Mass renormalization in light-front perturbation theory

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Motivation

- Low-x DIS at NLO in the dipole factorization picture

  $\rightarrow$ Calculations most conveniently done in light-front perturbation theory (LFPT)

  - Massless quarks case:
    G.B., PRD94 (2016) & PRD96 (2017)
    Hänninen, Lappi and Paatelainen, Annals Phys. 393 (2018)
  - Massive quarks case:
    G.B., Lappi and Paatelainen, *in preparation*

  See next talk by Risto Paatelainen

- Main new ingredient in the massive case: Quark mass renormalization LFPT

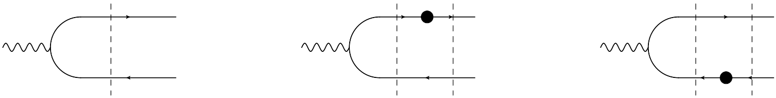
From bare to renormalized LFPT: kinetic mass c.t.

For the LO $\gamma \to q\bar{q}$ LFWF, the energy denominator with bare mass $m_0 = m\, Z_m$ writes

$$(ED_{LO})_{m_0} \equiv q^- - \frac{\mathbf{k}_0{}^2 + m_0{}^2}{2k_0^+} - \frac{\mathbf{k}_1{}^2 + m_0{}^2}{2k_1^+} = (ED_{LO}) + \frac{m^2 - m_0{}^2}{2k_0^+} + \frac{m^2 - m_0{}^2}{2k_1^+}$$

Kinetic mass counter-terms are then obtained from the bare LO graph as

$$\Rightarrow \frac{1}{(ED_{LO})_{m_0}} = \frac{1}{(ED_{LO})} + \frac{(Z_m - 1)m^2}{k_0^+ (ED_{LO})^2} + \frac{(Z_m - 1)m^2}{k_1^+ (ED_{LO})^2} + O(\alpha_s^2)$$



On-shell mass renormalization scheme: Choose Z_m such that all terms with an extra $1/(ED_{LO})$ factor cancel between counter-terms and one-loop graphs.

From bare to renormalized LFPT: vertex mass

On the LF, the *good* components of the spinors are independent of m, and the *bad* components are constrained as

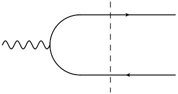
$$u_B(k,h) = \frac{\gamma^+}{2k^+}\,\left(\mathbf{k}^j\gamma^j + m\right)\,u_G(k^+,h)$$
$$v_B(k,h) = \frac{\gamma^+}{2k^+}\,\left(\mathbf{k}^j\gamma^j - m\right)\,v_G(k^+,h)$$

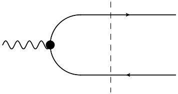
Hence, the $\gamma_L \to q\bar{q}$ vertex is mass independent:

$$\overline{u}(k_0,h_0)\,\gamma^+\,v(k_1,h_1) = \overline{u_G}(k_0^+,h_0)\,\gamma^+\,v_G(k_1^+,h_1)$$

And the $\gamma_T \to q\bar{q}$ vertex depends on the mass as

$$\left[\overline{u}(k_0,h_0)\,\not\epsilon_\lambda(q)\,v(k_1,h_1)\right]\Big|_{m_0} = \left[\overline{u}(k_0,h_0)\,\not\epsilon_\lambda(q)\,v(k_1,h_1)\right]\Big|_{m}$$
$$+\,(Z_m-1)m\,\frac{q^+}{2k_0^+k_1^+}\,\left[\overline{u}(k_0,h_0)\,\gamma^+\,\not\epsilon_\lambda(q)\,v(k_1,h_1)\right]\Big|_{m}$$





Earlier results on mass renormalization on the light-front

UV divergent one-loop corrections in QED and QCD on the light-front first calculated long ago
Mustaki, Pinsky, Shigemitsu and Wilson, PRD43 (1991)
Harindranath and Zhang, PRD48 (1993)

$\rightarrow$ Puzzling results:

- Same result for the quark vertex mass correction as in covariant PT
- But different result for the quark kinetic mass correction

  $\Rightarrow$ Do vertex mass and kinetic mass become different objects on the light front, with different counter-terms and different anomalous dimensions?
- Non-zero correction to the gluon mass

  $\Rightarrow$ Can the bare and the renormalized gluon masses vanish simultaneously in light-front quantization?

