



Istituto Nazionale di Fisica Nucleare
Laboratori Nazionali di Frascati

Lensing function relation in Hadrons

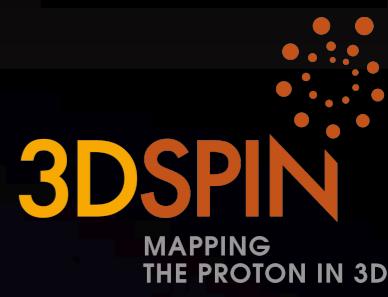
Simone Rodini

In collaboration with

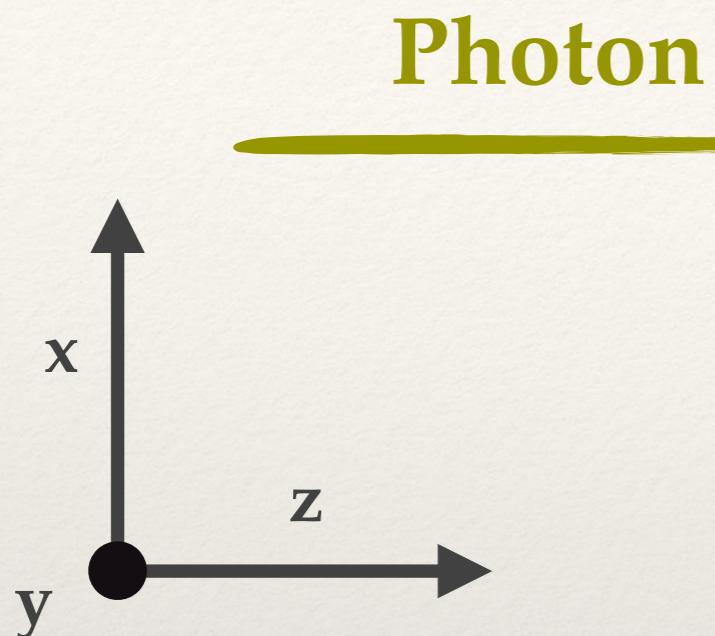
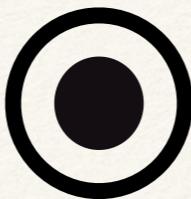
Barbara Pasquini and Alessandro Bacchetta



European Research Council
Established by the European Commission



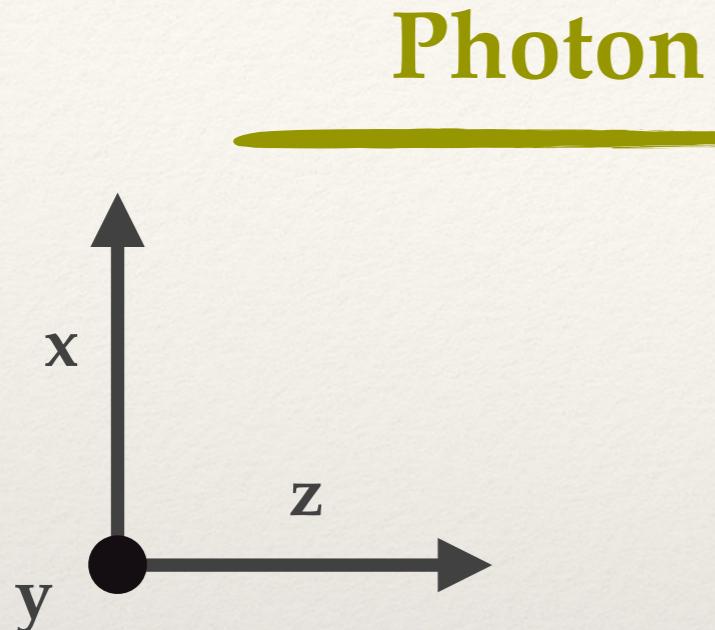
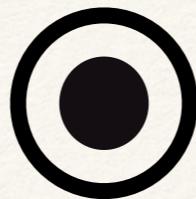
Proton Polarization



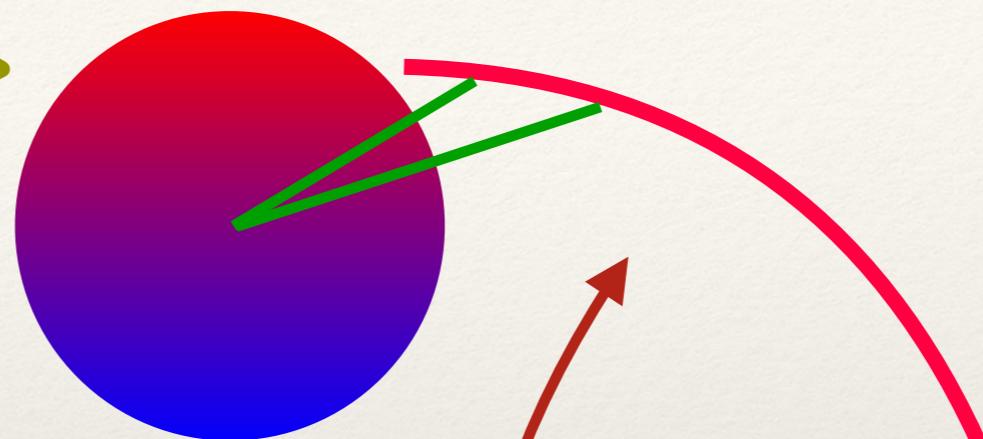
Gluons
 \approx
Final State Interactions

Detected hadron

Proton Polarization



Gluons
 \approx
Final State Interactions

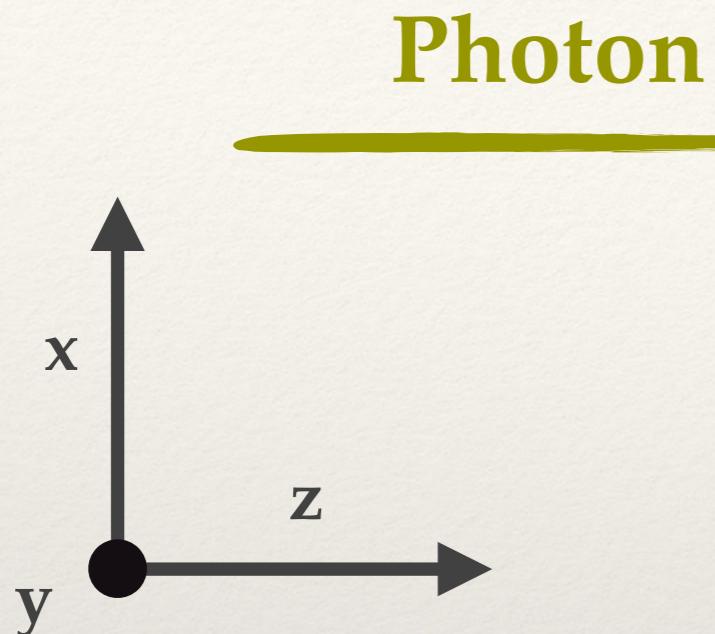
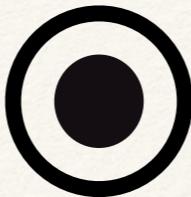


