Unequal Rapidity Correlators

in the dilute limit of JIMWLK

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arXiv:1904.00782[hep-ph]

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Motivation

- Understand QCD dynamics of particle correlations with large rapidity separation, in saturation regime

- Within Color Glass Condensate framework → find equivalence between two descriptions of

JIMWLK evolution

(Jalilian-Marian–Iancu–McLerran–Weigert–Leonidov–Kovner)

Fokker-Planck

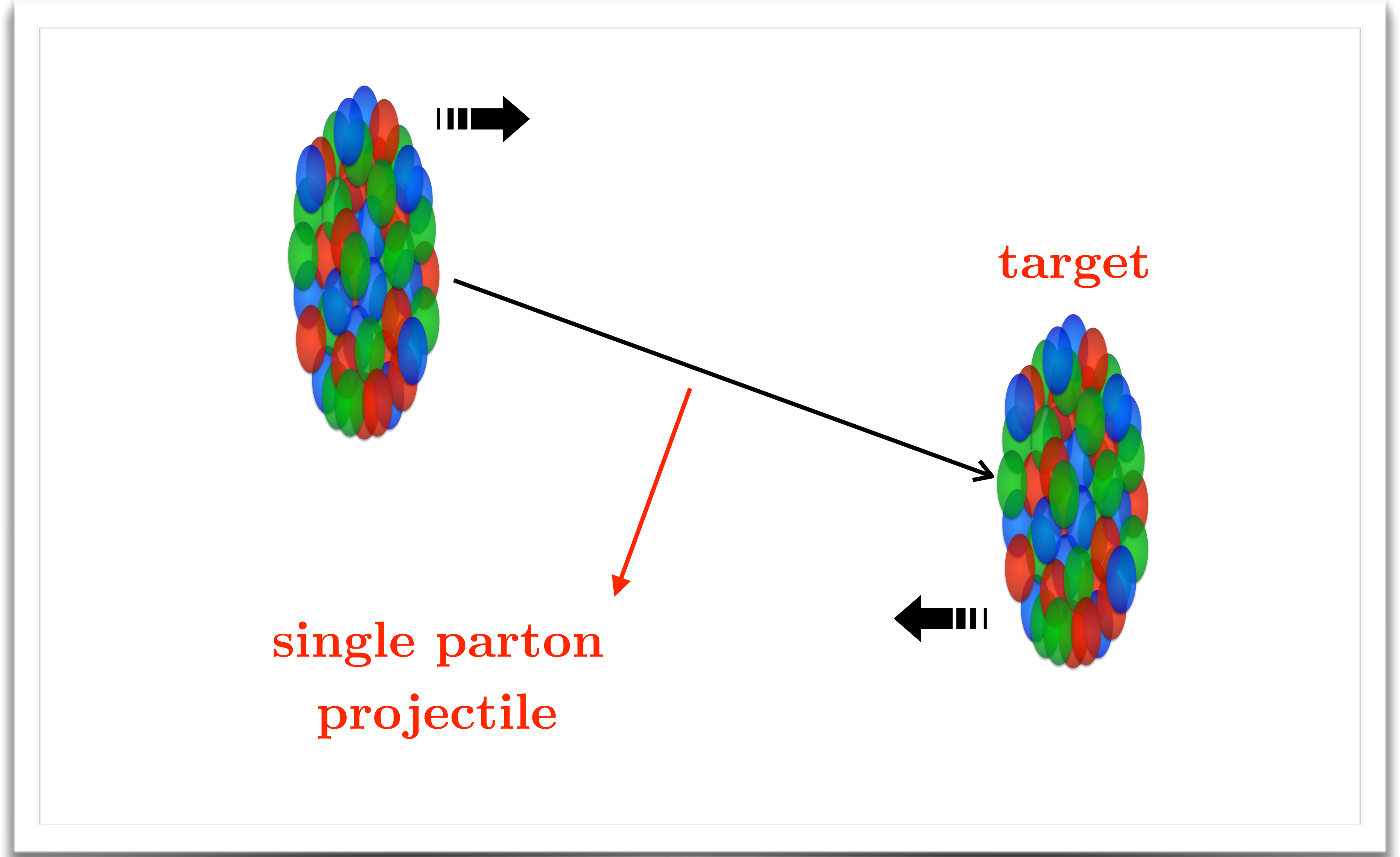
Langevin

- stochastic interpretation
- good for numerics

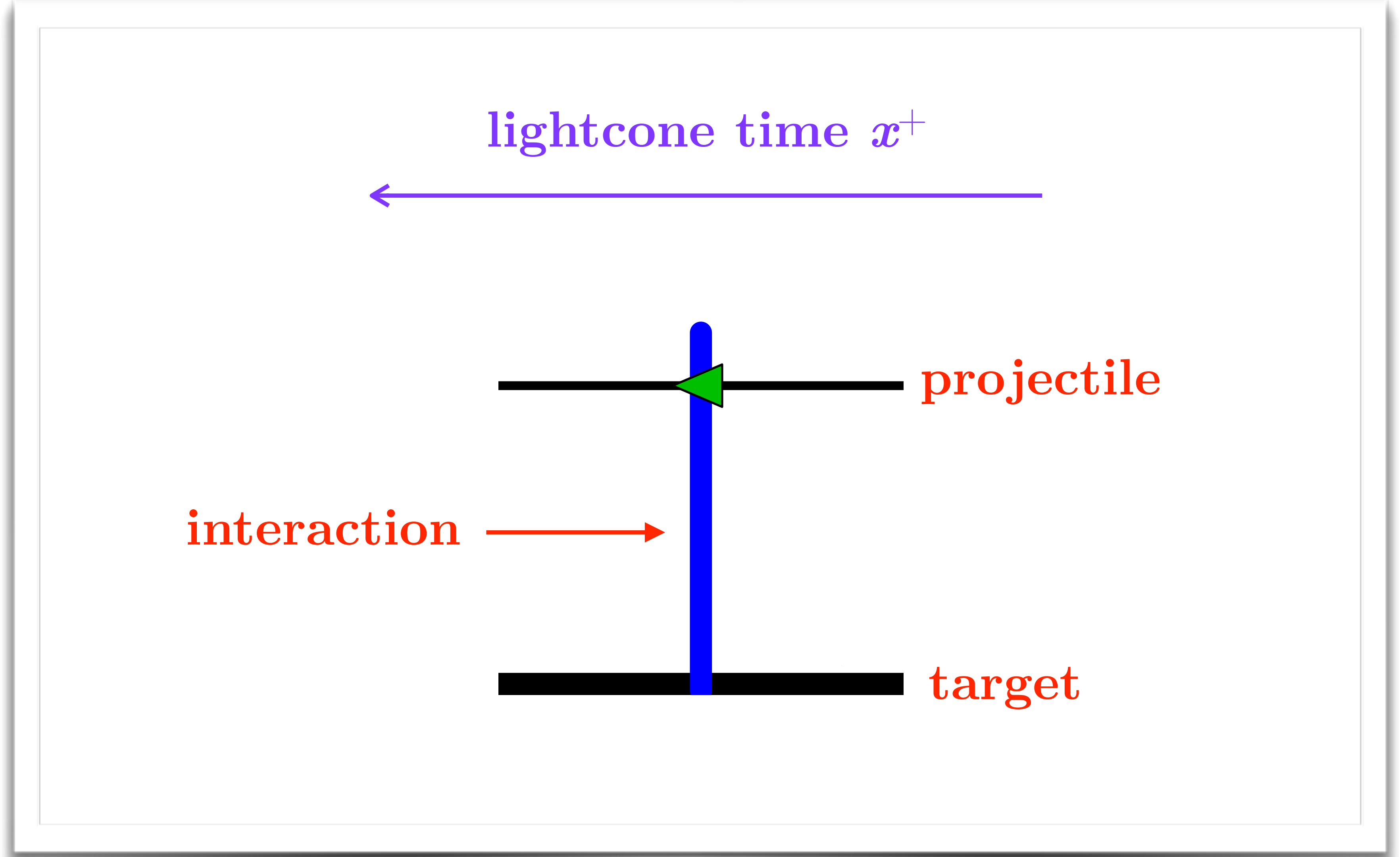
- Go to dilute limit → describe emergence of BFKL dynamics

Context

Ultra-relativistic nuclear collision →



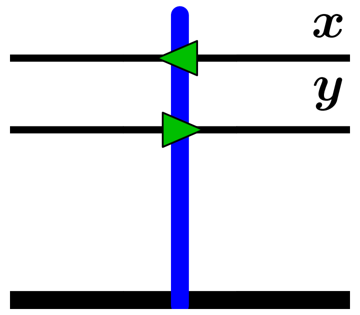
diagrammatic notation



Wilson line describes interaction (eikonal approximation)

$$U^\dagger_{\boldsymbol{x}} \equiv P \exp\left\{ig\int dx^+ \alpha^a_{\boldsymbol{x}}(x^+)t^a\right\}$$

