EMT densities in the bag model

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based on work in collaboration with:

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DIS 2019, Torino, 8-12 April 2019

April 11, 2019

Outline

- Energy Momentum Tensor (EMT) form factors
  - Why are they interesting?
  - How to measure?
  - What is known?
- Bag model
  - Lucid and internally consistent.
  - Use to test new theoretical concepts.
- 3D densities
  - Justified in large N_c limit.
  - Interesting new insights: forces, mechanical properties, stability
- Summary

EMT form factors of nucleon

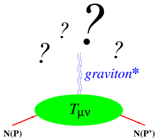
$$\langle p'|\hat{T}^a_{\mu\nu}|p\rangle = \bar{u}(p')\Big[\, \boldsymbol{A^a(t,\mu^2)}\,\frac{P_\mu P_\nu}{M} + \boldsymbol{J^a(t,\mu^2)}\,\frac{i(P_\mu\sigma_{\nu\rho}+P_\nu\sigma_{\mu\rho})\Delta^\rho}{2M} + \boldsymbol{D^a(t,\mu^2)}\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4M} + \bar{c}^a(t,\mu^2)g_{\mu\nu}\Big]u(p)$$

where $2P=(p'+p)$, $\Delta = p'-p$ and $t=\Delta^2$. (Kobzarev & Okun 1962, Pagels 1966)

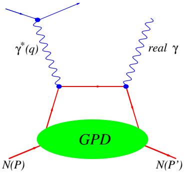
- conserved $\boldsymbol{\partial_\mu \hat{T}^{\mu\nu} = 0}$, $\hat{T}_{\mu\nu} = \sum_a \hat{T}^a_{\mu\nu}$ ($a = q, g$ all constituents)
- $\sum_a \bar{c}^a(t,\mu^2) = 0$
- constraints: **mass** $\Leftrightarrow$ $A(0)=1$ $\Leftrightarrow$ quarks + gluons carry 100 % of nucleon momentum
  spin $\Leftrightarrow$ $J(0) = \frac{1}{2}$ $\Leftrightarrow$ quarks + gluons carry 100 % of nucleon spin
  D-term $\Leftrightarrow$ $D(0) \equiv \boldsymbol{D}$ $\rightarrow$ unconstrained! What is it?

How to measure EMT form factors?

- direct probe: scattering of gravitons



- indirect access through: generalized parton distributions (GPDs)
  D.Müller, D.Robaschik, B.Geyer, F.-M.Dittes, J.Hořejši, Fortsch. Phys. **42**, 101 (1994)
  X.D.Ji, PRL **78**, 610 (1997); PRD **55**, 7114 (1997).
  A.V.Radyushkin, PLB **380**, 417 & **385**, 333 (1996).



Deeply Virtual Compton Scattering

How to measure EMT form factors?

- The second Mellin moments of unpolarized GPDs are related to EMT FFs by

$$\int_{-1}^{1} dx\; x\; H^a(x,\xi,t) = A^a(t) + \xi^2\; D^a(t)$$
$$\int_{-1}^{1} dx\; x\; E^a(x,\xi,t) = 2J^a(t) - A^a(t) - \xi^2\; D^a(t)$$

- bonus: gravity couples only to total EMT, but hard exclusive reactions distinguish q, g

Bag model Chodos, Jaffe, Johnson, Thorn, Weisskopf, PRD9 (1974) 3471

- Lagrangian $\mathcal{L} = \mathcal{L}_Q + \mathcal{L}_{surf} + \mathcal{L}_G$

$$\mathcal{L}_Q = \sum_q \left[\bar{\psi}_q \left(-\frac{i}{2}\overleftarrow{\partial} + \frac{i}{2}\overrightarrow{\partial} - m\right)\psi_q\right]\Theta_V ,$$

$$\mathcal{L}_{surf} = \frac{1}{2}\sum_q \bar{\psi}\,\psi\,\eta^\mu \partial_\mu \Theta_V ,$$

$$\mathcal{L}_G = -B\,\Theta_V$$

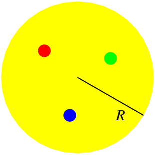
- $\mathcal{L}_G$ not really gluons, but
  (i) "non-fermionic degree of freedom",
  (ii) crucial for formation of bound states
  in this sense: "mimics" gluons of QCD

