

DIS structure functions at low- x_{Bj} in the dipole factorization: Including massive quarks at NLO

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- ▶ Motivation
- ▶ Dipole factorization framework for DIS
- ▶ DIS at NLO with massive quarks
 - ▶ One-loop QCD correction to the $\gamma_L^* \rightarrow q\bar{q}$ LCWF
 - ▶ Full DIS cross section for γ_L^* at NLO
- ▶ Conclusion

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 - ▶ Can be used to describe pp, pA and other DIS observables

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DIS structure functions in the dipole factorization beyond LO

- ▶ Fast theoretical progress towards NLO accuracy:
 - ▶ Massless quarks: $F_{T/L}$ at NLO computed
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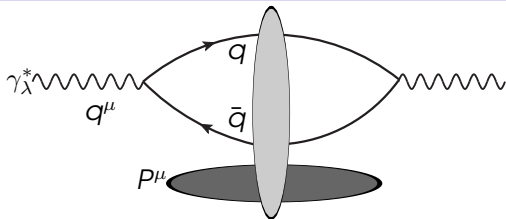
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Charm and bottom quark contributions are sizable at HERA:

- ▶ Need to calculate $F_{T/L}$ with massive quarks at NLO (this talk)

Dipole factorization for DIS



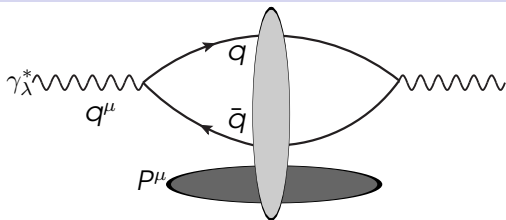
Dipole picture: A relativistic projectile γ^* fluctuates into a $q\bar{q}$ -pair which then **scatters** on a very dense **target**

- ▶ Kinematics in LC coordinates $(+, -, \perp)$

$$q^\mu = (q^+, \frac{\mathbf{q}^2 + q^2}{2q^+}, \mathbf{q}), \quad q^2 = -Q^2 > 0, \quad q^+ = \text{very large}$$

$$\text{Bjorken } x_{Bj} \equiv \frac{Q^2}{2P \cdot q} \in [0, 1]$$

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$$\text{Bjorken } x_{B_j} \equiv \frac{Q^2}{2P \cdot q} \in [0, 1]$$

- **High energy limit:** $x_{B_j} \rightarrow 0$ and Q^2 is "fixed"

$$F_{T,L}(x_{B_j}, Q^2) = \frac{Q^2}{(2\pi)^2 \alpha_{\text{em}}} \sigma_{T,L}^{\gamma^*}, \quad F_2 = F_T + F_L, \quad 2x_{B_j} F_1 = F_T$$

DIS cross section at NLO

The total cross section for γ^* scattering from a classical gluon field (optical theorem)

$$\sigma_{\lambda=T/L}^{\gamma^*} = \frac{2}{2q^+(2\pi)\delta(q'^+ - q^+)} \Re \left[i \langle \gamma_{\lambda}^*(\vec{q}, Q^2) | 1 - \hat{S}_E | \gamma_{\lambda}^*(\vec{q}', Q^2) \rangle_i \right]$$

- ▶ Full perturbative Fock state decomposition for γ^*

$$\begin{aligned} |\gamma_{\lambda}^*(\vec{q}, Q^2)\rangle_i &= |QED\rangle + \int PS_{(q\bar{q})} \psi^{\gamma^* \rightarrow q\bar{q}} |q\bar{q}\rangle \\ &\quad + \int PS_{(q\bar{q}g)} \psi^{\gamma^* \rightarrow q\bar{q}g} |q\bar{q}g\rangle + \dots \end{aligned}$$

where $\psi^{\gamma^* \rightarrow q\bar{q}/q\bar{q}g}, \dots$ are the **light cone wave-functions**

