Non-linear evolution in QCD at low-x beyond leading order

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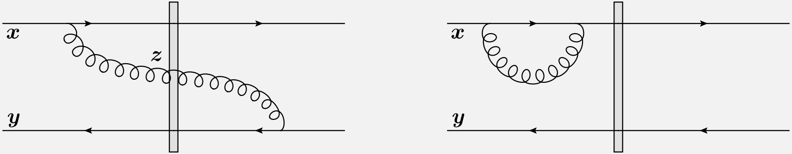
B. Ducloué, E. Iancu, A.H. Mueller, E. Iancu, DT, 1902.06637 (to appear in JHEP)

Outline

- LO BK equation and solutions
- NLO BK equation in projectile rapidity Y and instability
- Time ordering and resummation in Y and issues
- NLO BK equation in target rapidity η and instability
- Resummed evolution in η via shifts
- Fits, thoughts on saturation momentum Q_s^2
- Conclusion

BK Equation at LO

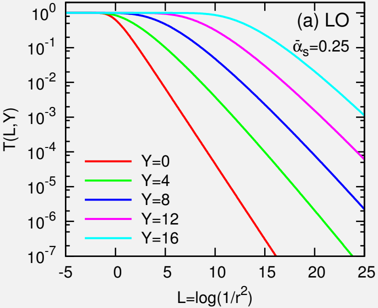
Scatter dipole $\boldsymbol{r} = (\boldsymbol{x}, \boldsymbol{y})$ and q^+ on hadron with Q_0^2 and $q_0^+ = Q_0^2/2q_0^-$. Typically $r^2 Q_0^2 < 1$. Emit soft gluon.



Resum powers of $\bar{\alpha}_s Y = \bar{\alpha}_s \ln q^+/q_0^+$ in presence of strong target field $\rightsquigarrow$ BK equation. $Y \equiv$ "projectile" rapidity.

$$\frac{\partial S_{\boldsymbol{xy}}(Y)}{\partial Y} = \frac{\bar{\alpha}_s}{2\pi} \int \frac{\mathrm{d}^2\boldsymbol{z}\, (\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2} \big[S_{\boldsymbol{xz}}(Y) S_{\boldsymbol{zy}}(Y) - S_{\boldsymbol{xy}}(Y)\big].$$

Solution to BK Equation at LO



Saturation line $T(Y, r^2 \sim 1/Q_s^2) = 1/2$

Asymptotically $\lambda_s = \mathrm{d}\ln Q_s^2/\mathrm{d}Y \simeq 4.88\bar{\alpha}_s$

Scaling $T(Y, r^2) \sim (r^2 Q_s^2)^{\gamma_0}$ with $\gamma_0 \simeq 0.63$

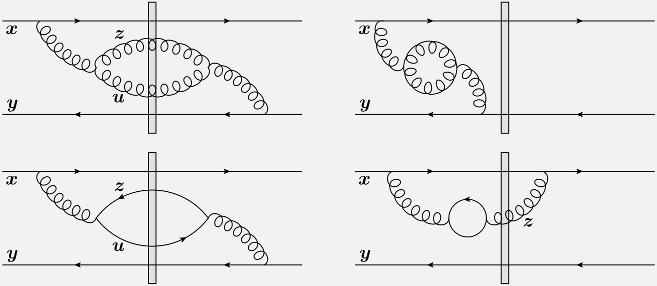
Can calculate preasymptotics

BK Evolution at NLO (I)

Emit soft + non-soft (in general)

NLO N_c

NLO N_f



+ many more

BK Evolution at NLO (II)

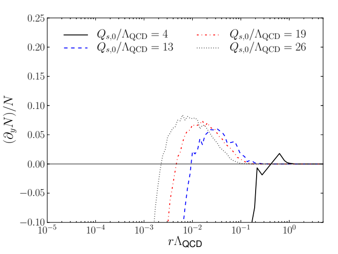
$$\frac{\partial S_{\boldsymbol{xy}}}{\partial Y} = \frac{\bar{\alpha}_s}{2\pi} \int \frac{\mathrm{d}^2\boldsymbol{z}\,(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2} \Bigg\{ 1 + \bar{\alpha}_s \Bigg[\underbrace{-\frac{1}{2} \ln \frac{(\boldsymbol{x}-\boldsymbol{z})^2}{(\boldsymbol{x}-\boldsymbol{y})^2} \ln \frac{(\boldsymbol{y}-\boldsymbol{z})^2}{(\boldsymbol{x}-\boldsymbol{y})^2}}_{\text{double logs}}$$