- The momentum space wave function in the bag is given by

$$\varphi(\vec{k}) = \sqrt{4\pi} N R^3 \begin{pmatrix} t_0(k)\chi_m \\ \vec{\sigma}\cdot\hat{k} \quad t_1(k)\chi_m \end{pmatrix}$$

where N is the normalization constant, R is the bag radius and $t_l(k) = \int_0^1 du\, u^2 j_l(ukR) j_l(u\omega_i)$

Bag model



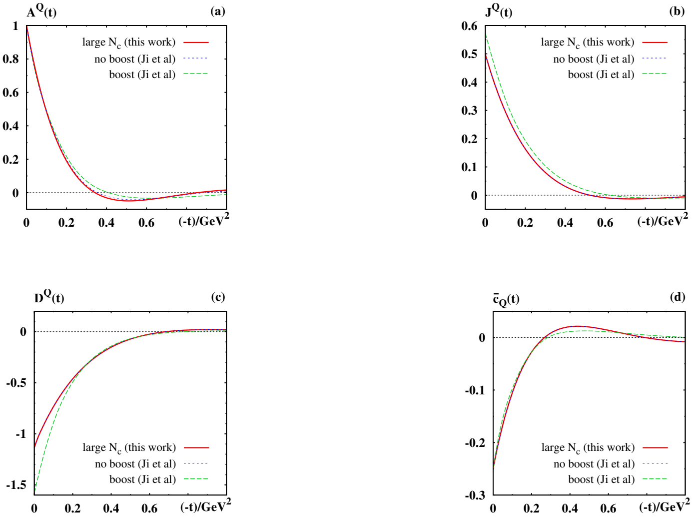
- not realistic: bag boundary violates chiral symmetry.
- but simple and lucid model, and internally consistent! valuable playground to shed light on theoretical concepts
- study of EMT form factors (and first model study of GPDs) [Ji, Melnitchouk, Song, PRD56, 5511 (1997)].
  here: revisit, learn new lessons

Large-N_c expansion

- In QCD, the coupling constant g is high at low energies. Hence is not a good expansion parameter.
- The only known expansion parameter valid in all regions in QCD is $1/N_c$ obtained by generalizing the color gauge group $SU(3) \to SU(N_c)$ [t Hooft G. Nucl. Phys. B 72 (1974)]
- As $N_c \to \infty$, QCD simplifies significantly and can be approached nonperturbatively; with an expansion parameter $1/N_c$
- In this picture, baryons appear as solitons in the background of weakly interacting mesons [Witten E. Nucl. Phys. B 160 (1979)]
- In Large-N_c limit, the nucleon mass scales with N_c while retaining a stable size

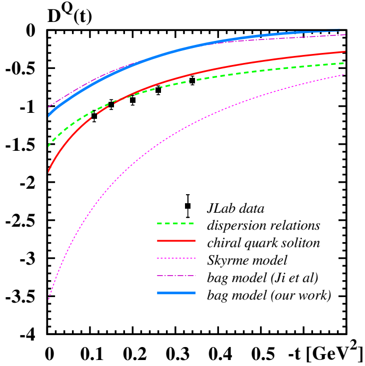
$$M_N \sim N_c$$
$$R \sim N_c^0$$

EMT form factors in *Large* − N_c limit



D-term

JLab: [V. D. Burkert, L. Elouadrhiri, and F. X. Girod, Nature 557, 396 (2018)]
Chiral quark soliton: [Goeke et al PRD71, 034011 (2005)]
Dispersion relation: [Pasquini et al PLB739, 133 (2014)]
Skyrme model: [Cebulla et al NPA 794, 87 (2007)]



For free spin $\frac{1}{2}$ fermion: $D = 0$ [Donoghue et al, PLB529, 132 (2002); Hudson, Schweitzer PRD97 (2018)]

