

With *increasing the basis size*, the magnetic form factor *approach to the experiment data*

*Anomalous magnetic moment*
- **Proton**: \( \mu_N = 2.42 \) (Exp.: 2.79)
- **Neutron**: \( \mu_N = -1.57 \) (Exp.: -1.91)
Electromagnetic radii

\[ \langle r^2_E \rangle^N = -6 \frac{dG^N_{E}(Q^2)}{dQ^2} \bigg|_{Q^2=0}, \]
\[ \langle r^2_M \rangle^N = -6 \frac{dG^N_{M}(Q^2)}{G^N_{M}(0) dQ^2} \bigg|_{Q^2=0} \]

The Sachs form factors are defined as

\[ G^N_{E}(Q^2) = F^N_{1}(Q^2) - \frac{Q^2}{4M^2_N} F^N_{2}(Q^2), \]
\[ G^N_{M}(Q^2) = F^N_{1}(Q^2) + F^N_{2}(Q^2). \]

<table>
<thead>
<tr>
<th>Quantity</th>
<th>$N_{\text{max}} = 6$</th>
<th>$N_{\text{max}} = 8$</th>
<th>Data from PDG</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r^p_E$ (fm)</td>
<td>0.966</td>
<td>0.877</td>
<td>0.877 ± 0.005</td>
</tr>
<tr>
<td>$r^p_M$ (fm)</td>
<td>1.094</td>
<td>1.008</td>
<td>0.777 ± 0.016</td>
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<tr>
<td>$\langle r^2_E \rangle^n$ (fm$^2$)</td>
<td>−0.1175</td>
<td>−0.1398</td>
<td>−0.1161 ± 0.0022</td>
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<tr>
<td>$r^n_M$ (fm)</td>
<td>1.278</td>
<td>1.2045</td>
<td>0.862$_{-0.008}^{+0.009}$</td>
</tr>
</tbody>
</table>
Generalized Parton Distribution functions (GPDs)

- **Deeply Virtual Compton Scattering (DVCS)**

  - **vector meson productions experiment:**

  ![DVCS Diagram](image)

  - **GPDs** appear in **DVCS** processes.
    - GPDs are functions of three variables:
      - Longitudinal momentum fraction \( \chi = \frac{k^+}{P'^+} \)
      - Longitudinal momentum transfer \( \Delta^+ \)
      - Skewness \( \xi = \frac{\Delta^+}{P'^+} = 0 \)
      - Square of total mom transfer \( t = \Delta^2 = (P' - P)^2 \)

  - Encode the information about:
    - Three dimensional spatial structure
    - The spin and orbital angular momentum

where the \( b \perp \) is transverse position of parton.
**Generalized Parton Distribution Functions (GPD)**

**GPD** encode the information about three dimensional spatial structure of nucleon, as well as the spin and orbital angular momentum of the constituents.

With **increasing momentum transfer** \((t)\), the peaks of distributions shift to larger \(x\);

At large \(x\), BLFQ results follow the quark-diquark model.

From BLFQ

\[
t = \Delta^2, \quad x = \frac{k^+}{P^+}, \quad \zeta = \frac{\Delta^+}{P^+} = 0
\]

[Work in progress, C. Mondal, Siqi Xu, et.al]
Generalized Parton Distribution Functions

With increasing momentum transfer ($t$), the peaks of distributions shift to larger $x$; At Large $x$, BLFQ results follow the quark-diquark model

$N_{\text{max}}=8$, $k_{\text{max}}=16$, $\alpha=0.8$, $\kappa_{L/T}=0.4$ GeV

From BLFQ

[Work in progress, C. Mondal, Siqi Xu, et.al]
Parton distribution functions in BLFQ:

\[ < x_q > = \int dx \, x f_1^q(x) \]

Average momentum is carried by the quark.

Here, we use the NNLO DGLAP equation to evolve the PDF. Qualitative behavior of PDF is more or less same with the global fit CTEQ 15 PDF.

**U Quark**

*Low x region*: agrees with global fit

*High x region*: deviates from the fit

**D Quark**

*Low x region*: deviates from the fit

*High x region*: more or less agrees with global fit

We need the contribution from higher Fock sector such as \(|qqqg\rangle, |qqq\, q\bar{q}\rangle\) to obtain better agreement, and to study the sea quark and gluon distributions.

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[Work in progress, C. Mondal, Siqi Xu, Jiangshan Lan, et.al]
Conclusions

- We have discussed the preliminary results of nucleon form factors, PDFs & GPDs in BLFQ approach.
- In the effective Hamiltonian, we have the kinetic energy & the confining potential in both the transverse and longitudinal direction and one gluon exchange with fixed coupling. Here, we consider only the leading Fock sector.
- BLFQ formalism provides promising results in order to understand the nucleon structure.

Outlook:

- Increase basis size
- Include the higher Fock component $|qqqq\rangle$.
- Investigate other nucleon properties.
- Investigate the structure of other baryons.

Thank You