High-energy effects in forward inclusive dijet and hadron-jet production

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in collaboration with

A.D. Bolognino, **D.Yu. Ivanov**, **M.M.A. Mohammed**, **A. Papa**

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HAS QCD
HADRONIC STRUCTURE AND QUANTUM CHROMODYNAMICS

INFN
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Outline

1. Introductory remarks
   - QCD and semi-hard processes
   - BFKL resummation
   - Motivation
2. Inclusive hadron-jet production
   - Hors d'œuvre
   - Theoretical setup
   - Numerical analysis
3. Conclusions and Outlook

Outline

QCD and the semi-hard sector

High energies reachable at the LHC and at future colliders:

- great opportunity in the search for long-waited signals of New Physics...
- ...faultless chance to test Standard Model in unprecedent kinematic ranges
- duality between non-perturbative and perturbative aspects (**confinement** and **asymptotic freedom** concurrent properties) makes QCD a challenging sector surrounded by a broad and constant interest in its phenomenology

QCD and the semi-hard sector

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Semi-hard processes

Collision processes with the following **scale hierarchy**: $s \gg Q^2 \gg \Lambda^2_{\rm QCD}$

- Q is the **hard scale** of the process (*e.g.* photon virtuality, heavy quark mass, jet/hadron transverse momentum, t, *etc.*)
- large $Q \implies \alpha_s(Q) \ll 1 \implies$ perturbative QCD
- large $s \implies$ large energy logs $\implies \alpha_s(Q) \log s \sim 1 \implies$ need to **resummation**

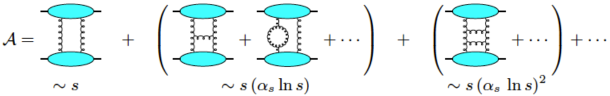
The BFKL resummation

pQCD, **semi-hard processes**: $s \gg Q^2 \gg \Lambda^2_{\rm QCD}$

- **BFKL resummation**: [V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975, 1976, 1977); Y.Y. Balitskii, L.N. Lipatov (1978)]

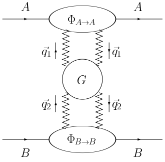
$\xrightarrow{\text{based on}}$ **gluon Reggeization**

leading logarithmic approximation (LLA): $\alpha_s^n (\ln s)^n$



next-to-leading logarithmic approximation (NLA): $\alpha_s^{n+1} (\ln s)^n$

total cross section for $A + B \to X$: $\sigma_{AB}(s) = \frac{\mathcal{I}m_s\{\mathcal{A}_{AB}^{AB}\}}{s} \Leftarrow$ ***optical theorem***



- $\mathcal{I}m_s\{\mathcal{A}_{AB}^{AB}\}$ factorization:

  convolution of the **Green's function** of two interacting Reggeized gluons with the **impact factors** of the colliding particles

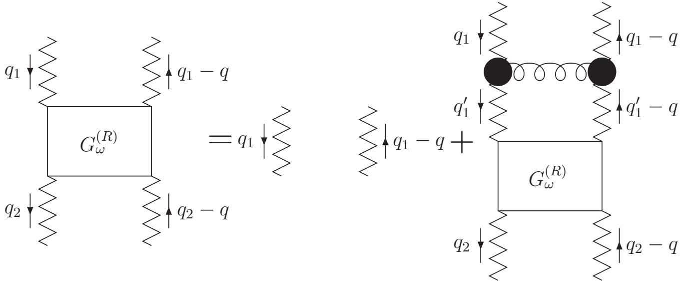
$$\mathcal{I}m_s\{\mathcal{A}\} = \frac{s}{(2\pi)^{D-2}}\int\frac{d^{D-2}q_1}{\vec{q}_1^{\,2}}\Phi_A(\vec{q}_1, s_0)\int\frac{d^{D-2}q_2}{\vec{q}_2^{\,2}}\Phi_B(-\vec{q}_2, s_0)\int_{\delta-i\infty}^{\delta+i\infty}\frac{d\omega}{2\pi i}\left(\frac{s}{s_0}\right)^{\omega} G_\omega(\vec{q}_1, \vec{q}_2)$$

- **Green's function** is **process-independent** and takes care of the **energy dependence**

$\longrightarrow$ determined through the **BFKL equation**

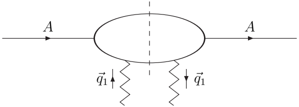
[Ya.Ya. Balitskii, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975)]

$$\omega\, G_\omega(\vec{q}_1, \vec{q}_2) = \delta^{D-2}(\vec{q}_1 - \vec{q}_2) + \int d^{D-2}q\, K(\vec{q}_1, \vec{q})\, G_\omega(\vec{q}, \vec{q}_1)\ .$$



- **Impact factors** are **process-dependent** and depend on the hard scale, but not on the energy

  $\longrightarrow$ known in the NLA just for few processes



◇ **colliding partons**

[V.S. Fadin, R. Fiore, M.I. Kotsky, A. Papa (2000)]
[M. Ciafaloni, G. Rodrigo (2000)]

◇ $\gamma^* \longrightarrow V$, with $V = \rho^0,\ \omega,\ \phi$, forward case

[D.Yu. Ivanov, M.I. Kotsky, A. Papa (2004)]

◇ forward jet production

[J. Bartels, D. Colferai, G.P. Vacca (2003)]
(exact IF) [F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa, A. Perri (2012)]
(small-cone IF) [D.Yu. Ivanov, A. Papa (2012)]
(several jet algorithms discussed) [D. Colferai, A. Niccoli (2015)]

◇ forward identified hadron production

[D.Yu. Ivanov, A. Papa (2012)]

◇ $\gamma^* \longrightarrow \gamma^*$

[J. Bartels *et al.* (2001), I. Balitsky, G.A. Chirilli (2011, 2013)]

Progress in high-energy phenomenology

So far, search for BFKL effects had these general drawbacks:

- ◇ too low $\sqrt{s}$ or rapidity intervals among tagged particles in the final state
- ◇ too inclusive observables, other approaches can fit them

Advent of LHC:

- → higher energies $\leftrightarrow$ larger rapidity intervals
- → unique opportunity to **test pQCD in the high-energy limit**
- → disentangle applicability region of energy-log resummation (**BFKL approach**)

[V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975, 1976, 1977)]
[Y.Y. Balitskii, L.N. Lipatov (1978)]

Last years:

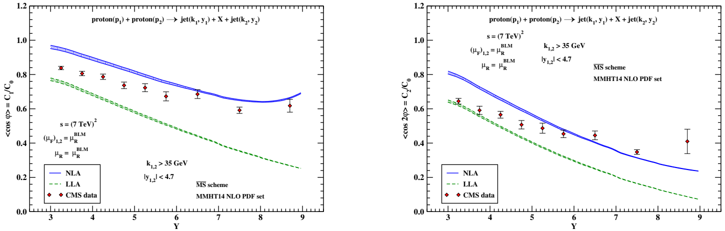
Mueller–Navelet jets

[B. Ducloué, L. Szymanowski, S. Wallon (2014)]
[F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa (2014); F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2015, 2016)]

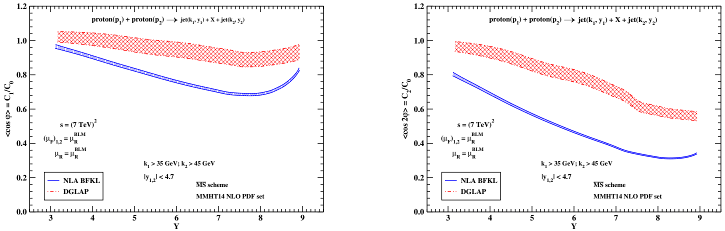
Inclusive di-hadron production

[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2016, 2017)]

Theory vs experiment vs fixed-order DGLAP (dijet)



(**7 TeV theory vs exp. + sym.**) [F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa (2014)]

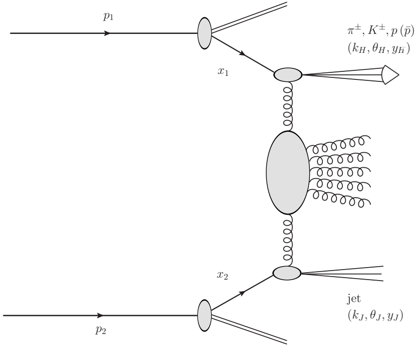


(**7 TeV BFKL vs DGLAP + asym.**) [F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2015)]
(**13 TeV BFKL vs DGLAP + asym. windows**) [A.D. Bolognino, F.G. C. (in progress)]

Outline

A hadron-jet final-state reaction

Process: $\text{proton}(p_1) + \text{proton}(p_2) \to \text{hadron}(k_H) + X + \text{jet}(k_J)$



$k_H^{\min} = 5$ GeV, $|y_H| < 2.4$ [CMS]

$k_J^{\min} = 35$ GeV, $|y_J| < 4.7$ [CMS]

$k_J^{\min} = 5$ GeV, $-6.6 < y_J < -5.2$ [CASTOR]

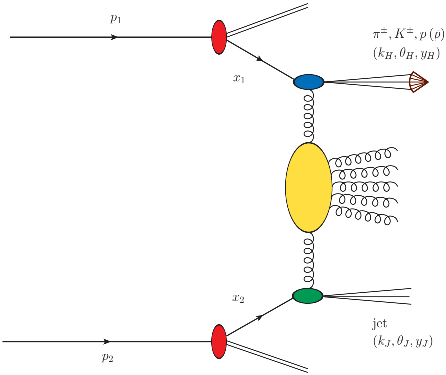
Why hadron-jet correlations?

- *asymmetric* cuts suppress Born, allowing to discriminate BFKL from DGLAP
- one-hadron detection quenches "minimum-bias" contaminations
- *linear* observables facilitate to compare different FF sets and jet algorithms

BFKL partonic cross section (hadron-jet)

$$\frac{d\sigma}{dy_H\, dy_J\, d^2\vec{k}_H\, d^2\vec{k}_J} = \sum_{r,s=q,g} \int_0^1 dx_1 \int_0^1 dx_2 \int_{x_1}^1 dx_H\, f_r(x_1,\mu_F)\, f_s(x_2,\mu_F)\, \frac{d\hat{\sigma}_{r,s}(x_1 x_2 s,\mu_F)}{dy_H\, dy_J\, d^2\vec{k}_J\, d^2\vec{k}_J}\, D_r^H\left(\frac{x_H}{x_1},\mu_F\right)$$

The expression for the **partonic cross section** in the BFKL approach reads:

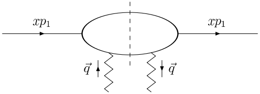


$$\frac{d\hat{\sigma}_{r,s}(x_1 x_2 s,\mu)}{dy_H\, dy_J\, d^2\vec{k}_H\, d^2\vec{k}_J} = \frac{1}{2\pi^2} \times \int \frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}}\, \mathcal{V}_H^{(r)}(\vec{q}_1, s_0, x_1; \vec{k}_H, x_H) \times \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{x_1 x_2 s}{s_0}\right)^{\omega} \mathcal{G}_\omega(\vec{q}_1,\vec{q}_2) \times \int \frac{d^2\vec{q}_2}{\vec{q}_2^{\,2}}\, \mathcal{V}_J^{(s)}(\vec{q}_2, s_0, x_2; \vec{k}_J, x_J)$$

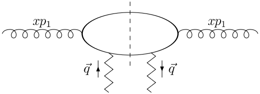
Forward hadron and jet impact factor

- take the impact factors for **colliding partons**

[V.S. Fadin, R. Fiore, M.I. Kotsky, A. Papa (2000); M. Ciafaloni and G. Rodrigo (2000)]

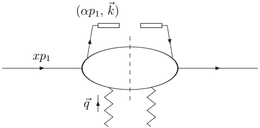


quark vertex

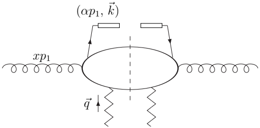


gluon vertex

- "open" one of the integrations over the phase space of the intermediate state to allow one parton to generate the hadron/jet



$\mathcal{V}_{H/J}^{(q)}$ (quark hadron/jet vertex)



$\mathcal{V}_{H/J}^{(g)}$ (gluon hadron/jet vertex)

