DIS 2019 Torino Italia

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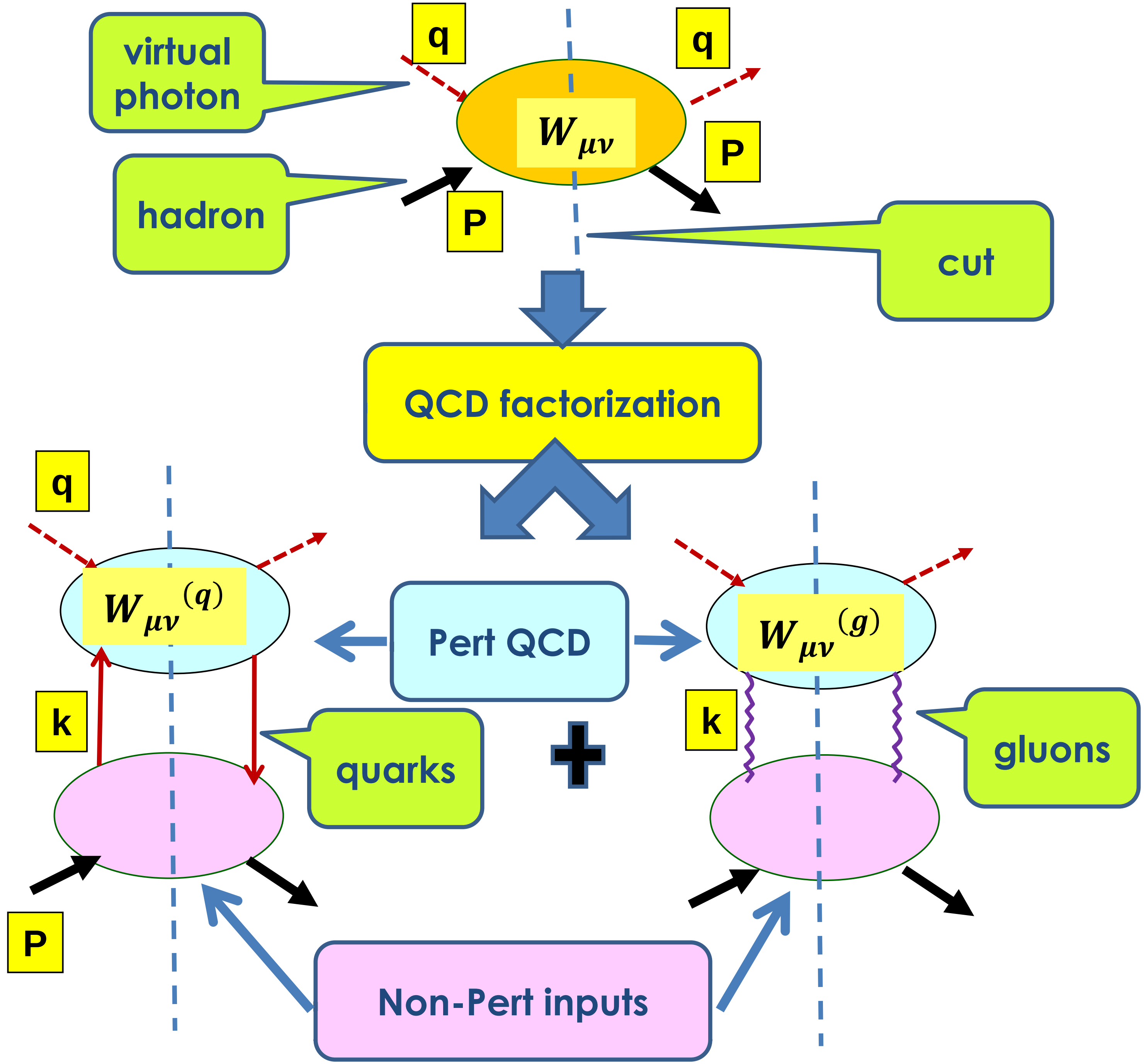
Contribution of Double-Logarithmic Pomeron to structure function F_1

talk based on results obtained in collaboration with S.I. Troyan


virtual photon
q
q
$W_{\mu\nu}$
P
hadron
P
cut
QCD factorization
q
$W_{\mu\nu}{}^{(q)}$
Pert QCD
$W_{\mu\nu}{}^{(g)}$
k
k
quarks
gluons
P
Non-Pert inputs

Hadronic tensor for unpolarized DIS

$$W_{\mu\nu}(p,q) = P^{(1)}{}_{\mu\nu}\, F_1\left(x, Q^2\right) + P^{(2)}{}_{\mu\nu}\, F_2\left(x, Q^2\right)$$

