Studying gluon TMDs with $J/\psi$ pair production at the LHC

F. Scarpa, D. Boer, J.-P. Lansberg, C. Pisano, M. Schlegel

10th of April 2019
TMD factorisation and gluon distributions
TMD correlators & distributions

- Proton beams at the LHC = unpolarised

- Relevant TMDs at twist-2:
  - $f_{g}^{1}$: TMD distribution of unpolarised gluons
  - $h_{g}^{1\perp}$: TMD distribution of linearly polarised gluons

- Parametrisation of the TMD correlator for an unpolarised proton:

$$\Phi_{g}^{\mu\nu}(x, \vec{k}_T) = -\frac{1}{2x} \left[ g_{T}^{\mu\nu} f_{g}^{1}(x, \vec{k}_T) - \left( \frac{k_1^{\mu} k_1^{\nu}}{M_H^2} + g_{T}^{\mu\nu} \frac{\vec{k}_T^2}{2M_H^2} \right) h_{g}^{1\perp}(x, \vec{k}_T) \right] \quad (1)$$
\[ d\sigma^{gg} \propto \]

\[ F_1 \, C[f_1^g f_1^g] \]

⇒ azimuthally independent

\[ + F_2 \, C[w_2 \times h_1^g h_1^g] \]

⇒ azimuthally independent

\[ + \left( F_3 \, C[w_3 \times f_1^g h_1^g] + F_3' \, C[w_3' \times h_1^g f_1^g] \right) \cos(2\phi_{CS}) \]

⇒ \( \cos(2\phi_{CS}) \)-modulation

\[ + \, F_4 \, C[w_4 \times h_1^g h_1^g] \cos(4\phi_{CS}) \]

⇒ \( \cos(4\phi_{CS}) \)-modulation

modulations in \( \phi_{CS} = (\vec{q}_T, \vec{p}_T) \)
TMD Gaussian modelling:

- $f_1^g$ modelled as a Gaussian in $\vec{k}_T$:
  $$f_1^g(x, \vec{k}_T^2) = \frac{g(x)}{\pi \langle k_T^2 \rangle} e^{-\vec{k}_T^2 / \langle k_T^2 \rangle}$$

- "Gaussian" $h_1^\perp g(x, \vec{k}_T^2) \Rightarrow$ Model 1:
  Boer, de Dunnen, Pisano, Schlegel, Vogelsang, PRL 108 (2012) 032002

- Positivity bound: $h_1^\perp g(x, \vec{k}_T^2) \leq \frac{2M_p^2}{k_T^2} f_1^g(x, \vec{k}_T^2) =$ maximal value (bound saturated) $\Rightarrow$ Model 2

- In order to isolate the different $\phi$-dependences, one can define:
  $$2\langle \cos(n\phi_{CS}) \rangle = \frac{\int_0^{2\pi} d\phi_{CS} \cos n\phi_{CS} d\sigma}{\int_0^{2\pi} d\phi_{CS} d\sigma} \simeq \frac{F_n C[w_n TMD_1 TMD_2]}{F_1 C[f_1^g f_1^g]}$$

  $$n = 2, 4$$

  = relative amplitude of the azimuthal modulations \hspace{1cm} (2)
Evolution in $b_T$ space

- TMDs contain UV and rapidity divergences $\Rightarrow$ introduction of a cut-off scale $\mu$ and a regulator $\zeta$:

$$f(x, \vec{k}^2_T) \rightarrow f(x, \vec{k}^2_T, \zeta, \mu) \quad (3)$$

- 3 evol. equations $\Rightarrow \mu$ and $\zeta$ dependence of the TMDs $\Rightarrow$ evaluated at natural scale to minimise large logs:

$$C[f^g_1 f^g_1] = \int d^2\vec{k}_{1T} d^2\vec{k}_{2T} \delta^{(2)}(\vec{k}_{1T} + \vec{k}_{2T} - \vec{q}_T) f^g_1(x_1, k^2_{1T}; \zeta_1, \mu) f^g_1(x_2, k^2_{2T}; \zeta_2, \mu)$$

$$= \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{b}_T \cdot \vec{q}_T} e^{-S_A(b_T; Q)} \tilde{f}^g_1(x_1, b^2_T; \mu^2_b, \mu_b) \tilde{f}^g_1(x_2, b^2_T; \mu^2_b, \mu_b) \quad (4)$$

\[\uparrow\]