Detected hadron



**Transverse Momentum PDF
Informations**

Proton Polarization



Gluons

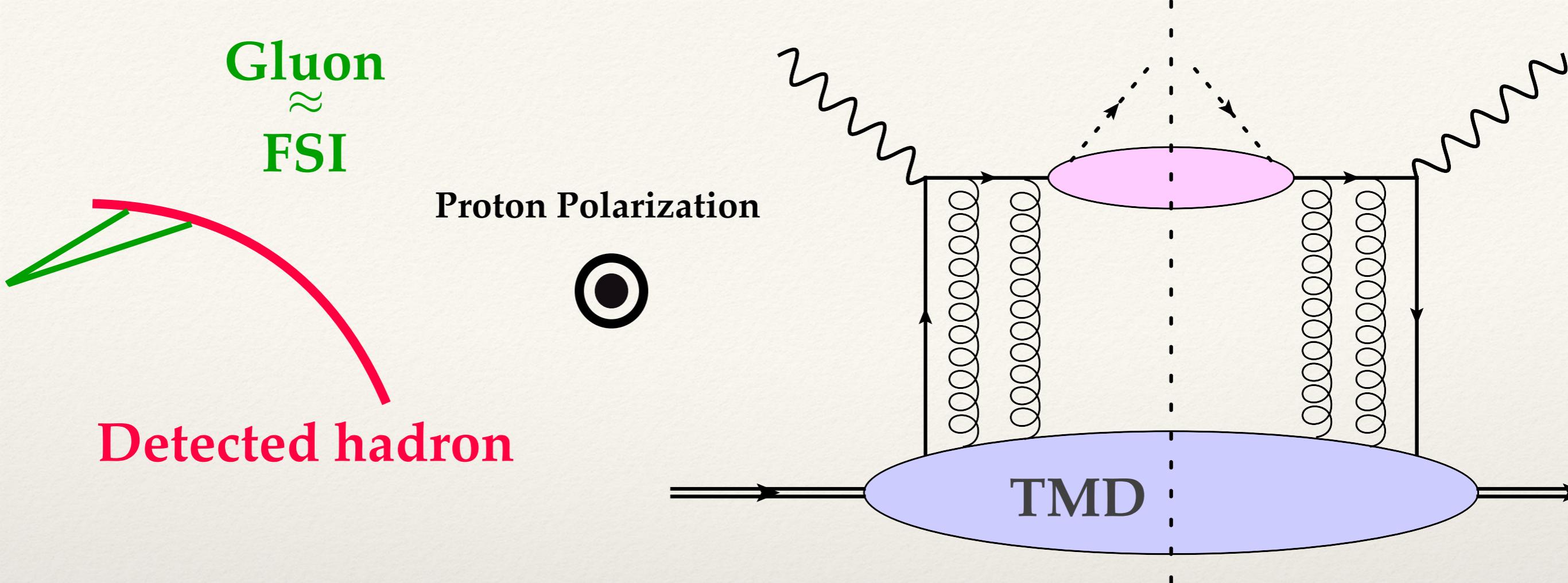
\approx

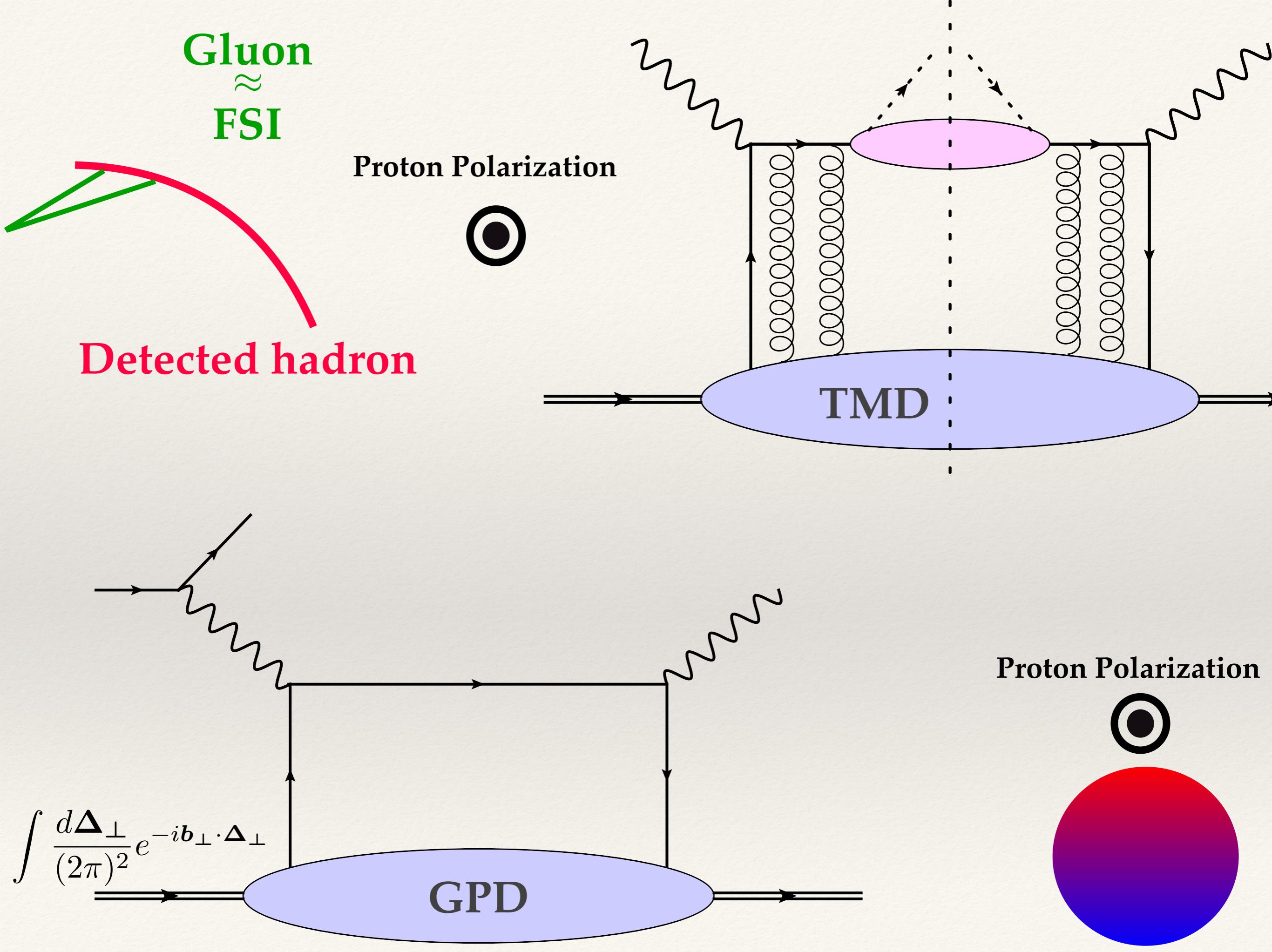
Final State Interactions

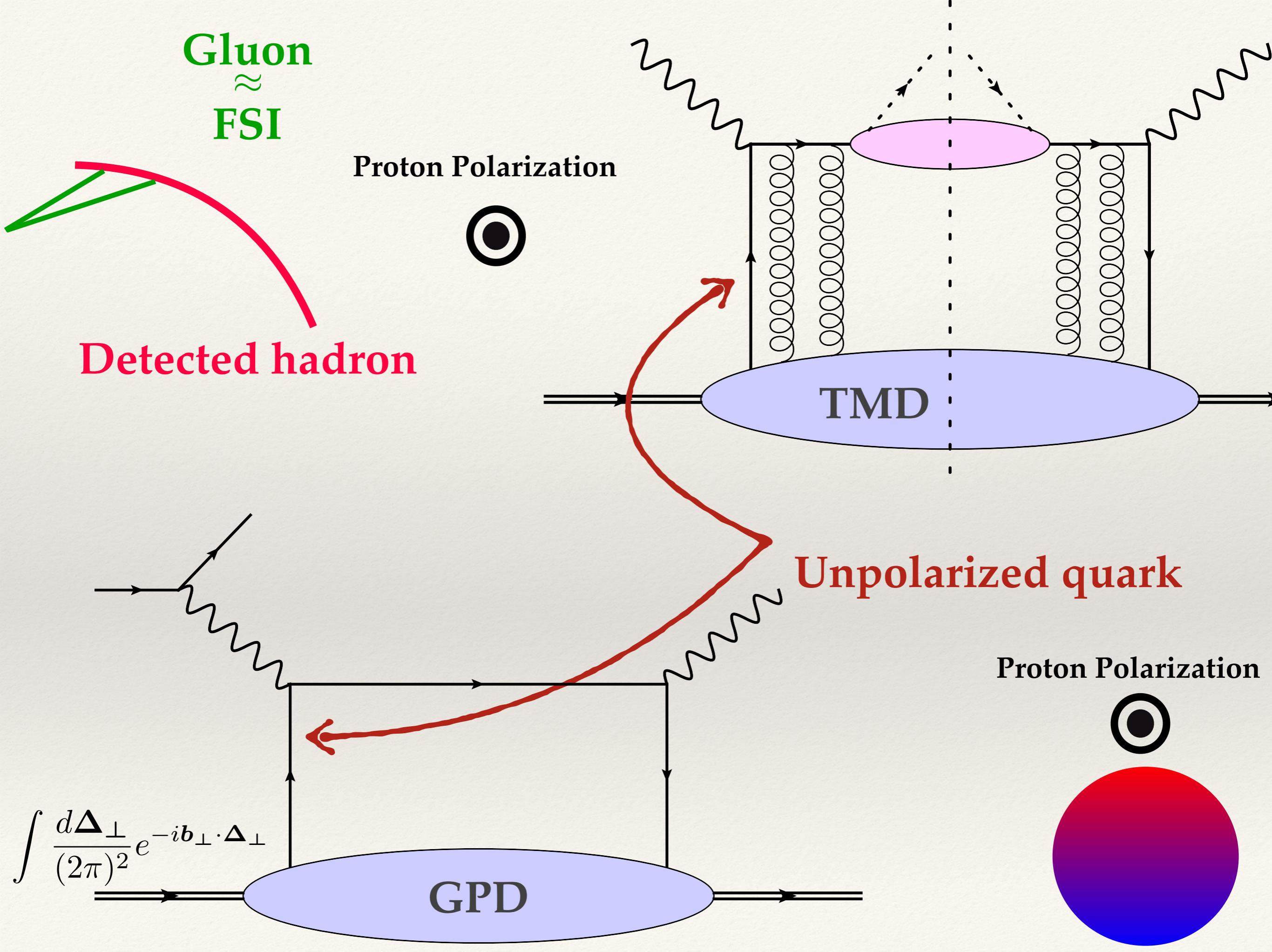
Detected hadron

**Generalized Parton Distribution
Informations**

**Transverse Momentum PDF
Informations**







Unpolarized quark in an transversely polarized proton

Unpolarized quark in an transversely polarized proton

$$f_{1T}^\perp\left(x,\textbf{\textit{k}}_\perp^2\right)\leftrightarrow-\mathcal{E}'\left(x,\textbf{\textit{b}}_\perp^2\right)$$