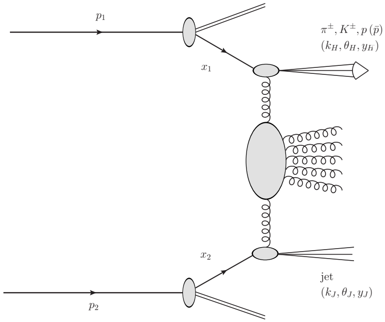
- use QCD collinear factorization

$$\text{hadron} \rightarrow \sum_{r=q,\bar{q}} f_r \otimes \mathcal{V}_H^{(r)} \otimes D_r^H + f_g \otimes \mathcal{V}_H^{(g)} \otimes D_g^H$$

$$\text{jet} \rightarrow \sum_{s=q,\bar{q}} f_s \otimes \mathcal{V}_J^{(s)} + f_g \otimes \mathcal{V}_J^{(g)}$$

BFKL cross section (hadron-jet)

$$\frac{d\sigma}{dy_H\, dy_J\, d^2\vec{k}_H\, d^2\vec{k}_J} = \sum_{r,s=q,g} \int_0^1 dx_1 \int_0^1 dx_2 \int_{x_1}^1 dx_H\, f_r(x_1,\mu_F) f_s(x_2,\mu_F) \frac{d\hat{\sigma}_{r,s}(x_1 x_2 s,\mu_F)}{dy_H\, dy_J\, d^2\vec{k}_J\, d^2\vec{k}_J} D_r^H\left(\frac{x_H}{x_1},\mu_F\right)$$



- slight change of variable in the final state
- project onto the eigenfunctions of the LO BFKL kernel, i.e. transfer from the reggeized gluon momenta to the (n,ν)-representation
- suitable definition of the **azimuthal coefficients**

$$\frac{d\sigma}{dx_H dx_J\, d|\vec{k}_H|\, d|\vec{k}_J| d\phi_H d\phi_J} = \frac{1}{(2\pi)^2}\left[\mathcal{C}_0 + \sum_{n=1}^{\infty} 2\cos(n\phi)\, \mathcal{C}_n\right]$$

with $\phi = \phi_H - \phi_J - \pi$

...useful definitions:

$$Y = \ln \frac{x_H x_J s}{|\vec{k}_H||\vec{k}_J|}, \qquad Y_0 = \ln \frac{s_0}{|\vec{k}_H||\vec{k}_J|}$$

Observables and kinematics (hadron-jet)

- **Observables:**

  ϕ-averaged cross section $\mathcal{C}_0$, $\langle \cos(n\phi) \rangle \equiv \frac{\mathcal{C}_n}{\mathcal{C}_0} \equiv R_{n0}$, with $n = 1, 2, 3$

  $\langle \cos(2\phi) \rangle / \langle \cos(\phi) \rangle \equiv \mathcal{C}_2/\mathcal{C}_1 \equiv R_{21}$, $\langle \cos(3\phi) \rangle / \langle \cos(2\phi) \rangle \equiv \mathcal{C}_3/\mathcal{C}_2 \equiv R_{32}$

◇ *Integrated coefficients* + BLM scale optimization:

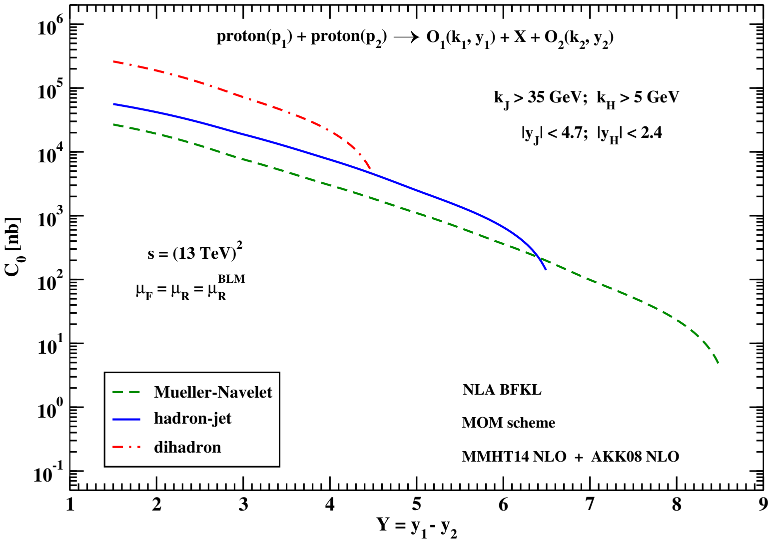
$$C_n = \int_{y_H^{\min}}^{y_H^{\max}} dy_H \int_{y_J^{\min}}^{y_J^{\max}} dy_J \int_{k_H^{\min}}^{k_H^{\max}} dk_H \int_{k_J^{\min}}^{k_J^{\max}} dk_J \, \delta\left(y_H - y_J - Y\right) \mathcal{C}_n\left(y_H, y_J, k_H, k_J\right)$$

- **Kinematic settings:**
  - ◇ $\sqrt{s} = 7,\ 13$ TeV
  - ◇ $|y_H| \leqslant 2.4$; $|y_J^{(\mathrm{CMS})}| \leqslant 4.7$; $-6.6 \leqslant y_J^{(\mathrm{CASTOR})} \leqslant -5.2$
  - ◇ $k_H \geqslant 5$ GeV; $k_J^{(\mathrm{CMS})} \geqslant 35$ GeV; $k_J^{(\mathrm{CASTOR})} \geqslant 5$ GeV

- **Phenomenological analysis:**
  - ◇ full **NLA** BFKL: NLA GGF ⊛ NLO collinear IFs
  - ◇ **JETHAD** (**HEP@WORK**, F95) + LHAPDF + native FF sets
  - ◇ (MMHT14, CT14, NNPDF3.0) ⊛ (AKK08, DSS07, HKNS07, NNFF1.0)

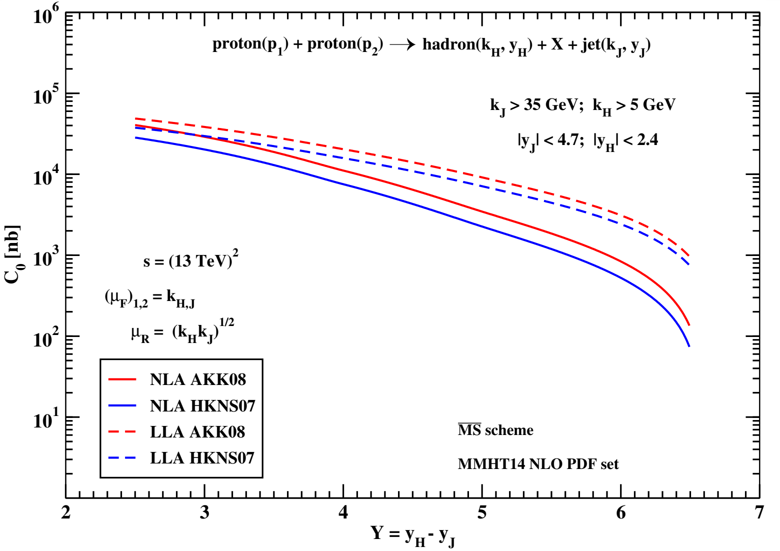
[A.D. Bolognino, F.G. C., D.Yu. Ivanov, A. Papa (under development)]

MN, hadron-jet and di-hadron C_0 vs Y, $\sqrt{s} = 13$ TeV



[A.D. Bolognino, F.G. Celiberto, D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2018)]

Hadron-jet C_0 vs Y, $\sqrt{s} = 13$ TeV, NS $\overline{\text{MS}}$ [*CMS-jet*]



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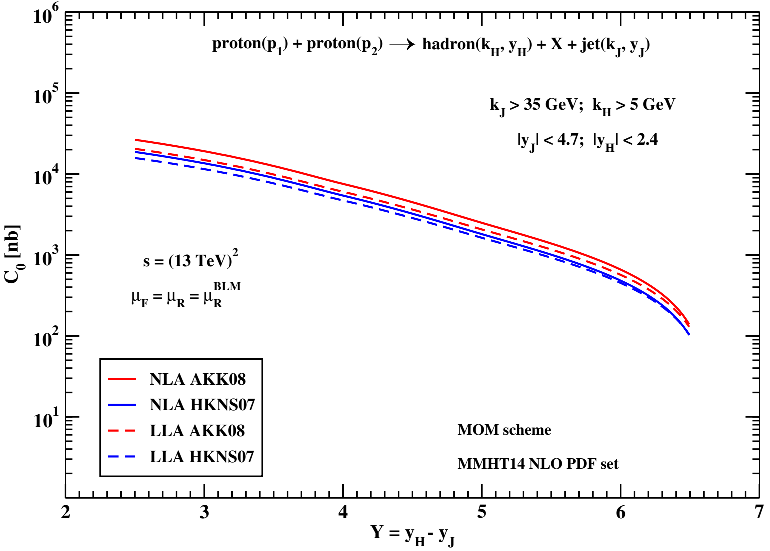
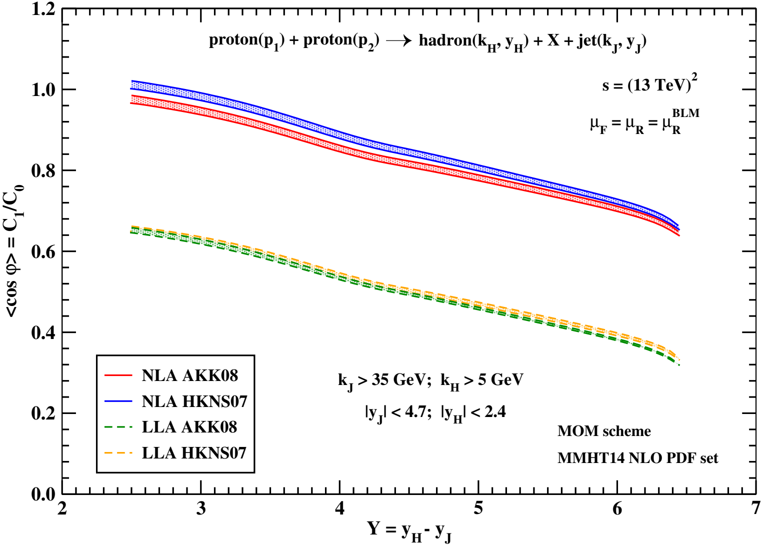
Hadron-jet C_0 vs Y, $\sqrt{s} = 13$ TeV [*CMS-jet*]



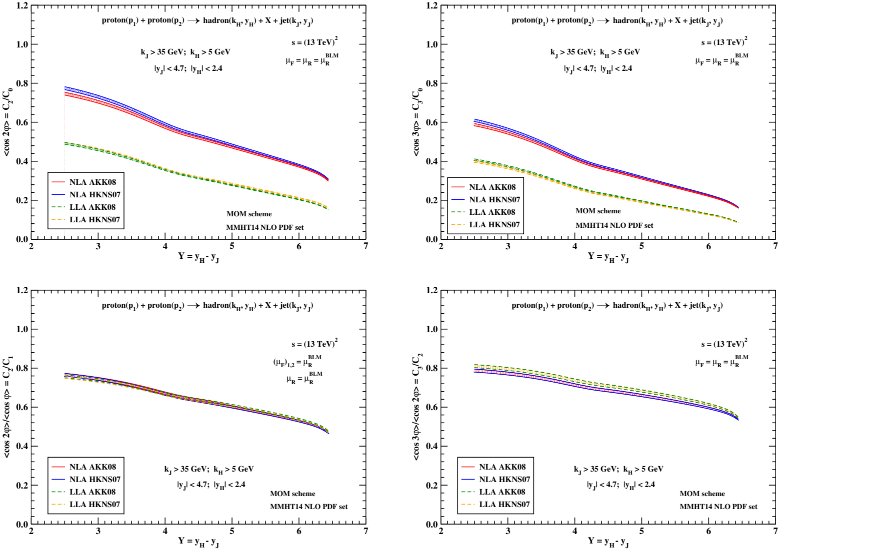
[A.D. Bolognino, F.G. Celiberto, D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2018)]

Hadron-jet R_{10} vs Y, $\sqrt{s} = 13$ TeV [*CMS-jet*]