- perturbative Sudakov factor

- natural scale: $\mu_b = \frac{b_0}{b_T}$
Large and small $b_T$ regulations

- **Small $b_T$**: $\mu_b > Q$ for small $b_T < b_0/Q$
  
  ⇒ regularisation: $\mu_b \rightarrow \mu'_b = \frac{Qb_0}{\sqrt{Q^2b_T^2 + b_0^2}}$

- **Large $b_T$**: $S_A$ expression only valid at $b_T \ll \Lambda_{QCD}^{-1}$

  ⇒ $b_*$ prescription: $\tilde{\mathcal{W}}(b_T) \equiv \tilde{\mathcal{W}}(b_*) e^{-S_{NP}(b_T)}$ with $b_* = \frac{b_T}{\sqrt{1 + \frac{b_T^2}{b_{T\text{max}}^2}}}$
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  $\Rightarrow b_\star$ prescription: $\tilde{W}(b_T) \equiv \tilde{W}(b_\star) e^{-S_{NP}(b_T)}$ with $b_\star = \frac{b_T}{\sqrt{1 + \frac{b_T^2}{b_{T\text{max}}^2}}}$

- $S_A$ analytical and identical for $f^g_1$ and $h^\perp g_1$

  $S_{NP}$ encodes all the nonperturbative (NP) information on the TMDs; different between $f^g_1$ and $h^\perp g_1$

- Perturbative component of the TMDs:

  \[
  \tilde{f}^g_1(x, b_T^2; \mu^2_b, \mu'_b) = f_{g/P}(x; \mu'_b) + O(\alpha_s)
  \]

  \[
  \tilde{h}^\perp g_1(x, b_T^2; \mu^2_b, \mu'_b) = \frac{\alpha_s(\mu'_b) C_A}{\pi} \int_x^1 \frac{d\hat{x}}{\hat{x}} \left( \frac{\hat{x}}{x} - 1 \right) f_{g/P}(\hat{x}; \mu'_b) + O(\alpha_s^2)
  \]
Processes of interest
\[ gg \rightarrow Q + \gamma \]

- \( q\bar{q} \) contributions negligible \( \Rightarrow \) gluon sensitive process

- The possibility of isolating the quarkonium can eliminate the CO contributions; the photon is isolated

- CO (orange) smaller than CS (blue); isolation not needed for \( \Upsilon \)

- At 14 TeV, \( \sigma(J/\psi|\Upsilon + \gamma)|Q > 20 \text{ GeV} \approx 100 \text{ fb} \); about half at 7 TeV

- With the \( \mathcal{L} \approx 20 \text{ fb}^{-1} \) of \( pp \) data on tape, one expects up to 2000 events

- ATLAS has looked for \( H^0 \rightarrow J/\psi(\Upsilon) + \gamma \) at \( Q \approx 125 \text{ GeV} \)

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Results with UGDs as Ansätze for TMDs

- Observable $S_{q_T}^{(n)} = \frac{F_n C[w_n TMD_1^{(n)} TMD_2^{(n)}]}{F_1} \int d^2 \vec{q}_T C[f_1^g f_1^g] \Rightarrow S_{q_T}^{(0)}, S_{q_T}^{(2)}, S_{q_T}^{(4)}$

- $S_{q_T}^{(2)}, S_{q_T}^{(4)} \neq 0 \Rightarrow$ nonzero gluon polarisation in unpolarised protons

- $S_{q_T}^{(0)} : f_1^g(x, k_T)$ from the $q_T$-dependence of the yield

- $\int d^2 \vec{q}_T S_{q_T}^{(2,4)}$ probably measurable (few percent : ok with 2000 events)
$gg \rightarrow \eta_c + \eta_c$