$$+ \underbrace{\bar{b}\, \ln(\boldsymbol{x}-\boldsymbol{y})^2\mu^2 - \bar{b}\, \frac{(\boldsymbol{x}-\boldsymbol{z})^2 - (\boldsymbol{y}-\boldsymbol{z})^2}{(\boldsymbol{x}-\boldsymbol{y})^2} \ln \frac{(\boldsymbol{x}-\boldsymbol{z})^2}{(\boldsymbol{y}-\boldsymbol{z})^2}}_{\text{running coupling: choose } \mu^2 \sim 1/r^2_{\min}} \Bigg] \Bigg\} [S_{\boldsymbol{xz}} S_{\boldsymbol{zy}} - S_{\boldsymbol{xy}}]$$

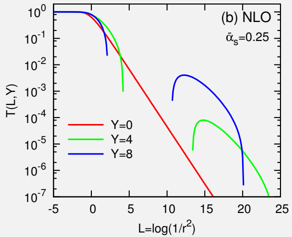
$$+ \frac{\bar{\alpha}_s^2}{8\pi^2} \int \mathrm{d}^2\boldsymbol{z}\, \mathrm{d}^2\boldsymbol{u}\, \underbrace{\mathcal{K}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z},\boldsymbol{u})}_{\text{single logs + reg}} [S_{\boldsymbol{xu}} S_{\boldsymbol{uz}} S_{\boldsymbol{zy}} - S_{\boldsymbol{xu}} S_{\boldsymbol{uy}}]$$

$$+ \mathcal{O}(N_f) + \mathcal{O}(1/N_c^2)$$

Solution to BK Equation at NLO: Unstable



NLO BK



LO BK + double log

Double logarithm main source of problem

Large Double Logarithm and Time Ordering

Strongly ordered large perturbative dipoles (DLA)

$$r^2 \ll (\boldsymbol{x}-\boldsymbol{z})^2 \simeq (\boldsymbol{z}-\boldsymbol{y})^2 \ll 1/Q_s^2$$

Large dipoles interact stronger, real terms dominate

$$\frac{\mathrm{d}T(Y,r)}{\mathrm{d}Y} = \bar{\alpha}_s \int_{r^2}^{1/Q_s^2} \frac{r^2 \mathrm{d}z^2}{z^4} \left(1 - \frac{\bar{\alpha}_s}{2} \ln^2 \frac{z^2}{r^2}\right) T(Y,z)$$

NLO $>$ LO, expansion in $\bar{\alpha}_s$ fails

Lifetime of fluctuation $\tau_{\text{fluct}} \sim k^+_{\text{fluct}} r^2_{\text{fluct}}$

Problem for large daughters: $k_2^+ < k_1^+ \not\Rightarrow \tau_2 < \tau_1$

LO does not care, NLO "tries" correcting $\rightsquigarrow$ big correction

Resum Double Logs in Y Evolution (I)

Local equation with resummed kernel

$$\frac{\partial S_{\boldsymbol{xy}}(Y)}{\partial Y} = \frac{\bar{\alpha}_s}{2\pi} \int \frac{\mathrm{d}^2\boldsymbol{z}\,(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2}\, \mathcal{K}_{\rm DLA}\left(\sqrt{\ln\frac{(\boldsymbol{x}-\boldsymbol{z})^2}{(\boldsymbol{x}-\boldsymbol{y})^2}\ln\frac{(\boldsymbol{y}-\boldsymbol{z})^2}{(\boldsymbol{x}-\boldsymbol{y})^2}}\right)$$

$$\times \left[S_{\boldsymbol{xz}}(Y)S_{\boldsymbol{zy}}(Y) - S_{\boldsymbol{xy}}(Y)\right].$$

with

$$\mathcal{K}_{\rm DLA}(\rho) = \frac{\mathrm{J}_1\left(2\sqrt{\bar{\alpha}_s\rho^2}\right)}{\sqrt{\bar{\alpha}_s\rho^2}} = 1 - \frac{\bar{\alpha}_s\rho^2}{2} + \frac{(\bar{\alpha}_s\rho^2)^2}{12} + \cdots$$

