Applications of the WW-type approximation to SIDIS

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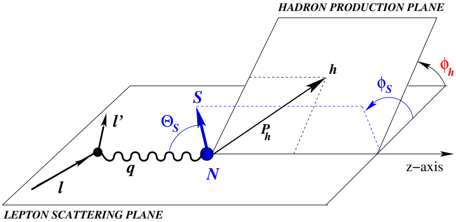
SIDIS Cross Section

- Consider the SIDIS process $l + N \to l' + h + X$



- In single photon exchange approximation, SIDIS cross section can be expressed by 18 structure functions (SFs) [Kotzinian '95, Mulders and Tangerman '96, Review: Bacchetta et al. JHEP 02 (2007)]

$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_h F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)F_{UU}^{\cos 2\phi_h}$$
$$+ \lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\sin\phi_h F_{LU}^{\sin\phi_h} + S_{||}\Big[\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_h F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)F_{UL}^{\sin 2\phi_h}\Big] + S_{||}\lambda_e\Big[\sqrt{1-\varepsilon^2}F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_h F_{LL}^{\cos\phi_h}\Big]$$
$$+ |\vec{S}_\perp|\Big[\sin(\phi_h-\phi_S)\big(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}\big) + \varepsilon\sin(\phi_h+\phi_S)F_{UT}^{\sin(\phi_h+\phi_S)}$$
$$+ \varepsilon\sin(3\phi_h-\phi_S)F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_S F_{UT}^{\sin\phi_S} + \sqrt{2\varepsilon(1+\varepsilon)}\sin(2\phi_h-\phi_S)F_{UT}^{\sin(2\phi_h-\phi_S)}\Big]$$
$$+ |\vec{S}_\perp|\lambda_e\Big[\sqrt{1-\varepsilon^2}\cos(\phi_h-\phi_S)F_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_S F_{LT}^{\cos\phi_S} + \sqrt{2\varepsilon(1-\varepsilon)}\cos(2\phi_h-\phi_S)F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Big\}$$

SIDIS Cross Section

- TMD factorization for $P_{h\perp} \ll Q$: Structure functions can be described by convolutions of TMDs and FFs. Generically

$$\mathcal{C}\Big[\omega\ f\ D\Big] = x \sum_a e_a^2 \int d^2\boldsymbol{k}_\perp d^2\boldsymbol{P}_\perp \delta^{(2)}(z\boldsymbol{k}_\perp + \boldsymbol{P}_\perp - \boldsymbol{P}_{h\perp})\ \omega f^a(x, \boldsymbol{k}_\perp^2)\ D^a(z, \boldsymbol{P}_\perp^2)$$

  1. 8 structure functions → twist-2 (factorization ✓)
  2. 8 structure functions → twist-3 (factorization assumed)

- SIDIS CS can be expressed by
  1. 8 twist-2 TMDs: $f_1, g_1, h_1, f_{1T}^\perp, g_{1T}^\perp, h_{1T}^\perp, h_{1L}^\perp, h_1^\perp$
  2. 16 twist-3 TMDs: $e, e_L, e_T, f^\perp, g_T, \ldots$
  3. 2 twist-2 FFs: $D_1, H_1^\perp$
  4. 4 twist-3 FFs: $D^\perp, G^\perp, H, E$

What is WW Approximation?

- First application of EoM [Wandzura and Wilczek '77]:

$$\underbrace{g_T^q(x)}_{twist-3} = \underbrace{< \bar{q}q >}_{related\ \ to\ \ g_1^q(x)} + \underbrace{< \bar{q}gq >}_{''tilde''\ term}$$

- The approximation is

$$\left|\frac{< \bar{q}gq >}{< \bar{q}q >}\right| \ll 1.$$

- Neglecting "tilde" terms is known to be WW approximation

$$g_T^q(x) = \int_x^1 \frac{dy}{y} g_1^q(y) + \tilde{g}_T^q(x) \overset{WW}{\approx} \int_x^1 \frac{dy}{y} g_1^q(y)$$

- Analogously [Jaffe and Ji '92]

$$h_L^q(x) = 2x \int_x^1 \frac{dy}{y^2} h_1^q(y) + \tilde{h}_L^q(x) \overset{WW}{\approx} 2x \int_x^1 \frac{dy}{y^2} h_1^q(y)$$

- WW approximation is supported by:
  Instanton model of QCD vacuum [Balla et al. NPB 510 (1998)]
  Lattice data [Göckeler et al. PRD 63 (2001)]
  Quark models [Bag, LFCM, Spectator, ...]