Wilson Line Correlators

- Wilson lines appear in cross sections inside correlators e.g. dipole operator $\hat{S}_{\boldsymbol{xy}} \equiv \frac{\mathrm{tr}\left\{U^\dagger_{\boldsymbol{x}} U_{\boldsymbol{y}}\right\}}{N_c} = \frac{1}{N_c}\mathrm{tr}\left\{ \right.$  $\left. \right\}$

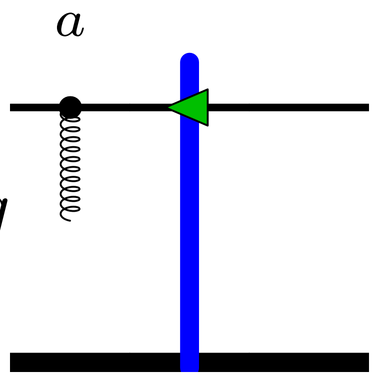
- JIMWLK equation describes rapidity evolution of correlators

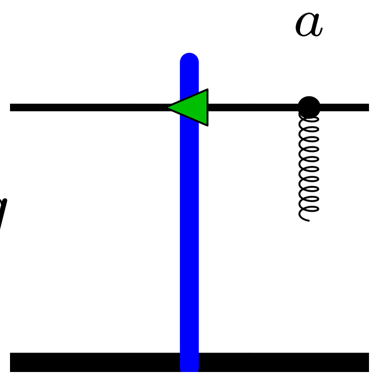
$$\left\langle \hat{\mathcal{O}} \right\rangle_Y \equiv \int [DU] W_Y[U] \hat{\mathcal{O}} \qquad\qquad \frac{\partial}{\partial Y} W_Y[U] = H W_Y[U]$$

$$H \equiv \frac{1}{8\pi^3} \int_{\boldsymbol{uvz}} \underbrace{\mathcal{K}_{\boldsymbol{uvz}}}_{\mathcal{K}^i_{\boldsymbol{uz}} \mathcal{K}^i_{\boldsymbol{vz}}} (L^a_{\boldsymbol{u}} - U^{\dagger ab}_{\boldsymbol{z}} R^b_{\boldsymbol{u}})(L^a_{\boldsymbol{v}} - U^{\dagger ac}_{\boldsymbol{z}} R^c_{\boldsymbol{v}})$$

$$\mathcal{K}^i_{\boldsymbol{uz}} \mathcal{K}^i_{\boldsymbol{vz}} \longrightarrow \mathcal{K}^i_{\boldsymbol{uz}} = \frac{(\boldsymbol{u} - \boldsymbol{z})^i}{(\boldsymbol{u} - \boldsymbol{z})^2}$$