Standard one-loop graphs for the gluon mass on the LF

Inserting a gluon or quark loop on an internal gluon line within any LFPT graph amounts to multiply this graph by

$$-\frac{(\mu^2)^{2-\frac{D}{2}}}{k_0^+(ED_{LO})}\int\frac{d^{D-2}\mathbf{K}}{(2\pi)^{D-2}}\int_0^1 d\xi\left\{\alpha_s C_A\left[\frac{1}{\xi^2}+\frac{1}{(1-\xi)^2}+\frac{(D_s-2)}{(D-2)}\right]\right.$$
$$\left.+\alpha_s T_F N_f\left[\frac{1}{\xi}+\frac{1}{(1-\xi)}-\frac{4}{(D-2)}+\frac{4}{(D-2)}\frac{m^2}{(\mathbf{K}^2+m^2)}\right]\right\}+O(1)$$

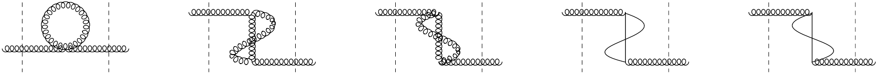
for $ED_{LO}\to 0$

ED_{LO} : Energy Denominator of the Fock state containing the parent gluon ($\rightsquigarrow$ off-shellness of the parent gluon)

- Nonzero result: need for gluon mass counter-term?
- Both quadratic and log UV divergences in the **K** integral
- Power and log divergences in the ξ momentum fraction integral

Normal-ordering graphs for the gluon mass on the LF

Some 2-points vertices are obtained in LFPT as leftover after extracting the 4-points vertices by normal ordering of the LF Hamiltonian $\hat{P}^-$



$$\frac{(\mu^2)^{2-\frac{D}{2}}}{k_0^+(ED_{LO})} \int \frac{d^{D-2}\mathbf{K}}{(2\pi)^{D-2}} \Bigg\{ \alpha_s C_A \Bigg[\int_0^1 d\xi \left(\frac{1}{\xi^2} + \frac{1}{(1-\xi)^2} \right) + \frac{(D_s-2)}{2} \int_0^{+\infty} \frac{dk^+}{k^+} \Bigg] + \alpha_s T_F N_f \left[\int_0^1 d\xi \left(\frac{1}{\xi} + \frac{1}{(1-\xi)} \right) - 2 \int_0^{+\infty} \frac{dk^+}{k^+} \right] \Bigg\}$$

- Only quadratic UV divergence w.r.t. $\mathbf{K}$
- New type of contributions with unbounded k^+ integration
- Needs new regulator for $k^+ \to +\infty$

Full one-loop terms for the gluon mass on the LF

Adding the two types of loop diagrams (non-instantaneous and instantaneous)

$$\frac{(\mu^2)^{2-\frac{D}{2}}}{k_0^+(ED_{LO})}\int\frac{d^{D-2}\mathbf{K}}{(2\pi)^{D-2}}\left\{\alpha_s C_A\frac{(D_s-2)}{(D-2)}\left[-1+\frac{(D-2)}{2}\int_0^{+\infty}\frac{dk^+}{k^+}\right]\right.$$
$$\left.-\alpha_s T_F N_f\frac{4}{(D-2)}\left[\frac{m^2}{(\mathbf{K}^2+m^2)}-1+\frac{(D-2)}{2}\int_0^{+\infty}\frac{dk^+}{k^+}\right]\right\}+O(1)$$

for $ED_{LO}\to 0$

Can the $\alpha_s C_A$ and the $\alpha_s T_F N_f$ terms both vanish, to ensure a zero gluon mass?

One-body phase space integral

The terms with unbounded k^+ integration correspond to one-body phase space integrals

$$\int \frac{d^D k}{(2\pi)^D}\,\theta(k^0)\,2\pi\delta(k^2 - m_i^2) = \int \frac{d^{D-2}\mathbf{k}}{(2\pi)^{D-2}} \int_0^{+\infty} \frac{dk^+}{(2\pi)(2k^+)}$$

But this light-cone expression a bit awkward and badly defined:

- Parton mass m_i dependence lost
- Divergences at $k^+ \to +\infty$ and at $k^+ \to 0$ (as $k^- \to +\infty$) are of the UV type, not regularized by transverse dim. reg.
- Anywhere else in LFPT, UV divergences come only from $\mathbf{K}$ integration

