Gradient corrections to the classical McLerran-Venugopalan model

Douglas Wertepny
Ben Gurion University of the Negev

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MORTIMER B.
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STEM LEADERSHIP
PROGRAM

Outline

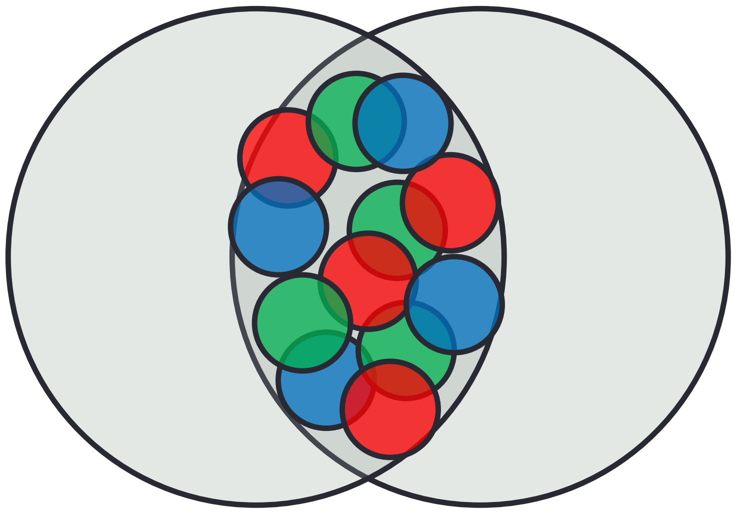
- Heavy-ion collisions, angular correlations, and anisotropies.

- Gluon dipole in the classical MV model.

- Gradient corrections in large and small systems.

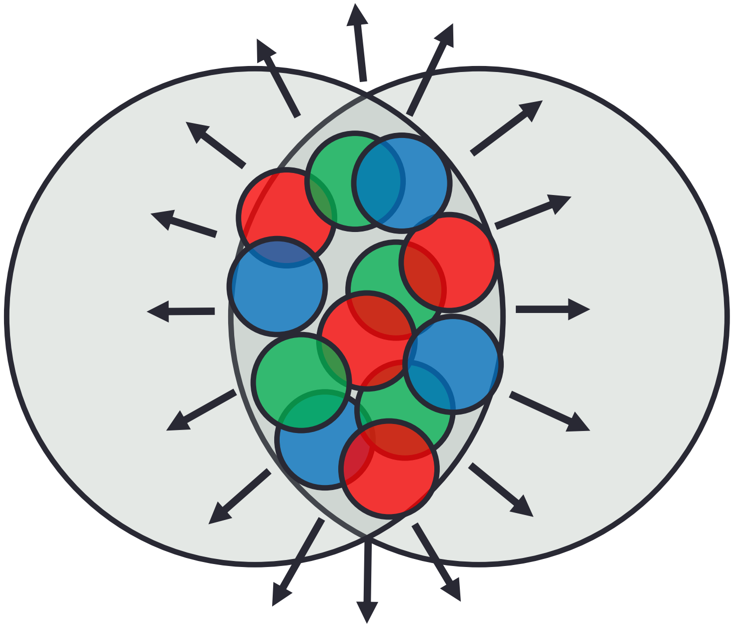
- Conclusions.

Initial Collisional Geometry



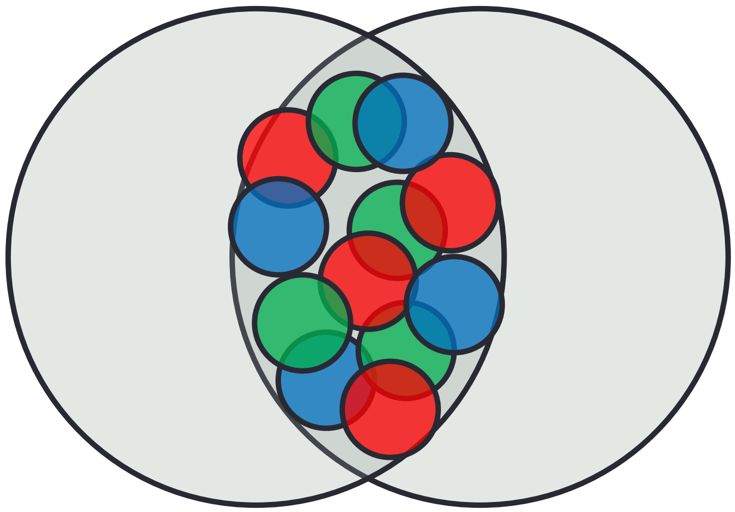
- In heavy-ion collisions two ions collide into each other and they create an initial transverse density profile.