WW Approximation

- Illustration

$$g_2(x) = \frac{1}{2}\sum_a e_a^2 \left(g_T^a(x) - g_1^a(x)\right)$$
$$\overset{WW}{\approx} \frac{d}{dx}\left[x\int_x^1 \frac{dy}{y} g_1(y)\right]$$

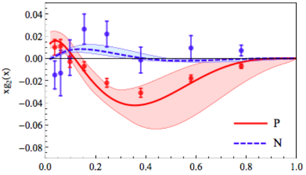


Figure: The structure function $xg_2(x)$ in WW approximation at $Q^2 = 7.1GeV^2$ vs. data E143[PRD 58 (1998)], E155[PLB 553 (2003)]

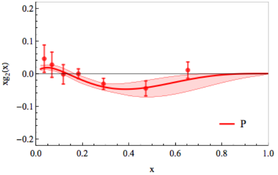


Figure: The structure function $xg_2(x)$ in WW approximation at $Q^2 = 2.4GeV^2$ vs. HERMES data[EPJC 72 (2012)].

- holds with 40% or better [Accardi et al. JHEP 11 (2009)]

WW-type Approximation

- EoM allow us to decompose twist-2 $h_{1L}^{\perp}$, $g_{1T}^{\perp}$ and all twist-3 TMDs, FFs into $\bar{q}q$ and $\bar{q}gq$-matrix elements.
- Assuming

$$|<\bar{q}gq>|\ll|<\bar{q}q>|$$

We obtain
twist-2:

$$g_{1T}^{\perp(1)}(x)\overset{WW-type}{\approx}x\int_x^1\frac{dy}{y}g_1(y),\quad h_{1L}^{\perp(1)}(x)\overset{WW-type}{\approx}-x^2\int_x^1\frac{dy}{y^2}h_1(y).$$

twist-3:

$$\begin{aligned}
xe^q(x,k_\perp^2)&\overset{\text{WW-type}}{\approx}0,\\
xf^{\perp q}(x,k_\perp^2)&\overset{\text{WW-type}}{\approx}f_1^q(x,k_\perp^2),\\
xg_L^{\perp q}(x,k_\perp^2)&\overset{\text{WW-type}}{\approx}g_1^q(x,k_\perp^2),\\
xg_T^{\perp q}(x,k_\perp^2)&\overset{\text{WW-type}}{\approx}g_{1T}^{\perp q}(x,k_\perp^2),\\
xg_T^q(x,k_\perp^2)&\overset{\text{WW-type}}{\approx}g_{1T}^{\perp(1)q}(x,k_\perp^2),\\
xh_L^q(x,k_\perp^2)&\overset{\text{WW-type}}{\approx}-2\,h_{1L}^{\perp(1)q}(x,k_\perp^2),\\
xh_T^q(x,k_\perp^2)&\overset{\text{WW-type}}{\approx}-h_1^q(x,k_\perp^2)-h_{1T}^{\perp(1)}(x,k_\perp^2),\\
xh_T^{\perp q}(x,k_\perp^2)&\overset{\text{WW-type}}{\approx}h_1^q(x,k_\perp^2)-h_{1T}^{\perp(1)}(x,k_\perp^2).
\end{aligned}$$