- ▶ \hat{S}_E describes the eikonal scattering on the gluon field

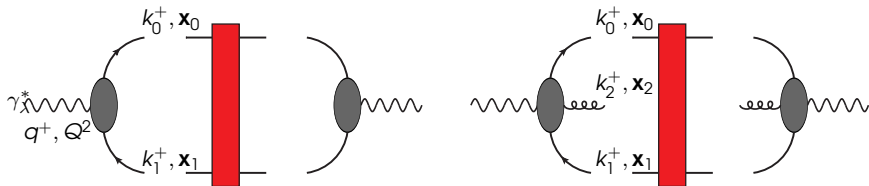
$$\hat{S}_E |q(\dots, \mathbf{x}_0, \alpha) \bar{q}(\dots, \mathbf{x}_1, \beta)\rangle = U_F(\mathbf{x}_0)_{\alpha\bar{\alpha}} U_F^\dagger(\mathbf{x}_1)_{\beta\bar{\beta}} |q(\dots, \bar{\alpha}) \bar{q}(\dots, \bar{\beta})\rangle$$

DIS cross section at NLO

In mixed space (k^+, \mathbf{x}) factorization between $|\psi|^2$ and target

$$\sigma_\lambda^{\gamma^*} = 2N_c \int \widetilde{PS}_{q\bar{q}} |\widetilde{\psi}^{\gamma_\lambda^* \rightarrow q\bar{q}}|^2 \Re[1 - S_{01}]$$

$$+ 2N_c C_F \int \widetilde{PS}_{q\bar{q}g} |\widetilde{\psi}^{\gamma_\lambda^* \rightarrow q\bar{q}g}|^2 \Re[1 - S_{012}]$$



- ▶ Dipole target: $S_{01} \equiv \frac{1}{N_c} \text{Tr} [U_F(\mathbf{x}_0) U_F^\dagger(\mathbf{x}_1)]$
- ▶ Dipole/g target: $S_{012} \equiv \frac{1}{N_c C_F} \text{Tr} [t^b U_F(\mathbf{x}_0) t^a U_F^\dagger(\mathbf{x}_1)] U_A(\mathbf{x}_2)_{ba}$

DIS computation at NLO: massless quark case

LCWF's for $\gamma^* \rightarrow q\bar{q}$ and $\gamma^* \rightarrow q\bar{q}g$ computed by using the LCPT

$\gamma^* \rightarrow q\bar{q}$ **LCWF at NLO with $m_q = 0$:**

$$\psi_{1-loop}^{\gamma^* \rightarrow q\bar{q}} \sim \sqrt{\alpha_{em}\alpha_s} \int_0^{k_{max}^+} \frac{dk^+}{k^+} \int \frac{d^{D-2}\mathbf{k}}{(2\pi)^{D-2}} \frac{num(\vec{k}, \vec{q}, Q^2, \dots)}{ED_{i,1}^- ED_{i,2}^- \dots ED_{i,f}^-}$$

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- ▶ Cut-off $k^+ > \alpha q^+$ introduced to regulate the $k^+ \rightarrow 0$ div's
 - ▶ $\log(\alpha)$'s have to cancel at full cross section level

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- ▶ Cut-off $k^+ > \alpha q^+$ introduced to regulate the $k^+ \rightarrow 0$ div's
 - ▶ $\log(\alpha)$'s have to cancel at full cross section level
- ▶ UV divergences from transverse \mathbf{k} integrals:
 - ▶ Transverse dimensional regularization in $D = 4 - 2\varepsilon_{UV}$

$$\frac{1}{\varepsilon_{UV}} + C_{scheme}, \quad \frac{1}{\varepsilon_{UV}} \log(\alpha)$$

- ▶ No UV renormalization at $\mathcal{O}(\alpha_{em}\alpha_s)$
 - ▶ UV divergences and finite regularization artifacts ($= C_{scheme}$) have to cancel at full cross section level

DIS computation at NLO: massive quark case

$\gamma^* \rightarrow q\bar{q}$ LCWF at NLO with $m_q \neq 0$:

$$\psi_{1-loop}^{\gamma^* \rightarrow q\bar{q}} \sim \sqrt{\alpha_{em}\alpha_s} \int_0^{k_{max}^+} \frac{dk^+}{k^+} \int \frac{d^{D-2}\mathbf{k}}{(2\pi)^{D-2}} \frac{\text{num}(\vec{k}, \vec{q}, Q^2, m_q \dots)}{ED_{i,1}^-(m_q^2) \dots ED_{i,f}^-(m_q^2)}$$