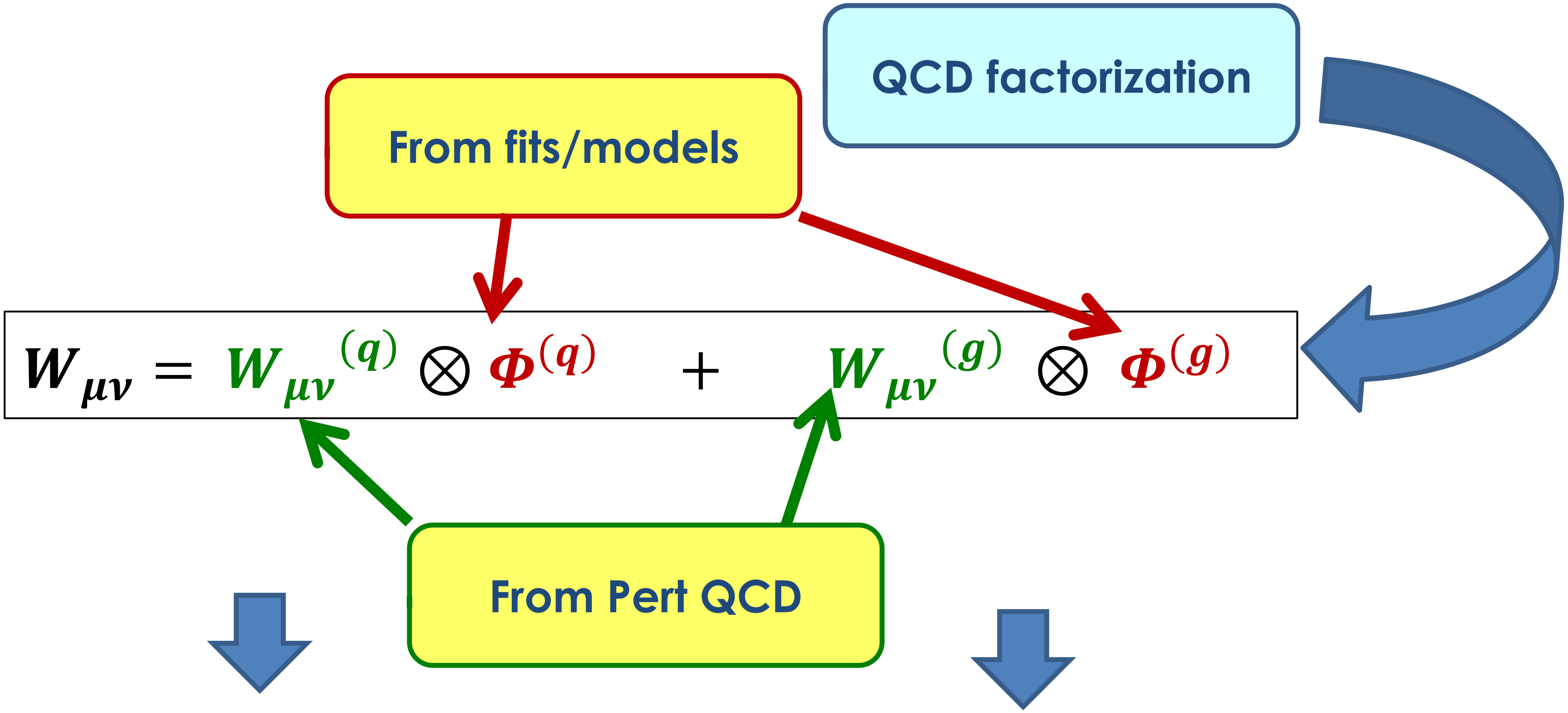
Projection operators

$$x = \frac{Q^2}{2pq}, \qquad Q^2 = -q^2$$

$$P^{(1)}{}_{\mu\nu} = -\left(g_{\mu\nu} - q_\mu q_\nu / q^2\right)$$

$$P^{(2)}{}_{\mu\nu} = \left(p_\mu - q_\mu \frac{pq}{q^2}\right)\left(p_\nu - q_\upsilon \frac{pq}{q^2}\right)\frac{1}{pq}$$

$$W_{\mu\nu} = W_{\nu\mu} \qquad q_\mu W_{\mu\nu} = q_\upsilon W_{\mu\nu} = 0$$



$$F_1 = F_1{}^{(q)} \otimes \Phi_q + F_1{}^{(g)} \otimes \Phi_g$$

Optical theorem:

$$W_{\mu\nu}{}^{(q)} = \frac{1}{2\pi}\, Im A_{\mu\nu}{}^{(q)}$$

Amplitude of Compton scattering off quark

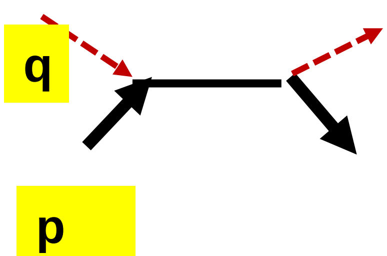
$$W_{\mu\nu}{}^{(g)} = \frac{1}{2\pi}\, Im A_{\mu\nu}{}^{(g)}$$

Amplitude of Compton scattering off gluon

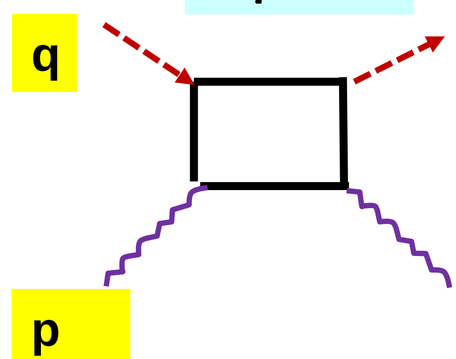
We first calculate these amplitudes and then obtain F_1 from them

In the lowest order in QCD coupling:

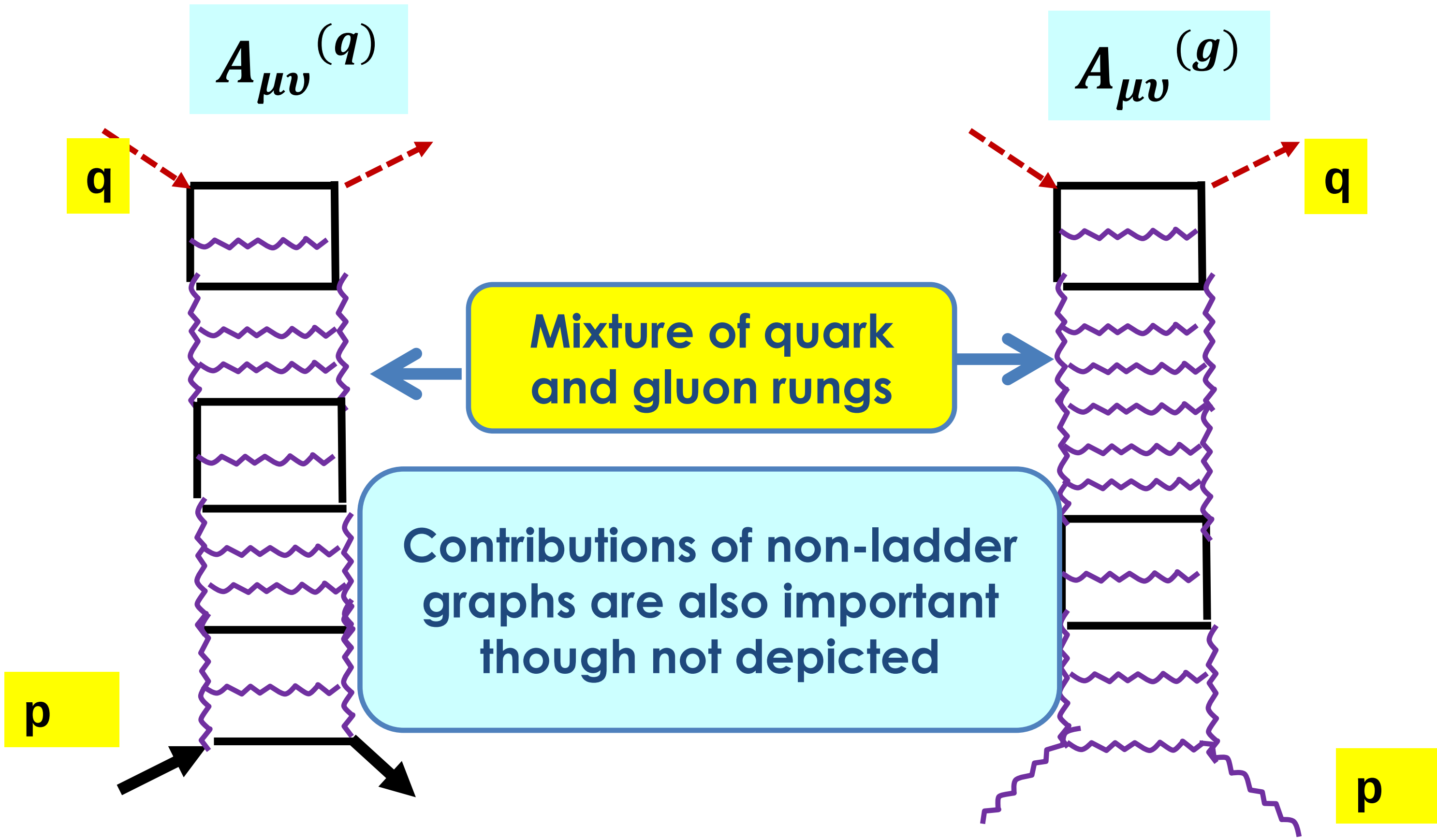
$$A_{\mu\nu}{}^{(q)}$$



$$A_{\mu\nu}{}^{(g)}$$



Higher orders in QCD coupling:



The standard instrument to describe F_1 at $x \sim 1$ is DGLAP (Dokshitzer-Gribov-Lipatov-Altarelli-Parisi) approach. It sums Contributions $\sim ln^n(Q^2/\mu^2)$ to all orders in the coupling

The x-evolution in this kinematics is not important. It is calculated in few first loops.

At small x contributions $\sim ln^n(1/x)$ become large, so it is important to account for them to all orders in the coupling,
which is absolutely beyond the DGLAP control

Calculating and summing double-logarithmic contributions together with logs of Q^2 , we construct a generalization of DGLAP to the small-x region. We account for evolution in both x and Q^2.

Many years ago we obtained similar generalizations of DGLAP for the spin structure function g_1 and non-singlet component of F_1
Now we extend our studying to the singlet F_1 which has been treated with approaches based on BFKL

There are known two methods to sum logarithmic contributions to all orders:
Leading Logarithmic Approximation (LLA) and Double Logarithmic Approximation (DLA)

Leading Logarithmic Approximation (LLA)

The instrument for total resummation of leading log contributions to F_1 is BFKL equation

V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1976); I.I. Balitsky, L.N. Lipatov (1978); V.S. Fadin, L.N. Lipatov (1998), G. Camici, M. Ciafaloni (1998)

where logs of x are calculated in Leading Logarithmic Approximation (LLA)

$$\left(\frac{1}{x}\right)\left[1 + c_1(\alpha_s ln(1/x)) + c_2(\alpha_s ln(1/x))^2 + c_3(\alpha_s ln(1/x))^3 + \cdots\right]$$

$X \to 0$

$$x^{-\omega_0}$$

Small –x asymptotics i.e. BFKL Pomeron

$$\omega_0 = 1 + \Delta$$

comes from the overall factor 1/x

comes from resummation

Solution to BFKL equation is written as the infinite series of asymptotics. The leading asymptotics in this series is called BFKL Pomeron

Double-Logarithmic Approximation (DLA)

V.V. Sudakov (1956)
V.G. Gorshkov, V.N. Gribov, L.N. Lipatov, G.V. Frolov (1967)

DL series for QED scattering amplitudes at *t=0*:

$$M_{DL} = M_0 \left[1 + c_1\left(\alpha\, ln^2\left(s/m^2\right)\right) + c_2\left(\alpha\, ln^2\left(s/m^2\right)\right)^2 + \cdots\right]$$

QCD scattering amplitudes:

$$M_{DL} = M_0 \left[1 + c_1\left(\alpha_s\, ln^2\left(s/m^2\right)\right) + c_2\left(\alpha_s\, ln^2\left(s/m^2\right)\right)^2 + \cdots\right]$$

In DLA, each power of the coupling is multiplied by two logs

Let us compare
Leading Logarithmic (LL) and Double-Logarithmic (DL) Approximations

LLA

$$F_1^{(LL)} \sim \frac{1}{x} \sum c_n(\alpha_s \; ln\,(1/x))^n$$

Large overall factor

DLA

$$F_1^{(DL)} \sim \sum c'_n(\alpha_s \; ln^2\,x)^n$$

overall factor $1/x$ is absent

and because of that, DLA contribution to F_1 etc. have commonly been neglected compared to LLA one

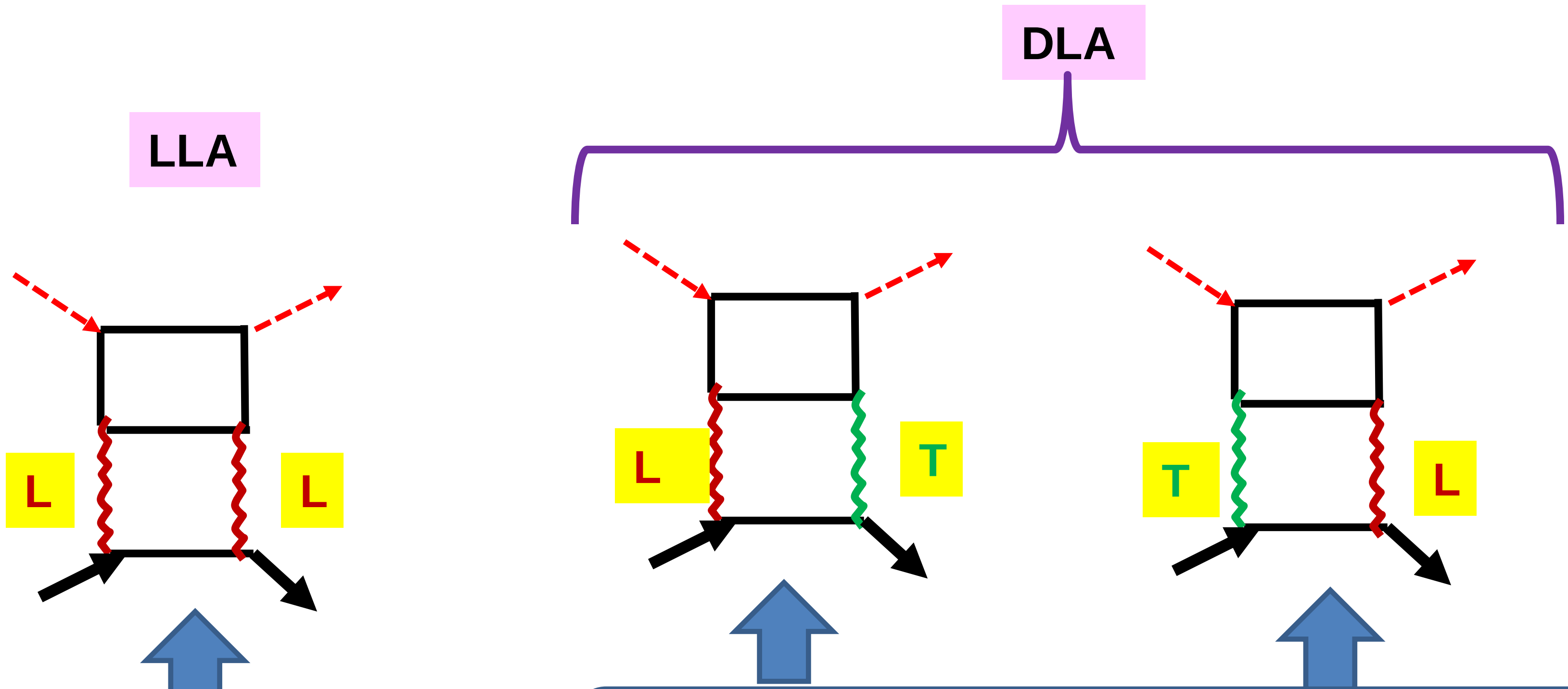
We account for DLA contribution to F_1 and prove that it is no less essential than LLA one