$$\begin{aligned}
xe_L^q(x,k_\perp^2)&\overset{\text{WW-type}}{\approx}0,\\
xe_T^q(x,k_\perp^2)&\overset{\text{WW-type}}{\approx}0,\\
xe_T^{\perp q}(x,k_\perp^2)&\overset{\text{WW-type}}{\approx}0,\\
xg^{\perp q}(x,k_\perp^2)&\overset{\text{WW-type}}{\approx}0,\\
xf_L^{\perp q}(x,k_\perp^2)&\overset{\text{WW-type}}{\approx}0,\\
xf_T^{\perp q}(x,k_\perp^2)&\overset{\text{WW-type}}{\approx}f_{1T}^{\perp q}(x,k_\perp^2),\\
xf_T^q(x,k_\perp^2)&\overset{\text{WW-type}}{\approx}-f_{1T}^{\perp(1)q}(x,k_\perp^2),\\
xh^q(x,k_\perp^2)&\overset{\text{WW-type}}{\approx}-2\,h_1^{\perp(1)}(x,k_\perp^2).
\end{aligned}$$

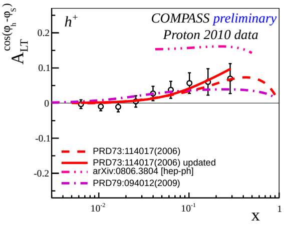
$$\begin{aligned}
E(z,P_\perp^2)&\overset{\text{WW-type}}{\approx}0,\\
G^\perp(z,P_\perp^2)&\overset{\text{WW-type}}{\approx}0,\\
D^\perp(z,P_\perp^2)&\overset{\text{WW-type}}{\approx}z\,D_1(z,P_\perp^2)\,,\\
H(z,P_\perp^2)&\overset{\text{WW-type}}{\approx}-\frac{P_\perp^2}{zm_h^2}\,H_1^\perp(z,P_\perp^2)\,.
\end{aligned}$$

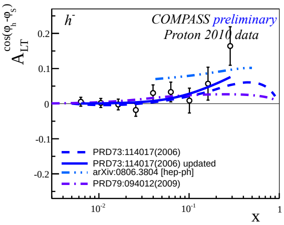
WW-type Approximation

- As a result of WW-type approximation, all SIDIS SFs (twist-2 & twist-3) can be expressed in terms of 8 basis functions:
  1. 6 leading-twist TMDs: $f_1, g_1, h_1, f_{1T}^{\perp}, h_1^{\perp}, h_{1T}^{\perp}$
  2. 2 leading-twist FFs: $D_1, H_1^{\perp}$
- Our goal is to check where the WW-type approximation works and where it does not
- We use state-of-the-art parametrizations for the basis functions [MSTW, DSS, Anselmino et al., Barone et al., Lefky and Prokudin; Gauss Ansatz used, parameters fixed using lattice data Hägler et al.]

WW-type Approximation at Leading Twist

- At leading twist, WW-type approximation is useful for 2 structure functions: $F_{LT}^{\cos(\phi_h-\phi_S)}$ and $F_{UL}^{\sin(2\phi_h)}$.
- $F_{LT}^{\cos(\phi_h-\phi_S)} = \mathcal{C}\left[\frac{\hat{h}\cdot\vec{k}_\perp}{M_N} g_{1T}^{\perp} D_1\right]$.
- First application of WW-type approximation [Kotzinian, Parsamyan, Prokudin, PRD 73 (2006)]
- Results are compatible with the recent preliminary COMPASS data [Parsamyan, PoS DIS2013 (2013) 231].

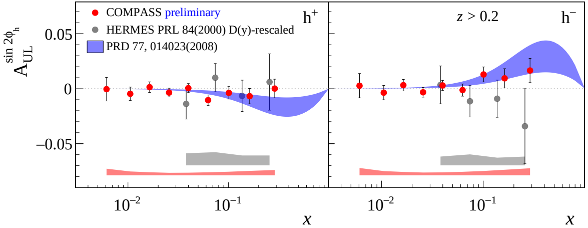


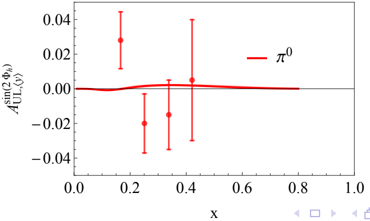


quark-diquark model [Kotzinian, arXiv:0806.3804] and light-cone quark constituent model [Boffi et al., Phys. Rev. D 79, 094012], are also displayed for comparison.