Oscillations cut “unphysically“ large daughter dipoles

Add remaining $\mathcal{O}(\bar{\alpha}_s^2)$ to fully match NLO BK

Resum Double Logs in Y Evolution (II)

"Delay" equation with LO dipole kernel

$$\frac{\partial S_{\boldsymbol{xy}}(Y)}{\partial Y} = \frac{\bar{\alpha}_s}{2\pi} \int \frac{\mathrm{d}^2\boldsymbol{z}\,(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2}\,\Theta(Y-\rho_{\min})$$

$$\times \left[S_{\boldsymbol{xz}}(Y-\Delta_{\boldsymbol{xz};r})S_{\boldsymbol{zy}}(Y-\Delta_{\boldsymbol{zy};r})-S_{\boldsymbol{xy}}(Y)\right]$$

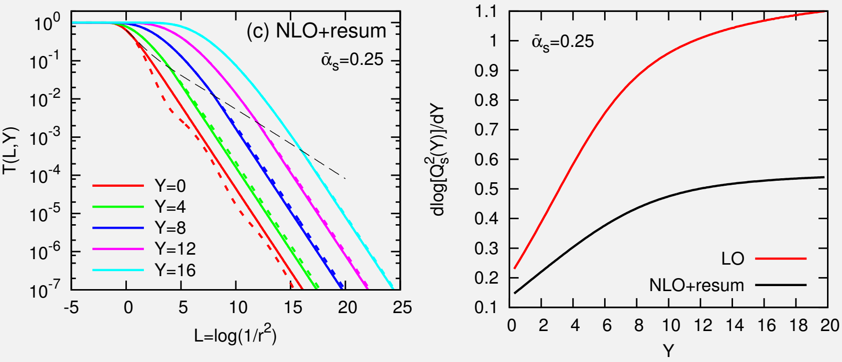
with $\rho_{\min} = \ln(1/r^2_{\min}Q^2_0)$ and

$$\Delta_{\boldsymbol{xz};r} \equiv \max\left\{0, \ln\frac{(\boldsymbol{x}-\boldsymbol{z})^2}{r^2}\right\} \qquad \text{(canonical shift)}$$

Big log shift for large daughters, no shift otherwise

Again can fully match to NLO BK

Solution to Resummed Equation: Stable



Everybody happy, go home? Not really ...

Issues/Comments in Y (Projectile) Evolution

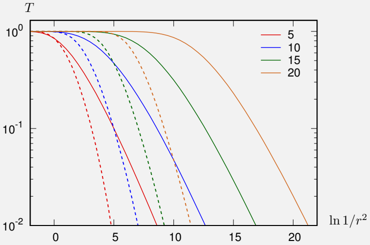
- λ_s very small (when including all NLO), $\gamma_s \sim 1$ too large
- Target rapidity determines kinematics

$$\eta \equiv \ln \frac{1}{x_{\rm Bj}} = \ln \frac{s}{Q^2} = \ln \frac{2q^+ q_0^-}{2q^+ q^-} = \ln \frac{q_0^-}{q^-}$$
$$= \ln \frac{Q_0^2}{2q_0^+} \frac{2q^+}{Q^2} \to Y - \ln \frac{1}{r^2 Q_0^2} \equiv Y - \rho$$

  Saturation is a target property, need $Q_s^2(\eta)$

- Not really an Initial Condition problem, but a Boundary one
  Extemely difficult to solve properly
- "Close eyes" and try various motivated Initial Conditions
  Need to express the results in terms of η (easy)