- Theoretically simple, low $q\bar{q}$ contribution
- No reason for significant CO: $\langle 0 | O_1^\eta (1 S_0) | 0 \rangle \Rightarrow$ factorisation expected to hold

<table>
<thead>
<tr>
<th>$Q(GeV)$ ((6.0, 10.0))</th>
<th>((10.0, 15.0))</th>
<th>((15.0, 20.0))</th>
<th>((20.0, 40.0))</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle 1 \rangle (pb)$</td>
<td>$2.3 \times 10^4$</td>
<td>$1.7 \times 10^3$</td>
<td>$1.8 \times 10^2$</td>
</tr>
<tr>
<td>$</td>
<td>\langle \cos 2\phi \rangle</td>
<td>(pb)$</td>
<td>$2.4 \times 10^3$</td>
</tr>
<tr>
<td>$\langle \cos 4\phi \rangle (pb)$</td>
<td>$0.20 \times 10^2$</td>
<td>$9.1$</td>
<td>$2.5$</td>
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Guang-Peng Zhang / Phys.Rev. D 90 (2014) 9, 094011. The $\eta_c$ weighted differential cross-sections obtained from Gaussian model at $\sqrt{s} = 7$ TeV and $\Delta y = 0$ with $\alpha_s = 0.15$ and $M_{\eta_c} = 3.0$ GeV

- At $\sqrt{s} = 14$ TeV, cross-sections will increase by a 2 factor ($\langle 1 \rangle \sim \sigma$)

\[
\langle 1, \cos 2\phi \rangle \times Br^2(\eta_c \rightarrow p\bar{p}) \simeq 1 - 50 \text{ fb} \ (\text{observable at LHC Run II ?})
\]

$\langle \cos 4\phi \rangle$ negligible

- However $\eta_c$ remain hard to see in experiments
$J/\psi$ pair production
$gg \rightarrow J/\psi + J/\psi : SPS$

- $J/\psi$ rather easy to detect $\Rightarrow$ Pair prod. already studied by 4 collaborations:

  - LHCb JHEP 1706 (2017) 047
  - CMS JHEP 1409 (2014) 094
  - D0 PRD 90 (2014) 111101
\( gg \rightarrow J/\psi + J/\psi : \text{SPS} \)

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- LO dominant for \( P_{\psi\psi_T} \leq 6 \text{ GeV} \)
- Larger \( P_{\psi\psi_T} \Rightarrow \text{NLO}^* \) dominates: the pair recoils against a real gluon
- CO channels are negligible \( \Rightarrow \) FSI do not break TMD factorisation
$gg \rightarrow J/\psi + J/\psi$ : DPS

- Double Parton Scattering (DPS) should compete with Single Parton Scattering (SPS) at LHCb
- Large uncertainties: a clear way to extract the DPS distribution is still lacking

- ATLAS: $p_T$-cut on the $J/\psi$'s; small DPS
- Large $P_{\psi\psi T}$: TM of the pair generated perturbatively from recoil against a real hard gluon $\Rightarrow$ wipes out TMD effects
2 → 2 processes computed in the TMD framework

- $gg \rightarrow \gamma\gamma$: J.W Qiu, M. Schlegel, W. Vogelsang, PRL 107, 062001 (2011)
2 → 2 processes computed in the TMD framework

- \( gg \rightarrow \gamma\gamma \): J.W Qiu, M. Schlegel, W. Vogelsang, PRL 107, 062001 (2011)
- \( H^0 + \text{jet} \): D. Boer, C. Pisano, Phys. Rev. D91 no. 7, (2015) 074024

Hard scattering coefficients bound:

\[
F_1 \geq F_{2,3,4} \quad (6)
\]

\[ gg \rightarrow J/\psi + J/\psi \] limit at \( M_{\psi\psi} \gg M_\psi \) and \( \cos(\theta_{CS}) \rightarrow 0 \):

\[
\begin{align*}
F_{1,4} & \rightarrow \frac{256N}{M_{QQ}^4 M_\psi^2}, \\
\frac{F_2}{F_1} & \rightarrow \frac{81M_Q^4 \cos(\theta_{CS})^2}{2M_{QQ}^4}, \\
\frac{F_3}{F_1} & \rightarrow -\frac{24M_Q^2 \cos(\theta_{CS})^2}{M_{QQ}^2}
\end{align*} \quad (7)
\]