EMT densities

Breit frame with $\Delta^\mu = (0, \vec{\Delta})$: static EMT [M. V. Polyakov, Phys. Lett. B 555, 57 (2003)]

$$T_{\mu\nu}(\vec{r}) = \int \frac{d^3\vec{\Delta}}{2E(2\pi)^3}\, e^{i\vec{\Delta}\cdot\vec{r}}\, \langle P'|\hat{T}_{\mu\nu}|P\rangle$$

$$\int d^3r\; T_{00}(\vec{r}) = M \qquad \text{known!}$$

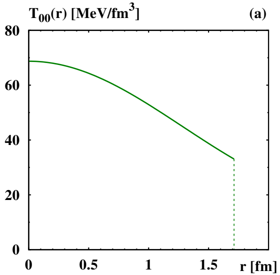
$$\int d^3r\; \varepsilon^{ijk}\, s_i\, r_j\, T_{0k}(\vec{r},\vec{s}) = \frac{1}{2} \qquad \text{known!}$$

$$-\frac{2}{5}\, M \int d^3r\; \left(r^i r^j - \frac{r^2}{3}\,\delta^{ij}\right)\, T_{ij}(\vec{r}) \equiv D \qquad \textbf{new!}$$

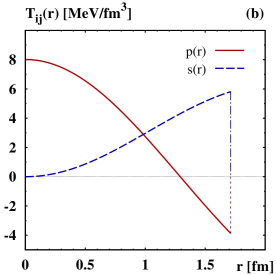
with: $\boldsymbol{T_{ij}(\vec{r}) = s(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\,\delta_{ij}\right) + p(r)\,\delta_{ij}}$ **stress tensor**

$s(r)$ related to distribution of shear forces
$p(r)$ distribution of pressure inside hadron
$\Big\} \longrightarrow$ "mechanical properties"

EMT densities



The energy density $T_{00}(r)$. The vertical lines mark the position of the bag boundary



The stress tensor $T_{ij}(r), s(r), p(r)$.

Stability

relation to stability: EMT conservation $\Leftrightarrow$ $\partial^\mu \hat{T}_{\mu\nu} = 0$ $\Leftrightarrow$ $\nabla^i T_{ij}(\vec{r}) = 0$

$\hookrightarrow$ necessary condition for stability $\int_0^\infty dr\, r^2\, p(r) = 0$ (von Laue, 1911)

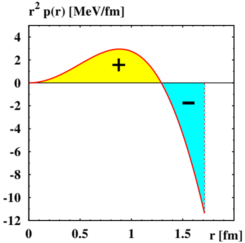


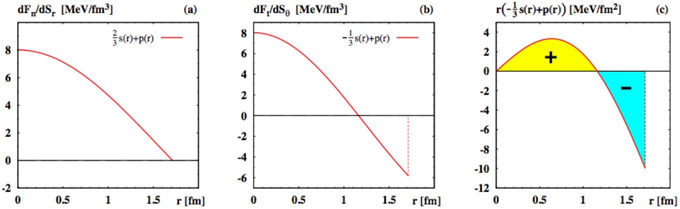
Illustration how the von Laue condition is realized in the bag model. The vertical line $R \approx 1.71 fm$ marks the position of the bag boundary. The areas above and below the r axis are equal to each other.

Stability

Normal and tangential forces in hadrons: In a spherically symmetric hadron

[M. V. Polyakov and P. Schweitzer, Int. J. Mod. Phys. A **33**, no. 26, 1830025 (2018)]

$$\frac{dF_n}{dS_r} = \frac{2}{3}s(r) + p(r), \quad \frac{dF_t}{dS_\theta} = -\frac{1}{3}s(r) + p(r).$$



Densities of (a) normal forces and (b) tangential forces in the bag model as functions of r. Mechanical stability requires the normal forces to be positive, i.e. directed towards outside, which the bag model complies with as panel (a) shows. Moreover the tangential forces must compensate according to $\int_0^R dr\ r\,[-\frac{1}{3}s(r) + p(r)] = 0$ which the bag model also satisfies as the panel (c) shows.

Summary

- Nucleon EMT form factors in bag model has been revisited
- We formulate the model in the $large - N_c$ limit in which the nucleon becomes non-relativistic
- The simplicity and the consistency of the model allows us to test in a lucid way many theoretical concepts: $3D$ EMT densities, normal and tangential forces, stability, ...