Slope and Speed for η-Front



Front in $Y \Leftrightarrow$ Front in η, physical

Scaling in Y: $-\ln T = \gamma_s(\rho - \lambda_s Y)$

Change variable $Y = \eta + \rho$

Scaling in η: $-\ln \bar{T} = \bar{\gamma}_s(\rho - \bar{\lambda}_s \eta)$

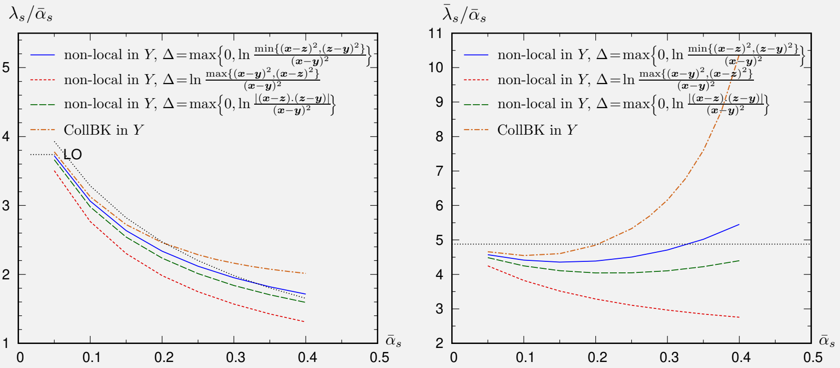
$$\boxed{\bar{\gamma}_s = \gamma_s(1 - \lambda_s)} \quad \text{and} \quad \boxed{\bar{\lambda}_s = \frac{\lambda_s}{1 - \lambda_s}}$$

$\lambda_s = \mathcal{O}(\bar{\alpha}_s)$, difference is NLO. In practice it is large.

Physical η-front less steep and faster

What we wanted, but not under control

Bad Scheme Dependence, Unphysical Results



Unphysical inflection point

Even larger than LO result

Canonical shift behaves properly (still, IC is not under control)

NLO BK in Target Rapidity η

- Evolve the projectile: dipole kernels in transverse space
- Variable change $\eta = Y - \rho$ in longitudinal space

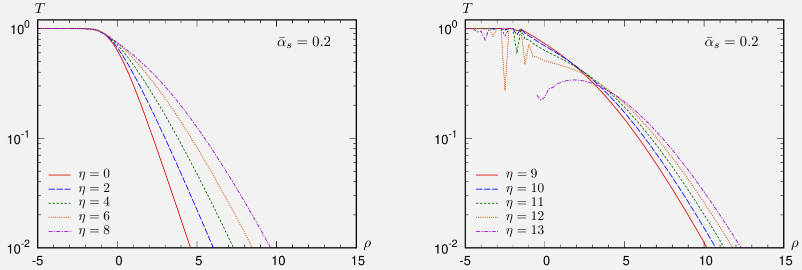
$$S(Y,r) = S\big(\eta + \ln(1/r^2 Q_0^2), r\big) \equiv \bar{S}(\eta, r)$$

$$S(Y,z) = S\big(\eta + \ln(1/z^2 Q_0^2) - \ln(r^2/z^2), z\big) = \bar{S}(\eta - \ln(r^2/z^2), z)$$

Non-local, expand at NLO. Extra term for NLO BK in η

$$\boxed{\frac{\bar{\alpha}_s^2}{2\pi^2} \int \frac{\mathrm{d}^2\boldsymbol{z}\, \mathrm{d}^2\boldsymbol{u}\, (\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{u})^2 (\boldsymbol{u}-\boldsymbol{z})^2 (\boldsymbol{z}-\boldsymbol{y})^2} \ln \frac{(\boldsymbol{u}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{y})^2}\, \bar{S}_{\boldsymbol{xu}} \left[\bar{S}_{\boldsymbol{uz}} \bar{S}_{\boldsymbol{zy}} - \bar{S}_{\boldsymbol{uy}}\right]}$$

Solution to NLO BK in Target Rapidity η



Milder but still problematic instability due to small dipoles

$$\frac{\partial \bar{T}(\eta, r)}{\partial \eta} \simeq \bar{\alpha}_s \int_0^{r^2} \frac{\mathrm{d}z^2}{z^2} \left(1 - \bar{\alpha}_s \ln^2 \frac{r^2}{z^2}\right) \bar{T}(\eta, z) \quad \text{for} \quad z \ll r$$

No double logs in solution (color transparency)