New features and technical complications appearing in the massive case:

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New features and technical complications appearing in the massive case:

- ▶ New spinor structures induced by quark LC helicity flip vertices

DIS computation at NLO: massive quark case

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New features and technical complications appearing in the massive case:

- ▶ New spinor structures induced by quark LC helicity flip vertices
- ▶ Quark mass renormalization has to be performed (LO result depends on m_q)

DIS computation at NLO: massive quark case

$\gamma^* \rightarrow q\bar{q}$ **LCWF at NLO with $m_q \neq 0$:**

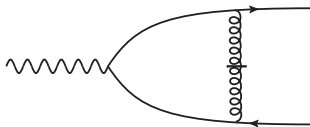
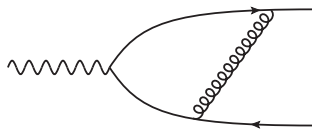
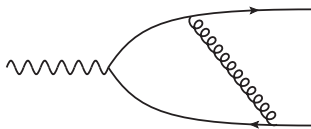
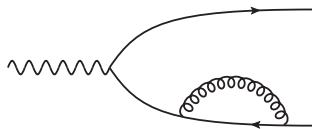
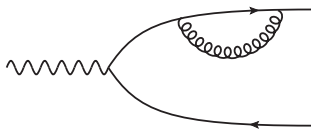
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New features and technical complications appearing in the massive case:

- ▶ New spinor structures induced by quark LC helicity flip vertices
- ▶ Quark mass renormalization has to be performed (LO result depends on m_q)
- ▶ Loop/Fourier transform integrals cannot fully be done analytically anymore:
 - ▶ Extract the **divergent** parts from the full expression (can be done analytically)
 - ▶ Some of the remaining **finite** parts given as integrals over Feynman/Schwinger parameters (can be done numerically)

One-loop LC diagrams for the $\gamma_L^* \rightarrow q\bar{q}$

At one-loop the following 5 LC diagrams must be calculated



Results: NLO $\gamma_L^* \rightarrow q\bar{q}$ LCWF in momentum space

$$\psi_{NLO}^{\gamma_L^* \rightarrow q\bar{q}} = \psi_{LO}^{\gamma_L^* \rightarrow q\bar{q}} \left(\frac{\alpha_s C_F}{2\pi} \right) \mathcal{K}^L + \psi_{hf}^{\gamma_L^* \rightarrow q\bar{q}} + \mathcal{O}(\alpha_{em}\alpha_s^2)$$

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where the NLO kernel

$$\begin{aligned} \mathcal{K}^L = & 2 \left[\log \left(\frac{\alpha}{\sqrt{z(1-z)}} \right) + \frac{3}{4} \right] \left\{ \frac{1}{\varepsilon_{\overline{MS}}} + \log \left(\frac{\mu^2}{M^2} \right) - 2 \log \left(\frac{\mathbf{P}^2 + M^2}{M^2} \right) \right\} \\ & + \log^2 \left(\frac{z}{1-z} \right) - \frac{\pi^2}{3} + \frac{5}{2} + C_{\text{scheme}} + \mathcal{G}^L(m_q; z) + \mathcal{J}^L(m_q; z) + \mathcal{O}(\varepsilon) \end{aligned}$$

- ▶ Notations: $z = k_0^+ / q^+$, $z \in [0, 1]$ and $\xi = k^+ / k_0^+$, $\xi \in [0, 1]$

$$\overline{Q}^2 = z(1-z)Q^2, \quad \mathbf{P} = \mathbf{k}_0 - z\mathbf{q}, \quad M^2 = \overline{Q}^2 + m_q^2$$

- ▶ Regularization artifact: $C_{\text{scheme}} = 1/2$ in CDR and 0 in FDH
- ▶ At NLO the UV finite hf-term vanishes on the cross section level

$$\psi_{\text{hf}}^{\gamma_L^* \rightarrow q\bar{q}} (\psi_{\text{LO}}^{\gamma_L^* \rightarrow q\bar{q}})^* \rightarrow 0$$