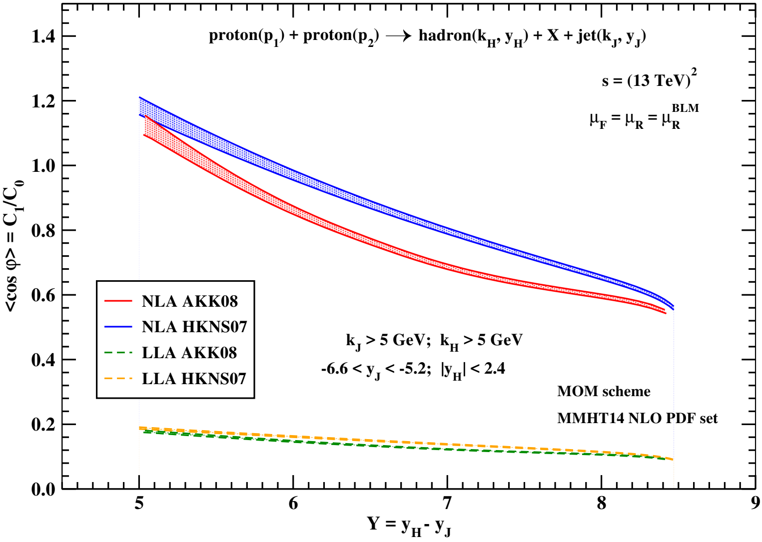
[A.D. Bolognino, F.G. Celiberto, D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2018)]

Hadron-jet R_{nm} vs Y, $\sqrt{s} = 13$ TeV [*CMS-jet*]



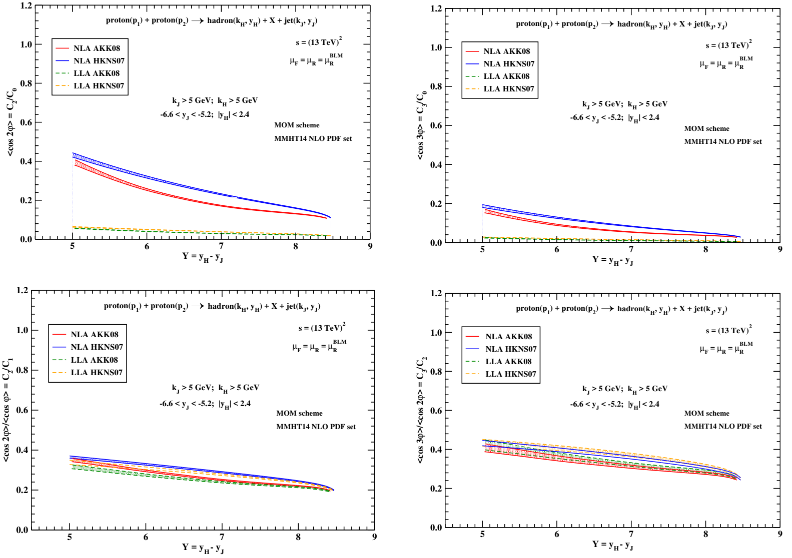
[A.D. Bolognino, F.G. Celiberto, D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2018)]

Hadron-jet R_{10} vs Y, $\sqrt{s} = 13$ TeV [*CASTOR-jet*]



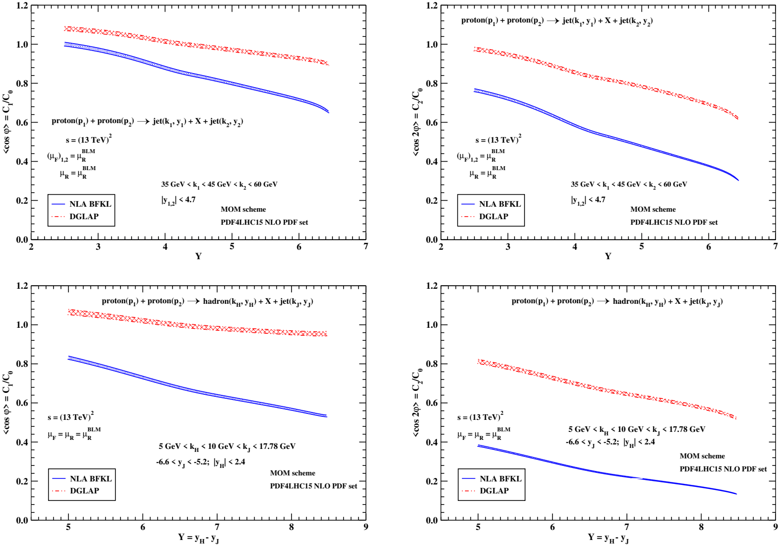
[A.D. Bolognino, F.G. Celiberto, D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2018)]

Hadron-jet R_{nm} vs Y, $\sqrt{s} = 13$ TeV [*CASTOR-jet*]



[A.D. Bolognino, F.G. Celiberto, D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2018)]

BFKL vs high-energy DGLAP [*CMS-, CASTOR-jet*]



(preliminary results) [A.D. Bolognino, F.G. Celiberto (in progress)]

Outline

Conclusions...

New candidate probe of BFKL dynamics at the LHC in the process for the inclusive production of an identified charged light hadron and a jet well separed in rapidity

◇ *asymmetric* kinematics + "minimum-bias" suppression

◇ expressions *linear* in FFs and jet algorithms

◇ ...new and complementary study of strong interactions at high energies

- scale optimization: *exact* implementation of the **BLM method**
- three distinct kinematic ranges: **[*CMS-jet*]** @(7, 13) TeV; **[*CASTOR-jet*]** @13 TeV

⇒ **[*CMS-jet*]**: usual trend for cross section and azimuthal correlations as in **Mueller–Navelet** and **di-hadron** production channels

⇒ **[*CASTOR-jet*]**: new features → different FFs lead to clearly distinct results

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⇒ **[*CASTOR-jet*]**: new features → different FFs lead to clearly distinct results

...Outlook

- ◇ further investigation of our observables in the **[*CASTOR-jet*]** ranges
- ◇ comparison with fixed-order DGLAP, inclusion of other resummation effects
- ◇ probe BFKL through other processes: **heavy-quark pair** production

[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2018); A.D. Bolognino, F.G. C., M. Fucilla, D.Yu. Ivanov, A. Papa (in progress)]

...thanks! ☻

BACKUP slides

BACKUP slides

Gluon Reggeization in perturbative QCD

◇ Gluon quantum numbers in the t-channel: 8^- representation

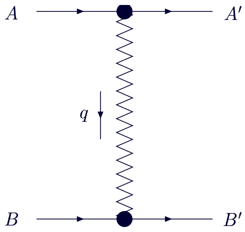
◇ Regge limit: $s \simeq -u \to \infty$, t not growing with s

$\to$ amplitudes governed by **gluon Reggeization** $\to$ $D_{\mu\nu} = -i\frac{g_{\mu\nu}}{q^2}\left(\frac{s}{s_0}\right)^{\alpha_g(q^2)-1}$

$\xrightarrow{\text{feature}}$ all-order resummation: **LLA** $[\alpha_s^n(\ln s)^n]$ + **NLA** $[\alpha_s^{n+1}(\ln s)^n]$

$\xrightarrow{\text{consequence}}$ factorization of elastic and real part of inelastic amplitudes

$\xrightarrow{\text{example}}$ Elastic scattering process: $A + B \longrightarrow A' + B'$



$$(\mathcal{A}_8^-)_{AB}^{A'B'} = \Gamma^c_{A'A}\left[\left(\frac{-s}{-t}\right)^{j(t)} - \left(\frac{s}{-t}\right)^{j(t)}\right]\Gamma^c_{B'B}$$

$$j(t) = 1 + \omega(t)\,, \quad j(0) = 1$$

$\omega(t)$ $\to$ Reggeized gluon trajectory

$\Gamma^c_{A'A} = g\langle A'|T^c|A\rangle\Gamma_{A'A}$ $\to$ PPR vertex

T^c $\to$ fundamental (q) or adjoint (g)

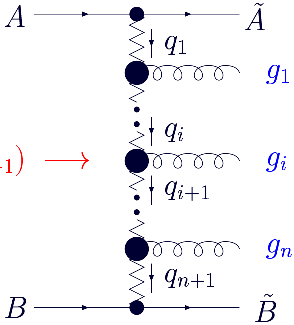
- QCD is the unique SM theory where all elementary particles reggeize
- Possible extensions: N=4 SYM, AdS/CFT,...

BACKUP slides

BFKL in the LLA (I)

Inelastic scattering process $A + B \;\; \rightarrow \;\; \tilde{A} + \tilde{B} + n$ in the LLA

$\gamma^{G_i}_{c_i c_{i+1}}(q_i, q_{i+1}) \longrightarrow$



multi-Regge kinematics

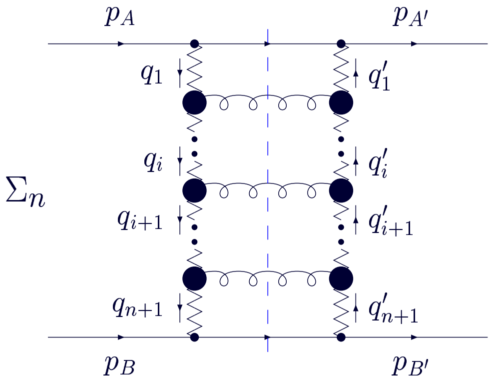
$$\mathrm{Re}\mathcal{A}^{\tilde{A}\tilde{B}+n}_{AB} = 2s\,\Gamma^{c_1}_{\tilde{A}A} \left(\prod_{i=1}^{n} \gamma^{P_i}_{c_i c_{i+1}}(q_i, q_{i+1}) \left(\frac{s_i}{s_R}\right)^{\omega(t_i)} \frac{1}{t_i} \right) \frac{1}{t_{n+1}} \left(\frac{s_{n+1}}{s_R}\right)^{\omega(t_{n+1})} \Gamma^{c_{n+1}}_{\tilde{B}B}$$

$\gamma^{P_i}_{c_i c_{i+1}}(q_i, q_{i+1}) \;\; \rightarrow$ RRG vertex

$s_R \;\; \rightarrow$ energy scale, irrelevant in the LLA

BFKL in the LLA (II)

Elastic amplitude $A + B \longrightarrow A' + B'$ in the LLA via s-channel unitarity



$$\mathcal{A}_{AB}^{A'B'} = \sum_{\mathcal{R}} (\mathcal{A}_{\mathcal{R}})_{AB}^{A'B'} \,, \qquad \mathcal{R} = 1 \text{ (singlet)}, \, 8^- \text{ (octet)}, \ldots$$

The 8^- color representation is important for the bootstrap, i.e. the consistency between the above amplitude and that with one Reggeized gluon exchange

BACKUP slides

How could we further and deeply probe BFKL?

1. Study less inclusive two-body final states...

[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2016, 2017, 2018)]

Di-hadron production

- inclusive di-hadron detection with large k_T and rapidity separation
- hadrons detected at <u>much smaller</u> k_T than jets!
- possibility to constrain not only the PDFs, <u>but also</u> the FFs!

Heavy-quark pair photoproduction

- quark masses play the role of hard scale
- e^+e^- at LEP2 and future lepton colliders

BACKUP slides

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2. Study three- and four-body final-state processes...