To calculate F_1 in DLA we use Infra-Red Evolution Equations (IREE)
This method was invented by L.N. Lipatov and applied to both Gravity (Lipatov, 1982) and quark-quark elastic scattering (Kirschner-Lipatov, 1982)
After that IREE approach has been applied to many problems of QED, QCD and EW interactions

Essence of applying IREE method to scattering amplitudes is first to introduce an IR cut-off and then evolve the amplitudes with respect to the cut-off.

The keystone of the IREE appoach is the observation that DL contributions of the partons with minimal transverse momenta can be factorized. This was first shown by V.N. Gribov in the QED context.

LLA and DLA deal with the same graphs but account for different polarizations of the ladder (vertical) gluons. For instance, such difference for amplitude of the Compton scattering off quark $A^{(q)}$ is In polarizations of the ladder gluons:



Both gluons have longitudinal polarizations (L)

One of the gluons has longitudinal (L) polarization while the other has transverse (T)

IREE equations involve convolutions, so it is convenient to use the Mellin transform to simplify them

It is convenient to use the logarithmic variables

$$\rho = ln(s/\mu^2) \qquad y = ln(Q^2/\mu^2)$$

μ^2 Infrared cut-off

So, the Mellin transform is

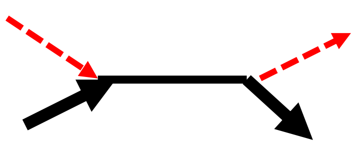
positive signature factor

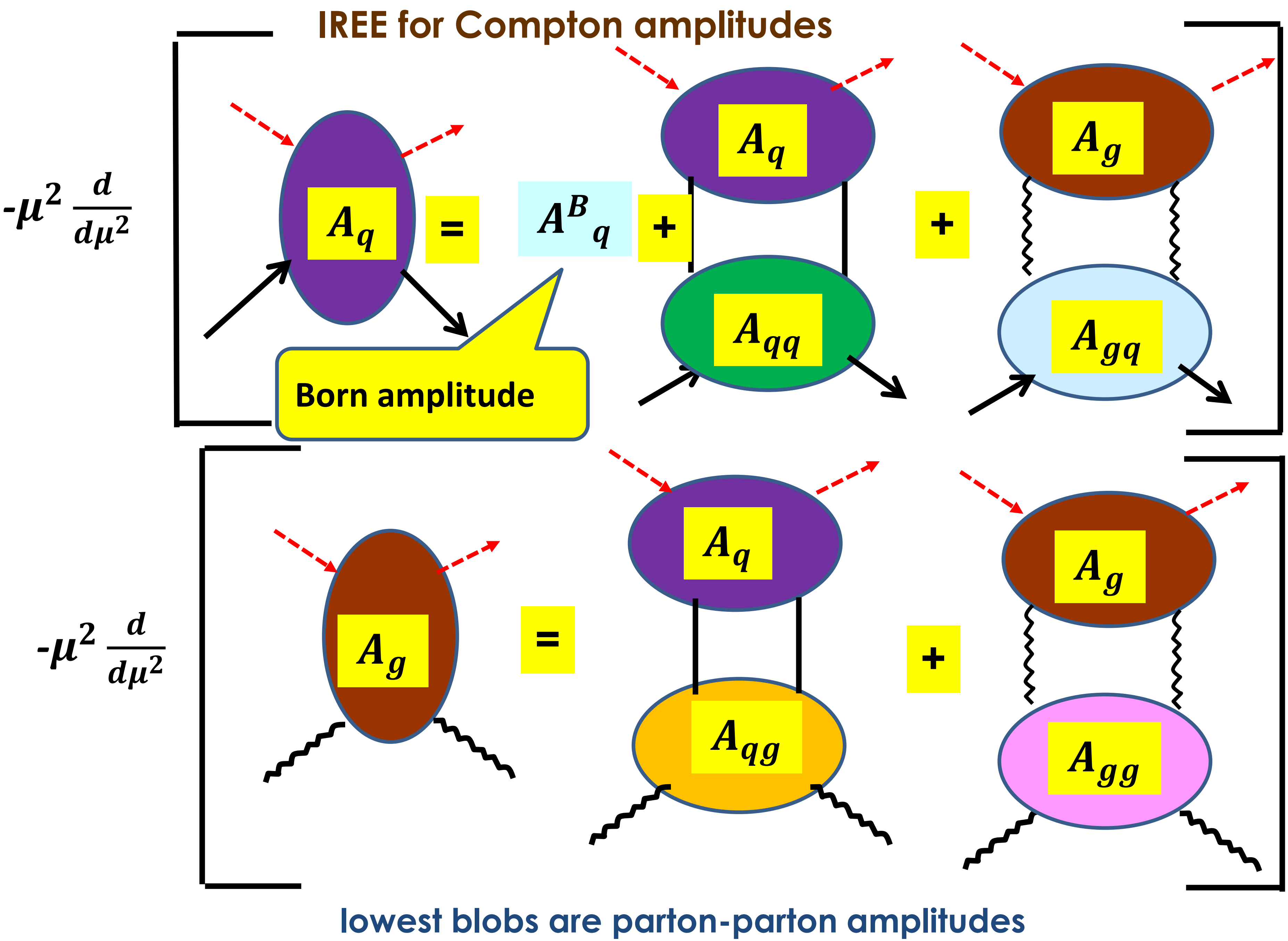
$$A_{q,g}(x, Q^2) = \int_{-i\infty}^{i\infty} \frac{d\,\omega}{2\pi i}\, e^{\omega\rho}\ \ \xi^{(+)}(\omega)\ F_{q,g}(\omega, y)$$

$$\xi^{(+)} = -\left(1 + e^{-i\pi\omega}\right)/2$$

Born approximation

$A_g = 0$,

$$A_q = -e^2\, \frac{s}{s - \mu^2 + i\,\varepsilon} =$$ 



IREE for Compton amplitudes
$-\mu^2 \frac{d}{d\mu^2}$
A_q
=
$A^B{}_q$
+
A_q
A_{qq}
+
A_g
A_{gq}
Born amplitude
$-\mu^2 \frac{d}{d\mu^2}$
A_g
=
A_q
A_{qg}
+
A_g
A_{gg}
lowest blobs are parton-parton amplitudes