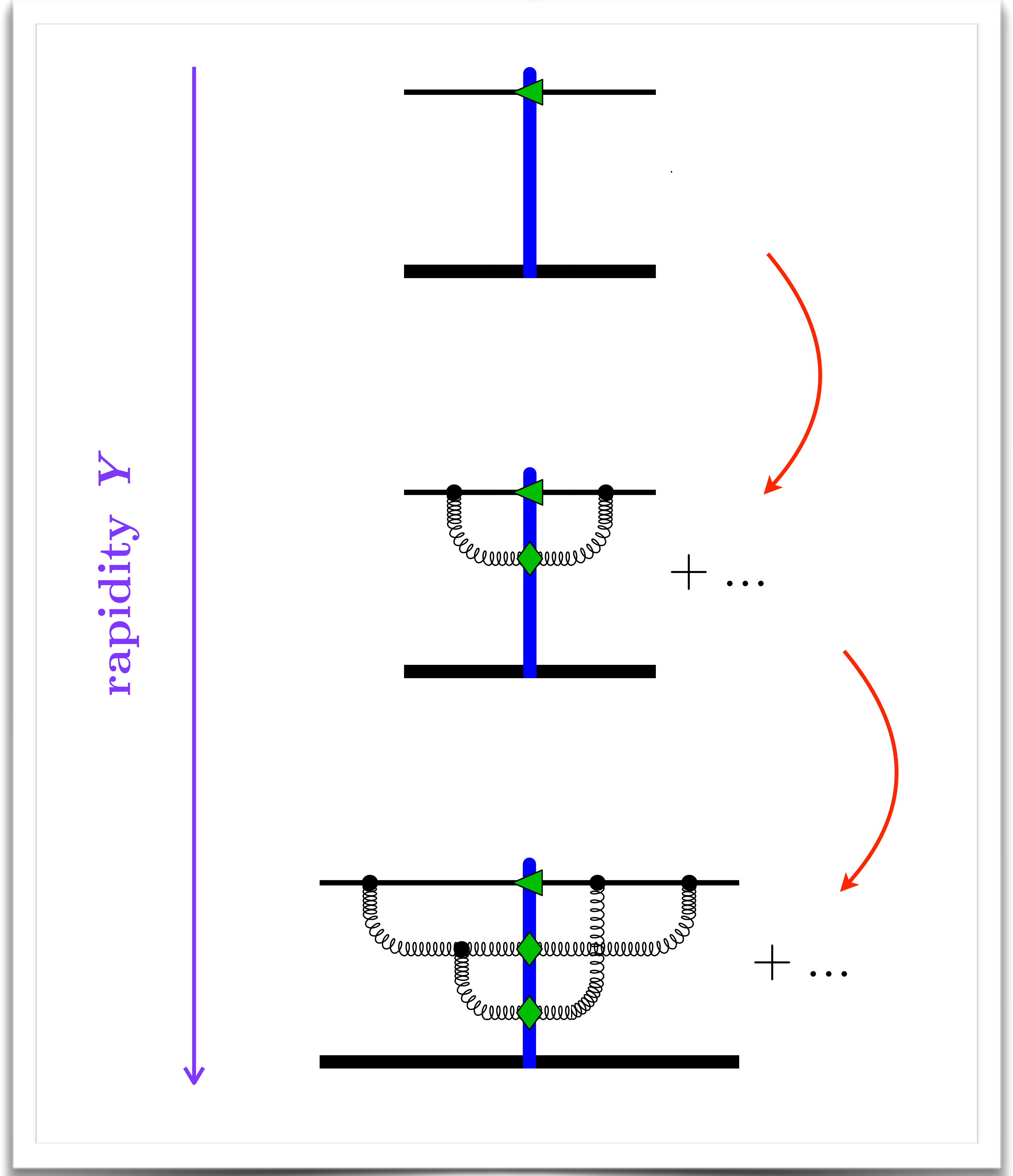
- Lie derivatives

$R^a_{\boldsymbol{u}} U^\dagger_{\boldsymbol{x}} = ig$ 

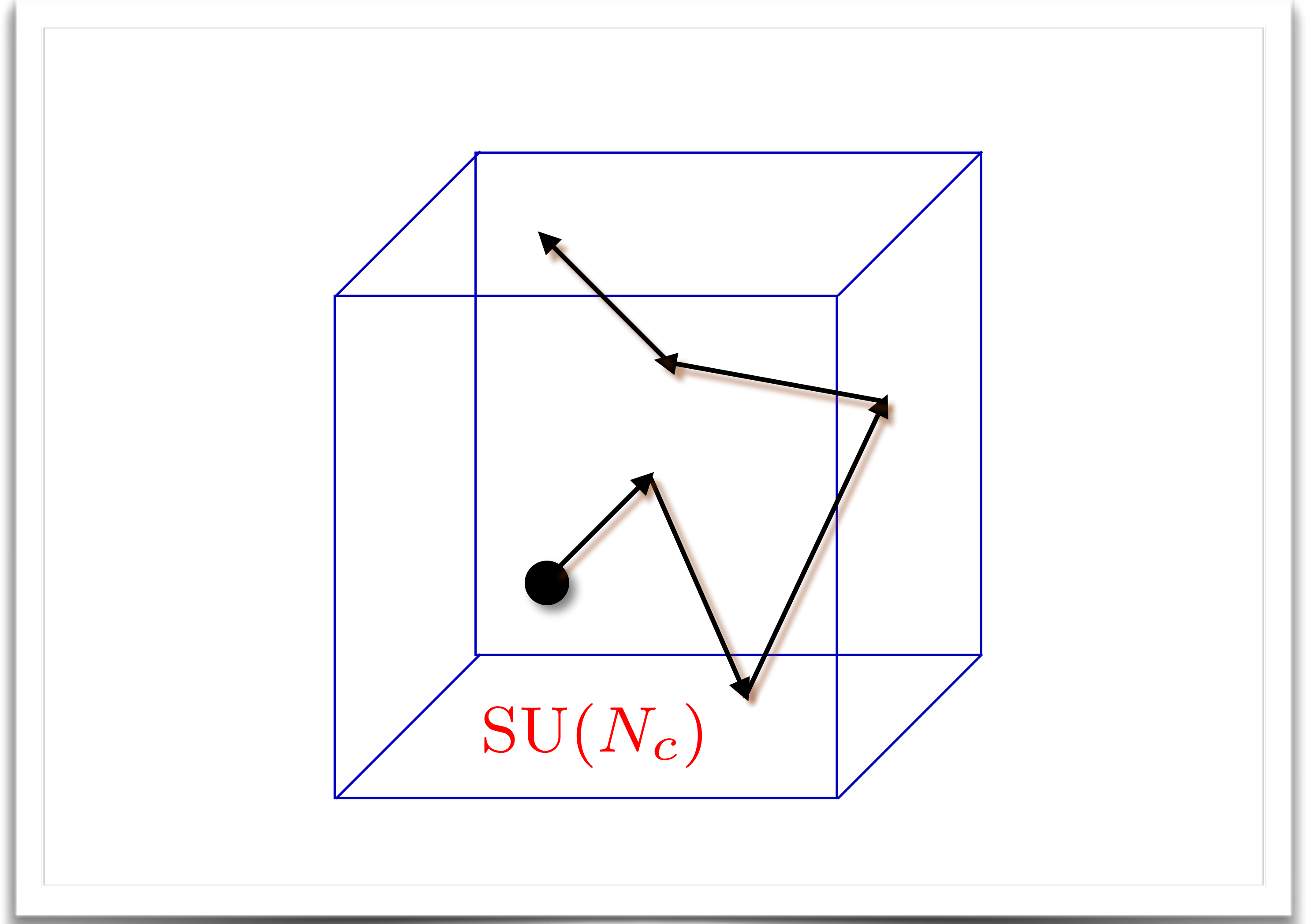
$L^a_{\boldsymbol{u}} U^\dagger_{\boldsymbol{x}} = ig$ 

JIMWLK Evolution

Fokker-Planck $\Longleftrightarrow$ Langevin



stochastic diffusive process
in space of Wilson lines



Blaizot, Iancu & Weigert [Nucl.Phys. A713 (2003) 441-469],
Kovner & Lublinsky [JHEP 0611 (2006) 083],
Iancu & Triantafyllopoulos [JHEP 1311 (2013) 067],