[F. Caporale, F.G. C., G. Chachamis, A. Sabio Vera (2016); F. Caporale, F.G. C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2016, 2017)]

Multi-jet production

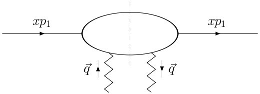
- definition of new, **suitable BFKL observables...**
- ...in order to further investigate the azimuthal distribution of the final state

Mueller–Navelet jets

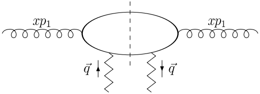
Forward jet impact factor

- take the impact factors for **colliding partons**

  [V.S. Fadin, R. Fiore, M.I. Kotsky, A. Papa (2000)]
  [M. Ciafaloni and G. Rodrigo (2000)]

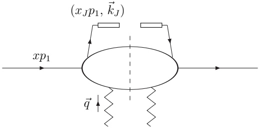


quark vertex

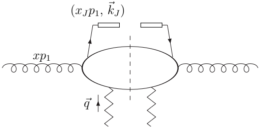


gluon vertex

- "open" one of the integrations over the phase space of the intermediate state to allow one parton to generate the jet



quark jet vertex



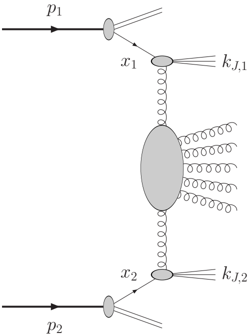
gluon jet vertex

- use QCD collinear factoriz.: $\sum_{s=q,\bar{q}} f_s \otimes$ [quark vertex] $+ f_g \otimes$ [gluon vertex]

BACKUP slides

BFKL cross section (MN jets)...

$$\frac{d\sigma}{dx_{J_1}dx_{J_2}d^2k_{J_1}d^2k_{J_2}} = \sum_{i,j=q,\bar{q},g}\int_0^1 dx_1 \int_0^1 dx_2 f_i(x_1,\mu) f_j(x_2,\mu) \frac{d\hat{\sigma}_{i,j}(x_1x_2s,\mu)}{dx_{J_1}dx_{J_2}d^2k_{J_1}d^2k_{J_2}}$$



- slight change of variable in the final state
- project onto the eigenfunctions of the LO BFKL kernel, i.e. transfer from the reggeized gluon momenta to the (n,ν)-representation
- suitable definition of the **azimuthal coefficients**

$$\frac{d\sigma}{dx_{J_1}dx_{J_2}\,d|\vec{k}_{J_1}|\,d|\vec{k}_{J_2}|d\phi_{J_1}d\phi_{J_2}} = \frac{1}{(2\pi)^2}\left[\mathcal{C}_0 + \sum_{n=1}^{\infty} 2\cos(n\phi)\,\mathcal{C}_n\right]$$

with $\phi = \phi_{J_1} - \phi_{J_2} - \pi$

...useful definitions:

$$Y = \ln\frac{x_{J_1}x_{J_2}s}{|\vec{k}_{J_1}||\vec{k}_{J_2}|}, \qquad Y_0 = \ln\frac{s_0}{|\vec{k}_{J_1}||\vec{k}_{J_2}|}$$

...and azimuthal coefficients (MN jets)

$$\mathcal{C}_n = \int_{-\infty}^{+\infty} d\nu \, e^{(Y-Y_0)\left[\bar{\alpha}_s(\mu_R)\chi(n,\nu)+\bar{\alpha}_s^2(\mu_R)\mathcal{K}^{(1)}(n,\nu)\right]} \alpha_s^2(\mu_R)$$
$$\times c_1(n,\nu)\, c_2(n,\nu) \left[1 + \alpha_s(\mu_R)\left(\frac{c_1^{(1)}(n,\nu)}{c_1(n,\nu)} + \frac{c_2^{(1)}(n,\nu)}{c_2(n,\nu)}\right)\right]$$

where

$$\chi(n,\nu) = 2\psi(1) - \psi\left(\frac{n}{2} + \frac{1}{2} + i\nu\right) - \psi\left(\frac{n}{2} + \frac{1}{2} - i\nu\right)$$

$$\mathcal{K}^{(1)}(n,\nu) = \bar{\chi}(n,\nu) + \frac{\beta_0}{8N_c}\chi(n,\nu)\left(-\chi(n,\nu) + \frac{10}{3} + i\frac{d}{d\nu}\ln\left(\frac{c_1(n,\nu)}{c_2(n,\nu)}\right) + 2\ln\left(\mu_R^2\right)\right)$$

$$c_1(n,\nu,|\vec{k}|,x) = 2\sqrt{\frac{C_F}{C_A}}(\vec{k}^2)^{i\nu-1/2}\left(\frac{C_A}{C_F}f_g(x,\mu_F) + \sum_{a=q,\bar{q}} f_a(x,\mu_F)\right)$$

...several NLA-equivalent expressions can be adopted for $\mathcal{C}_n$!

$\longrightarrow$ ...we use the *exponentiated* one

[F. Caporale, D.Yu Ivanov, B. Murdaca, A. Papa (2014)]

BACKUP slides

The BFKL BLM azimuthal coefficients (MN jets)

a) $\left(\mu_R^{BLM}\right)^2 = k_1 k_2 \ \exp\left[2\left(1+\frac{2}{3}I\right) - f(\nu) - \frac{5}{3}\right] \sim 5^2 k_1 k_2$

b) $\left(\mu_R^{BLM}\right)^2 = k_1 k_2 \ \exp\left[2\left(1+\frac{2}{3}I\right) - 2f(\nu) - \frac{5}{3} + \frac{1}{2}\chi(\nu, n)\right] < (11.5)^2 k_1 k_2$

$$\mathcal{C}_n^{\mathrm{BFKL}(a)} = \frac{x_{J_1} x_{J_2}}{|\vec{k}_{J_1}||\vec{k}_{J_2}|} \int_{-\infty}^{+\infty} d\nu \, e^{(Y-Y_0)\left[\bar{\alpha}_s(\mu_R)\chi(n,\nu) + \bar{\alpha}_s^2(\mu_R)\left(\bar{\chi}(n,\nu) - \frac{T^\beta}{C_A}\chi(n,\nu) - \frac{\beta_0}{8C_A}\chi^2(n,\nu)\right)\right]}$$

$$\times \, \alpha_s^2(\mu_R)\, c_1(n,\nu,|\vec{k}_{J_1}|, x_{J_1}) c_2(n,\nu,|\vec{k}_{J_2}|, x_{J_2})$$

$$\times \left[1 - \frac{2}{\pi}\alpha_s(\mu_R)\, T^\beta + \alpha_s(\mu_R)\left(\frac{\bar{c}_1^{(1)}(n,\nu,|\vec{k}_{J_1}|, x_{J_1})}{c_1(n,\nu,|\vec{k}_{J_1}|, x_{J_1})} + \frac{\bar{c}_2^{(1)}(n,\nu,|\vec{k}_{J_2}|, x_{J_2})}{c_2(n,\nu,|\vec{k}_{J_2}|, x_{J_2})}\right)\right]$$

$$\mathcal{C}_n^{\mathrm{BFKL}(b)} = \frac{x_{J_1} x_{J_2}}{|\vec{k}_{J_1}||\vec{k}_{J_2}|} \int_{-\infty}^{+\infty} d\nu \, e^{(Y-Y_0)\left[\bar{\alpha}_s(\mu_R)\chi(n,\nu) + \bar{\alpha}_s^2(\mu_R)\left(\bar{\chi}(n,\nu) - \frac{T^\beta}{C_A}\chi(n,\nu)\right)\right]}$$

$$\times \, \alpha_s^2(\mu_R)\, c_1(n,\nu,|\vec{k}_{J_1}|, x_{J_1}) c_2(n,\nu,|\vec{k}_{J_2}|, x_{J_2})$$

$$\times \left[1 + \alpha_s(\mu_R)\left(\frac{\beta_0}{4\pi}\chi(n,\nu) - 2\frac{T^\beta}{\pi}\right) + \alpha_s(\mu_R)\left(\frac{\bar{c}_1^{(1)}(n,\nu,|\vec{k}_{J_1}|, x_{J_1})}{c_1(n,\nu,|\vec{k}_{J_1}|, x_{J_1})} + \frac{\bar{c}_2^{(1)}(n,\nu,|\vec{k}_{J_2}|, x_{J_2})}{c_2(n,\nu,|\vec{k}_{J_2}|, x_{J_2})}\right)\right]$$

The DGLAP BLM cross section (MN jets)

a) $\left(\mu_R^{BLM}\right)^2 = k_1 k_2 \ \exp\left[2\left(1+\frac{2}{3}I\right) - f\left(\nu\right) - \frac{5}{3}\right] \sim 5^2 k_1 k_2$

b) $\left(\mu_R^{BLM}\right)^2 = k_1 k_2 \ \exp\left[2\left(1+\frac{2}{3}I\right) - 2f\left(\nu\right) - \frac{5}{3} + \frac{1}{2}\chi\left(\nu, n\right)\right] < (11.5)^2 k_1 k_2$

$$\mathcal{C}_n^{\text{DGLAP (a)}} = \frac{x_{J_1} x_{J_2}}{|\vec{k}_{J_1}||\vec{k}_{J_2}|} \int_{-\infty}^{+\infty} d\nu \ \alpha_s^2\left(\mu_R\right) c_1(n,\nu,|\vec{k}_{J_1}|,x_{J_1}) c_2(n,\nu,|\vec{k}_{J_2}|,x_{J_2})$$
$$\times \left[1 - \frac{2}{\pi}\alpha_s\left(\mu_R\right) T^\beta + \bar{\alpha}_s\left(\mu_R\right)\left(Y - Y_0\right)\chi\left(n,\nu\right)\right.$$
$$\left. + \alpha_s\left(\mu_R\right)\left(\frac{\bar{c}_1^{(1)}(n,\nu,|\vec{k}_{J_1}|,x_{J_1})}{c_1(n,\nu,|\vec{k}_{J_1}|,x_{J_1})} + \frac{\bar{c}_2^{(1)}(n,\nu,|\vec{k}_{J_2}|,x_{J_2})}{c_2(n,\nu,|\vec{k}_{J_2}|,x_{J_2})}\right)\right]$$

$$\mathcal{C}_n^{\text{DGLAP (b)}} = \frac{x_{J_1} x_{J_2}}{|\vec{k}_{J_1}||\vec{k}_{J_2}|} \int_{-\infty}^{+\infty} d\nu \ \alpha_s^2\left(\mu_R\right) c_1(n,\nu,|\vec{k}_{J_1}|,x_{J_1}) c_2(n,\nu,|\vec{k}_{J_2}|,x_{J_2})$$
$$\times \left[1 + \alpha_s\left(\mu_R\right)\left(\frac{\beta_0}{4\pi}\chi\left(n,\nu\right) - 2\frac{T^\beta}{\pi}\right) + \bar{\alpha}_s\left(\mu_R\right)\left(Y - Y_0\right)\chi\left(n,\nu\right)\right.$$
$$\left. + \alpha_s\left(\mu_R\right)\left(\frac{\bar{c}_1^{(1)}(n,\nu,|\vec{k}_{J_1}|,x_{J_1})}{c_1(n,\nu,|\vec{k}_{J_1}|,x_{J_1})} + \frac{\bar{c}_2^{(1)}(n,\nu,|\vec{k}_{J_2}|,x_{J_2})}{c_2(n,\nu,|\vec{k}_{J_2}|,x_{J_2})}\right)\right]$$

BACKUP slides

The "exact" BLM cross section (MN jets)

$$\mathcal{C}_n^{\rm BLM} = \frac{x_{J_1} x_{J_2}}{|\vec{k}_{J_1}||\vec{k}_{J_2}|} \int_{-\infty}^{+\infty} d\nu \, e^{(Y-Y_0)\,\bar{\alpha}_s^{\rm MOM}(\mu_R^{\rm BLM})\left[\chi(n,\nu)+\bar{\alpha}_s^{\rm MOM}(\mu_R^{\rm BLM})\left(\bar{\chi}(n,\nu)+\frac{T^{\rm conf}}{N_c}\chi(n,\nu)\right)\right]}$$

$$\times\,(\alpha_s^{\rm MOM}(\mu_R^{\rm BLM}))^2 c_1(n,\nu,|\vec{k}_{J_1}|,x_{J_1}) c_2(n,\nu,|\vec{k}_{J_2}|,x_{J_2})$$

$$\times \left[1 + \alpha_s^{\rm MOM}(\mu_R^{\rm BLM}) \left\{ \frac{\bar{c}_1^{(1)}(n,\nu,|\vec{k}_{J_1}|,x_{J_1})}{c_1(n,\nu,|\vec{k}_{J_1}|,x_{J_1})} + \frac{\bar{c}_2^{(1)}(n,\nu,|\vec{k}_{J_2}|,x_{J_2})}{c_2(n,\nu,|\vec{k}_{J_2}|,x_{J_2})} + \frac{2T^{\rm conf}}{N_c} \right\} \right] ,$$

with the $\mu_R^{\rm BLM}$ scale chosen as the solution of the following integral equation...

$$\mathcal{C}_n^{\beta} \equiv \frac{x_{J_1} x_{J_2}}{|\vec{k}_{J_1}||\vec{k}_{J_2}|} \int_{-\infty}^{\infty} d\nu \left(\frac{s}{s_0}\right)^{\bar{\alpha}_s^{\rm MOM}(\mu_R^{\rm BLM})\chi(n,\nu)} \left(\alpha_s^{\rm MOM}(\mu_R^{\rm BLM})\right)^3$$

$$\times\, c_1(n,\nu) c_2(n,\nu) \frac{\beta_0}{2N_c} \left[\frac{5}{3} + \ln \frac{(\mu_R^{\rm BLM})^2}{Q_1 Q_2} - 2\left(1+\frac{2}{3}I\right) \right.$$

$$\left. + \bar{\alpha}_s^{\rm MOM}(\mu_R^{\rm BLM}) \ln\frac{s}{s_0} \frac{\chi(n,\nu)}{2} \left(-\frac{\chi(n,\nu)}{2} + \frac{5}{3} + \ln \frac{(\mu_R^{\rm BLM})^2}{Q_1 Q_2} - 2\left(1+\frac{2}{3}I\right) \right) \right] \stackrel{!}{=} 0$$

BACKUP slides
...choosing the μ_R^{BLM} scale (MN jets)

...which represents the condition that terms proportional to β_0 in C_n disappear

$$\alpha^{\mathrm{MOM}} = -\frac{\pi}{2T}\left[1-\sqrt{1+4\alpha_s\left(\mu_R\right)\frac{T}{\pi}}\right],$$

with $T = T^{\beta} + T^{\mathrm{conf}}$,

$$T^{\beta} = -\frac{\beta_0}{2}\left(1+\frac{2}{3}I\right),$$

$$T^{\mathrm{conf}} = \frac{C_A}{8}\left[\frac{17}{2}I+\frac{3}{2}\left(I-1\right)\xi+\left(1-\frac{1}{3}I\right)\xi^2-\frac{1}{6}\xi^3\right],$$

where $I = -2\int_0^1 dx \frac{\ln(x)}{x^2-x+1} \simeq 2.3439$ and ξ is a gauge parameter.