Applying the Feynman rules to the graphs we write IREEs in the analytic way:

$$\frac{\partial F_q}{\partial y} + \omega F_q = F_q{}^B + \frac{1}{8\pi^2}\left[F_q f_{qq} + F_g f_{gq}\right]$$

Born contribution

Parton-parton amplitudes

$$\frac{\partial F_g}{\partial y} + \omega F_g = \frac{1}{8\pi^2}\left[F_q f_{qg} + F_g f_{gg}\right]$$

All parton-parton amplitudes can also be found with IREEs

They do not depend on y, so IREEs for them are algebraic though non-linear

Solving the IREEs, we express the Compton amplitudes through parton-parton amplitudes f_{ik}

$$A_q = \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} \quad x^{-\omega}\left[C_{(+)} e^{y\Omega_{(+)}} + C_{(-)} e^{y\Omega_{(-)}}\right]$$

$$A_g = \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} \quad x^{-\omega}\left[C_{(+)} \frac{h_{gg} - h_{qq} + \sqrt{R}}{2h_{qg}} e^{y\Omega_{(+)}} + C_{(-)} \frac{h_{gg} - h_{qq} - \sqrt{R}}{2h_{qg}} e^{y\Omega_{(-)}} \right]$$

where

$$h_{ik} = \left(1/8\pi^2\right) f_{ik}$$

$$\Omega_{(\pm)} = \frac{1}{2}\left[h_{gg} + h_{qq} \pm \sqrt{R}\right] \qquad R = \left(h_{gg} - h_{qq}\right)^2 + h_{gq} h_{qg}$$

and

$$C_{(\pm)} = \pm a_q \frac{h_{qg}h_{gq} - (\omega - (\pm)h_{gg})(h_{gg} - h_{qq} - (\pm)\sqrt{R})}{2G\sqrt{R}}$$

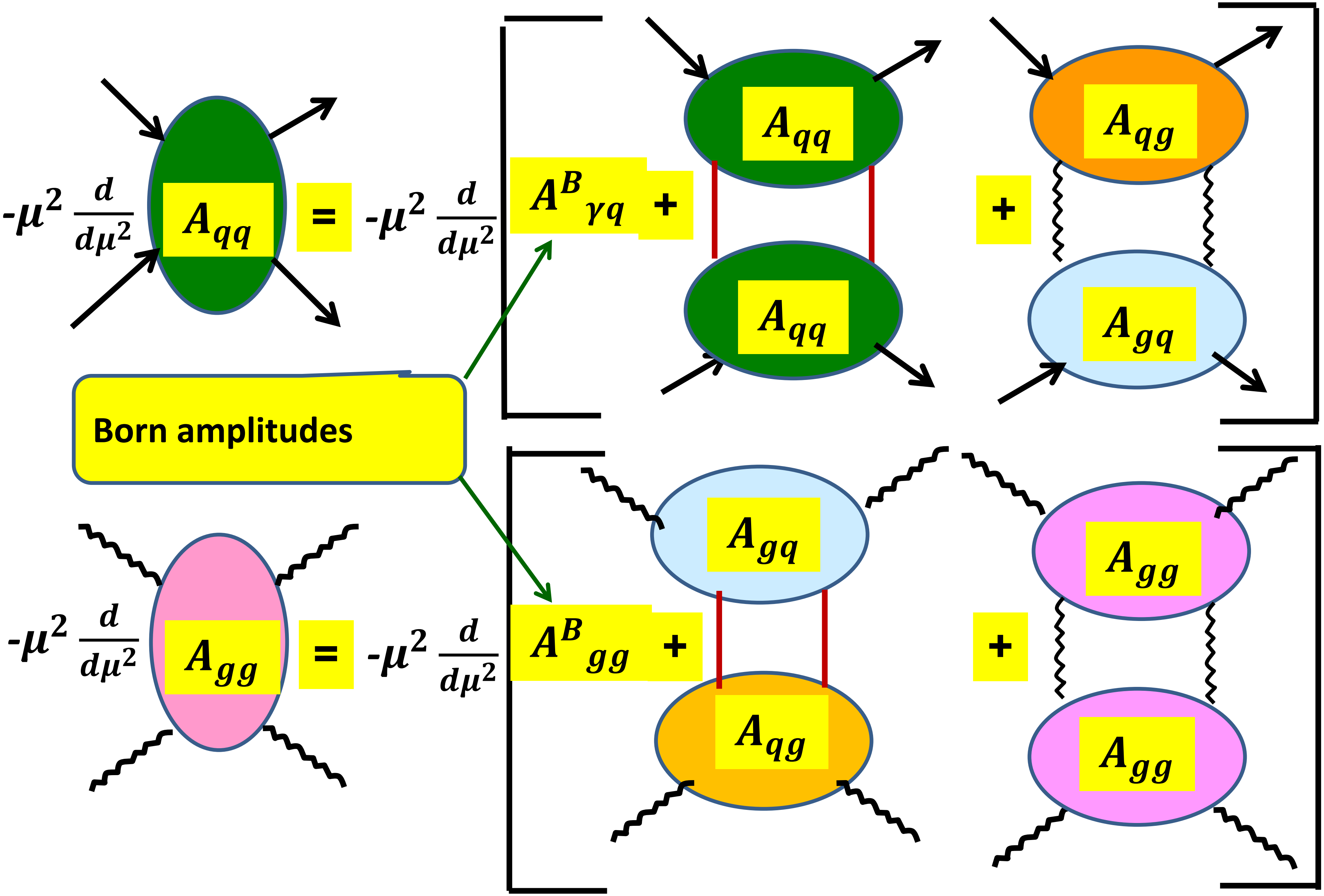
Born amplitude

$$G = (\omega - h_{gg})(\omega - h_{qq}) - h_{qg} h_{gq}$$

All parton-parton amplitudes can also be found with IREEs

They do not depend on y, so IREEs for them are algebraic though non-linear

IREE for quark-quark and gluon-gluon amplitudes



IREE for the parton-parton amplitudes

$$\omega h_{qq} = b_{qq} + h_{qq}h_{qq} + h_{qg}h_{gq}$$
$$\omega h_{qg} = b_{qg} + h_{qq}h_{qg} + h_{qg}h_{gg}$$
$$\omega h_{gq} = b_{gq} + h_{gq}h_{qq} + h_{gg}h_{gq}$$
$$\omega h_{gg} = b_{gg} + h_{gq}h_{qg} + h_{gg}h_{gg}$$