Hatta & Iancu [JHEP 1608 (2016) 083]

Langevin Picture

Blaizot, Iancu & Weigert
[Nucl.Phys. A713 (2003) 441-469]

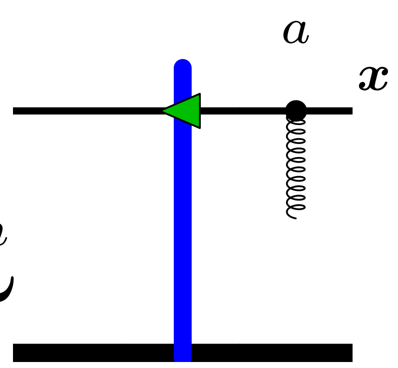
- Discretize rapidity $\quad Y - Y_0 = N\epsilon, \qquad \epsilon \to 0, \qquad n = 0, ..., N$

- Langevin equation for Wilson line

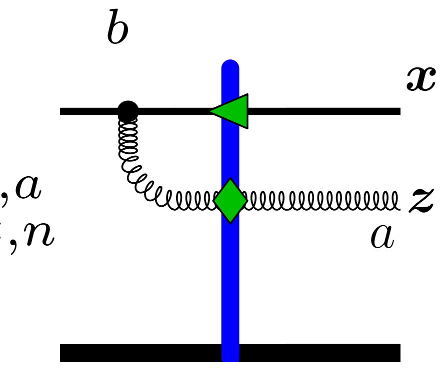
$$U^{\dagger}_{\boldsymbol{x},n+1} = \exp\left\{i\epsilon g \alpha^{L}_{\boldsymbol{x},n}\right\} U^{\dagger}_{\boldsymbol{x},n} \exp\left\{-i\epsilon g \alpha^{R}_{\boldsymbol{x},n}\right\}$$