WW-type Approximation at Leading Twist

- Similarly, in WW-type approximation: $F_{UL}^{sin(2\phi_h)} = \mathcal{C}\left[\omega \; h_{1L}^{\perp} H_1^{\perp}\right]$
- Results [Avakian et al., PRD 77 (2008)] are compatible with preliminary COMPASS [Parsamyan, PoS DIS2017 (2018) 259] and JLab π^0 [Jawalkar et al., PLB 782 (2018)] data





WW-type Approximation at Subleading Twist

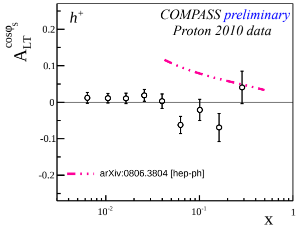
- Subleading twist → Complexity. Example (expression symbolic):

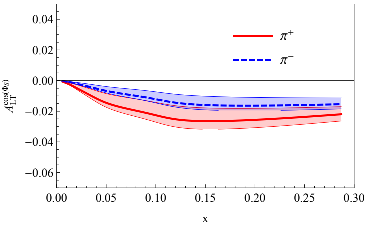
$$F_{LT}^{cos(\phi_S)} = \frac{M_N}{Q}\left[g_T\ D_1 + h_1\ \tilde{E} + e_T\ H_1^{\perp} + g_{1T}^{\perp}\ \tilde{D}^{\perp} + e_T^{\perp}\ H_1^{\perp} + f_{1T}^{\perp}\ \tilde{G}^{\perp}\right]$$

- WW-type approximation → Crucial simplification

$$F_{LT}^{cos(\phi_S)} \overset{WW-type}{\approx} \frac{M_N}{Q}\left[g_T\ D_1\right]\Bigg|_{g_T \to g_1}$$

- Results are compatible with COMPASS data [Parsamyan, PoS DIS2013 (2013) 231].





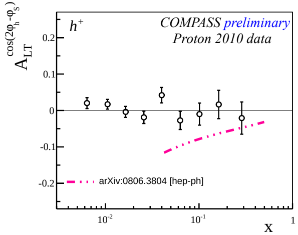
WW-type Approximation at Subleading Twist

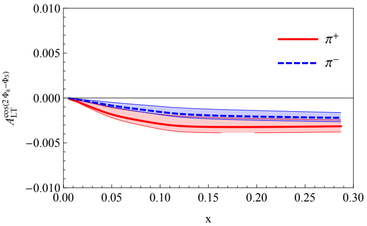
$$F_{LT}^{\cos(2\phi_h-\phi_S)} = \frac{M_N}{Q}\left[e_T\ H_1^{\perp} + g_{1T}^{\perp}\ \tilde{D}^{\perp} + e_T^{\perp}\ H_1^{\perp} + f_{1T}^{\perp}\ \tilde{G}^{\perp} + g_T^{\perp}\ D_1 + h_{1T}^{\perp}\ \tilde{E}\right]$$

- WW-type approximation → Crucial simplification

$$F_{LT}^{\cos(2\phi_h-\phi_S)} \overset{WW-type}{\approx} \frac{M_N}{Q}\left[g_T^{\perp}\ D_1\right]\Big|_{g_T^{\perp}\to g_1}$$

- Results are not in contradiction with COMPASS data [Parsamyan, PoS DIS2013 (2013) 231].





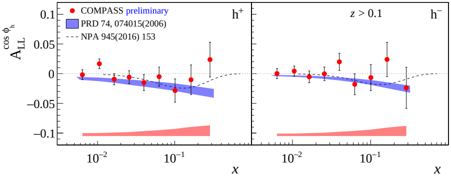
WW-type Approximation at Subleading Twist

$$F_{LL}^{\cos(\phi_h)} = \frac{M_N}{Q}\left[e_L\ H_1^{\perp} + g_1\ \tilde{D}^{\perp} + g_L^{\perp}\ D_1 + h_{1L}^{\perp}\ \tilde{E}\right]$$

- In WW-type approximation

$$F_{LL}^{\cos\phi_h} \overset{WW-type}{\approx} \frac{M_N}{Q}\left[g_L^{\perp}\ D_1\right]\Big|_{g_L^{\perp}\to g_1}$$

- WW-type results [Anselmino et al., Phys. Rev. D74 (2006)] are compatible with COMPASS preliminary data [Parsamyan, PoS DIS2017 (2018) 259].