BACKUP slides

Observables and kinematics (MN jets)

- **Observables:**

ϕ-averaged cross section $\mathcal{C}_0$, $\langle \cos [n(\phi_{J_1} - \phi_{J_2} - \pi)] \rangle \equiv \frac{\mathcal{C}_n}{\mathcal{C}_0}$, with $n = 1, 2, 3$

$$\frac{\langle \cos [2(\phi_1 - \phi_2 - \pi)] \rangle}{\langle \cos (\phi_1 - \phi_2 - \pi) \rangle} \equiv \frac{\mathcal{C}_2}{\mathcal{C}_1} \equiv R_{21}, \quad \frac{\langle \cos [3(\phi_1 - \phi_2 - \pi)] \rangle}{\langle \cos [2(\phi_1 - \phi_2 - \pi)] \rangle} \equiv \frac{\mathcal{C}_3}{\mathcal{C}_2} \equiv R_{32}.$$

◇ *Integrated coefficients:*

$$C_n = \int_{y_1^{\min}}^{y_1^{\max}} dy_1 \int_{y_2^{\min}}^{y_2^{\max}} dy_2 \int_{k_{J_1}^{\min}}^{k_{J_1}^{\max}} dk_{J_1} \int_{k_{J_2}^{\min}}^{k_{J_2}^{\max}} dk_{J_2} \, \delta(y_1 - y_2 - Y) \, \mathcal{C}_n(y_{J_1}, y_{J_2}, k_{J_1}, k_{J_2})$$

- **Kinematic settings:**
  - ◇ $R = 0.5$ and $\sqrt{s} = 7, 13$ TeV
  - ◇ $y_{\max}^{C} \leqslant |y_{J_{1,2}}| \leqslant 4.7$
  - ◇ symmetric and asymmetric choices for k_{J_1} and k_{J_2} ranges

- **Numerical tools: JETHAD (HEP@WORK, F95)** + LHAPDF

[A.D. Bolognino, F.G. C., D.Yu. Ivanov, A. Papa (under development)]

High-energy DGLAP

◇ NLA BFKL expressions for the observables truncated to $\mathcal{O}\left(\alpha_s^3\right)$ **!**

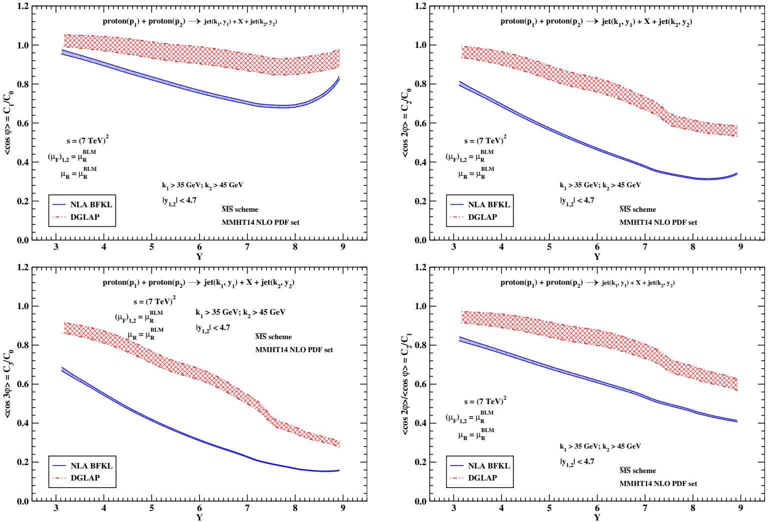
Why asymmetric cuts?

- suppress Born contribution to ϕ-averaged cross section C_0 (back-to-back jets)
  - ◇ avoid instabilities observed in NLO fixed-order calculations

    [J.R. Andersen, V. Del Duca, S. Frixione, C.R. Schmidt, W.J. Stirling (2001)]
    [M. Fontannaz, J.P. Guillet, G. Heinrich (2001)]
  - ◇ **enhance effects of additional hard gluons** $\xrightarrow{\text{emphasize}}$ **BFKL effects**

BACKUP slides

R_{nm} for $k_{J_1} > 35$ GeV, $k_{J_2} > 45$ GeV at $\sqrt{s} = 7$ TeV

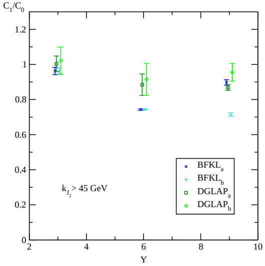


[A.D. Bolognino, F.G. C. (in progress)]

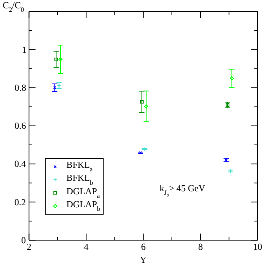
BACKUP slides

R_{nm} for $k_{J_1} > 35$ GeV, $k_{J_2} > 45$ GeV at $\sqrt{s} = 7$ TeV

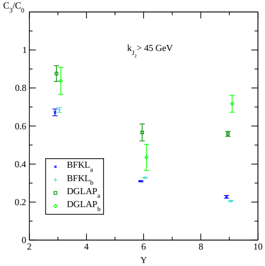
$\langle \cos(\phi) \rangle$



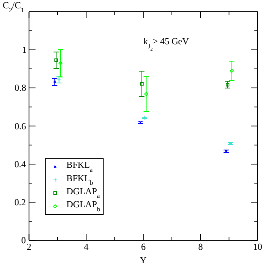
$\langle \cos(2\phi) \rangle$



$\langle \cos(3\phi) \rangle$



$$\frac{\langle \cos(2\phi) \rangle}{\langle \cos(\phi) \rangle}$$

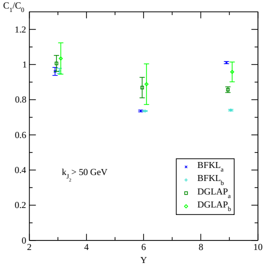


[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2015)]

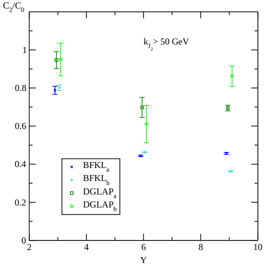
BACKUP slides

R_{nm} for $k_{J_1} > 35$ GeV, $k_{J_2} > 50$ GeV at $\sqrt{s} = 7$ TeV

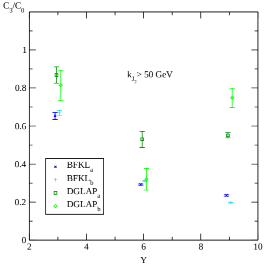
$\langle \cos(\phi) \rangle$



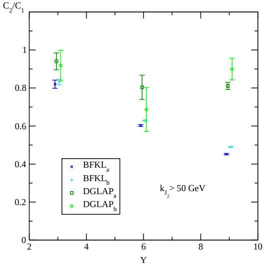
$\langle \cos(2\phi) \rangle$



$\langle \cos(3\phi) \rangle$



$\dfrac{\langle \cos(2\phi) \rangle}{\langle \cos(\phi) \rangle}$



[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2015)]

BACKUP slides

Exclusion of central jet rapidities (MN jets)

Motivation...

- ◇ At given $Y = y_{J_1} - y_{J_2}$ ...
- → $|y_{J_i}|$ could be so small ($\lesssim 2$), that the jet i is actually produced in the central region, rather than in one of the two forward regions
- → longitudinal momentum fractions of the parent partons $x \sim 10^{-3}$
- → for $|y_{J_i}|$ and $|k_{J_i}| < 100$ GeV $\Rightarrow$ increase of C_0 by 25% due to NNLO PDF effects

[J. Currie, A. Gehrmann-De Ridder, E. W. N. Glover, J. Pires (2014)]

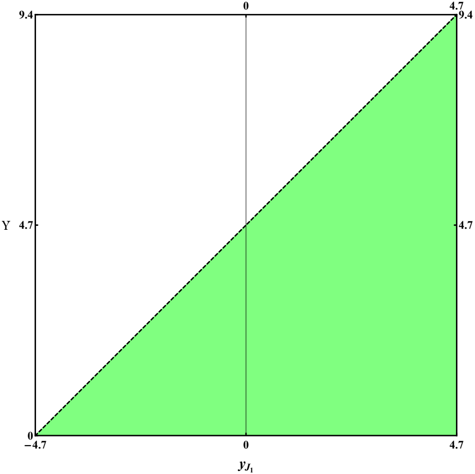
! Our BFKL description of the process could be not so accurate...

...let's return to the original Mueller–Navelet idea!

- ◇ remove regions where jets are produced at central rapidities...
- → ...in order to reduce as much as possible theoretical uncertainties

Rapidity range (MN jets)

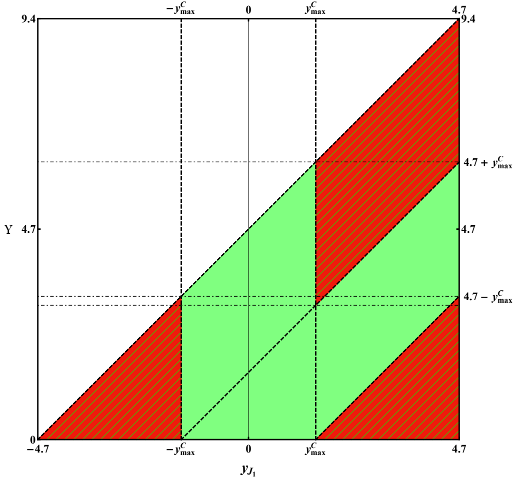
$$\int_{-4.7}^{4.7} dy_1 \int_{-4.7}^{4.7} dy_2 \; \delta\left(y_1 - y_2 - Y\right) \; \theta\left(|y_1| - y_{max}^C\right) \; \theta\left(|y_2| - y_{max}^C\right) \; \mathcal{C}_n\left(y_{J_1}, y_{J_2}, k_{J_1}, k_{J_2}\right)$$



$$Y = y_{J_1} - y_{J_2}$$

Rapidity range (MN jets)

$$\int_{-4.7}^{4.7} dy_1 \int_{-4.7}^{4.7} dy_2 \; \delta(y_1 - y_2 - Y) \; \theta\left(|y_1| - y^C_{max}\right) \theta\left(|y_2| - y^C_{max}\right) \mathcal{C}_n \left(y_{J_1}, y_{J_2}, k_{J_1}, k_{J_2}\right)$$

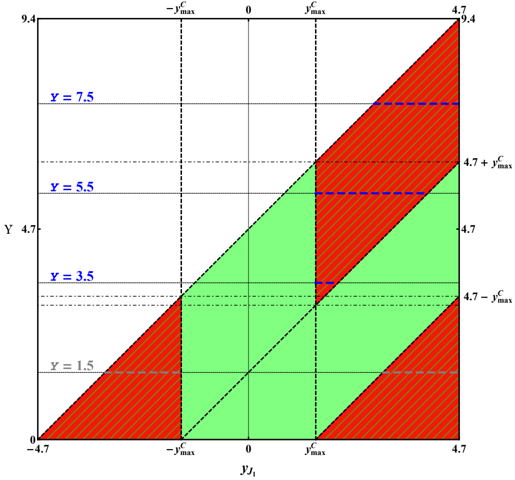


$$Y = y_{J_1} - y_{J_2}$$

BACKUP slides

Rapidity range (MN jets)

$$\int_{-4.7}^{4.7} dy_1 \int_{-4.7}^{4.7} dy_2\ \delta(y_1 - y_2 - Y)\ \theta\left(|y_1| - y_{max}^C\right)\ \theta\left(|y_2| - y_{max}^C\right)\ \mathcal{C}_n\left(y_{J_1}, y_{J_2}, k_{J_1}, k_{J_2}\right)$$