\( F_4 = F_1 \) at large \( M_{\psi\psi} \) ⇒ unique feature of di-\( J/\psi \)
The $P_{\psi\psi_T}$-spectrum and azimuthal asymmetries

- $\langle k_T^2 \rangle$ fit over normalised LHCb’s $d\sigma/dP_{\psi\psi_T}$ data using the analytical expression of $C[f_1^g f_1^g]$:

- Integration over $\phi \Rightarrow \cos(n\phi)$-contributions cancelled out

- $F_2 \ll F_1 \Rightarrow$ only $C[f_1^g f_1^g]$ contributes to the cross-section

- Large value of $\langle k_T^2 \rangle$: indication for evolution effects?
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Large value of $\langle k_T^2 \rangle$ : indication for evolution effects ?

$\cos 4\phi$-mod. up to 50% !
Evolution : the $P_{\psi\psi_T}$-spectrum

- The TMD scale $\mu'_b$, the perturbative and NP Sudakov factors contribute to the broadening of the $P_{\psi\psi_T}$-spectrum

- Narrow $b_T$-spectrum $\leftrightarrow$ broad $P_{\psi\psi_T}$-spectrum

- $S_{NP}$ unconstrained for gluons
  $\Rightarrow S_{NP} = A \ln(M_{\psi\psi}) \ b_T^2$

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### Graphs

- Gaussian $\langle k_T^2 \rangle = 3.3 \pm 0.8$ GeV$^2$
- Evolved $f_1g$, $b_{T\text{lim}} \in [2;8]$ GeV$^{-1}$
Evolution : the $P_{\psi \psi T}$-spectrum

- The TMD scale $\mu_b'$, the perturbative and NP Sudakov factors contribute to the broadening of the $P_{\psi \psi T}$-spectrum

- Narrow $b_T$-spectrum $\leftrightarrow$ broad $P_{\psi \psi T}$-spectrum

- $S_{NP}$ unconstrained for gluons
  $\Rightarrow$ $S_{NP} = A \ln(M_{\psi \psi}) b_T^2$

- Large $M_{\psi \psi}$: $S_A$ strongly suppresses large $b_T$ values
  $\Rightarrow$ broader spectrum, weaker NP Sudakov influence

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Evolution: Azimuthal asymmetries

- Asymmetries quite smaller: $\alpha_s$-suppression of $h_1^\perp g$

- Double $\alpha_s$-suppression in $C[w_4 h_1^\perp g h_1^\perp g]$ compensated by large hard-scattering coefficient $F_4$
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- Double $\alpha_s$-suppression in $C[w_4 h_1^\perp g h_1^\perp g]$ compensated by large hard-scattering coefficient $F_4$

- Ratios increasing with $P_{\psi\psi_T}$

\[ \downarrow \]

Convolutions involving $h_1^\perp g$ fall slower with $P_{\psi\psi_T}$ than $C[f_1^g f_1^g]$

- Size of asymmetries sensitive to width of $S_{NP} \Rightarrow$ constraint on $\langle k_T^2 \rangle$ for both TMDs
Evolution: Azimuthal asymmetries

- \( C[w_3 f_1^g h_1^g] \) and \( C[w_4 h_1^g h_1^g] \) more Sudakov-suppressed than \( C[f_1^g f_1^g] \)

- Convol. ratios and asymmetries fall with \( M_{\psi\psi} \)

\[ \langle \cos(2\phi_{CS}) \rangle \text{ in [%]} \]

\[ M_{\psi\psi} \text{ [GeV]} \]

\[ |\cos \theta_{CS}| < 0.25 \]
\[ x_1 = x_2 = 10^{-2} \]

\[ P_{\psi\psi} = 4 \text{ GeV}, 7 \text{ GeV}, 10 \text{ GeV} \]