Initial Collisional Geometry - Hydro



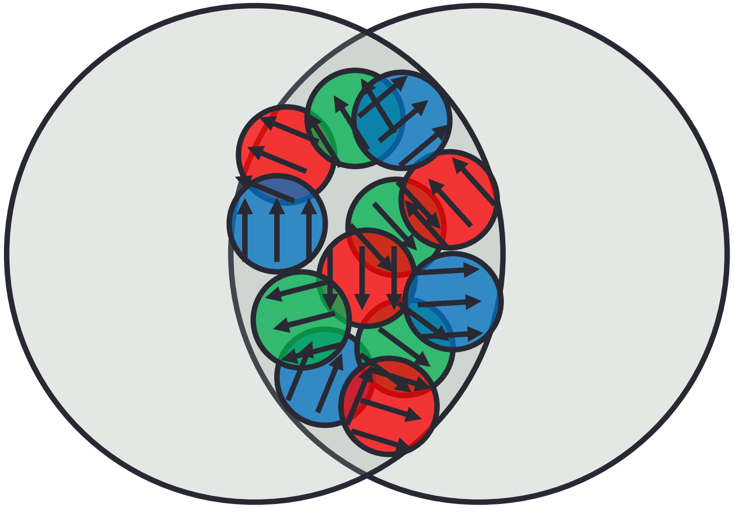
- In the hydrodynamical picture the shape of the initial distribution drives the behavior of the system.
- The system will flow and produce particles whose angular distribution is related to the initial shape.

Initial Collisional Geometry - CGC



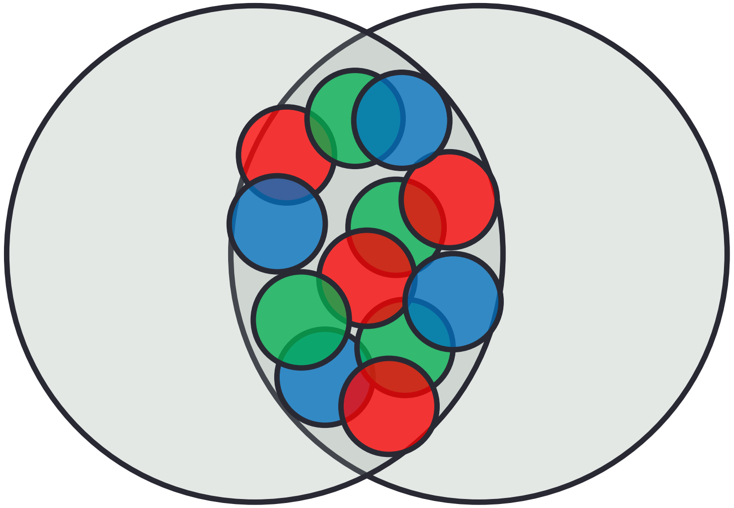
- In a CGC picture there are two ways to generate angular correlations:
  - Interactions between two particles
  - Individual particles couple to some inherent distribution.
- The talk focuses on the second case.

Initial Collisional Geometry - CGC



- In the second case there can only be angular correlations if there is some sort of anisotropy that the particles couple to.
- An example would be if the various color patches had a direction that the produced particles could couple to.
- These have been studied in various papers:
  - A. Kover, M. Lublinsky (2011,2013)
  - A. Dumitru, A. Giannini (2014)
  - A. Dumitru, V. Skokov (2014)
  - Iancu, Rezaeian (2017)
  - Kovner, Skokov (2018)