Unpolarized quark in an transversely polarized proton

$$f_{1T}^\perp(x, k_\perp^2) \leftrightarrow -\mathcal{E}'(x, b_\perp^2)$$



$$b_\perp \times S_\perp$$

Unpolarized quark in an transversely polarized proton

$$f_{1T}^{\perp}(x, k_{\perp}^2) \leftrightarrow -\mathcal{E}'(x, b_{\perp}^2)$$



$$k_{\perp} \times S_{\perp}$$

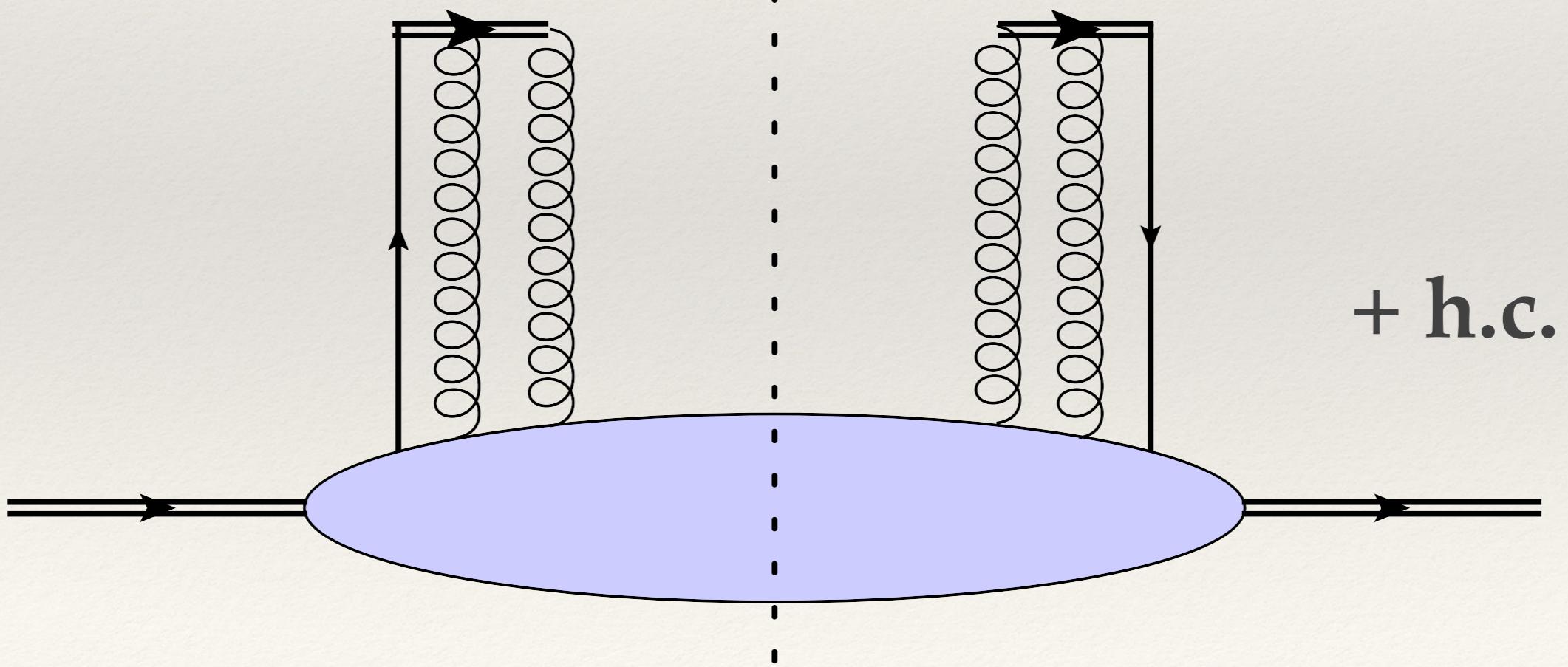


$$b_{\perp} \times S_{\perp}$$

$$\begin{aligned}\Phi^{[\Gamma]}\left(x,\boldsymbol{k}_{\perp},S\right) &= \frac{1}{2}\int\frac{dz^-d\boldsymbol{z}_{\perp}}{(2\pi)^3}e^{ixp^+z^--i\boldsymbol{k}_{\perp}\cdot\boldsymbol{z}_{\perp}} \\ &\times\langle p,S|\overline{\psi}\left(-\frac{z}{2}\right)\Gamma\mathcal{W}\left(-\frac{z}{2},\frac{z}{2}\right)\psi\left(\frac{z}{2}\right)|p,S\rangle\mid_{z^+=0}\end{aligned}$$

$$\begin{aligned} \Phi^{[\Gamma]}(x, \mathbf{k}_\perp, S) = & \frac{1}{2} \int \frac{dz^- dz_\perp}{(2\pi)^3} e^{ixp^+ z^- - i\mathbf{k}_\perp \cdot \mathbf{z}_\perp} \\ & \times \langle p, S | \bar{\psi} \left(-\frac{z}{2} \right) \Gamma \mathcal{W} \left(-\frac{z}{2}, \frac{z}{2} \right) \psi \left(\frac{z}{2} \right) | p, S \rangle |_{z^+=0} \end{aligned}$$

$$\langle k_\perp^i(x) \rangle_{UT} = \frac{1}{2} \int d^2 \mathbf{k}_\perp k_\perp^i \left[\Phi^{[\gamma^+]}(x, \mathbf{k}_\perp, \mathbf{S}_\perp) - \Phi^{[\gamma^+]}(x, \mathbf{k}_\perp, -\mathbf{S}_\perp) \right]$$



$$\left\langle k_{\perp}^i(x)\right\rangle _{UT}\approx\int d^2\boldsymbol{b}_{\perp}\mathcal{I}^{q,i}(x,\boldsymbol{b}_{\perp})\mathcal{F}^{[\gamma^{+}]}(x,\boldsymbol{b}_{\perp},S)$$

$$\left\langle k_{\perp}^i(x)\right\rangle _{UT}\approx\int d^2\boldsymbol{b}_{\perp}\mathcal{I}^{q,i}(x,\boldsymbol{b}_{\perp})\mathcal{F}^{[\gamma^{+}]}(x,\boldsymbol{b}_{\perp},S)$$

$$\begin{aligned}\mathcal{F}^{[\Gamma]}(x,\boldsymbol{b}_{\perp},S) = & \frac{1}{2}\int\frac{dz^-}{2\pi}e^{ixp^+z^-}\\&\times\left\langle p^+,\boldsymbol{R}_{\perp}=\boldsymbol{0}_{\perp},S|\overline{\psi}(z_1)\Gamma\psi\left(z_2\right)|p^+,\boldsymbol{R}_{\perp}=\boldsymbol{0}_{\perp},S\right\rangle\end{aligned}$$

$$\mathcal{F}^{[\gamma^{+}]} \left(x, \boldsymbol{b}_{\perp}, S \right) \propto \mathcal{E}' \left(x, \xi=0, b_{\perp}^2 \right) \qquad \qquad z_{1,2} = \left(0^{+}, \mp \frac{z^{-}}{2}, \boldsymbol{b}_{\perp} \right)$$