we remind that

$$h_{ik} = (1/8\,\pi^2) f_{ik}$$

$$b_{ik}(\omega) = a_{ik}(\omega) + V_{ik}(\omega)$$

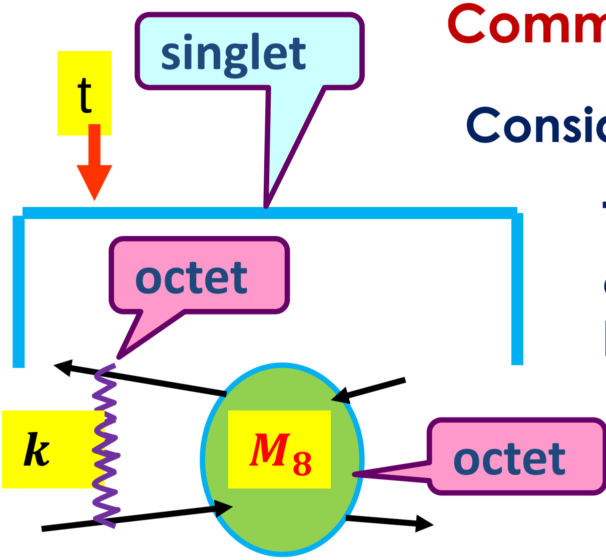
Born contributions. They are independent of ω when QCD coupling is fixed but depend on it when the coupling is running

Contributions of the color octet (non-ladder graphs)

$a_{ik}(\omega)$ coincide with analogous factors for g_1 singlet, see e.g. Bartels-Ermolaev-Ryskin (1996) or Blumlein (1997)

However such coincidence does not take place for $V_{ik}(\omega)$

Comment on the color octets $V_{ik}(\omega)$



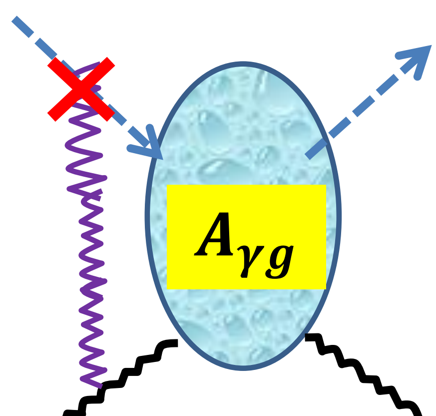
Consider a term of IREE with the factorized gluon.

The gluon belongs to the vector representation of the color group SU(3). Hence, the amplitude M_8 belongs to the octet representation too

$$8 \otimes 8 = 1 \oplus 8_s \oplus 8_A \oplus \ \ldots$$

We have to compose IREEs for the parton-parton octet amplitudes.
It is done absolutely in the same way

Fortunately, we need not octet components for photon-parton amplitudes:
Photon-gluon vertices are absent in QCD



Approximation of fixed QCD coupling

$$a_{qq} = (\alpha_s/2\pi)\ C_F\ , \qquad a_{qg} = (\alpha_s/\pi) C_F,$$
$$a_{gq} = -(\alpha_s/2\pi)\ n_f\ , \qquad a_{gg} = (\alpha_s/\pi)\ 2N$$

In this case one should set the scale of α_s . We use Principle of Minimal Sensitivity (P.M.Stevenson)

Similarly, the scale of α_s for BFKL Pomeron intercept was set (Brodsky-Fadin-Kim-Lipatov-Pivovarov) with using Principle of Maximum Conformality (Brodsky-Di Giustino)

Running QCD coupling

The standard the DGLAP parametrization $\alpha_s = \alpha_s\,(Q^2)$ cannot be used at small x. We use the parametrization more adequate in the small-x limit.

Ermolaev-Greco-Troyan

for space-like momentum

$$A'(\omega) = (1/b)\left[\eta^{-1} - \int_0^\infty dz\, e^{-z\omega}\left((z+\ \eta)^{-2}\right)\right]$$

$$\eta = ln\left(\mu^2/\Lambda_{QCD}{}^2\right), \qquad b = (11N\ - 2\,n_f)/(12\,\pi)$$

for time-like momentum

$$A(\omega) = (1/b)\left[\eta\left(\eta^2+\pi^2\right)^{-1} - \int_0^\infty dz\, e^{-z\omega}\left((z+\eta)^2+\pi^2\right)^{-1}\right]$$

The system of non-linear algebraic equations for can be solved exactly for both the case of fixed and running QCD coupling. Explicit expressions for the parton-parton amplitudes h_{ik} are

$$f_{qq} = 4\pi^2 \left[\omega - Z - \frac{b_{gg} - b_{qq}}{Z}\right] \qquad f_{qg} = \frac{8\pi^2 b_{qg}}{Z}$$

$$f_{gq} = \frac{8\pi^2 b_{gq}}{Z} \qquad f_{gg} = 4\pi^2 \left[\omega - Z + \frac{b_{gg} - b_{qq}}{Z}\right]$$