Initial Collisional Geometry - CGC



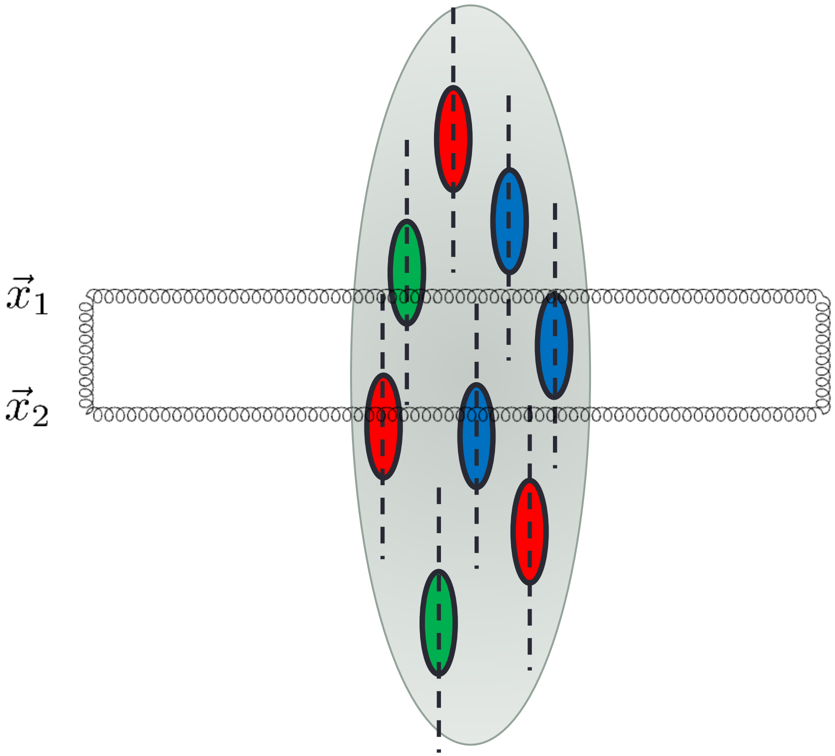
- This talk finds and quantifies the anisotropies introduced by looking at the gradient expansion of the distribution.
- To do this we will examine the gluon dipole.

Gluon dipole

- A gluon dipole passing through the target can be represented with Wilson lines.
- The survival probability for the gluon dipole is:

$$S_G(\vec{x}_1, \vec{x}_2) \equiv \frac{1}{N_c^2 - 1} \left\langle Tr[U_{\vec{x}_1} U^{\dagger}_{\vec{x}_2}] \right\rangle$$

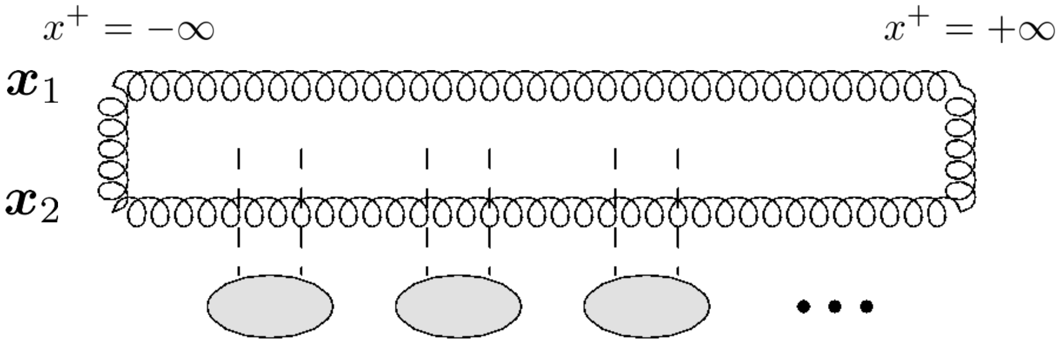
- Angle brackets represent averaging over all possible charge configurations.



Gluon dipole – survival probability

- The gluon dipole interacts with many valence charges in the target, exchanging two gluons each time (McLerran Venugopalan (MV) model).

$$S_G(\vec{x}_1, \vec{x}_2) \equiv \frac{1}{N_c^2 - 1} \left\langle Tr[U_{\vec{x}_1} U^\dagger_{\vec{x}_2}] \right\rangle$$