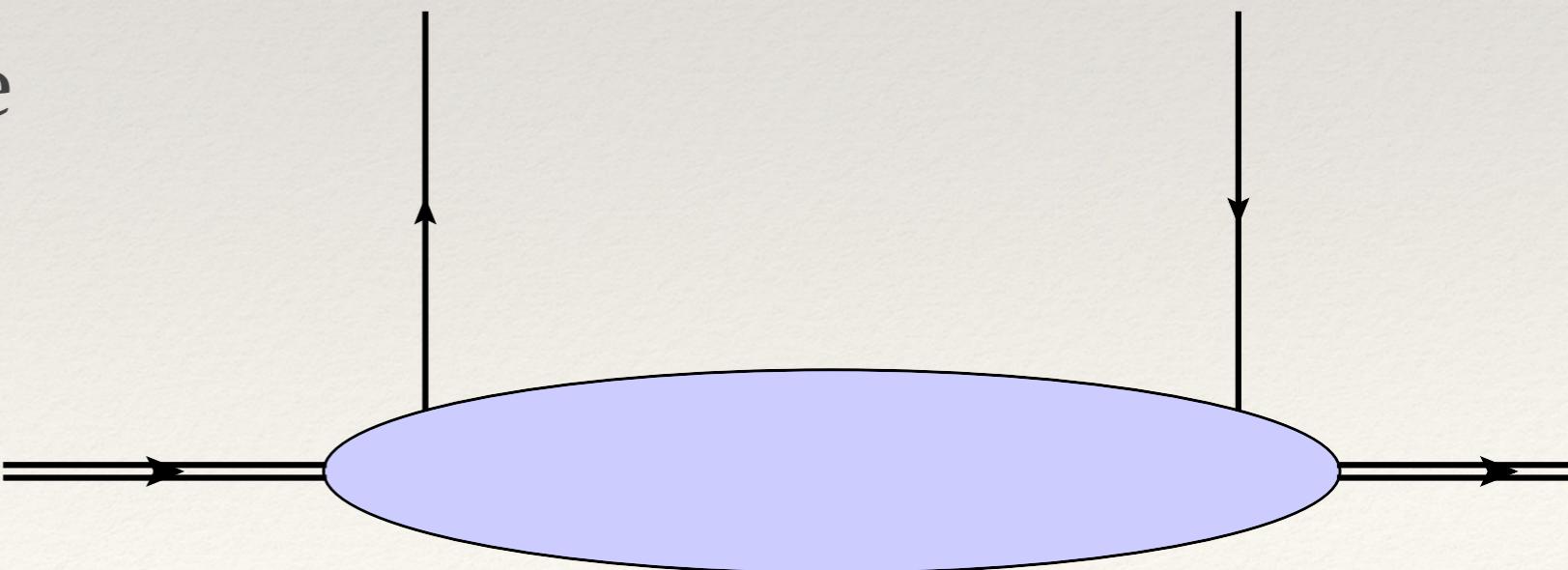
$$\langle k_{\perp}^i(x) \rangle_{UT} \approx \int d^2 \mathbf{b}_{\perp} \mathcal{I}^{q,i}(x, \mathbf{b}_{\perp}) \mathcal{F}^{[\gamma^+]}(x, \mathbf{b}_{\perp}, S)$$

$$\begin{aligned} \mathcal{F}^{[\Gamma]}(x, \mathbf{b}_{\perp}, S) &= \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixp^+z^-} \\ &\times \langle p^+, \mathbf{R}_{\perp} = \mathbf{0}_{\perp}, S | \bar{\psi}(z_1) \Gamma \psi(z_2) | p^+, \mathbf{R}_{\perp} = \mathbf{0}_{\perp}, S \rangle \end{aligned}$$

$$\mathcal{F}^{[\gamma^+]}(x, \mathbf{b}_{\perp}, S) \propto \mathcal{E}'(x, \xi = 0, b_{\perp}^2) \quad z_{1,2} = \left(0^+, \mp \frac{z^-}{2}, \mathbf{b}_{\perp}\right)$$

In light cone gauge

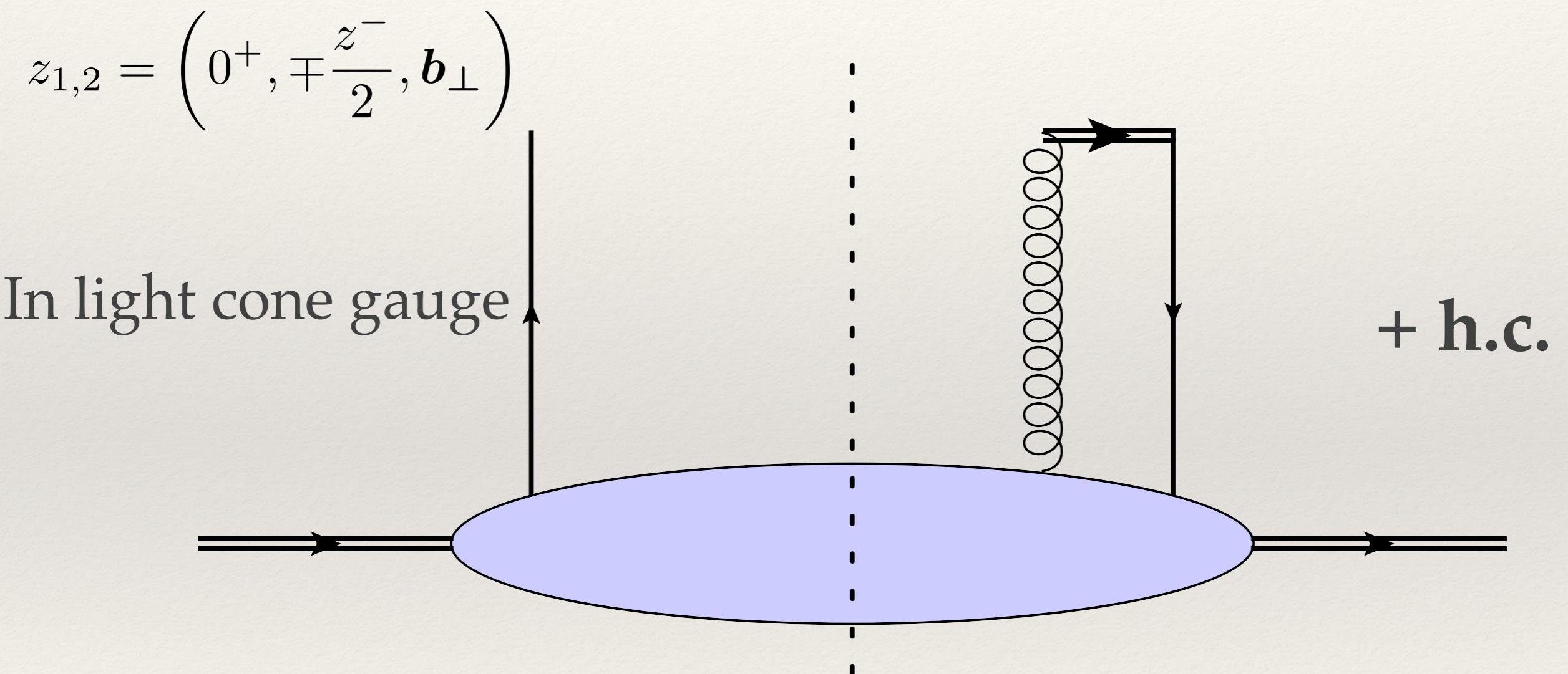
$$\int \frac{d\Delta_{\perp}}{(2\pi)^2} e^{-i\mathbf{b}_{\perp} \cdot \Delta_{\perp}}$$



$$\begin{aligned} \langle k_{\perp}^i(x) \rangle_{UT} = & \frac{1}{2} \int d^2 \boldsymbol{b}_{\perp} \int \frac{dz^-}{2\pi} e^{ixp^+z^-} \\ & \times \langle p^+, \boldsymbol{R}_{\perp} = \mathbf{0}_{\perp}, \boldsymbol{S}_{\perp} | \bar{\psi}(z_1) \gamma^+ I^{q,i}(z_2) \psi(z_2) | p^+, \vec{R}_{\perp} = \mathbf{0}_{\perp}, \boldsymbol{S}_{\perp} \rangle \Big|_{\substack{z_{\perp}^+ = 0 \\ \boldsymbol{z}_{\perp} = \mathbf{0}_{\perp}}} \end{aligned}$$