\( b_{\text{lim}} = 2 \text{ GeV}^{-1}, 4 \text{ GeV}^{-1}, 8 \text{ GeV}^{-1} \)
Evolution: Azimuthal asymmetries

- \( C[w_3 f \perp g h \perp g] \) and \( C[w_4 h \perp g h \perp g] \) more Sudakov-suppressed than \( C[f \perp g f \perp g] \)

- Convol. ratios and asymmetries fall with \( M_{\psi \psi} \)

- Combined with \( M_{\psi \psi} \)-dependence of hard-scattering coefficients ratios

- \( M_{\psi \psi} \)-shape of asymmetries = test of evolution, access to the NP component
Summary

- The features of quarkonia make them good probes for the study of poorly known gluon TMDs

- $gg \rightarrow J/\psi + J/\psi$ is a promising channel to investigate:
  
  - LHC data already available to realise an extraction of the gluon TMDs
  
  - Hard-scattering coefficients are large enough to allow for various sizeable azimuthal asymmetries
  
  - Including evolution effects allows extracting the intrinsically non-perturbative component of TMDs; asymmetries suffer suppression but remain reasonably large
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Backup slides
Beyond collinear factorisation

- Observed final-state $q_T$ generated from "intrinsic" $k_T$ from initial partons

- TMD factorisation: collinear partonic scattering amplitude factorised with $k_T$-dependent correlators in the cross-section for $q_T \ll Q$
Beyond collinear factorisation

- Observed final-state $q_T$ generated from "intrinsic" $k_T$ from initial partons

- TMD factorisation: *collinear partonic scattering amplitude* factorised with $k_T$-dependent correlators in the cross-section for $q_T \ll Q$

\[d\sigma = \int dx_1 dx_2 d^2\vec{k}_{T1} d^2\vec{k}_{T2} \delta^{(2)}(\vec{k}_{T1} + \vec{k}_{T2} - \vec{q}_T)\]

\[\times \Phi_{\mu\nu}^g(x_1, \vec{k}_{T1})\Phi_{\rho\sigma}^g(x_2, \vec{k}_{T2}) \left[ \hat{M}_{\mu\rho} \hat{M}^*_{\nu\sigma} \right]_{k_1=x_1P_1}^{k_2=x_2P_2}\]

\[+ \mathcal{O}\left(\frac{q_T}{Q}\right)\] (8)
TMD correlators

Naive definition of the (fully unintegrated) gluon correlator:

$$\phi_{g}^{\mu\nu}(k, k', P, S) = \int d^4 \xi d^4 \eta e^{i (k. \xi - k'. \eta)} \langle P, S | A^{\mu}(\xi) A^{\nu}(\eta) | P, S \rangle$$  (9)
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\[
\phi^{\mu\nu}_{g}(k, k', P, S) = \int d^{4}\xi d^{4}\eta e^{i(k\cdot\xi - k'\cdot\eta)} \langle P, S | A^{\mu}(\xi) A^{\nu}(\eta) | P, S \rangle
\]  

(9)

- Additional contributions of arbitrary numbers of soft gluons: LO in 1/Q expansion \(\Rightarrow\) need to be resummed to make the correlator gauge-invariant

- Insertion of gauge links in the correlator:

\[
U^{[n]}_{a, b} \equiv \mathcal{P} e^{-ig \int_{a \cdot P}^{b \cdot P} d(\eta \cdot P) A^{n}(\eta)}
\]  

(10)
TMD correlators

Naive definition of the (fully unintegrated) gluon correlator:

\[ \phi_{g}^{\mu\nu}(k, k', P, S) = \int d^4\xi d^4\eta e^{i(k, \xi - k', \eta)} \langle P, S | A^{\mu}(\xi)A^{\nu}(\eta) | P, S \rangle \] (9)

Additional contributions of arbitrary numbers of soft gluons: LO in $1/Q$ expansion ⇒ need to be resummed to make the correlator gauge-invariant

Insertion of gauge links in the correlator:

\[ U_{a,b}^{[n]} \equiv \mathcal{P} e^{-ig \int_{a.P}^{b.P} d(\eta.P)A^{n}(\eta)} \] (10)