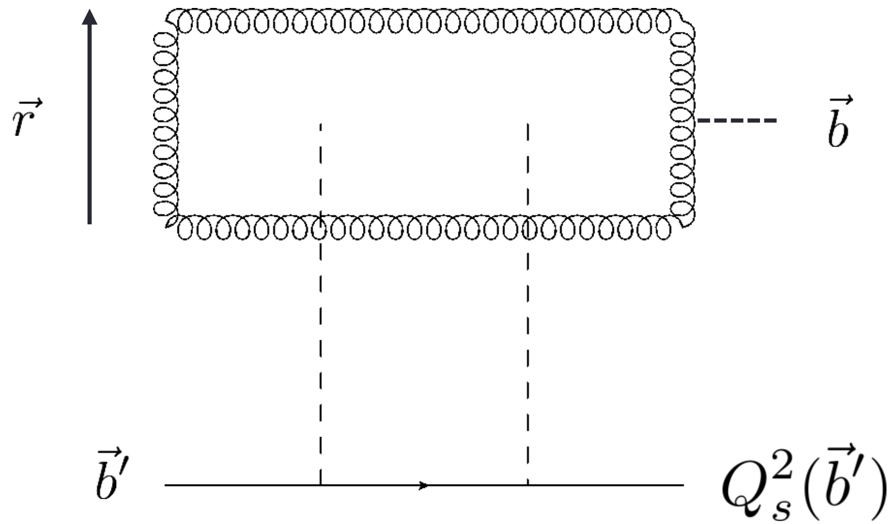


- This results in an exponentiated expression.

$$S_G(\vec{x}_1, \vec{x}_2) = \exp\left[-n_G\left(\vec{x}_1 - \vec{x}_2, \frac{1}{2}(\vec{x}_1 + \vec{x}_2)\right)\right]$$

Gluon Dipole

- The expression in the exponent is just the result due to the gluon dipole interacting with a single charge.
- In this expression we integrate over all positions of the charge weighted by the saturation scale.



$$n_G(\vec{r},\vec{b}) = \frac{1}{4\pi}\int d^2b'\, Q_s^2(\vec{b}')\ln^2\left(\frac{|\vec{b}-\frac{1}{2}\vec{r}-\vec{b}'|_\perp}{|\vec{b}+\frac{1}{2}\vec{r}-\vec{b}'|_\perp}\right)$$

Gluon dipole

- It is helpful to change the integration variable such that we have.

$$n_G(\vec{r},\vec{b}) = \frac{1}{4\pi}\int d^2z\, Q_s^2(\vec{b}+\vec{z})\ln^2\left(\frac{|\frac{1}{2}\vec{r}+\vec{z}|_\perp}{|\frac{1}{2}\vec{r}-\vec{z}|_\perp}\right)$$

- Since we are relaxing the translational invariance approximation we need find the various terms associated with a gradient expansion of the saturation scale.

$$Q_s^2(\vec{b}+\vec{z}) = Q_s^2(\vec{b}) + \vec{z}\cdot\vec{\nabla}_{\vec{b}}Q_s^2(\vec{b}) + \frac{1}{2}\vec{z}_i\vec{z}_j\vec{\nabla}_{\vec{b},i}\vec{\nabla}_{\vec{b},j}Q_s^2(\vec{b}) + \cdots$$

- When one inserts the gradient expansion in the equation the integral associated with the LO term is known. However the higher order terms are associated with highly non-trivial integrals. To help with this we first need to expand the log squared term into a series of various angular harmonics between r and z.

$$\cos(n\phi_{\vec{r},\vec{z}})$$

Series Expansion

- After much algebra you end up with a series expansion.

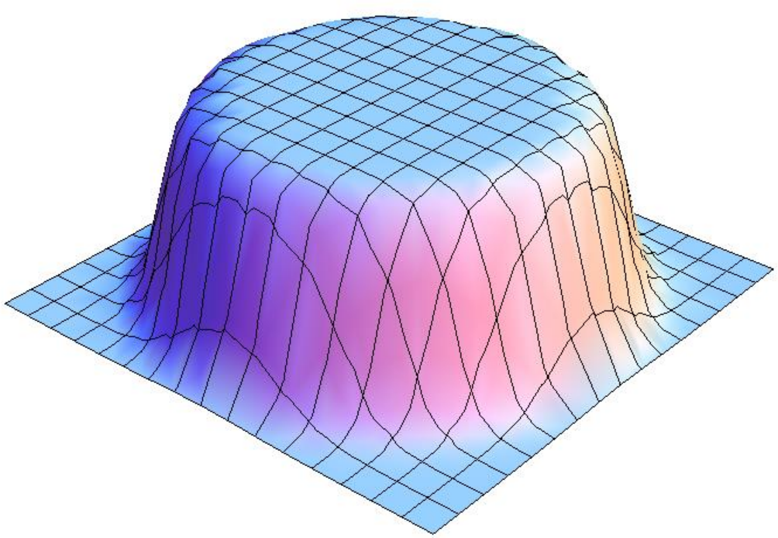
$$n_G(\vec{r},\vec{b}) = \frac{1}{4\pi}\int d^2z\, Q_s^2(\vec{b}+\vec{z}) \sum_{n=0}^{\infty} \cos(2n\phi_{\vec{r},\vec{z}})$$
$$\times \left(\Theta(z_\perp - \frac{1}{2}r_\perp)\, g_n\left(\frac{r_\perp}{2z_\perp}\right) + \Theta(\frac{1}{2}r_\perp - z_\perp)\, g_n\left(\frac{2z_\perp}{r_\perp}\right)\right)$$