But solution develops anomalous dimension → big NLO correction

Resum Double Logs in η Evolution

Lifetimes τ, or k^-, automatically ordered

But ordering in k^+ is not guaranteed any more

Enforcing it resums double logs for small dipoles to all orders

$$\frac{\partial \bar{S}_{\boldsymbol{xy}}(\eta)}{\partial \eta} = \frac{\bar{\alpha}_s}{2\pi} \int \frac{\mathrm{d}^2\boldsymbol{z}\,(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2}\, \Theta\big(\eta - \delta^{>}_{\boldsymbol{xyz}}\big)$$

$$\times \big[\bar{S}_{\boldsymbol{xz}}(\eta - \delta_{\boldsymbol{xz};r})\bar{S}_{\boldsymbol{zy}}(\eta - \delta_{\boldsymbol{zy};r}) - \bar{S}_{\boldsymbol{xy}}(\eta)\big]$$

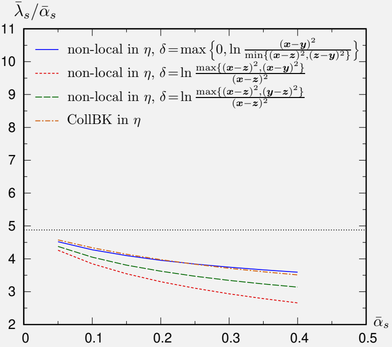
$$\text{with} \qquad \delta_{\boldsymbol{xz};r} \equiv \max\left\{0, \ln\frac{r^2}{(\boldsymbol{x}-\boldsymbol{z})^2}\right\} \qquad \text{(canonical shift)}$$

Typically hard to soft, but BFKL/BK is diffusive, cut small dipoles

IC problem, directly the result in $\eta \sim \ln(1/x_{\mathrm{Bj}})$

Can fully match to NLO BK

Solution to Resummed Evolution in η

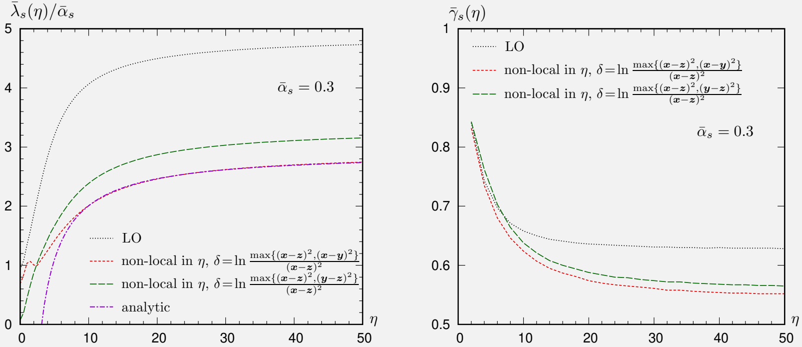


Correction of order $\mathcal{O}(\bar{\alpha}_s)$

Physically acceptable results

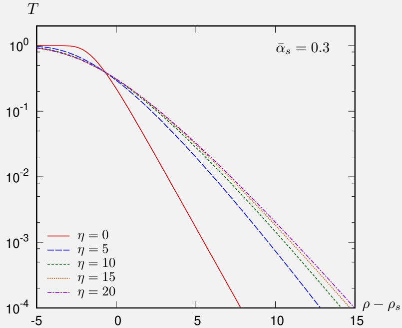
Inclusion of all NLO terms would reduce uncertainty by a factor $\bar{\alpha}_s$.