- Color rotations

$$\alpha^{L}_{\boldsymbol{x},n} U^{\dagger}_{\boldsymbol{x},n} = \frac{1}{\sqrt{4\pi^3}} \int_{\boldsymbol{z}} \mathcal{K}^{i}_{\boldsymbol{xz}} \nu^{i,a}_{\boldsymbol{z},n}$$



$$\alpha^{R}_{\boldsymbol{x},n} U^{\dagger}_{\boldsymbol{x},n} = \frac{1}{\sqrt{4\pi^3}} \int_{\boldsymbol{z}} \mathcal{K}^{i}_{\boldsymbol{xz}} \nu^{i,a}_{\boldsymbol{z},n}$$



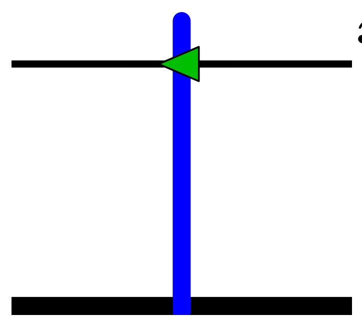
$$\left\langle \nu^{i,a}_{\boldsymbol{x},m} \nu^{jb}_{\boldsymbol{y},n} \right\rangle = \frac{1}{\epsilon} \delta^{ij} \delta^{ab} \delta_{mn} \delta_{\boldsymbol{xy}}$$

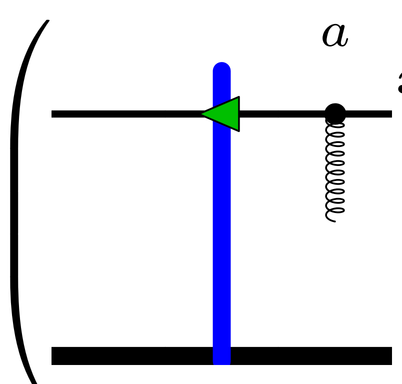
Gaussian white noise $\longleftrightarrow$ **rotated noise** $\tilde{\nu}^{i,a}_{\boldsymbol{z},n} \equiv U^{\dagger ab}_{\boldsymbol{z},n} \nu^{i,b}_{\boldsymbol{z},n}$

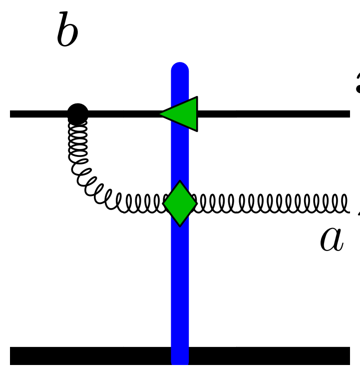
First Expansion

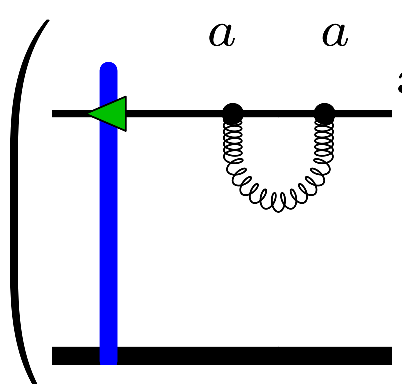
- Expand Langevin equation in rapidity step epsilon

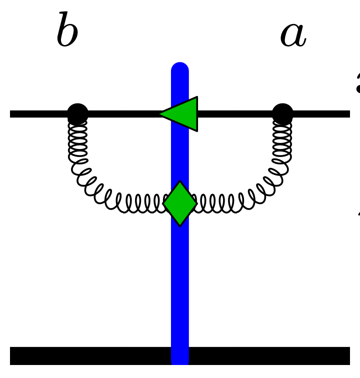
$$U^{\dagger}_{\boldsymbol{x},n+1} = U^{\dagger}_{\boldsymbol{x},n} + \frac{i\epsilon g}{\sqrt{4\pi^3}} \int_{\boldsymbol{z}} \mathcal{K}^i_{\boldsymbol{x}\boldsymbol{z}} \nu^{i,a}_{\boldsymbol{z},n} (t^a U^{\dagger}_{\boldsymbol{x},n} - U^{\dagger}_{\boldsymbol{x},n} U^{\dagger ab}_{\boldsymbol{z},n} t^b) - \frac{\epsilon g^2}{4\pi^3} \int_{\boldsymbol{z}} \mathcal{K}_{\boldsymbol{x}\boldsymbol{x}\boldsymbol{z}} t^a (t^a U^{\dagger}_{\boldsymbol{x},n} - U^{\dagger}_{\boldsymbol{x},n} U^{\dagger ab}_{\boldsymbol{z},n} t^b) + \mathcal{O}(\epsilon^{3/2})$$