One-body phase space integral

Calculation in cartesian coordinates, with full dim. reg. (using an IBP relation):

$$\int \frac{d^D k}{(2\pi)^D}\,\theta(k^0)\,2\pi\delta(k^2-m_i^2) = \int \frac{d^{D-2}\mathbf{k}}{(2\pi)^{D-2}} \int \frac{dk_L}{2\pi}\, \frac{1}{2\sqrt{{k_L}^2+\mathbf{k}^2+m_i^2}}$$

$$= \frac{1}{2\pi(D-2)} \int \frac{d^{D-2}\mathbf{k}}{(2\pi)^{D-2}} \left[1-\frac{m_i^2}{\mathbf{k}^2+m_i^2}\right]$$

In this way:

- No need for an extra regulator
- No breaking of Poincaré symmetry by the UV regulator
- Depends on the mass of the parton

Prescription to deal with the unbounded k^+ integrals

1. Split the instantaneous loop contributions into 1-body phase space integrals and integrals with internal k^+s bounded by the parent k_0^+
2. Replace the 1-body phase space integral with unbounded k^+ by its better defined expression

$$\int \frac{d^{D-2}\mathbf{k}}{(2\pi)^{D-2}} \int_0^{+\infty} \frac{dk^+}{k^+} \mapsto \frac{2}{(D-2)} \int \frac{d^{D-2}\mathbf{k}}{(2\pi)^{D-2}} \left[1 - \frac{m_i^2}{\mathbf{k}^2 + m_i^2}\right],$$

with the appropriate mass m_i for the corresponding parton.

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with the appropriate mass m_i for the corresponding parton.

In the gluon loop contribution (taking $m_i = 0$):

$$\alpha_s C_A \int \frac{d^{D-2}\mathbf{K}}{(2\pi)^{D-2}} \left[-1 + \frac{(D-2)}{2} \int_0^{+\infty} \frac{dk^+}{k^+}\right] \mapsto 0$$

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$$\int \frac{d^{D-2}\mathbf{k}}{(2\pi)^{D-2}} \int_0^{+\infty} \frac{dk^+}{k^+} \mapsto \frac{2}{(D-2)} \int \frac{d^{D-2}\mathbf{k}}{(2\pi)^{D-2}} \left[1 - \frac{m_i^2}{\mathbf{k}^2 + m_i^2}\right] ,$$

with the appropriate mass m_i for the corresponding parton.

In the quark loop contribution (taking $m_i = m$):

$$\alpha_s T_F N_f \int \frac{d^{D-2}\mathbf{K}}{(2\pi)^{D-2}} \left[\frac{m^2}{(\mathbf{K}^2 + m^2)} - 1 + \frac{(D-2)}{2} \int_0^{+\infty} \frac{dk^+}{k^+}\right] \mapsto 0$$

Prescription to deal with the unbounded k^+ integrals

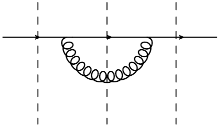
1. Split the instantaneous loop contributions into 1-body phase space integrals and integrals with internal k^+s bounded by the parent k_0^+
2. Replace the 1-body phase space integral with unbounded k^+ by its better defined expression

$$\int \frac{d^{D-2}\mathbf{k}}{(2\pi)^{D-2}} \int_0^{+\infty} \frac{dk^+}{k^+} \mapsto \frac{2}{(D-2)} \int \frac{d^{D-2}\mathbf{k}}{(2\pi)^{D-2}} \left[1 - \frac{m_i^2}{\mathbf{k}^2 + m_i^2}\right] ,$$

with the appropriate mass m_i for the corresponding parton.

- Cancelation of both the $\alpha_s C_A$ and the $\alpha_s T_F N_f$ corrections to the gluon mass, both in CDR and FDH
- Both the bare mass and the on-shell mass for the gluon can now be zero !
- Also applies to QED: massless photon on the light-front

Standard one-loop graphs for the quark mass on the LF



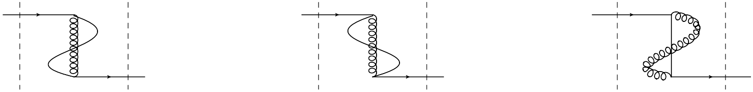
Inserting a gluon or quark loop on an internal quark line within any LFPT graph amounts to multiply this graph by

$$\alpha_s C_F \frac{(\mu^2)^{2-\frac{D}{2}}}{k_0^+(ED_{LO})} \int \frac{d^{D-2}\mathbf{K}}{(2\pi)^{D-2}} \int_0^1 d\xi \left\{ -\frac{2}{\xi^2} - \frac{(D_s-2)}{2(1-\xi)} + \frac{2m^2}{(\mathbf{K}^2+\xi^2 m^2)} \right\} + O(1)$$