$$z_{1,2}=\left(0^+,\mp\frac{z^-}{2},\boldsymbol{b}_{\perp}\right)$$

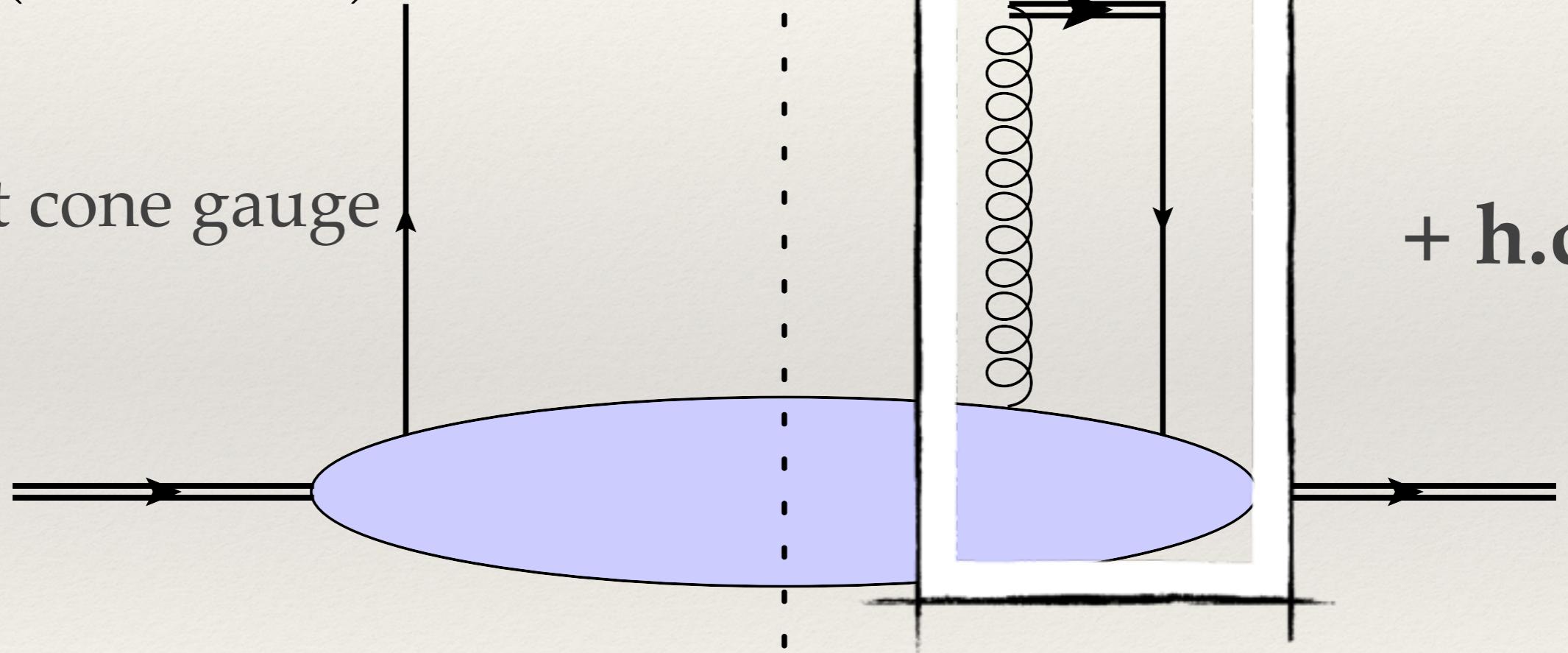
$$\langle k_\perp^i(x) \rangle_{UT} = \frac{1}{2} \int d^2 \mathbf{b}_\perp \int \frac{dz^-}{2\pi} e^{ixp^+ z^-} \\ \times \langle p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \mathbf{S}_\perp | \bar{\psi}(z_1) \gamma^+ I^{q,i}(z_2) \psi(z_2) | p^+, \vec{R}_\perp = \mathbf{0}_\perp, \mathbf{S}_\perp \rangle \Big|_{\substack{z_1^+ = 0 \\ \mathbf{z}_\perp = \mathbf{0}_\perp}}$$



$$\langle k_\perp^i(x) \rangle_{UT} = \frac{1}{2} \int d^2 \mathbf{b}_\perp \int \frac{dz^-}{2\pi} e^{ix p^+ z^-} \times \langle p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \mathbf{S}_\perp | \bar{\psi}(z_1) I^{q,i}(z_2) (z_2) | p^+, \vec{R}_\perp = \mathbf{0}_\perp, \mathbf{S}_\perp \rangle \Big|_{\substack{z_+^+ = 0 \\ \mathbf{z}_\perp = \mathbf{0}_\perp}}$$

$$z_{1,2} = \left(0^+, \mp \frac{z^-}{2}, \mathbf{b}_\perp \right)$$

In light cone gauge



$$I^{q,i}(z_2) = \frac{g_s}{2} A_\perp^i (\infty^-, 0^+, \mathbf{b}_\perp)$$

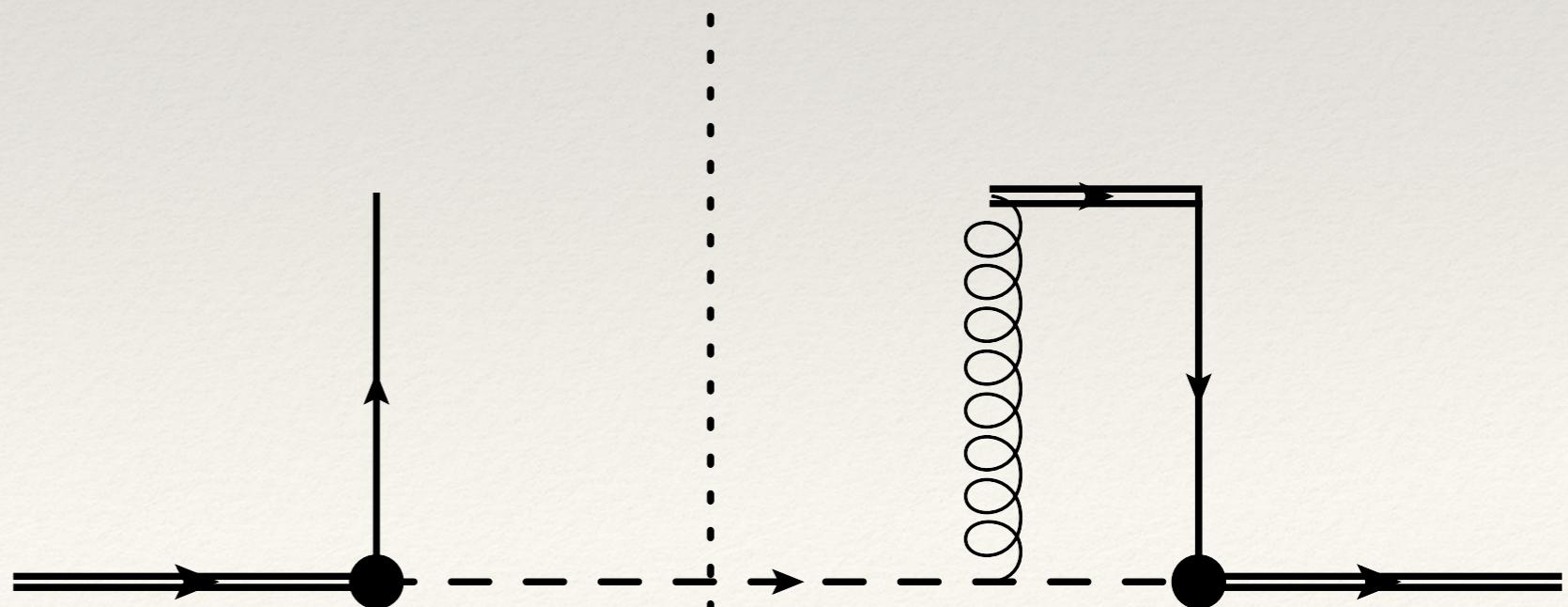
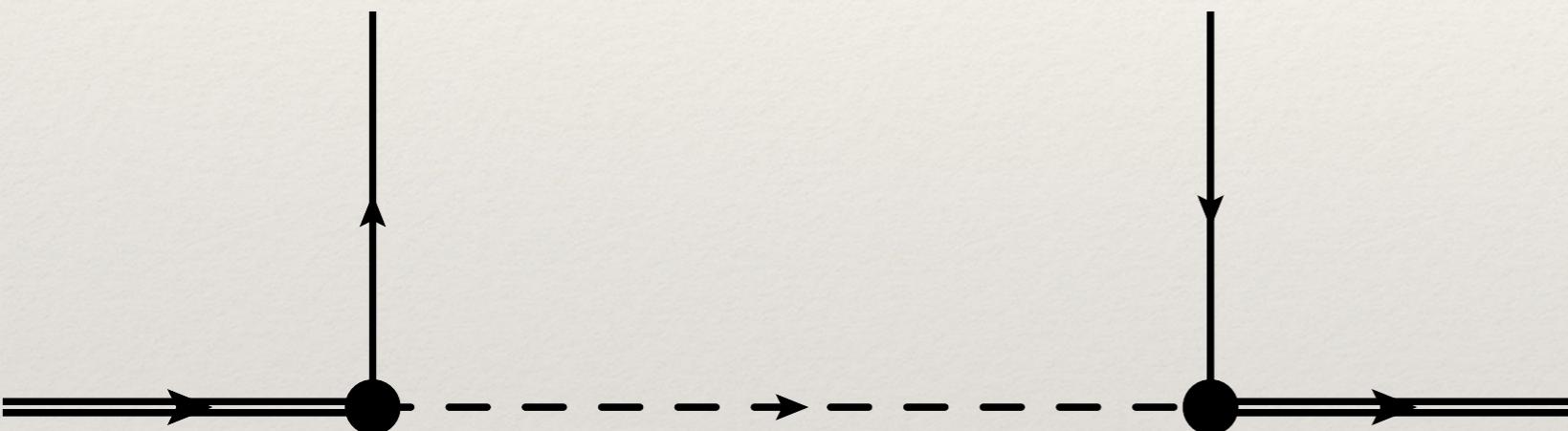
Proton remnant as a