$=$ 

$+ \frac{i\epsilon g}{\sqrt{4\pi^3}} \int_{\boldsymbol{z}} \mathcal{K}^i_{\boldsymbol{x}\boldsymbol{z}} \nu^{i,a}_{\boldsymbol{z},n} \Bigg($ 

$-$ 

$\Bigg) - \frac{\epsilon g^2}{4\pi^3} \int_{\boldsymbol{z}} \mathcal{K}_{\boldsymbol{x}\boldsymbol{x}\boldsymbol{z}} \Bigg($ 

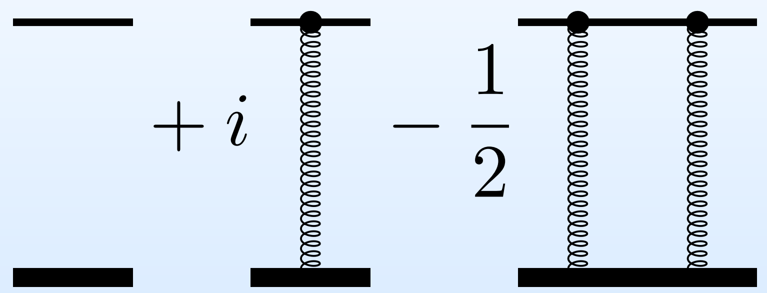
$-$ 

$\Bigg) + \mathcal{O}(\epsilon^{3/2})$

Second Expansion - Dilute Limit

- Expand in elements of group algebra

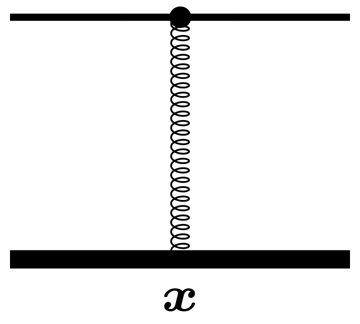
$$U^{\dagger}_{\boldsymbol{x},n} \equiv e^{i\lambda_{\boldsymbol{x},n}} = \mathbb{1} + i\lambda_{\boldsymbol{x},n} - \frac{1}{2}\lambda^2_{\boldsymbol{x},n} + \mathcal{O}(\lambda^3) =$$