$Y = y_{J_1} - y_{J_2}$

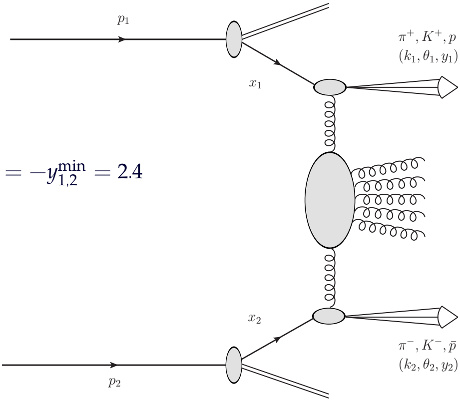
di-hadron production

BACKUP slides

Di-hadron production

Process: proton(p_1) + proton(p_2) $\rightarrow$ hadron(k_1) + X + hadron(k_2)

$k_{1,2}^{\text{min}} = 5$ GeV, $y_{1,2}^{\text{max}} = -y_{1,2}^{\text{min}} = 2.4$



(NLO impact factor) [D.Yu. Ivanov, A. Papa (2012)]
[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2016, 2017)]

Di-hadron production

Process: $\text{proton}(p_1) + \text{proton}(p_2) \to \text{hadron}(k_1) + X + \text{hadron}(k_2)$

$$\frac{d\sigma}{dy_1 dy_2 d^2\vec{k}_1 d^2\vec{k}_2} = \sum_{i,j=q,g} \int_0^1 \int_0^1 dx_1 dx_2 f_i(x_1,\mu) f_j(x_2,\mu) \frac{d\hat{\sigma}(x_1 x_2 s,\mu)}{dy_1 dy_2 d^2\vec{k}_1 d^2\vec{k}_2}$$

- large hadron transverse momenta: $\vec{k}_1^2 \sim \vec{k}_2^2 \gg \Lambda^2_{\rm QCD} \;\Rightarrow$ *pQCD allowed*
- QCD collinear factorization
- large rapidity intervals between hadrons (high energies) $\Rightarrow \; \Delta y = \ln \frac{x_1 x_2 s}{|\vec{k}_1||\vec{k}_2|}$

  $\Rightarrow$ BFKL resummation: $\sum_n \left(a_n^{(0)} \; \alpha_s^n \ln^n s + a_n^{(1)} \; \alpha_s^n \ln^{n-1} s \right)$

- Collinear fragmentation of the parton i into a hadron h
  $\Rightarrow$ convolution of D_i^h with a coefficient function C_i^h

$$d\sigma_i = C_i^h(z) dz \to d\sigma^h = d\alpha_h \int_{\alpha_h}^1 \frac{dz}{z} D_i^h\left(\frac{\alpha_h}{z},\mu\right) C_i^h(z,\mu)$$

where α_h is the momentum fraction carried by the hadron

BACKUP slides

The BFKL BLM cross section (di-hadrons)

$$C_n^{\rm BLM} = \frac{e^Y}{s} \int_{y_{\rm min}}^{y_{\rm max}} dy_1 \int_{k_{1,\rm min}}^{\infty} dk_1 \int_{k_{2,\rm min}}^{\infty} dk_2 \int_{-\infty}^{+\infty} d\nu \; \exp\left[(Y - Y_0)\, \bar\alpha_s^{\rm MOM}(\mu_R^*) \left\{ \chi(n,\nu) \right.\right.$$

$$\left.\left. + \; \bar\alpha_s^{\rm MOM}(\mu_R^*) \left(\bar\chi(n,\nu) + \frac{T^{\rm conf}}{C_A} \chi(n,\nu) \right) \right\} \right] 4 (\alpha_s^{\rm MOM}(\mu_R^*))^2 \frac{C_F}{C_A} \frac{1}{|\vec{k}_1||\vec{k}_2|} \left(\frac{\vec{k}_1^2}{\vec{k}_2^2} \right)^{i\nu}$$

$$\times \int_{\alpha_1}^{1} \frac{dx}{x} \left(\frac{x}{\alpha_1} \right)^{2i\nu - 1} \left[\frac{C_A}{C_F} f_g(x) D_g^h\left(\frac{\alpha_1}{x} \right) + \sum_{a=q,\bar q} f_a(x) D_a^h \left(\frac{\alpha_1}{x} \right) \right]$$

$$\times \int_{\alpha_2}^{1} \frac{dz}{z} \left(\frac{z}{\alpha_2} \right)^{-2i\nu - 1} \left[\frac{C_A}{C_F} f_g(z) D_g^h\left(\frac{\alpha_2}{z} \right) + \sum_{a=q,\bar q} f_a(z) D_a^h \left(\frac{\alpha_2}{z} \right) \right]$$

$$\times \left[1 + \bar\alpha_s^{\rm MOM}(\mu_R^*) \left(\frac{\bar c_1^{(1)}(n,\nu)}{c_1(n,\nu)} + \frac{\bar c_2^{(1)}(n,\nu)}{c_2(n,\nu)} + 2 \frac{T^{\rm conf}}{C_A} \right) \right]$$

Observables and kinematics (di-hadrons)

- **Observables:**

  ϕ-averaged cross section $\mathcal{C}_0$, $\langle \cos(n\phi) \rangle \equiv \frac{\mathcal{C}_n}{\mathcal{C}_0} \equiv R_{n0}$, with $n = 1, 2, 3$

  $\langle \cos(2\phi) \rangle / \langle \cos(\phi) \rangle \equiv \mathcal{C}_2/\mathcal{C}_1 \equiv R_{21}$, $\langle \cos(3\phi) \rangle / \langle \cos(2\phi) \rangle \equiv \mathcal{C}_3/\mathcal{C}_2 \equiv R_{32}$

◇ *Integrated coefficients:*

$$C_n = \int_{y_1^{\min}}^{y_1^{\max}} dy_1 \int_{y_2^{\min}}^{y_2^{\max}} dy_2 \int_{k_1^{\min}}^{k_1^{\max}} dk_1 \int_{k_2^{\min}}^{k_1^{\max}} dk_2 \delta\left(y_1 - y_2 - Y\right) \mathcal{C}_n\left(y_1, y_2, k_1, k_2\right)$$

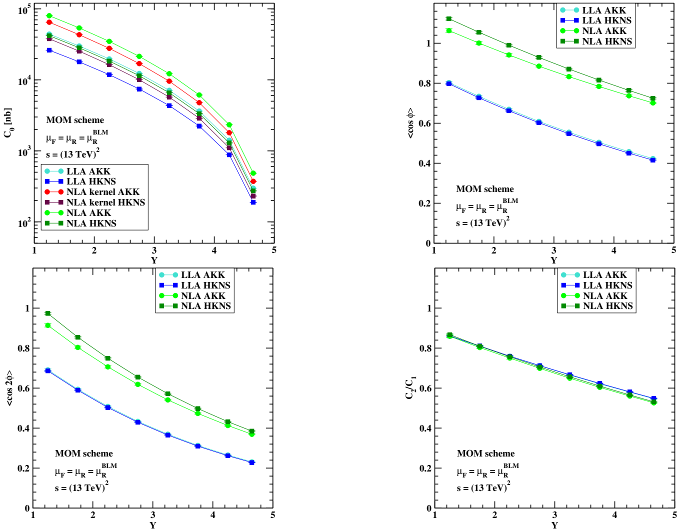
- **Kinematic settings:**
  - ◇ $\sqrt{s} = 7,\ 13$ TeV
  - ◇ $|y_i| \leqslant 2.4, 4.7$, with $i = 1, 2$
  - ◇ $k_{1,2} \geqslant$ **5 GeV** ...vs $k_{J_{1,2}}^{(\mathrm{MN\,jets})} \geqslant 35$ GeV! $\to$ more secondary gluon emissions!

- **Phenomenological analysis:**
  - ◇ full **NLA** BFKL
  - ◇ **JETHAD** (**HEP@WORK**, F95) + native FF sets
  - ◇ (MSTW08, MMHT14, CT14) PDFs ⊛ (AKK08, DSS07, HKNS07) FFs

[F.G. C., D.Yu Ivanov, B. Murdaca, A. Papa (2017)]

BACKUP slides

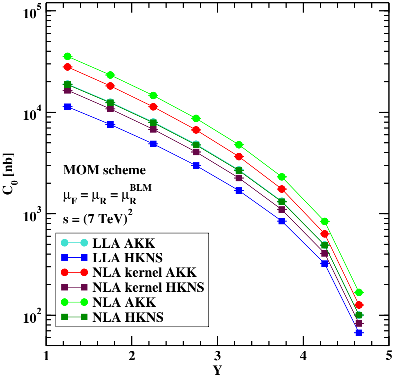
C_0 and R_{nm} at $\sqrt{s} = 13$ TeV, $Y \leqslant 4.8$, $\mu_F = \mu_R^{\rm BLM}$



[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2017)]

BACKUP slides

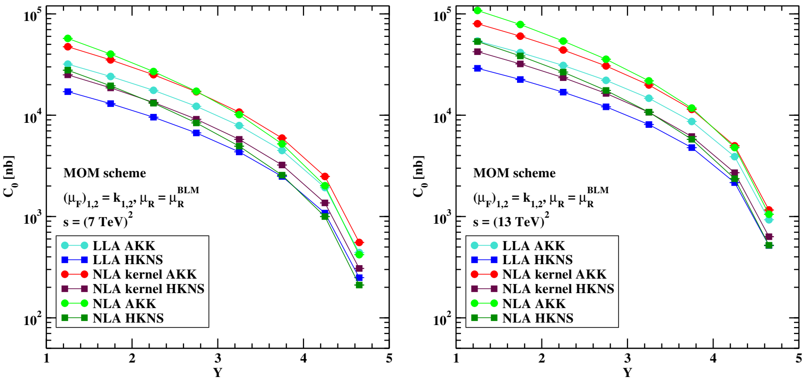
C_0 at $\sqrt{s} = 7, 13$ TeV, $Y \leqslant 4.8$, $\mu_F = \mu_R^{\rm BLM}$



[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2017)]

BACKUP slides

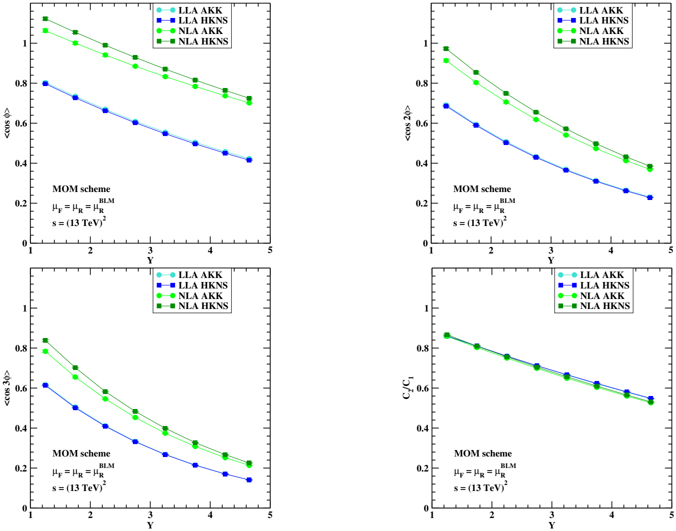
C_0 at $\sqrt{s} = 7, 13$ TeV, $Y \leqslant 4.8$, $(\mu_F)_{1,2} = |\vec{k}_{1,2}|$



[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2017)]

BACKUP slides

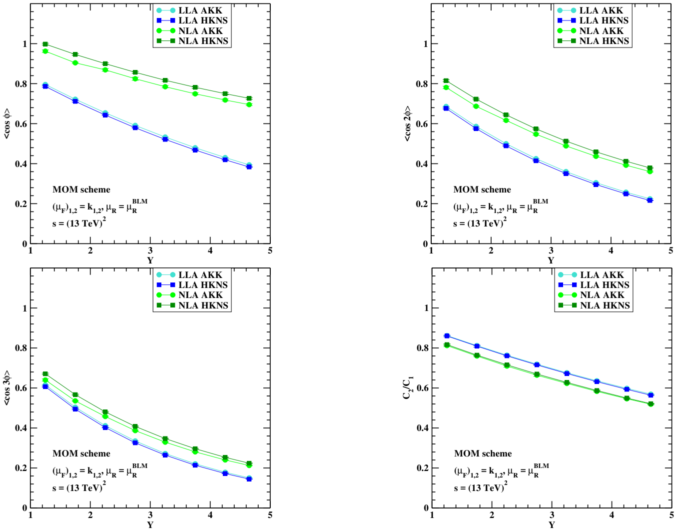
R_{nm} at $\sqrt{s} = 13$ TeV, $Y \leqslant 4.8$, $\mu_F = \mu_R^{\rm BLM}$