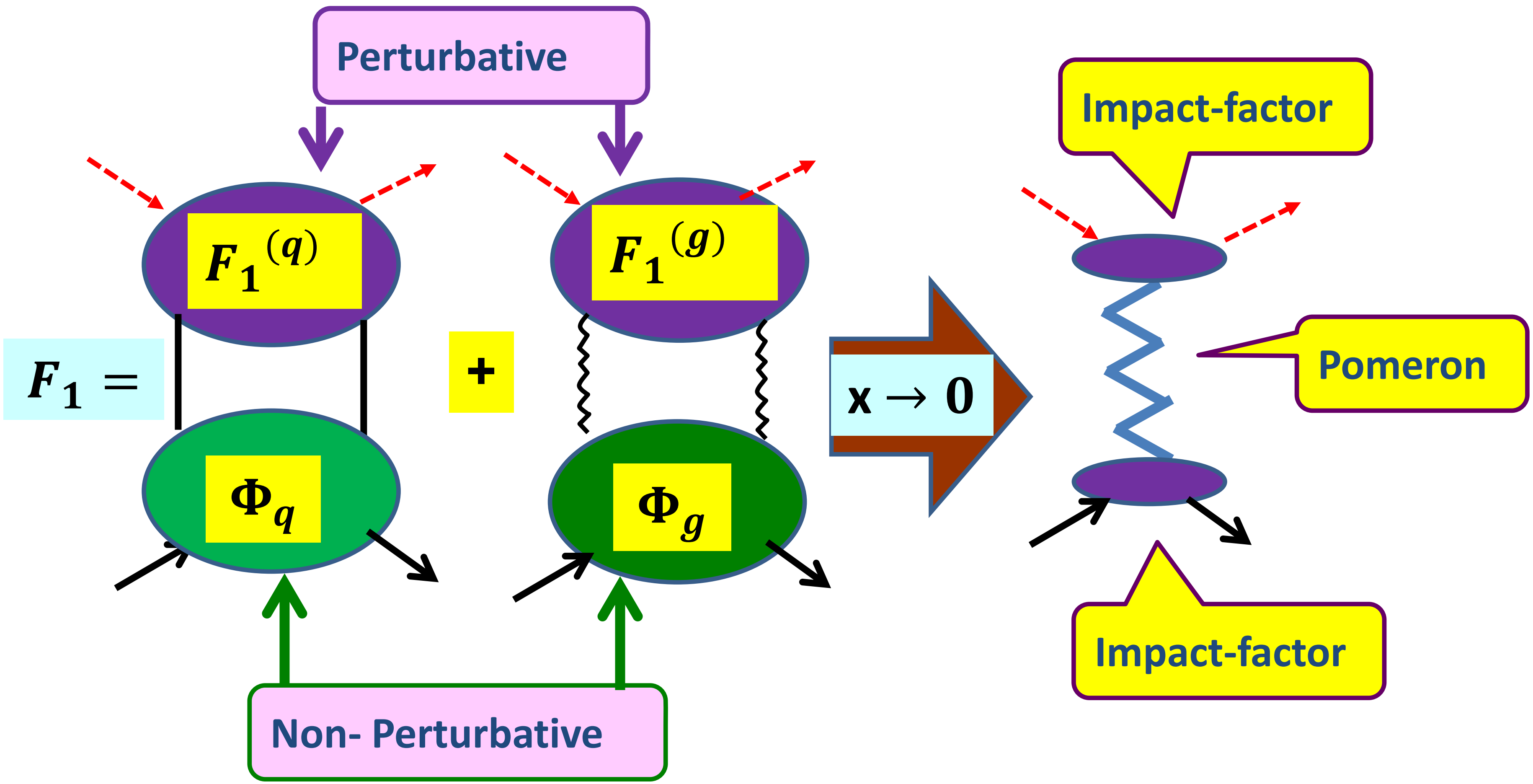
where

$$Z = \frac{1}{\sqrt{2}} [U + W] \qquad U = \omega^2 - 2\left(b_{qq} + b_{gg}\right)$$

$$W = \left[\left(\omega^2 - 2b_{gg} - 2b_{qq}\right)^2 - 4\left(b_{gg} - b_{qq}\right)^2 - 16 b_{qg} b_{gq}\right]^{1/2}$$

These expressions explicitly represent parton-parton scattering amplitudes in DLA at both fixed and running coupling Using them, we obtain explicit expressions for Compton amplitudes and for perturbative part of F_1

Small-x asymptotics of F_1



In DLA, small-x asymptotics of F_1 can be obtained by regular mathematical means

Saddle-Point method:

$$A(s, {Q_1}^2, {Q_2}^2) = -\int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} e^{\omega\rho} F(\omega) = -\int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} e^{\omega\rho + \Psi(\omega)}$$

We remind that $\rho = ln(s/\mu^2)$

Asymptotics: At $s \to \infty$, $$A \sim \frac{e^{\Psi(\omega_0)}}{\sqrt{2\pi\Psi''(\omega_0)}} \left(\frac{s}{\mu^2}\right)^{\omega_0}$$

$\rho + \Psi'(\omega_0) = 0$

$\rho \to \infty$ $\quad$ $\Psi'(\omega) \to -\infty$ when $\omega \to \omega_0$

Singularity point

There can be many singularities but the most important is the rightmost singularity. It can be a pole or a branching point of $\Psi'(\omega)$

Using explicit expression for $\Psi'(\omega)$ we conclude that ω_0 is the rightmost branching point . It corresponds to the largest root of

$$\left(\omega^2 - 2b_{gg} - 2b_{qq}\right)^2 - 4\left(b_{gg} - b_{qq}\right)^2 - 16 b_{qg} b_{gq} = 0$$

Stationary point equation

This equation can be solved analytically when QCD coupling is fixed.
If it runs, only numerical solution can be obtained

Applying the saddle-point method to F_1 we obtain its small-x asymptotics

We remind that in terms of the Mellin transform

Non-perturbative inputs

$$F_1 = \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} x^{-\omega} \left[F_q(\omega, y)\, \Phi_q(\omega) + F_g(\omega, y)\, \Phi_g(\omega) \right]$$

Applying the saddle-point method to F_1, we arrive at its asympotics:

Small-x asymptotics has the Regge form

includes non-perturbate parton distributions $\Phi_q(\omega_0)$ and $\Phi_g(\omega_0)$

intercept

Q^2 -dependence

$x \to 0$ ⇒ $$F_1 \sim \overline{F}_1 = \frac{\Pi(\omega_0)}{ln^{3/2}(1/x)} \left(\frac{1}{x}\right)^{\omega_0} \left(\frac{Q^2}{\mu^2}\right)^{\omega_{0/2}}$$

asymptotics

New Pomeron

Small-x asymptotics of F_1 in DGLAP

DGLAP fits for parton densities $\delta q = N x^{-a}(1-x)^r\left(1 + c x^d\right)$

From fits

LO anomalous dimension

$x \to 0$ $\Rightarrow$ $F_1^{DGLAP} \sim x^{-a}\left(\frac{Q^2}{\mu^2}\right)^{\gamma(a)/b}$

First coefficient of Gell-Mann –Low function

Singular terms in DGLAP fits generate the Regge asymptotics.
The intercepts are introduced as free parameters fixed from exp data

In contrast, we obtain the Regge asymptotics, applying the saddle-point method to total resummation of double logs of x and the intercepts in our approach are calculated

After the resummation has been accounted for, the factors x^{-a} in the fits become irrelevant

Asymptotic scaling

Normally, $F_1 = F_1(x, Q^2)$ — two independent variables

when $x \to 0$ ⇒ $F_1 \sim \left(\frac{\zeta}{\mu^2}\right)^{\omega_{0/2}}$ with $\zeta = Q^2/x^2$

Dependsnce on the single variable

Scaling variable

We arrive at Asymptotic Scaling which can be confirmed/disproved with analysis of available experimental data. As $F_2 = 2xF_1$ at small x, Asymptotic Scaling holds for F_2/x either.