Scalar

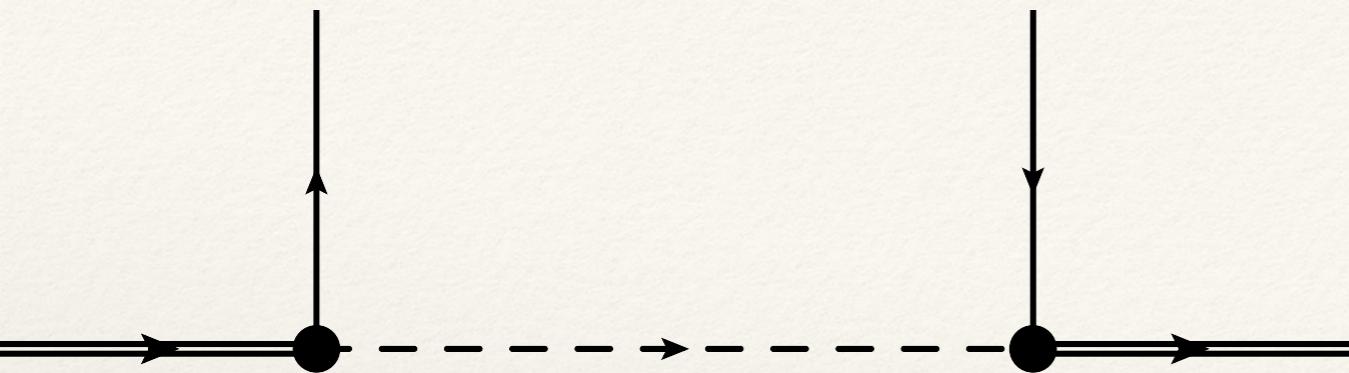
Point-like

Colored

Particle



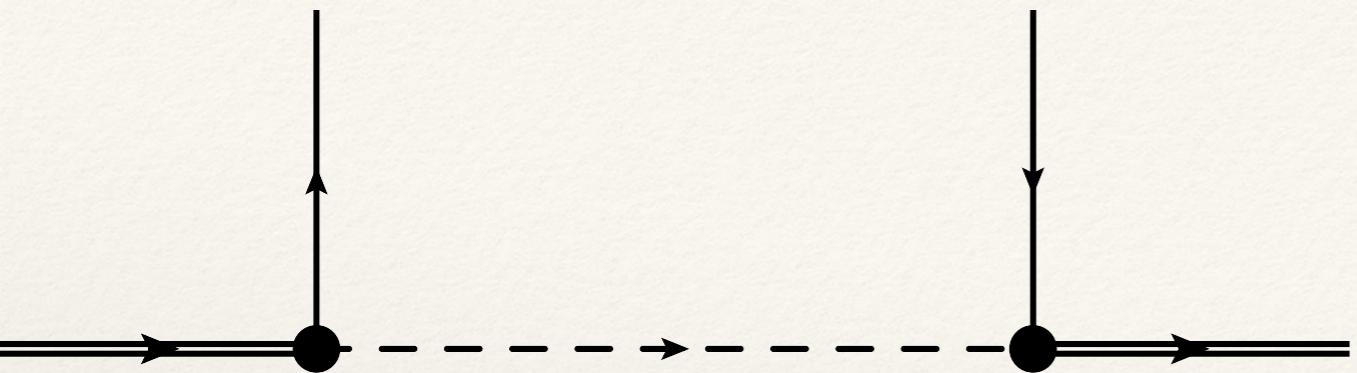
$$\mathcal{E}'(x, \xi = 0, b_\perp^2) \propto \mathcal{S}\left(x, \frac{-b_\perp}{1-x} \middle| x, \frac{-b_\perp}{1-x}\right)$$



Phys.Rev. D69 (2004) 074032

Phys.Lett. B530 (2002) 99-107

$$\mathcal{E}'(x, \xi = 0, b_\perp^2) \propto \mathcal{S}\left(x, \frac{-\mathbf{b}_\perp}{1-x} \middle| \left|x, \frac{-\mathbf{b}_\perp}{1-x}\right|\right)$$

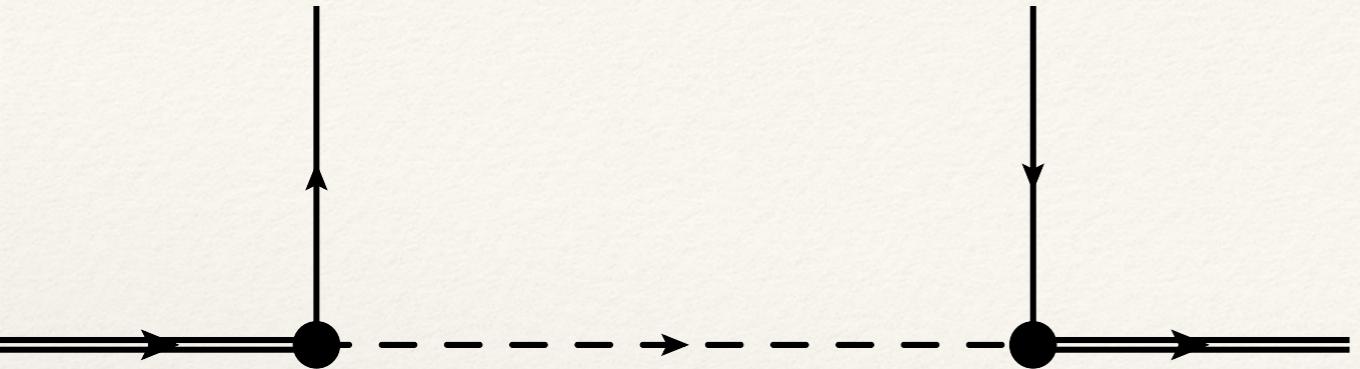


Phys.Rev. D69 (2004) 074032

Phys.Lett. B530 (2002) 99-107

$$\langle k_\perp^i(x) \rangle_{UT} \propto \int d\mathbf{k}_\perp k_\perp^i \int \frac{d\mathbf{q}_\perp}{q_\perp^2} S(x, \mathbf{k}_\perp | |x, \mathbf{k}_\perp - \mathbf{q}_\perp)$$

$$\mathcal{E}'(x, \xi = 0, b_\perp^2) \propto \mathcal{S}\left(x, \frac{-\mathbf{b}_\perp}{1-x} \middle| \middle| x, \frac{-\mathbf{b}_\perp}{1-x}\right)$$



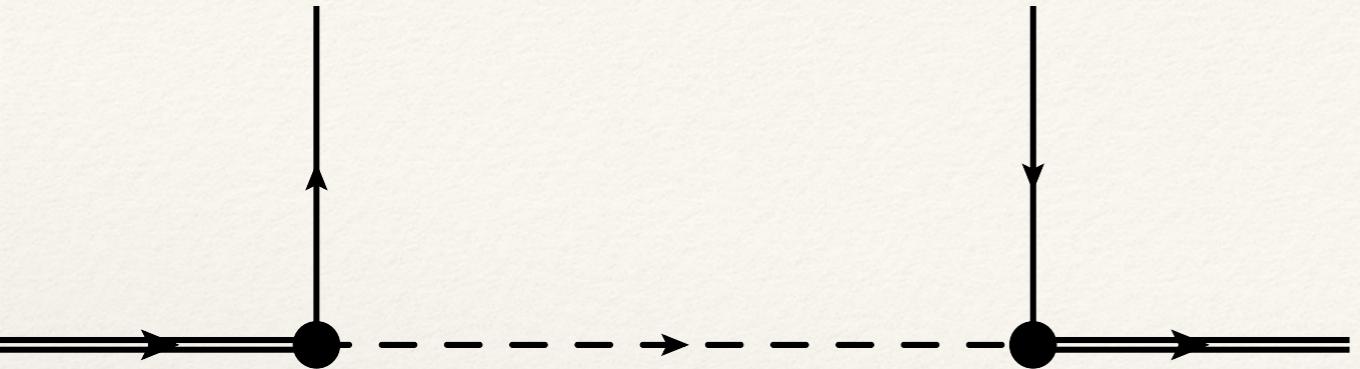
Phys.Rev. D69 (2004) 074032

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$$\langle k_\perp^i(x) \rangle_{UT} = \phi(x) \int d\mathbf{b}_\perp \int \frac{d\mathbf{q}_\perp}{q_\perp^2} q_\perp^i e^{-i\mathbf{b}_\perp \cdot \mathbf{q}_\perp} \mathcal{S}\left(x, \frac{-\mathbf{b}_\perp}{1-x} \middle| \middle| x, \frac{-\mathbf{b}_\perp}{1-x}\right)$$

$$\mathcal{E}'(x, \xi = 0, b_\perp^2) \propto \mathcal{S}\left(x, \frac{-b_\perp}{1-x} \middle| \middle| x, \frac{-b_\perp}{1-x}\right)$$