- Where the first two terms are

$$g_0(x) = 2Li_2(x^2) - \frac{1}{2}Li_2(x^4)$$
$$g_1(x) = 2(1 + x^2 + (x^2 - x^{-2})\mathrm{ArcTanh}(x^2))$$

- The way to further manipulate the above expression depending on whether or not the system is large or small.
- We will start by analyzing the large system.

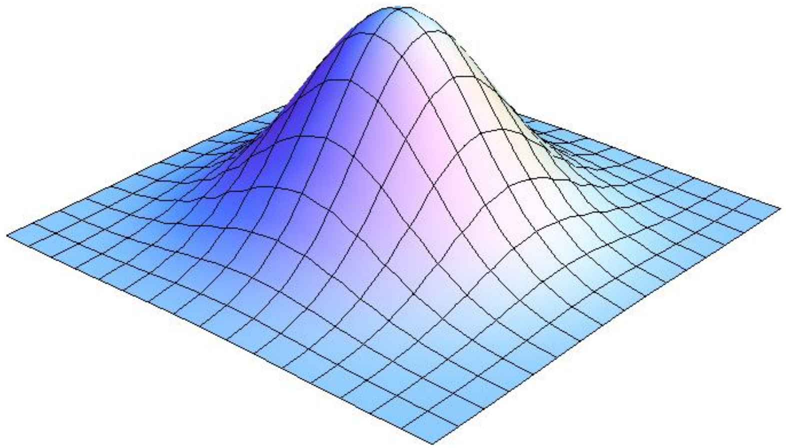
Large Systems



- The long distance behavior of QCD cuts off the long distance interactions.
- This hard cutoff effectively removes all but the translationally invariant contribution.

$$n_G(\vec{r},\vec{b}) = \frac{1}{4}\, Q_s^2(\vec{b})\, r_\perp^2\, \ln\left(\frac{1}{r_\perp \Lambda}\right)$$

Small Systems



- The size of the system cuts off the long distance behavior before the long distance QCD effects.
- The problem has only a single expansion parameter:

$$\frac{r_{\perp}}{l_{\text{fluc.}}}$$

Momentum Space Solution

- Since the long distance behavior is regulated by the system size this information is needed to control the integration.
- We can regulate the integration by working with the distribution's Fourier transform.

$$Q_s^2(\vec{b}) = \int \frac{d^2p}{(2\pi)^2} \widetilde{Q}_s^2(\vec{p}) e^{i\vec{p}\cdot\vec{b}}$$

- The gradient expansion now is represented with powers of the conjugate momentum.

$$\vec{\nabla} Q_s^2(\vec{b}) = \int \frac{d^2p}{(2\pi)^2} (-i\vec{p}) \widetilde{Q}_s^2(\vec{p}) e^{i\vec{p}\cdot\vec{b}}$$

- The relevant expansion parameter: $r_\perp p_\perp$

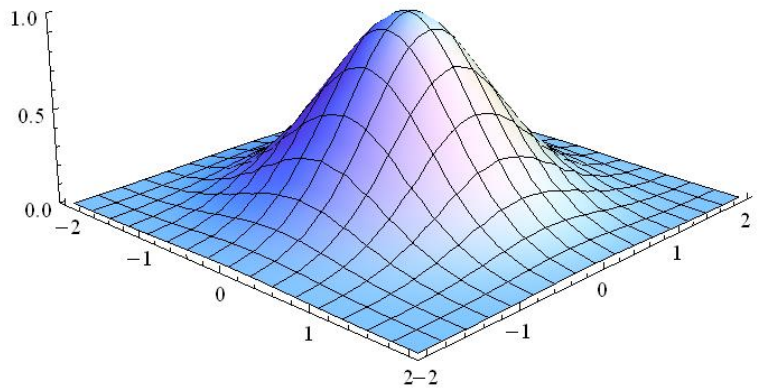
Momentum Space Solution

- Plugging in the Fourier transform into our main equation, evaluating the integral, and keeping only the LO contributions we arrive at the final result:

$$n_G(\vec{r},\vec{b}) = \int \frac{d^2p}{(2\pi)^2} e^{i\vec{p}\cdot\vec{b}} \frac{1}{4} \widetilde{Q}_s^2(\vec{p}) \vec{r}_i \vec{r}_j \left(\delta_{ij} \left(\ln \frac{2}{p_\perp r_\perp} - \gamma_E + 1 \right) - \left(\frac{\vec{p}_i \vec{p}_j}{p_\perp^2} - \frac{1}{2}\delta_{ij} \right) \right)$$