TMD case: Process dependence of gauge links. The $k_T$-dependent correlator for gluon fusion reads:

\[ \phi_{g}^{\mu\nu[-,-]}(x, k_T, P, S) = \int d(\xi.P)d^{2}\xi_{T} e^{ik,\xi} \times 2\text{Tr}_{c} \left[ \langle P, S | F_{\mu\nu}(0) U_{[-]}^{0,\xi} F_{n\mu}(\xi) U_{[-]}^{0,\xi} | P, S \rangle \right]_{LF} \] (11)
Staple-like links and process dependence

- TMD correlators require "staple-like" gauge links with a transverse component in their path:

\[ \int d^2 q_T \]

The way these links transform under time reversal allows for the existence of T-odd distributions in the correlator's parametrisation.

Sign change between T-odd TMDs extracted in processes with Initial-State Interactions (ISI) or Final State Interactions (FSI) ⇒ important prediction of the TMD formalism under investigation.
Staple-like links and process dependence

- TMD correlators require "staple-like" gauge links with a transverse component in their path:

\[ \int d^2 q_T \Rightarrow \]

- The way these links transform under time reversal $T$ allows for the existence of $T$-odd distributions in the correlator’s parametrisation:

<table>
<thead>
<tr>
<th>nucleon pol.</th>
<th>U</th>
<th>circ.</th>
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<tbody>
<tr>
<td>U</td>
<td>$f_1^g$</td>
<td></td>
<td>$h_{1L}^g$</td>
</tr>
<tr>
<td>L</td>
<td>$g_1^g$</td>
<td></td>
<td>$h_{1L}^g$</td>
</tr>
<tr>
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<td>$h_{1L}^g$, $h_{1T}^g$</td>
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black: survives $q_T$-integration
blue: $T$-even
red: $T$-odd
Staple-like links and process dependence

TMD correlators require "staple-like" gauge links with a transverse component in their path:

\[
\int d^2 \vec{q}_T
\]

The way these links transform under time reversal \( T \) allows for the existence of \( T \)-odd distributions in the correlator’s parametrisation.

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- **Sign change** between \( T \)-odd TMDs extracted in processes with Initial-State Interactions (ISI) or Final State Interactions (FSI) ⇒ important prediction of the TMD formalism under investigation.
Factorisation breaking and universality

- **Non-perturbative** interactions of the active quark/gluon with soft spectator partons in the hadron before (Initial-State Interactions) or after (Final-State Interactions) the scattering

- These can make $h_1^g$ process dependent and even break factorisation

- Gluon fusion: ISI can be encapsulated in the TMDs

- Colourless final state $\Rightarrow$ no FSI: leptons/photons/Higgs or Colour-Singlet (CS) hadronisation
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- **Non-perturbative** interactions of the active quark/gluon with soft spectator partons in the hadron before (Initial-State Interactions) or after (Final-State Interactions) the scattering

- These can make $h_1g$ process dependent and even break factorisation

- Gluon fusion: ISI can be encapsulated in the TMDs

- Colourless final state $\Rightarrow$ no FSI: leptons/photons/Higgs or Colour-Singlet (CS) hadronisation
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Transverse weights

The transverse momentum weights are given by

\[ w_2 \equiv \frac{2(\vec{k}_{1T} \cdot \vec{k}_{2T})^2 - \vec{k}_{1T}^2 \vec{k}_{2T}^2}{4 M_P^4} \]  \quad (12)

\[ w_3 \equiv \frac{q_T^2 \vec{k}_{2T}^2 - 2(q_T \cdot \vec{k}_{2T})^2}{2 M_P^2 q_T^2} \]  \quad (13)

\[ w'_3 \equiv \frac{q_T^2 \vec{k}_{1T}^2 - 2(q_T \cdot \vec{k}_{1T})^2}{2 M_P^2 q_T^2} \]  \quad (14)

\[ w_4 \equiv 2 \left[ \frac{\vec{k}_{1T} \cdot \vec{k}_{2T}}{2 M_P^2} - \frac{(\vec{k}_{1T} \cdot q_T)(\vec{k}_{2T} \cdot q_T^*)}{M_P^2 q_T^2} \right]^2 \frac{\vec{k}_{1T}^2 \vec{k}_{2T}^2}{4 M_P^4} \]  \quad (15)
Advantages of $2 \rightarrow 2$ processes