The finite functions \mathcal{G}^L and \mathcal{J}^L simplifies to ($\gamma \equiv \sqrt{1 + 4m_q^2/Q^2}$):

$$\begin{aligned} \mathcal{G}^L = & \sum_{k=\pm 1} \left[\text{Li}_2 \left(\frac{1}{1 - \frac{1}{2z}(1 + k\gamma)} \right) + \text{Li}_2 \left(\frac{1}{1 - \frac{1}{2(1-z)}(1 + k\gamma)} \right) \right] \\ & + \frac{1}{2z} \left[\log(1 - z) + \gamma \log \left(\frac{1 + \gamma}{1 + \gamma - 2z} \right) \right] \\ & + \frac{1}{2(1-z)} \left[\log(z) + \gamma \log \left(\frac{1 + \gamma}{1 + \gamma - 2(1-z)} \right) \right] \\ & + \frac{\gamma - 1}{4z(1-z)} \log \left(\frac{\bar{Q}^2 + m_q^2}{m_q^2} \right) + \frac{m_q^2}{2\bar{Q}^2} \log \left(\frac{\bar{Q}^2 + m_q^2}{m_q^2} \right) \end{aligned}$$

$$\begin{aligned} \mathcal{J}^L = & \int_0^1 \frac{d\xi}{\xi} \left(-\frac{2 \log(\xi)}{1 - \xi} + \frac{1 + \xi}{2} \right) \left[1 - \frac{\mathbf{P}^2 + M^2}{\mathbf{P}^2 + M^2 + \frac{z\xi}{1-\xi} m_q^2} \right] \\ & + m_q^2 \int_0^1 d\xi \int_0^1 dx \frac{C^L(x, \xi, z)}{x(1-x)(1-\xi)^2 \mathbf{P}^2 + (1-x)\Delta_1 + x\Delta_2} + [z \rightarrow 1 - z] \end{aligned}$$

$$\Delta_1 = \frac{\xi(1-\xi)}{1-z} [\mathbf{P}^2 + M^2] + \xi^2 m_q^2, \quad \Delta_2 = (1-\xi) \left[1 + \frac{z\xi}{1-z} \right] \bar{Q}^2 + m_q^2$$

Results: full $q\bar{q}$ contribution to $\sigma_L^{\gamma^*}$ at NLO

Fourier transformation to mixed space:

$$\tilde{\psi}^{\gamma_L^* \rightarrow q\bar{q}} = \int \frac{d^{D-2}\mathbf{P}}{(2\pi)^{D-2}} e^{+i\mathbf{P}\cdot\mathbf{x}_{01}} \psi^{\gamma_L^* \rightarrow q\bar{q}}$$

where $\mathbf{x}_{01} \equiv \mathbf{x}_0 - \mathbf{x}_1$

$$\sigma_L^{\gamma^*} \Big|_{q\bar{q}} \equiv 2N_c \int \tilde{PS}_{q\bar{q}} |\tilde{\psi}^{\gamma_L^* \rightarrow q\bar{q}}|^2 \Re[1 - \mathcal{S}_{01}]$$

gives

$$\begin{aligned} \sigma_L^{\gamma^*} \Big|_{q\bar{q}} &= 4N_c \frac{4\alpha_{em} e_f^2 Q^2}{(2\pi)^2} \int_{\mathbf{x}_0, \mathbf{x}_1} \int_0^1 dz [z(1-z)]^2 \left(\frac{M}{2\pi|\mathbf{x}_{01}|} \right)^{-\epsilon} K_\epsilon(|\mathbf{x}_{01}|M) \\ &\times \left\{ \left(\frac{M}{2\pi|\mathbf{x}_{01}|} \right)^{-\epsilon} K_\epsilon(|\mathbf{x}_{01}|M) + \left(\frac{\alpha_s C_F}{\pi} \right) \tilde{\mathcal{K}}^L \right\} \Re[1 - \mathcal{S}_{01}] + \mathcal{O}(\alpha_{em}\alpha_s^2) \end{aligned}$$