[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2017)]

BACKUP slides

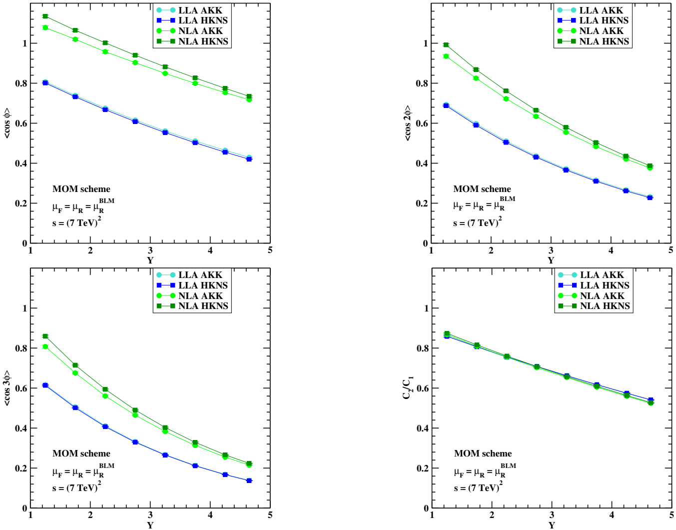
R_{nm} at $\sqrt{s} = 13$ TeV, $Y \leqslant 4.8$, $(\mu_F)_{1,2} = |\vec{k}_{1,2}|$



[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2017)]

BACKUP slides

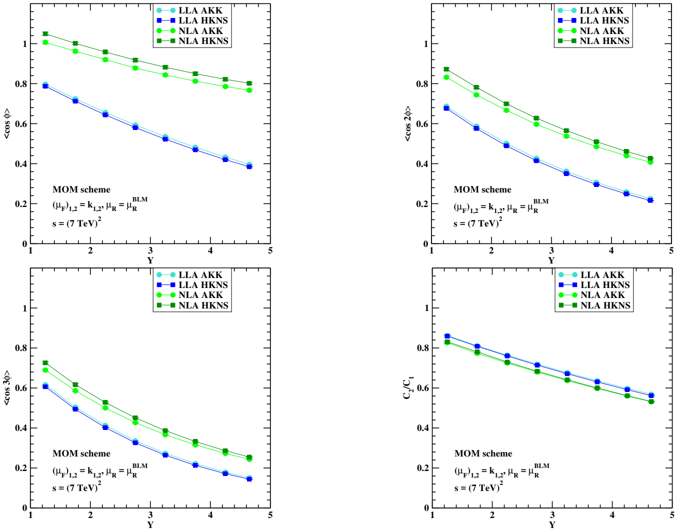
R_{nm} at $\sqrt{s} = 7$ TeV, $Y \leqslant 4.8$, $\mu_F = \mu_R^{\rm BLM}$



[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2017)]

BACKUP slides

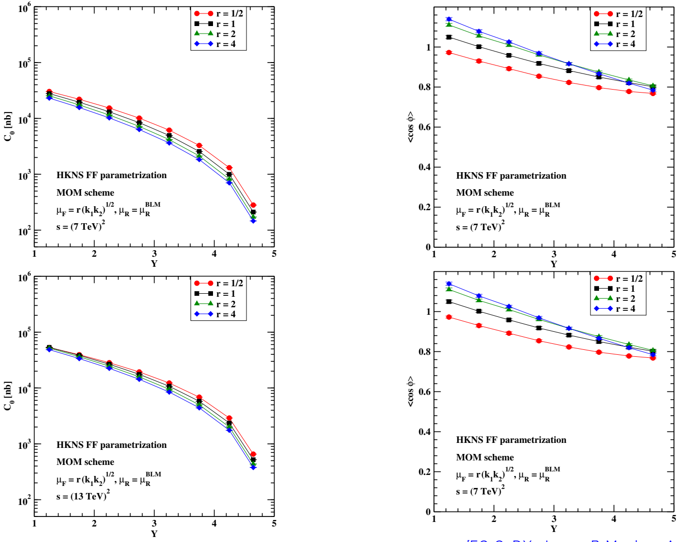
R_{nm} at $\sqrt{s} = 7$ TeV, $Y \leqslant 4.8$, $(\mu_F)_{1,2} = |\vec{k}_{1,2}|$



[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2017)]

BACKUP slides

C_0, R_{10} at $\sqrt{s} = 7, 13$ TeV, $Y \leqslant 4.8$, $\mu_F = r\sqrt{|\vec{k}_1||\vec{k}_2|}$



[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2017)]

hadron-jet correlations

Azimuthal coefficients (hadron-jet)

$$\mathcal{C}_n = \int_{-\infty}^{+\infty} d\nu \left(\frac{x_H x_J s}{s_0}\right)^{\bar{\alpha}_s(\mu_R)\{\chi(n,\nu)+\bar{\alpha}_s(\mu_R)\mathcal{K}^{(1)}(n,\nu)\}}$$
$$\times \frac{e^Y}{s} \alpha_s^2(\mu_R)\, c_H(n,\nu,|\vec{k}_H|,x_H)\, [c_J(n,\nu,|\vec{k}_J|,x_J)]^*$$
$$\times \left\{1+\alpha_s(\mu_R)\left[\frac{c_H^{(1)}(n,\nu,|\vec{k}_H|,x_H)}{c_H(n,\nu,|\vec{k}_H|,x_H)}+\left[\frac{c_J^{(1)}(n,\nu,|\vec{k}_J|,x_J)}{c_J(n,\nu,|\vec{k}_J|,x_J)}\right]^*\right]\right.$$
$$\left. + \bar{\alpha}_s^2(\mu_R)\ln\left(\frac{x_H x_J s}{s_0}\right)\frac{\beta_0}{4N_c}\chi(n,\nu)f(\nu)\right\}$$

where

$$\chi(n,\nu) = 2\psi(1) - \psi\left(\frac{n}{2}+\frac{1}{2}+i\nu\right) - \psi\left(\frac{n}{2}+\frac{1}{2}-i\nu\right)$$

$$\mathcal{K}^{(1)}(n,\nu) = \bar{\chi}(n,\nu) + \frac{\beta_0}{8N_c}\chi(n,\nu)\left[-\chi(n,\nu)+\frac{10}{3}+2\ln\left(\frac{\mu_R^2}{\sqrt{\vec{k}_H^2\vec{k}_J^2}}\right)\right]$$

...several NLA-equivalent expressions can be adopted for $\mathcal{C}_n$!

$\longrightarrow$ ...we use the *exponentiated* one

[F. Caporale, D.Yu Ivanov, B. Murdaca, A. Papa (2014)]

LO forward hadron and jet impact factors

- LO forward hadron impact factor in the (n, ν)-representation

$$c_H(n, \nu, |\vec{k}_H|, x_H) = 2\sqrt{\frac{C_F}{C_A}} (\vec{k}_H^2)^{i\nu - 1/2} \int_{x_H}^{1} \frac{dx}{x} \left(\frac{x}{x_H}\right)^{2i\nu - 1}$$
$$\times \left[\frac{C_A}{C_F} f_g(x) D_g^h \left(\frac{x_H}{x}\right) + \sum_{r=q,\bar{q}} f_r(x) D_r^h \left(\frac{x_H}{x}\right) \right]$$

- LO forward jet impact factor in the (n, ν)-representation

$$c_J(n, \nu, |\vec{k}_J|, x_J) = 2\sqrt{\frac{C_F}{C_A}} (\vec{k}_J^2)^{i\nu - 1/2} \left(\frac{C_A}{C_F} f_g(x_J) + \sum_{s=q,\bar{q}} f_s(x_J) \right)$$

- $f(\nu)$ function

$$i \frac{d}{d\nu} \ln \left(\frac{c_H}{[c_J]^*} \right) = 2 \left[f(\nu) - \ln \left(\sqrt{\vec{k}_H^2 \vec{k}_J^2} \right) \right]$$

BACKUP slides

The BFKL BLM azimuthal coefficients (hadron-jet)

$$C_n = \int_{y_H^{\rm min}}^{y_H^{\rm max}} dy_H \int_{y_J^{\rm min}}^{y_J^{\rm max}} dy_J \int_{k_H^{\rm min}}^{\infty} dk_H \int_{k_J^{\rm min}}^{\infty} dk_J \int_{-\infty}^{\infty} d\nu \, \frac{e^Y}{s} \, \left(\alpha_s^{\rm MOM}(\mu_R^{\rm BLM})\right)^2$$

$$\times \, e^{Y \bar\alpha_s^{\rm MOM}(\mu_R^{\rm BLM}) \left[\chi(n,\nu) + \bar\alpha_s^{\rm MOM}(\mu_R^{\rm BLM}) \left(\bar\chi(n,\nu) + \frac{T^{\rm conf}}{3}\chi(n,\nu)\right)\right]} c_H(n,\nu)[c_J(n,\nu)]^*$$

$$\times \left\{1 + \bar\alpha_s^{\rm MOM}(\mu_R^{\rm BLM}) \left[\frac{\bar c_H^{(1)}(n,\nu)}{c_H(n,\nu)} + \left[\frac{\bar c_J^{(1)}(n,\nu)}{c_J(n,\nu)}\right]^* + \frac{2T^{\rm conf}}{3}\right]\right\} ,$$

with the $\mu_R^{\rm BLM}$ scale chosen as the solution of the following integral equation...

$$C_n^\beta \propto \int_{y_H^{\rm min}}^{y_H^{\rm max}} dy_H \int_{y_J^{\rm min}}^{y_J^{\rm max}} dy_J \int_{k_H^{\rm min}}^{\infty} dk_H \int_{k_J^{\rm min}}^{\infty} dk_J \int_{-\infty}^{\infty} d\nu \, e^{Y \bar\alpha_s^{\rm MOM}(\mu_R^{\rm BLM})\chi(n,\nu)}$$

$$c_H(n,\nu)[c_J(n,\nu)]^* \left[\frac{5}{3} + \ln\frac{(\mu_R^{\rm BLM})^2}{|\vec k_H||\vec k_J|} + f(\nu) - 2\left(1 + \frac{2}{3}I\right)\right.$$

$$\left. + \bar\alpha_s^{\rm MOM}(\mu_R^{\rm BLM}) Y \, \frac{\chi(n,\nu)}{2}\left(-\frac{\chi(n,\nu)}{2} + \frac{5}{3} + \ln\frac{(\mu_R^{\rm BLM})^2}{|\vec k_H||\vec k_J|} + f(\nu) - 2\left(1 + \frac{2}{3}I\right)\right)\right] \stackrel{!}{=} 0$$

On the scale optimization: BLM method

NLA BFKL corrections to cross section with opposite sign with respect to the leading order (LO) result and large in absolute value...