Preasymptotic dependence on η

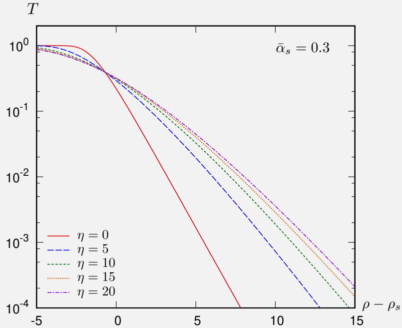


Analytic expression valid down to $\eta = 6 \div 8$

Geometric scaling in η Evolution

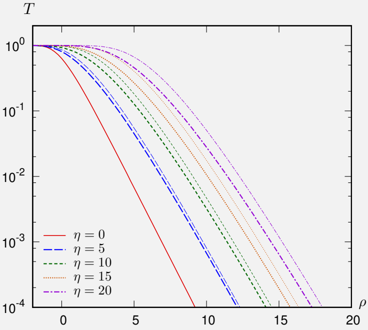


LO BK in η

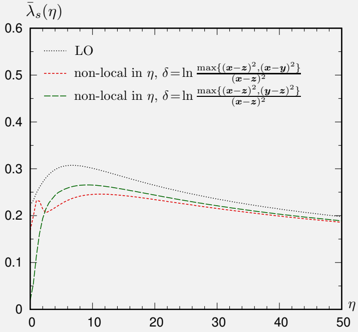


Resummed BK in η

Running Coupling in η Evolution



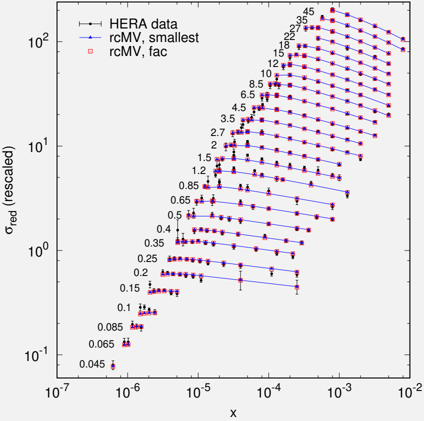
LO + RC vs. Resummed + RC

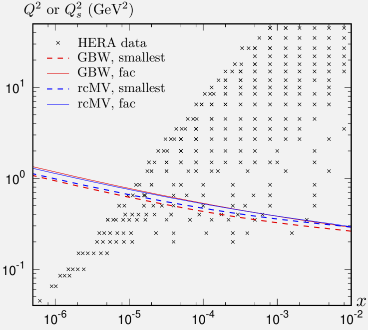


LO + RC vs. Resummed + RC

HERA Data Fit (I)

LO impact factor (γ^* WF) + Resummed in η (DL's, SL's and RC)





HERA Data Fit (II)

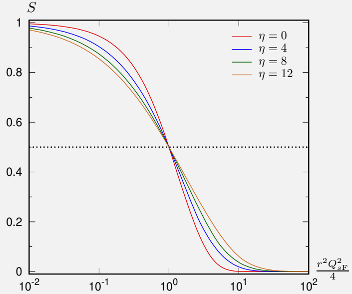
IC	RC	χ^2/npts	R_p[fm]	Q_0[GeV]	C_α	p
GBW	small	1.44	0.791	0.364	4.93	4
GBW	fac	1.35	0.756	0.381	0.830	4
rcMV	small	1.28	0.779	0.596	5.31	1.50
rcMV	fac	1.23	0.759	0.549	0.935	1.05

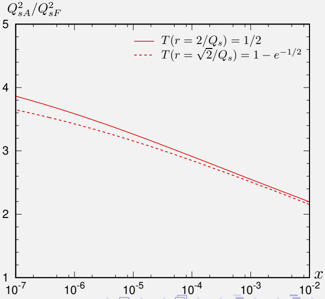
Going to higher Q^2

IC	RC	50	100	200	400
GBW	small	1.44	1.46	1.72	1.83
GBW	fac	1.35	1.36	1.59	1.68
rcMV	small	1.28	1.29	1.29	1.27
rcMV	fac	1.23	1.26	1.24	1.22

$Q_s^2(\eta)$ marginally perturbative, BUT:

- x - dependent scale
- $\boldsymbol{b}$ - averaged, in principle higher $Q_s^2(\eta)$ in "center"
- Saturation determines form even for $Q^2 > Q_s^2(\eta)$
- Extrapolate to nuclei $\sim A^{1/3}$ (maybe a factor of 3)
- $Q_s^2(\eta)$ in adjoint, factor of $\sim 3 > C_\mathrm{A}/C_\mathrm{F} = 9/4$





Conclusions

- Resummed evolution in target rapidity $\eta \sim \ln 1/x_{\rm Bj}$
  $\rightsquigarrow$ physical front directly
- Stable results, front in η faster and less steep
- Compared to LO: $\delta\lambda_s/\lambda_s \simeq \mathcal{O}(\bar{\alpha}_s)$, roughly the same γ_s
- Can match to full NLO BK evolution
- Fit to DIS HERA data
- Higher than expected Q_s^2 in adjoint