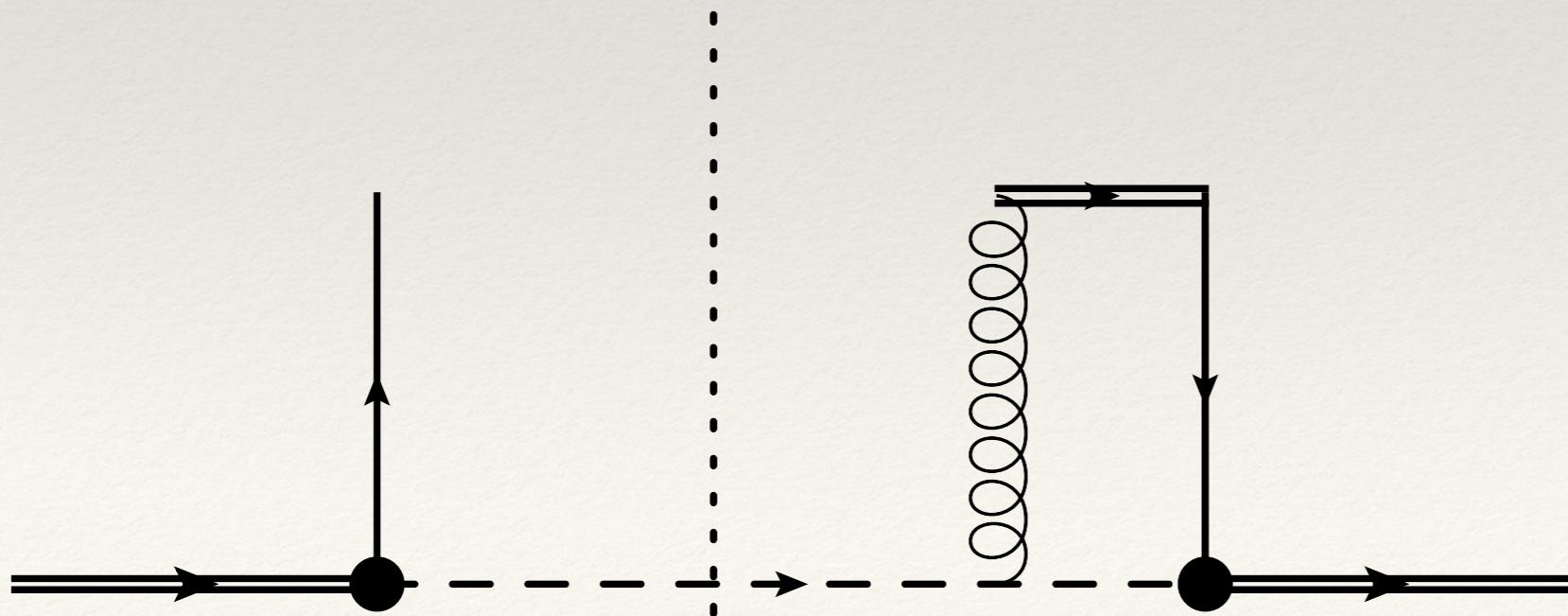


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$$\langle k_\perp^i(x) \rangle_{UT} \propto \int d\mathbf{k}_\perp k_\perp^i \int \frac{d\mathbf{q}_\perp}{q_\perp^2} S(x, \mathbf{k}_\perp | | x, \mathbf{k}_\perp - \mathbf{q}_\perp)$$

$$\langle k_\perp^i(x) \rangle_{UT} = \phi(x) \int d\mathbf{b}_\perp \int \frac{d\mathbf{q}_\perp}{q_\perp^2} q_\perp^i e^{-i\mathbf{b}_\perp \cdot \mathbf{q}_\perp} \mathcal{S}\left(x, \frac{-b_\perp}{1-x} \middle| \middle| x, \frac{-b_\perp}{1-x}\right)$$



+ h.c.

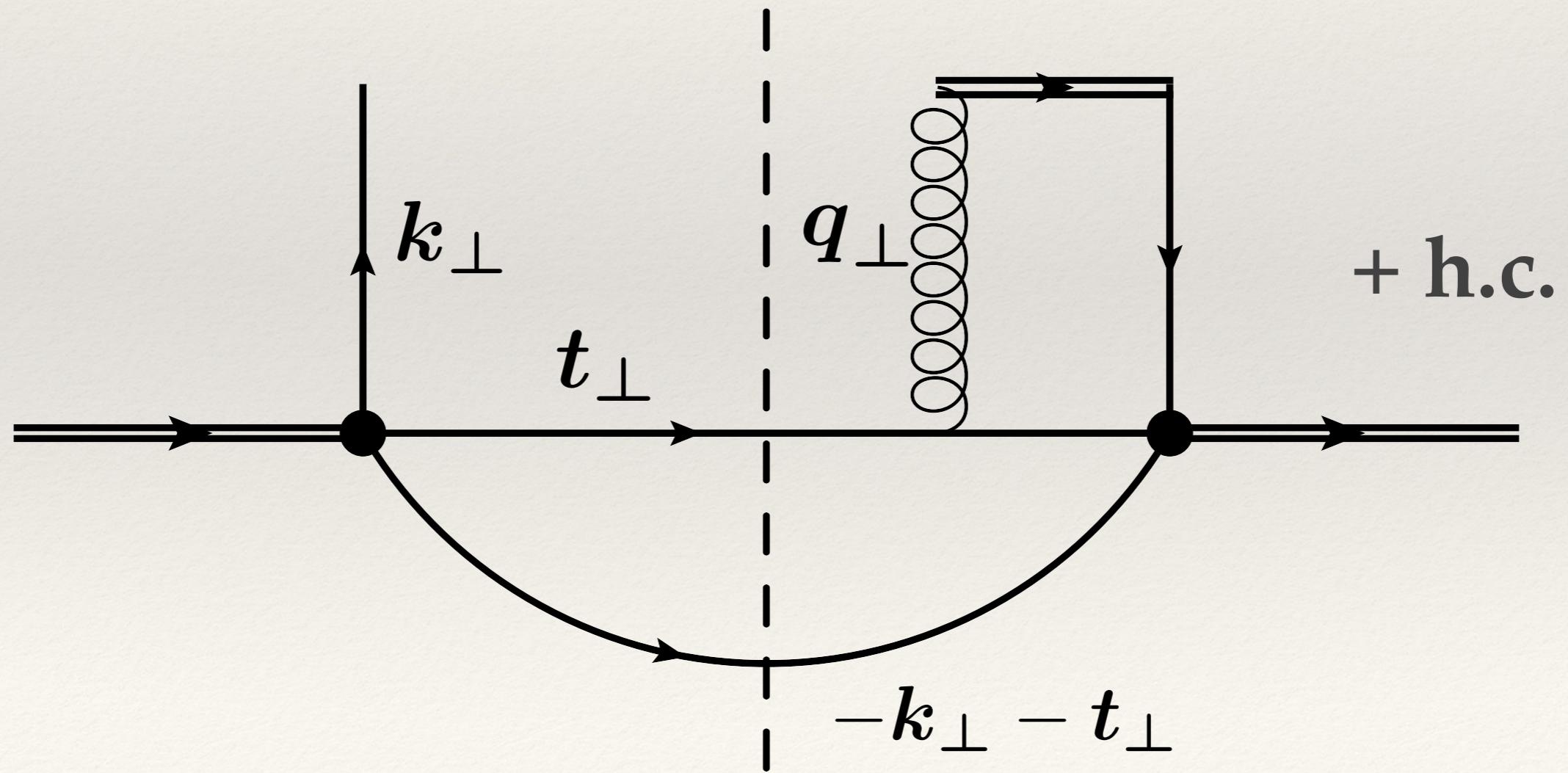
Three quark remnant

Three quark remnant

$$\begin{aligned} \langle k_{\perp}^i(x) \rangle_{UT} &\propto \int d\mathbf{k}_{\perp} k_{\perp}^i \int \frac{dq_{\perp}}{q_{\perp}^2} \int_0^x dy \int dt_{\perp} \\ &\times G(x, \mathbf{k}_{\perp}; y, t_{\perp} | |x, \mathbf{k}_{\perp} - \mathbf{q}_{\perp}; y, t_{\perp} + q_{\perp}) \end{aligned}$$