A model study [Mao et al., Nucl. Phys. A945 (2016)] is also displayed for comparison.

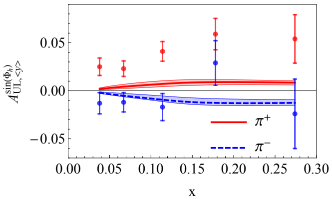
WW-type Approximation at Subleading Twist

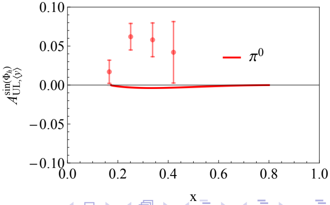
$$F_{UL}^{sin\phi_h} = \frac{M_N}{Q}\left[h_L\ H_1^{\perp} + g_1\ \tilde{G}^{\perp} + f_L^{\perp}\ D_1 + h_{1L}^{\perp}\ \tilde{H}\right]$$

- In WW-type approximation

$$F_{UL}^{sin\phi_h} \overset{WW-type}{\approx} \frac{M_N}{Q}\left[h_L\ H_1^{\perp}\right]\Big|_{h_L \to h_1}$$

- We observe discrepancies with HERMES $\pi^{\pm}$ [Airapetian et al., Phys. Lett. B622 (2005)] and JLAB π^0 [Jawalkar et al., Phys. Lett. B782 (2018)] data (underestimate π^+, can not describe π^0).





WW-type Approximation at Subleading Twist

- $F_{LU}^{\sin\phi_h} = \frac{M}{Q}\left[e\, H_1^{\perp} + f_1\, \tilde{G}^{\perp} + g^{\perp}\, D_1 + h_1^{\perp}\, \tilde{E}\right] \overset{WW-type}{\approx} 0$

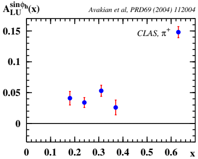


Figure: CLAS data shows a non-zero asymmetry

- $F_{UU}^{\cos\phi_h} = \frac{M}{Q}\left[hH_1^{\perp} + f_1\tilde{D}^{\perp} + f^{\perp}D_1 + h_1^{\perp}\tilde{H}\right] \overset{WW}{\approx} \frac{M}{Q}\left[hH_1^{\perp} + f^{\perp}D_1\right]\Big|_{h\to h_1^{\perp}, f^{\perp}\to f_1}$

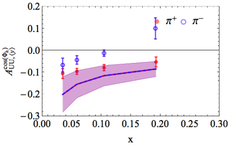


Figure: HERMES data [Airapetian et al., Phys. Rev. D87 (2013)]

Conclusions

- twist-3 PDFs: WW approximation works, supported by experiment for g_T, instanton calculus, lattice QCD
- twist-2 TMDs: applicable for gear worms $g_{1T}^{\perp}$, $h_{1L}^{\perp}$, in agreement with data
- Due to the complexity of SIDIS SFs at subleading-twist, a guideline is much needed. Predictions were made for all SIDIS asymmetries at subleading twist.
- Observations: compatible with data in several cases (e.g. $A_{LT}^{cos\phi_S}$, ...)
- Not in a agreement with data in some cases (e.g. $A_{LU}^{sin\phi_h}$, ...)
- More work is needed to reliably test where WW-type approximation works
- If we find it works: it will motivate theoretical studies to explain the suppression mechanism for $\bar{q}gq$. If we find it does not work: it will help to identify large $\bar{q}gq$ terms, and stimulate studies to find out why they are large. We learn in any case!
- Mathematica package: https://github.com/prokudin/WW-SIDIS