- ...call for some optimization procedure...
- ...choose scales to mimic the most relevant subleading terms

- **BLM** [S.J. Brodsky, G.P. Lepage, P.B. Mackenzie (1983)]
  - ✓ preserve the conformal invariance of an observable...
  - ✓ ...by making vanish its β_0-dependent part

* "Exact" BLM:

suspress NLO IFs + NLO Kernel β_0-dependent factors

* Partial (approximated) BLM:

a) $\left(\mu_R^{BLM}\right)^2 = k_1 k_2 \, \exp\left[2\left(1+\frac{2}{3}I\right) - f(\nu) - \frac{5}{3}\right] \leftarrow$ NLO IFs β_0

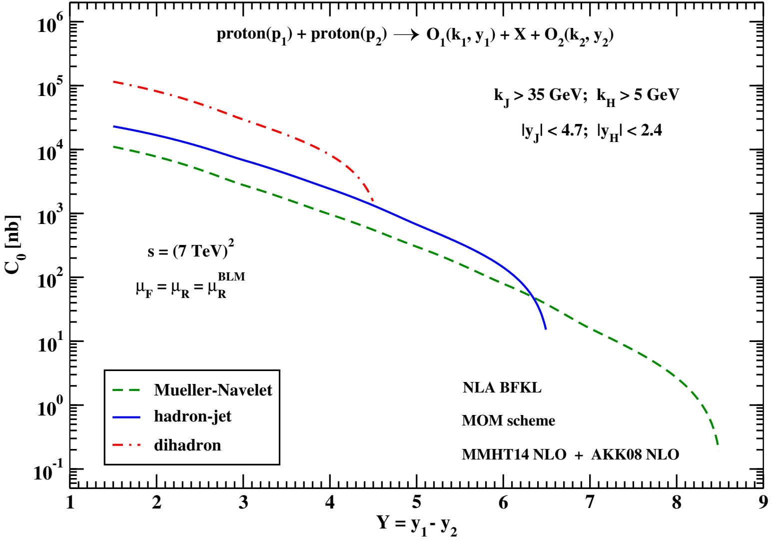
b) $\left(\mu_R^{BLM}\right)^2 = k_1 k_2 \, \exp\left[2\left(1+\frac{2}{3}I\right) - 2f(\nu) - \frac{5}{3} + \frac{1}{2}\chi(\nu,n)\right] \leftarrow$ NLO Kernel β_0

$$\text{with} \quad i\frac{d}{d\nu}\ln\left(\frac{c_1}{c_2}\right) = 2\left[f(\nu) - \ln\left(\sqrt{\vec{k}_1^2\vec{k}_2^2}\right)\right]$$

[F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa (2015)]

BACKUP slides

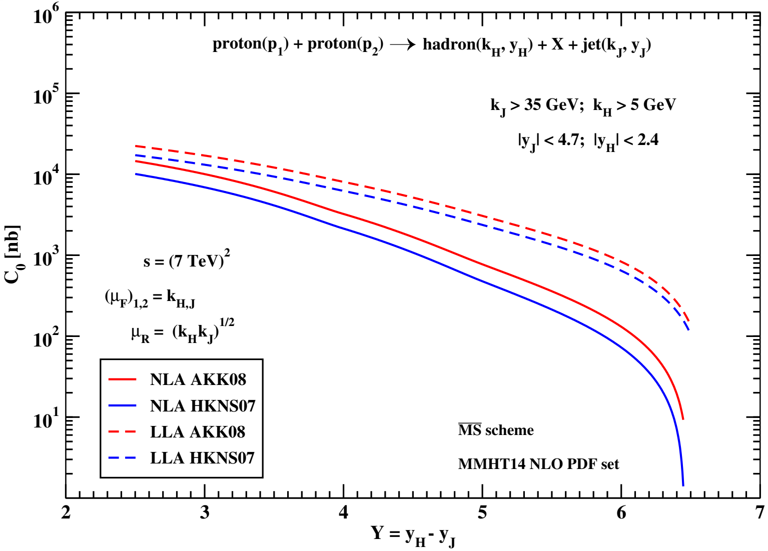
MN, hadron-jet and di-hadron C_0 vs Y, $\sqrt{s} = 7$ TeV



[A.D. Bolognino, F.G. Celiberto, D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2018)]

BACKUP slides

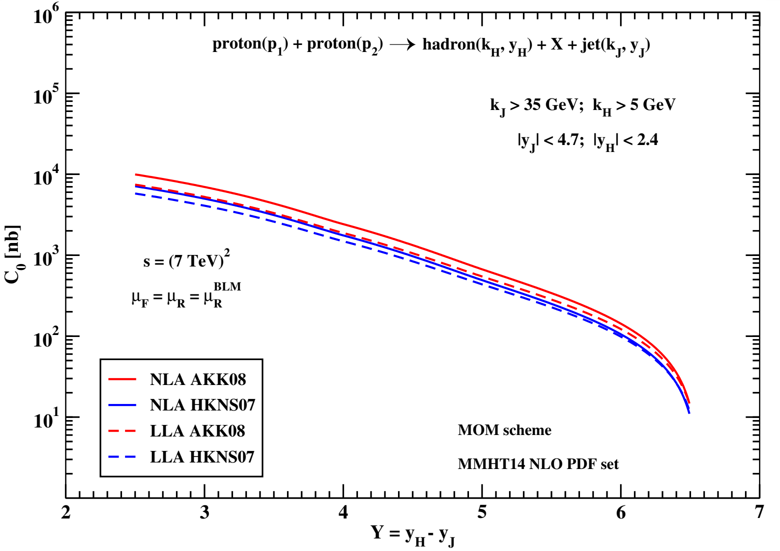
Hadron-jet C_0 vs Y, $\sqrt{s} = 7$ TeV, NS $\overline{\text{MS}}$ [*CMS-jet*]



[A.D. Bolognino, F.G. Celiberto, D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2018)]

BACKUP slides

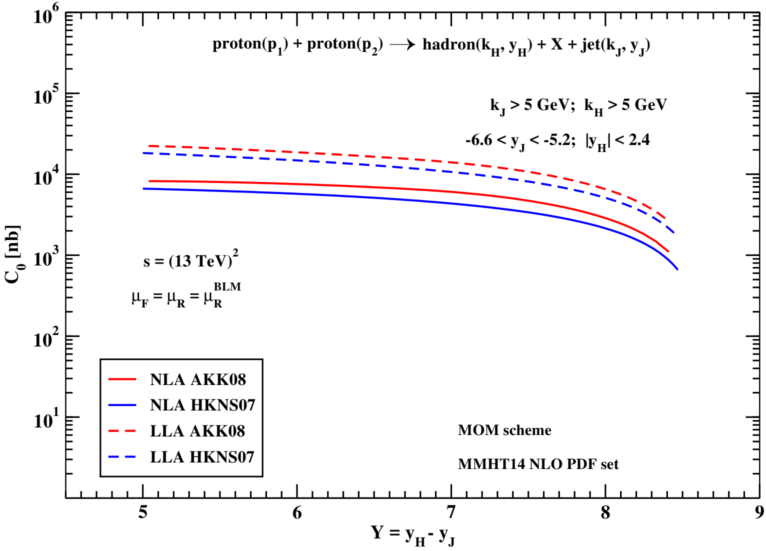
Hadron-jet C_0 vs Y, $\sqrt{s} = 7$ TeV [*CMS-jet*]



[A.D. Bolognino, F.G. Celiberto, D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2018)]

BACKUP slides

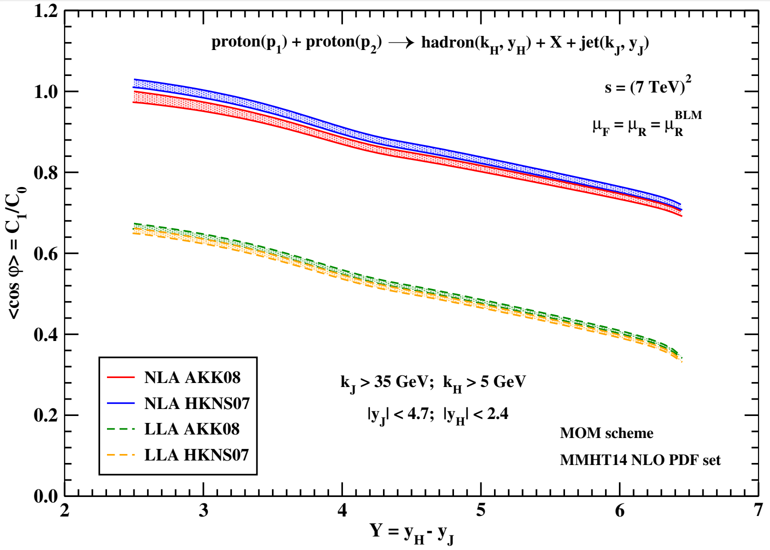
Hadron-jet C_0 vs Y, $\sqrt{s} = 13$ TeV [*CASTOR-jet*]



[A.D. Bolognino, F.G. Celiberto, D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2018)]

BACKUP slides

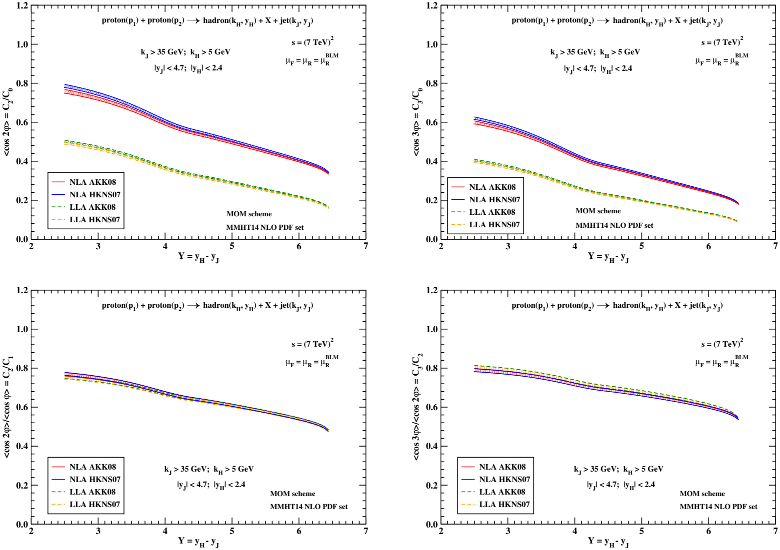
Hadron-jet R_{10} vs Y, $\sqrt{s} = 7$ TeV [*CMS-jet*]



[A.D. Bolognino, F.G. Celiberto, D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2018)]

BACKUP slides

Hadron-jet R_{nm} vs Y, $\sqrt{s} = 7$ TeV [*CMS-jet*]



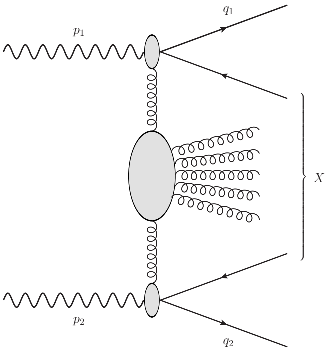
[A.D. Bolognino, F.G. Celiberto, D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2018)]

heavy-quark pair photoproduction

Heavy-quark pair photoproduction

Process: $\gamma(p_1) + \gamma(p_2) \rightarrow Q(q_1) + X + Q(q_2)$

...Q stands for a charm/bottom quark or antiquark

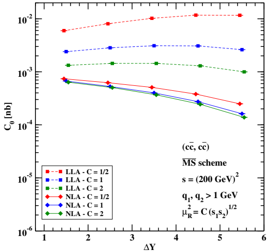


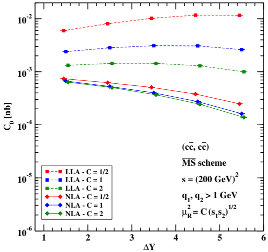
- photoproduction channel
- collision of (quasi-)real photons
- equivalent photon flux approximation
- quark masses play the role of hard scale
- first predictions within partial NLA BFKL (NLA Green's function + LO impact factors)
  - ◇ LEP2 and future e^+e^- colliders

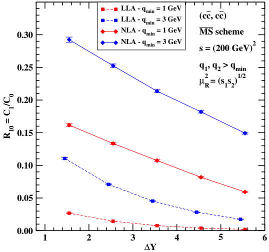
[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2018)]

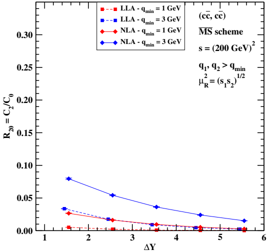
BACKUP slides

C_0 and R_{n0} vs Y at LEP2 (heavy quarks)







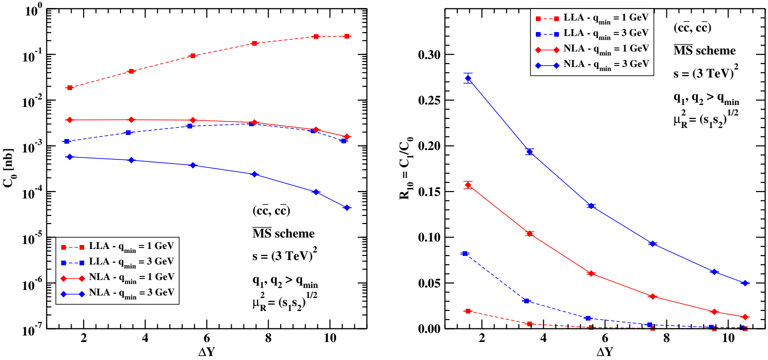


$$s_{1,2} = m_{1,2}^2 + q_{1,2}^2$$

[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2018)]

BACKUP slides

C_0 and R_{10} vs Y at e^+e^- future colliders (heavy quarks)



$$s_{1,2} = m_{1,2}^2 + q_{1,2}^2$$

[F.G. C., D.Yu. Ivanov, B. Murdaca, A. Papa (2018)]