- We have the usual isotropic term where the log is now controlled by the momentum.
- New anisotropic term which is not parametrically suppressed.
- The size of the effect is heavily model dependent.
- Need to look at a specific example.

Small System Example



$$Q_s^2(b_\perp) = Q_0^2 e^{-\frac{b_\perp^2}{R^2}}$$

$$\widetilde{Q}_s^2(p_\perp) = Q_0^2 \pi R^2 e^{-\frac{1}{4}p_\perp^2 R^2}.$$

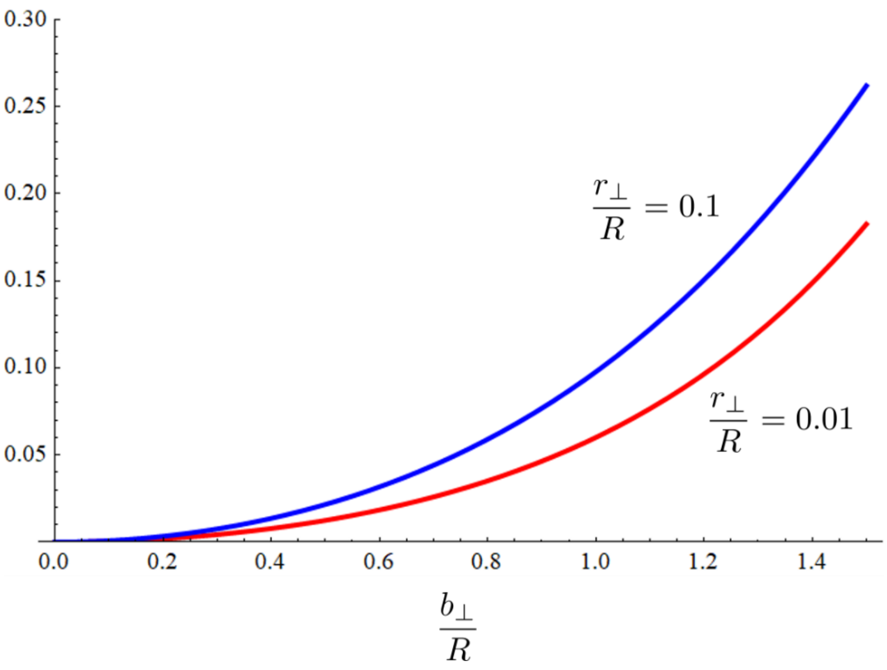
- Model the system as a gaussian distribution.

$$n_G(\vec{r},\vec{b}) = \frac{1}{4} Q_s^2(\vec{b}) r_\perp^2 \left(\left(\ln \frac{R}{r_\perp} + M_{R^{-1}}(b_\perp) \right) - \frac{1}{2} A_{\cos}(b_\perp) \cos(2\theta) \right)$$

$$M_{R^{-1}}(b_\perp) = \ln \frac{R}{b_\perp} + \frac{1}{2} Ei\left(\frac{b_\perp^2}{R^2} \right) - \gamma_E + 1$$

$$A_{cos}(b_\perp) = \left(1 - \frac{R^2}{b_\perp^2} \left(e^{\frac{b_\perp^2}{R^2}} - 1 \right) \right)$$

Small System Results



- Here we plot the ratio of the isotropic term to the anisotropic term.