The Fourier transformed $\tilde{\mathcal{K}}^L$ simplifies to:

$$\tilde{\mathcal{K}}^L = \left\{ 2 \left[\log \left(\frac{\alpha}{\sqrt{z(1-z)}} \right) + \frac{3}{4} \right] \left\{ \frac{1}{\varepsilon_{MS}} + \log \left(\frac{\mathbf{x}_{01}^2 \mu^2}{4} \right) - 2\Psi_0(1) \right\} + \log^2 \left(\frac{z}{1-z} \right) - \frac{\pi^2}{3} + \frac{5}{2} + C_{scheme} + \mathcal{G}^L \right\} \left(\frac{M}{2\pi|\mathbf{x}_{01}|} \right)^{-\varepsilon} K_\varepsilon(|\mathbf{x}_{01}|M) + \tilde{\mathcal{J}}^L$$

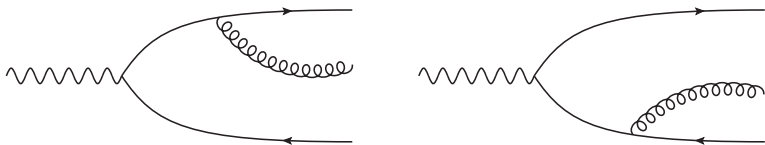
where

$$\begin{aligned} \tilde{\mathcal{J}}^L &= \int_0^1 \frac{d\xi}{\xi} \left[\frac{-2 \log(\xi)}{(1-\xi)} + \frac{(1+\xi)}{2} \right] \left\{ K_0(|\mathbf{x}_{01}|M) - K_0 \left(|\mathbf{x}_{01}| \sqrt{M^2 + \frac{\xi(1-z)m_q^2}{(1-\xi)}} \right) \right\} \\ &+ m_q^2 \int_0^1 \int_0^1 dx d\xi \left\{ K_0(|\mathbf{x}_{01}|M) - K_0 \left(|\mathbf{x}_{01}| \sqrt{\frac{M^2}{(1-x)} + \kappa} \right) \right\} \\ &\times \frac{(1-\xi)^{-1}(1-x)^{-1} C^L}{\left[x(1-\xi) + \frac{\xi}{(1-z)} \right] \left[\frac{xM}{(1-x)} + \kappa \right]} + [z \rightarrow 1-z] \end{aligned}$$

with $\kappa = m_q^2 \tilde{\kappa}(z, \xi, x)$

Tree-level $\gamma_L^* \rightarrow q\bar{q}g$ contribution

For the full NLO $\sigma_L^{\gamma^*}$ we need to compute the $\gamma_L^* \rightarrow q\bar{q}g$ LCWF



3 Steps:

- ▶ Compute the corresponding $\psi^{\gamma_L^* \rightarrow q\bar{q}g}$
- ▶ Fourier transform $\psi^{\gamma_L^* \rightarrow q\bar{q}g}$ into mixed space

$$\sigma_L^{\gamma^*} \Big|_{q\bar{q}g} = 2N_c C_F \int \widetilde{PS}_{q\bar{q}g} |\widetilde{\psi}^{\gamma^* \rightarrow q\bar{q}g}|^2 \Re[1 - S_{012}]$$

- ▶ UV subtraction
 - ▶ Add the $q\bar{q}$ and $q\bar{q}g$ terms together: cancellation of UV div's and C_{scheme}

Full NLO result for $\sigma_L^{\gamma^*}$

$$\sigma_L^{\gamma^*} = \sigma_L^{\gamma^*} \Big|_{q\bar{q},fin} + \sigma_L^{\gamma^*} \Big|_{q\bar{q}g,fin}$$

where the UV finite dipole contribution

$$\begin{aligned} \sigma_L^{\gamma^*} \Big|_{q\bar{q},fin} &= 4N_c \frac{4\alpha_{em}e_f^2 Q^2}{(2\pi)^2} \int_{\mathbf{x}_0, \mathbf{x}_1} \int_0^1 dz [z(1-z)]^2 \left\{ [K_0(|\mathbf{x}_{01}|M)]^2 \right. \\ &\times \left[1 + \left(\frac{\alpha_s C_F}{\pi} \right) \left\{ \log^2 \left(\frac{z}{1-z} \right) - \frac{\pi^2}{6} + \frac{5}{2} + \mathcal{G}^L \right\} \right] \\ &\left. + K_0(|\mathbf{x}_{01}|M) \left(\tilde{\mathcal{J}}^L + [z \rightarrow 1-z] \right) \right\} \Re[1 - \mathcal{S}_{01}] + \mathcal{O}(\alpha_{em}\alpha_s^2) \end{aligned}$$