- $2 \rightarrow 1$ process:
  - Resulting particle has to be at small $\vec{P}_{Q_T}$
    $\Rightarrow$ likely difficult to measure
  - Hard scale has to be the particle mass: $Q^2 = M_Q^2$ $\Rightarrow$ doesn’t help to study TMD evolution
Advantages of $2 \to 2$ processes

- **2 $\to$ 1 process:**
  - Resulting particle has to be at small $\vec{P}_{QT}$
  - $\Rightarrow$ likely difficult to measure
  - **Hard scale** has to be the particle mass:
    $Q^2 = M_Q^2 \Rightarrow$ doesn’t help to study TMD evolution

- **2 $\to$ 2 process:** $\vec{P}_{Q1T} \simeq -\vec{P}_{Q2T}$
  - Produced particles can each have a large $\vec{P}_{QT}$ adding up to a small $\vec{P}_{QQT}$ for the pair $\Rightarrow$ TMDs relevant for a wide range of final-state momenta
  - **Hard scale** $Q^2 = M_Q^2$ can be tuned to study TMD evolution
$Q + \gamma$ at AFTER@LHC

AFTER@LHC: a fixed-target experiment using the LHC beams

- $\sqrt{2 \times m_N \times E_p^{7 TeV}} = 115$ GeV

- Experimental coverage of ALICE or LHCb is about $y_{\text{cms}} \in [-3:0]$

- For $\psi + \gamma$, smaller yield ($14 \text{ TeV} \rightarrow 115 \text{ GeV}$) compensated by an access to lower $P_T$

- At $Y_{\text{cms}} \simeq -2$, $x_2 \simeq 10/115 \times e^2 \simeq 0.65$. Yet, $g - g > q - \bar{q}$!
\( gg \rightarrow \eta_Q \) and \( gg \rightarrow \chi_{Q_0,2} \)

- Low \( q_T \) \( C \)-even quarkonium production is a good probe of \( h_1^{\perp g} \)

- Very clean action on the low \( q_T \) spectra:

\[
\frac{1}{\sigma} \frac{d\sigma(\eta_Q)}{d\vec{q}_T^2} \propto 1 - R(\vec{q}_T^2) \quad \text{and} \quad \frac{1}{\sigma} \frac{d\sigma(\chi_{Q_0})}{d\vec{q}_T^2} \propto 1 + R(\vec{q}_T^2)
\]

\[
\left( R = \frac{F_2}{F_1} \frac{C[w_2 h_1^{\perp g} h_1^{\perp g}]}{C[f_1^g f_1^g]} \right)
\]
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R = \frac{F_2}{F_1} \frac{C[w_2 h_{1}^\perp g h_{1}^\perp g]}{C[f_1^g f_1^g]}
\]

- Cannot tune \( Q \Rightarrow Q \simeq M_Q \) (Higgs prod. \( \sim \eta_c \) prod. at \( Q = 125 \text{ GeV} \))
- Low \( q_T \): experimentally very difficult

\[
[\text{Only one } \eta_c \text{ production study at collider, for } q_T^{\eta_c} > 6 \text{ GeV } \text{LHCb, 1409.3612}]\]
Prospects for the future measurements

- Fixed-Target Program for the LHC (FTP4LHC):
  - Drell-Yan Single Transverse Spin Asymmetry (STSA) can constraint the quark Sivers effect, especially at large $x$
  - Other TMDs can be accessed: $h^q_1$, $h_1^⟂q$, $h_1^⟂T$
  - Gluon Sivers effect can also be measured in associated quarkonium production: $p^↑p \rightarrow J/ψ γX$, $p^↑p \rightarrow J/ψ J/ψ X$, $p^↑p \rightarrow γjet X$