DGLAP and BFKL do not predict such scaling

Many years ago we obtained Asymptotic Scaling for spin structure Function g_1, with $\omega_0 = 0.86$ and for non-singlet compone $\omega_0 = 0.42$ n

Now we focus in detail on the intercept ω_0 for F_1 singlet which conventionally has been given by BFKL

$$\left(\frac{1}{x}\right)\left[1 + c_1(\alpha_s ln(1/x)) + c_2(\alpha_s ln(1/x))^2 + c_3(\alpha_s ln(1/x))^3 + \cdots\right]$$

Asymptotics $x^{-\omega_0}$

$\omega_0 =$ 1 + Δ

comes from resummation

comes from the overall factor 1/x

DL Pomeron is asymptotics of the DL series:

$$1 + c'_1\left(\alpha_s ln^2(1/x)\right) + c'_2\left(\alpha_s ln^2(1/x)\right)^2 + c'_3\left(\alpha_s ln^2(1/x)\right)^3 + \cdots$$

The factor 1/x is absent, so the whole ω_0 comes entirely from calculations

For comparison with BFKL, it is convenient to introduce $\Delta \equiv \omega_0 - 1$

we calculate the intercept for several particular cases:

1. QCD coupling is fixed Equation for the stationary point is algebraic, it can be solved analytically:

$$\omega_{0_{fix}} = (\alpha_s/\pi)^{1/2}\left[4N + C_F + \sqrt{(4N - C_F)^2 - 8n_f C_F}\right]^{1/2}$$

α_s=0.24

Ermolaev-Greco-Troyan

A. quark contributions neglected, i.e. purely gluonic Pomeron Δ_{fix}= 0.35 ← close to LO BFKL intercept

B. both gluon and quark contributions accounted for Δ_{fix}= 0.29

2. Accounting for the running α_s effects

C. Purely gluonic Pomeron $\quad \boldsymbol{\Delta = 0.25}$

D. Both gluon and quark contributions are taken into account

$$\boldsymbol{\Delta = 0.066}$$

← Close to NLO BFKL intercept

We think that there is no a physical reason whatsoever for DL intercepts be close to BFKL ones and consider it as coincidence

OBSERVATION: The higher accuracy, the smaller the Pomeron intercept

$\Delta_{fix} = 0.35$	$\Delta_{fix} = 0.29$	$\Delta = 0.25$	$\Delta = 0.07$
Fixed coupling, gluons only	Fixed coupling, gluons and quarks	Running coupling, gluons only	Running coupling, gluons and quarks

SUGGESTION: this tendency suggests that eventually the intercept will go down to zero

Regge asymptotics are given by simple and elegant expressions. However the applicability regions of the asymptotics are unknown both in phenomenological Regge theory and in BFKL approach

Applicability region of BFKL Pomeron has not been estimated

However, in DLA we can estimate the applicability region for DL contribution to Pomeron

Asymptotics of F_1

We introduce $R_{as} = \frac{\overline{F}_1}{F_1}$ and study its x=dependence

Asymptotics reliably represents F_1 when R_{as} is close to 1.

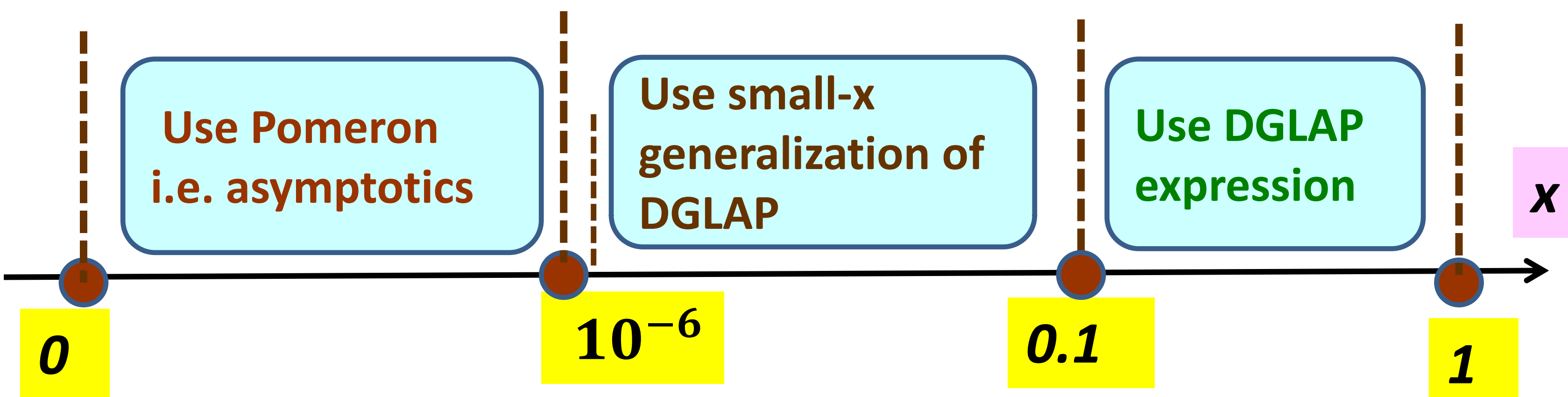
Numerical analyisis yields that $R_{as} > 0.9$ at $x < x_{max} = 10^{-6}$

Appicability region for DL Pomeron

At $x > x_{max}$ asymptotical expression $\overline{F}_1$ cannot be used

In this region one should use F_1

We think that x_{max} for BFKL Pomeron is even less



How to treat DL Pomeron in HEP

First, let us notice that intercepts of BFKL and DL Pomerons nearly coincide.

1. Straightforward/theoretical way:
To account for both DL Pomeron contribution and BFKL one in each hadronic reaction

2. Practical/phenomenological way:
As description of hadronic reactions involve phenomenological factors/fits with parameters fixed to meet experiment, one can absorb impact of BFKL into the fits and work with DL Pomeron entirely, using either small-x asymptotics of F_1 or the parent expressions , depending on value of x

3. Combining DLA and DGLAP: It is easy to combine the DL contribution to F_1 with the DGLAP expression and obtain an universal formula for F_1 valid at any $0<x<1$

The same reasoning applies to any hadronic reaction at high energies

CONCLUSIONS

We have obtained explicit expressions for Compton scattering amplitudes in DLA, with fixed and running QCD coupling and, using Optical theorem, we obtained F_1

Applying Saddle-Point method, we arrived at the Regge asymptotics for F_1, with the Reggeon being a new, DL contribution to Pomeron.

Although intercepts of DL Pomeron are not far from the ones of BFKL, DL Pomeron has nothing in common with BFKL Pomeron because they are asymptotics of total resummation of different kinds of logarithmic contributions:
double logarithms and single logarithms respectively.

We proved Asymptotic Scaling for F_1 and F_2

We obtained that the asymptotics is reliable at $x < x_{max} = 10^{-6}$

DL contribution to Pomeron can be used in HEP either together with BFKL Pomeron or instead of it