for $ED_{LO} \to 0$

Normal-ordering graphs for the quark mass on the LF

Contribution of the 2-points quark vertices obtained in LFPT as leftover after normal ordering of the LF Hamiltonian $\hat{P}^-$:



$$\alpha_s C_F \frac{(\mu^2)^{2-\frac{D}{2}}}{k_0^+(ED_{LO})} \int \frac{d^{D-2}\mathbf{K}}{(2\pi)^{D-2}} \left\{ \int_0^1 d\xi \left(\frac{2}{\xi^2} + \frac{(D_s-2)}{2(1-\xi)} \right) + \frac{(D_s-2)}{2} \int dk_2^+ \left(\frac{\theta(k_2^+)}{k_2^+} - \frac{\theta(k_2^+-k_0^+)}{k_2^+-k_0^+} \right) \right\}$$

- k_2^+ corresponds to the gluon line in the loop
- $k_0^+ - k_2^+$ or $k_2^+ - k_0^+$ corresponds to the quark line in the loop

The 2 terms with unbounded k_2^+ integrations would naively cancel
$\rightarrow$ but not when using the previous prescription with different masses.

Full one-loop terms for the quark mass on the LF

Sum of the one loop graphs for the quark mass correction, after applying the prescription:

$$\alpha_s C_F \frac{(\mu^2)^{2-\frac{D}{2}}}{k_0^+ (ED_{LO})} \int \frac{d^{D-2}\mathbf{K}}{(2\pi)^{D-2}} \left\{ \int_0^1 d\xi \, \frac{2m^2}{(\mathbf{K}^2 + \xi^2 m^2)} + \frac{(D_s - 2)}{(D-2)} \left[1 - \left(1 - \frac{m^2}{(\mathbf{K}^2 + m^2)} \right) \right] \right\} + O(1)$$

for $ED_{LO} \to 0$

- Quadratic UV divergences cancel
- But leave a new log UV divergent term as leftover

That result should be combined with the (kinetic) quark mass counterterm

Quark mass counter-term

Kinetic quark mass counter-term insertion: multiplication of LO graph by

$$\frac{(Z_m - 1) m^2}{k_0^+ (ED_{LO})}$$

In the on-shell mass scheme

$$(Z_m^{OS} - 1) = -\,\alpha_s C_F\,(\mu^2)^{2-\frac{D}{2}} \int \frac{d^{D-2}\mathbf{K}}{(2\pi)^{D-2}} \left\{ 2\int_0^1 d\xi\,\frac{1}{(\mathbf{K}^2 + \xi^2 m^2)} + \frac{(D_s - 2)}{(D - 2)}\,\frac{1}{(\mathbf{K}^2 + m^2)} \right\}$$

in order to cancel the $1/(ED_{LO})$ contributions from the one-loop graphs.

Quark mass counter-term

Taking $D_s = D$ in CDR or $D_s = 4$ in FDH, and then expanding in $\epsilon = 2 - \frac{D}{2}$:

$$\left.(Z_m^{OS}-1)\right|_{CDR} = -\frac{\alpha_s C_F}{\pi}\left\{\frac{3}{4}\left[\frac{1}{\bar{\epsilon}} - \log\left(\frac{m^2}{\mu^2}\right)\right] + 1 + O(\epsilon)\right\}$$

$$\left.(Z_m^{OS}-1)\right|_{FDH} = -\frac{\alpha_s C_F}{\pi}\left\{\frac{3}{4}\left[\frac{1}{\bar{\epsilon}} - \log\left(\frac{m^2}{\mu^2}\right)\right] + \frac{5}{4} + O(\epsilon)\right\}$$

- Consistent with the vertex mass renormalization
  $\Rightarrow$ Only one quark mass parameter
- Consistent with the results in covariant PT, including the finite term both for CDR and FDH

Conclusion

- Longstanding issues with mass renormalization in LFPT finally resolved

  $\rightarrow$ Expected results finally obtained:

  - The gluon mass stays zero without needing a counter-term
  - The quark mass stays the same in the energy denominators and in the vertices
  - The quark mass renormalization constant is the same as in covariant PT, including the finite terms in the on-shell scheme

- Reformulation of LFPT with Lorentz-invariant UV regularization