Three quark remnant

$$\langle k_{\perp}^i(x) \rangle_{UT} \propto \int d\mathbf{k}_{\perp} k_{\perp}^i \int \frac{dq_{\perp}}{q_{\perp}^2} \int_0^x dy \int dt_{\perp}$$
$$\times G(x, \mathbf{k}_{\perp}; y, t_{\perp} | |x, \mathbf{k}_{\perp} - \mathbf{q}_{\perp}; y, t_{\perp} + q_{\perp})$$



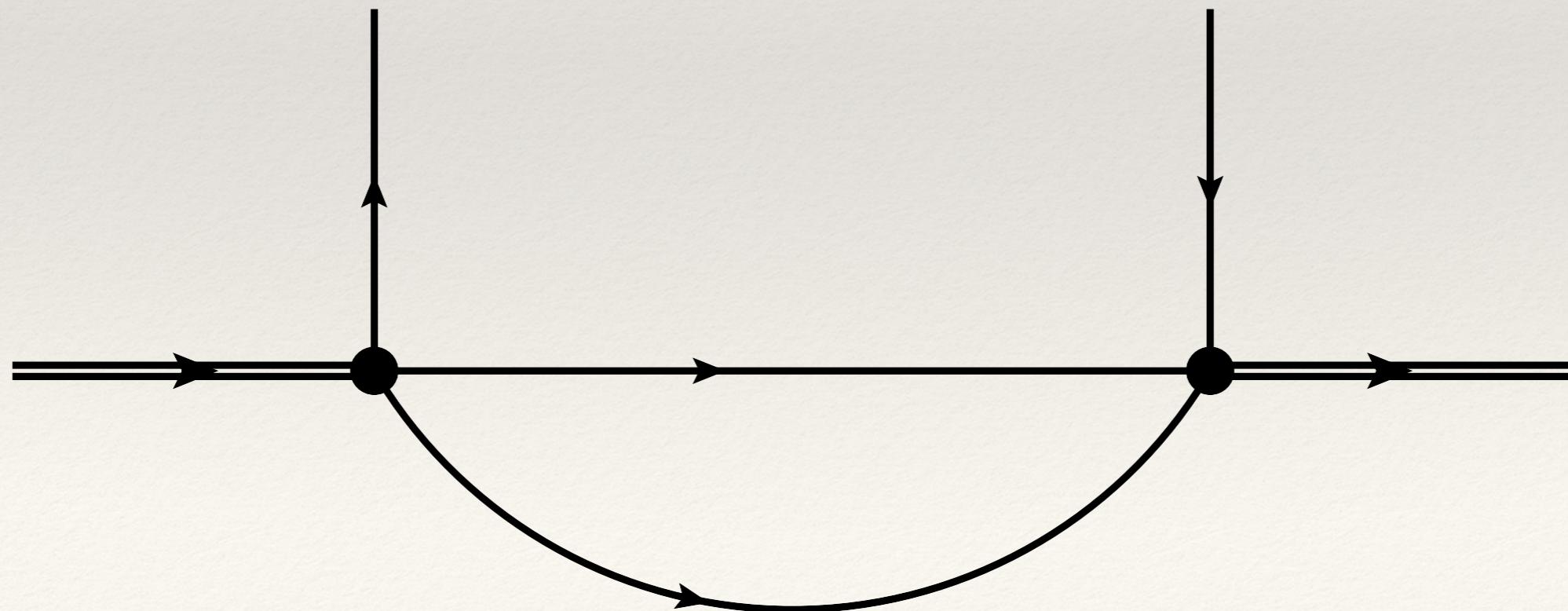
$$\begin{aligned} \left\langle k_{\perp}^i(x)\right\rangle_{UT} &\propto \int_0^x dy \int d\boldsymbol{b}_{\perp} d\boldsymbol{c}_{\perp} \frac{b_{\perp}^i}{b_{\perp}^2} \\ &\times \mathcal{G}\left(x,\frac{\boldsymbol{c}_{\perp}-\boldsymbol{b}_{\perp}}{1-x};y,\boldsymbol{c}_{\perp}\right|\left|x,\frac{\boldsymbol{c}_{\perp}-\boldsymbol{b}_{\perp}}{1-x};y,\boldsymbol{c}_{\perp}\right) \end{aligned}$$

$$\langle k_{\perp}^i(x) \rangle_{UT} \propto \int_0^x dy \int d\mathbf{b}_{\perp} d\mathbf{c}_{\perp} \frac{b_{\perp}^i}{b_{\perp}^2}$$

$$\times \mathcal{G} \left(x, \frac{\mathbf{c}_{\perp} - \mathbf{b}_{\perp}}{1-x}; y, \mathbf{c}_{\perp} \middle| \middle| x, \frac{\mathbf{c}_{\perp} - \mathbf{b}_{\perp}}{1-x}; y, \mathbf{c}_{\perp} \right)$$

$$\mathcal{E}'(x, b_{\perp}^2) \propto \int_0^x dy \int d\mathbf{c}_{\perp}$$

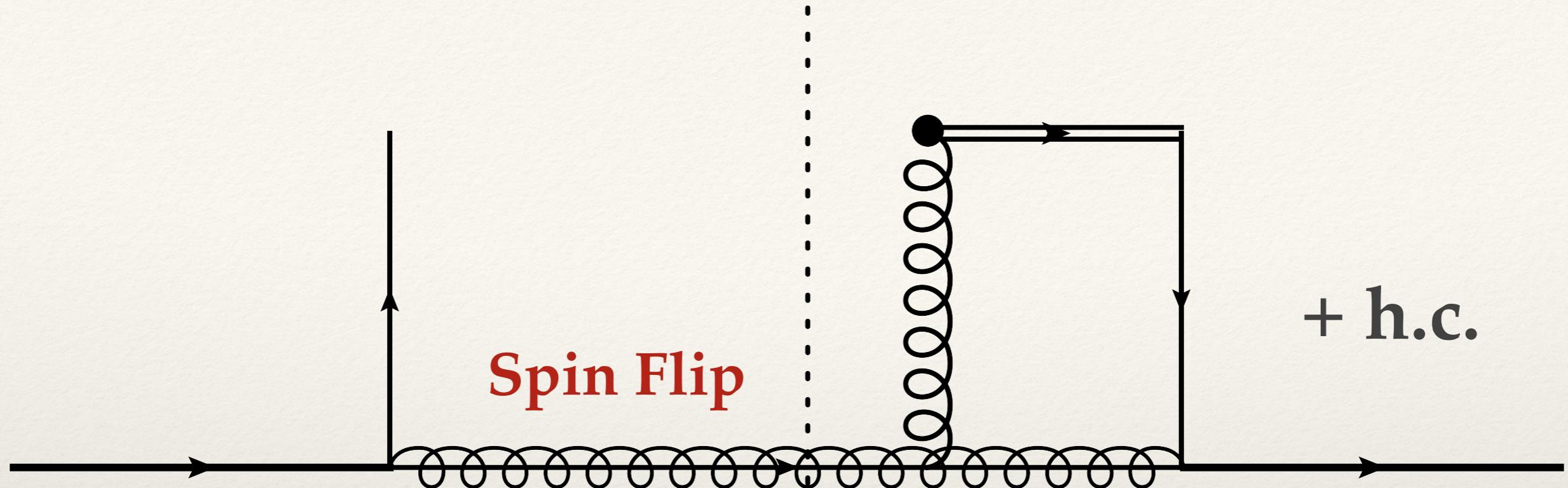
$$\times \mathcal{G} \left(x, \frac{y\mathbf{c}_{\perp} - \mathbf{b}_{\perp}}{1-x}; y, \mathbf{c}_{\perp} \middle| \middle| x, \frac{y\mathbf{c}_{\perp} - \mathbf{b}_{\perp}}{1-x}; y, \mathbf{c}_{\perp} \right)$$



**Many-partons remnant breaks the lensing
relation!**

**Many-partons remnant breaks the lensing
relation!**

But not only...



Conclusions

Model studies are useful to get insight on complex physics phenomena

Model-dependent relations between distributions can be useful

But they should not be extrapolated to different models