$$-\frac{1}{2}\frac{A_{\cos}(b_\perp)}{\ln\frac{R}{r_\perp} + M_R(b_\perp)}$$

- Starts off at zero but increases exponentially as the center of the dipole moves towards the edge of the system.
- When b = R we have the following percent corrections:

$$\frac{r_\perp}{R} = 0.1 \qquad 9\%$$

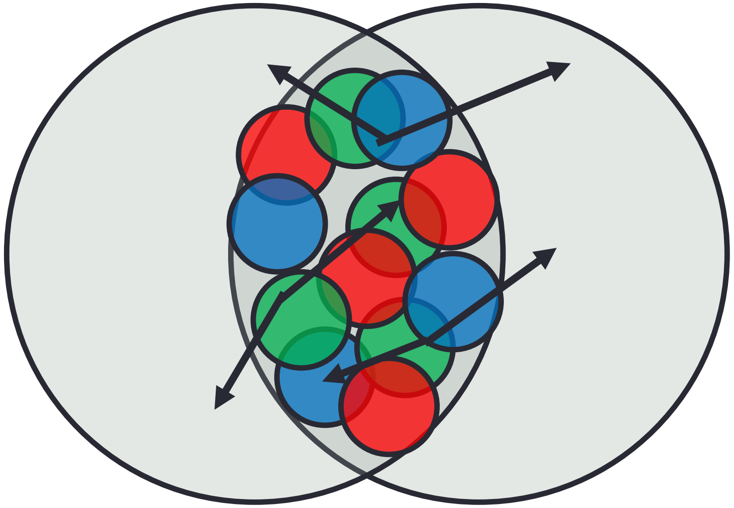
$$\frac{r_\perp}{R} = 0.01 \qquad 6\%$$

Conclusions

- In CGC calculations coupling to the gradients of the distribution is normally considered suppressed.
- While this is true in large systems we find that this is not generally true in small systems.
- The 2nd harmonic is not parametrically suppressed and its effects are heavily model dependent.
- We examined a small gaussian distribution to see the effects it has and while it was not the dominant contribution we found that it does contain anisotropies.

BACKUP SLIDES

Initial Collisional Geometry - CGC



- In the first case it is normally assumed that the fluctuations in the system are small and the charge distribution can be approximated as translationally invariant.
- This means that the only contributions can come from the interaction between particles.

Classical Field

- We write the vectors in the logarithm in terms of their magnitudes and the angle between them.

$$n_G\left(\vec{r},\vec{b}\right) = \frac{1}{4\pi}\int d^2z\, Q_s^2(\vec{b}+\vec{z})\ln^2\left(\frac{\sqrt{z_\perp^2 + z_\perp r_\perp \cos\phi_{\vec{r},\vec{z}} + \frac{1}{4}r_\perp^2}}{\sqrt{z_\perp^2 - z_\perp r_\perp \cos\phi_{\vec{r},\vec{z}} + \frac{1}{4}r_\perp^2}}\right)$$

- We then expand both of the logarithms in terms of the usual expansion.

$$\ln\left(\frac{1}{\sqrt{1-2x\cos\phi + x^2}}\right) = \sum_{n=1}^{\infty}\cos(n\phi)\frac{1}{n}x^n \qquad 1 > x > 0$$

- From here the expression is manipulated to create a single power series.

Multipole Expansion – Full Result

$$n_G(\vec{r},\vec{b}) = \frac{1}{4\pi}\int d^2z\, Q_s^2(\vec{b}+\vec{z}) \sum_{n=0}^{\infty} \cos(2n\phi_{\vec{r},\vec{z}})$$
$$\times\left(\Theta(z_\perp - \frac{1}{2}r_\perp)\, g_n\left(\frac{r_\perp}{2z_\perp}\right) + \Theta(\frac{1}{2}r_\perp - z_\perp)\, g_n\left(\frac{2z_\perp}{r_\perp}\right)\right)$$

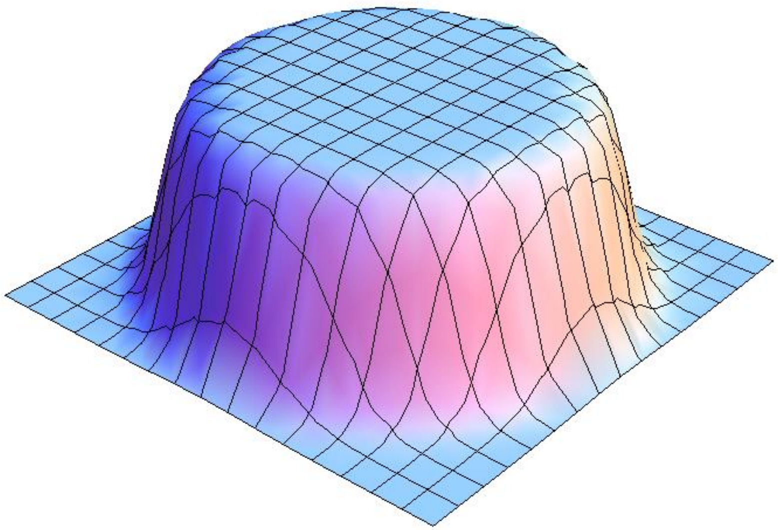
$$g_0(x) = 2\sum_{m=0}^{\infty}\frac{1}{(2m+1)^2}x^{2(2m+1)}.$$