and the UV finite $q\bar{q}g$ contribution

$$\begin{aligned}
\sigma_L^{\gamma*} \Big|_{q\bar{q}g,fin} &= 4N_c \frac{4\alpha_{em}e_f^2 Q^2}{(2\pi)^3} \left(\frac{\alpha_s C_F}{\pi} \right) \int_{\mathbf{x}_0 \mathbf{x}_1 \mathbf{x}_2} \int_0^\infty dk_0^+ dk_1^+ \int_0^\infty \frac{dk_2^+}{k_2^+} \frac{\delta(q^+ - \sum_{i=0}^2 k_i^+)}{(q^+)^5} \\
&\left\{ F_{(1)} \left[\frac{\mathbf{x}_{20}^2}{64} \mathcal{G}_{(1)}^2 \Re e[1 - \mathcal{S}_{012}] - \left[K_0(|\mathbf{r}_{01}| M_1) \right]^2 \frac{e^{-\frac{\mathbf{x}_{20}^2}{(\mathbf{x}_{01}^2 e^{\gamma E})}}}{\mathbf{x}_{20}^2} \Re e[1 - \mathcal{S}_{01}] \right] + [k_0^+ \leftrightarrow k_1^+, \mathbf{x}_0 \leftrightarrow \mathbf{x}_1] \right. \\
&- \frac{k_0^+ k_1^+}{32} \left[2 \left[(k_0^+ + k_2^+) k_0^+ + (k_2^+ + k_1^+) k_1^+ \right] (\mathbf{x}_{20} \cdot \mathbf{x}_{21}) \mathcal{G}_{(1)} \mathcal{G}_{(2)} \right. \\
&\left. \left. + \frac{m^2 (k_2^+)^4}{16} \left[\frac{(k_1^+)^2}{(k_0^+ + k_2^+)^2} \bar{\mathcal{G}}_{(1)}^2 + \frac{(k_0^+)^2}{(k_1^+ + k_2^+)^2} \bar{\mathcal{G}}_{(2)}^2 - 2 \frac{k_0^+ k_1^+}{(k_0^+ + k_2^+)(k_2^+ + k_1^+)} \bar{\mathcal{G}}_{(1)} \bar{\mathcal{G}}_{(2)} \right] \Re e[1 - \mathcal{S}_{012}] \right\}
\end{aligned}$$

where

$$\begin{aligned}
\mathcal{G}_{(i)} &= \int_0^\infty \frac{du}{u} e^{-uM_i^2} e^{-\mathbf{x}_{102}^2/(4u)} \int_0^{u/\omega_i} \frac{dt}{t^2} e^{-t\omega_i \lambda_i m_q^2} e^{-\mathbf{x}_{20}^2/(4t)} \\
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\end{aligned}$$

with

$$F_{(i)} = F_{(1)}(k_j^+), \quad M_i^2 = \bar{Q}_i^2 + m_q^2, \quad \lambda_i = \lambda_i(k_j^+, Q^2), \quad \omega_i = \omega_i(k_j^+),$$

- ▶ The $m_q \rightarrow 0$ limit fully recovers the massless result
- ▶ Remaining UV/IR finite integrals computed numerically

Conclusion

This talk:

- ▶ One-loop computation of $\gamma_L^* \rightarrow q\bar{q}$ LCWF with massive quarks
- ▶ Full NLO correction to DIS F_L : combination of massive $q\bar{q}$ and $q\bar{q}g$ contributions

Theory/Phenomenology outlook:

- ▶ Full NLO correction for F_T (**in preparation**)
 - ▶ Understand the LCPT mass renormalization in QCD (see G. Beuf talk)
- ▶ Application of the NLO $\gamma_{T/L}^* \rightarrow q\bar{q}$ LCWFs to calculate other DIS observables at NLO (vector meson production,)
- ▶ Numerical implementation of $F_{L/T}$: Fits to HERA data at NLO+LL/NLL accuracy