$$g_n(x)\big|_{n,\mathrm{odd}} = 2c_n x^{2n} + 4\sum_{m=\frac{1}{2}(n+1)}^{\infty}\frac{1}{4m^2-n^2}x^{4m}$$

$$g_n(x)\big|_{n,\mathrm{even}} = 2c_n x^{2n} + 4\sum_{m=\frac{1}{2}n}^{\infty}\frac{1}{(2m+1)^2-n^2}x^{2(2m+1)}$$

$$c_n = \sum_{m=-n+1,-n+3,\ldots}^{n-1}\frac{1}{n^2-m^2}$$

Large Systems



$l_{\text{fluc.}}$

Λ_{QCD}^{-1}

$r_\perp$

- The long distance behavior of QCD cuts off the long distance interactions.
- The problem has two expansion parameters:

$$\frac{1}{l_{\text{fluc.}}\Lambda_{QCD}} \qquad \frac{r_\perp}{l_{\text{fluc.}}}$$

Large Systems

- We insert the gradient expansion of the distribution and regulate the z integration with a hard cut off.

$$Q_s^2(\vec{b}+\vec{z}) = Q_s^2(\vec{b}) + \vec{z}\cdot\vec{\nabla}_{\vec{b}} Q_s^2(\vec{b}) + \frac{1}{2}\vec{z}_i\vec{z}_j\vec{\nabla}_{\vec{b},i}\vec{\nabla}_{\vec{b},j} Q_s^2(\vec{b}) + \cdots$$

- We arrive at the following results:

$$n_G(\vec{r},\vec{b})\Big|_{\cos 0} \approx \frac{1}{4} Q_s^2(\vec{b}) r_\perp^2 \left(\ln\frac{1}{r_\perp \Lambda_{QCD}} + 1\right) + \frac{1}{32}\nabla_{\vec{b}}^2 Q_s^2(\vec{b}) r_\perp^2 \left(\frac{1}{\Lambda_{QCD}^2} - \frac{r_\perp^2}{8}\right)$$

$$n_G(\vec{r},\vec{b})\Big|_{\cos 2\theta} \approx \frac{1}{64}\nabla_{\vec{b}}^2 Q_s^2(\vec{b}) r_\perp^2 \cos(2\theta)\left(\frac{1}{\Lambda_{QCD}^2} + \frac{r_\perp^2}{3}\left(\ln\frac{1}{r_\perp\Lambda_{QCD}} + \frac{5}{12}\right)\right)$$

- All but the LO isotropic term is suppressed.
- Have no control over higher-order corrections.

Momentum Space Solution

- A more convient way to write this for computational purposes is:

$$n_G(\vec{r},\vec{b}) = \frac{1}{4}Q_s^2(\vec{b})r_\perp^2\left(\left(\ln\frac{1}{r_\perp\Lambda} + M_\Lambda(\vec{b})\right) - \frac{1}{2}A_{\cos}(\vec{b})\cos(2\theta) - \frac{1}{2}A_{\sin}(\vec{b})\sin(2\theta)\right)$$

- Where

$$M_\Lambda(\vec{b}) = Q_s^{-2}(\vec{b})\int\frac{d^2p}{(2\pi)^2}e^{i\vec{p}\cdot\vec{b}}\widetilde{Q}^2(\vec{p})\left(\ln\frac{2\Lambda}{p_\perp} - \gamma_E + 1\right)$$

$$A_{\cos}(\vec{b}) = Q_s^{-2}(\vec{b})\int\frac{d^2p}{(2\pi)^2}e^{i\vec{p}\cdot\vec{b}}\,\widetilde{Q}_s^2(\vec{p})\cos(2\phi_{\vec{b},\vec{p}})$$

$$A_{\sin}(\vec{b}) = Q_s^{-2}(\vec{b})\int\frac{d^2p}{(2\pi)^2}e^{i\vec{p}\cdot\vec{b}}\,\widetilde{Q}_s^2(\vec{p})\sin(2\phi_{\vec{b},\vec{p}})$$

- We can explicitly see the angular dependence in the final solution.