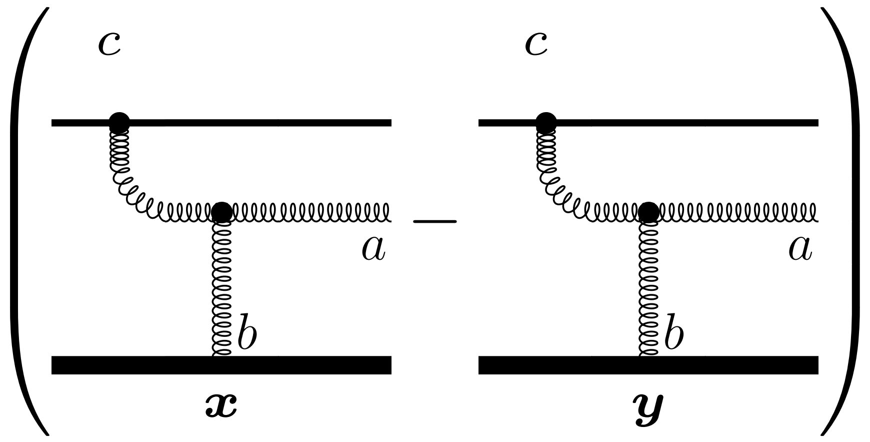
$$+\,\mathcal{O}(\lambda^3)$$

- Evolution equation

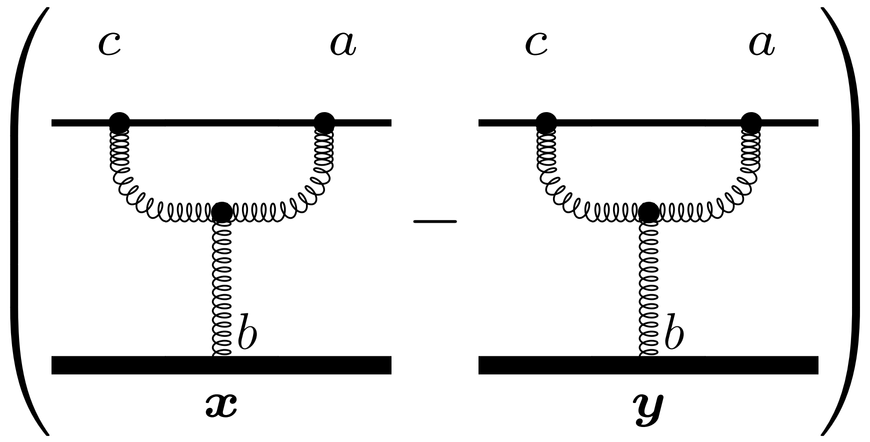
$$\lambda_{\boldsymbol{x},n+1} = \lambda_{\boldsymbol{x},n} + \frac{i\epsilon g}{\sqrt{4\pi^3}}\int_{\boldsymbol{y}} \mathcal{K}^i_{\boldsymbol{xy}}\nu^{i,a}_{\boldsymbol{y},n} i f^{abc} t^c(\lambda^b_{\boldsymbol{x},n} - \lambda^b_{\boldsymbol{y},n}) - \frac{\epsilon g^2}{4\pi^3}\int_{\boldsymbol{y}} \mathcal{K}_{\boldsymbol{xxy}} t^a i f^{abc} t^c (\lambda^b_{\boldsymbol{x},n} - \lambda^b_{\boldsymbol{y},n}) + \mathcal{O}(\epsilon^{3/2}, \lambda^2)$$

$=$ 

$$+\frac{i\epsilon g}{\sqrt{4\pi^3}}\int_{\boldsymbol{y}} \mathcal{K}^i_{\boldsymbol{xy}}\nu^{i,a}_{\boldsymbol{y},n} i$$



$$-\frac{\epsilon g^2}{4\pi^3}\int_{\boldsymbol{y}} \mathcal{K}_{\boldsymbol{xxy}}$$



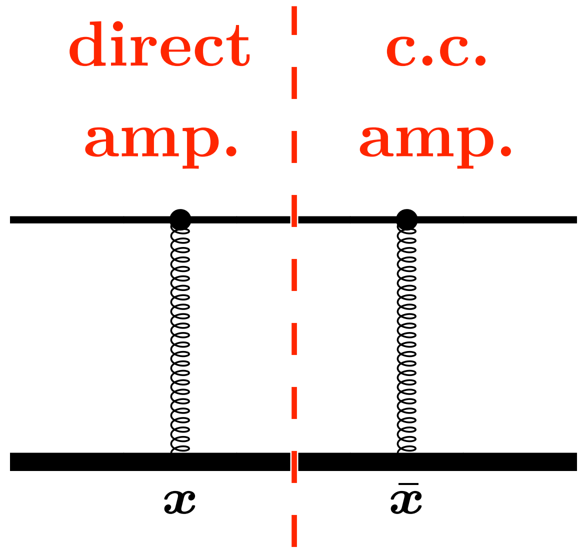
$$+\mathcal{O}(\epsilon^{3/2}, \lambda^2)$$

(this Reggeizes - backup slide)

BFKL Equation

- Define unintegrated gluon distribution

$$\phi^n_{\boldsymbol{x}\bar{\boldsymbol{x}}} \equiv \langle \lambda^a_{\boldsymbol{x},n} \bar{\lambda}^a_{\bar{\boldsymbol{x}},n} \rangle \sim$$



- Evolution equation for lambda gives

$$\phi^{n+1}_{\boldsymbol{x}\bar{\boldsymbol{x}}} - \phi^n_{\boldsymbol{x}\bar{\boldsymbol{x}}} = -\frac{N_c}{2}\frac{\epsilon\alpha_s}{\pi^2}\int_{\boldsymbol{z}} [\mathcal{K}_{\boldsymbol{x}\boldsymbol{x}\boldsymbol{z}}(\phi^n_{\boldsymbol{x}\bar{\boldsymbol{x}}} - \phi^n_{\boldsymbol{z}\bar{\boldsymbol{x}}}) + \mathcal{K}_{\bar{\boldsymbol{x}}\bar{\boldsymbol{x}}\boldsymbol{z}}(\phi^n_{\boldsymbol{x}\bar{\boldsymbol{x}}} - \phi^n_{\boldsymbol{x}\boldsymbol{z}}) - 2\mathcal{K}_{\boldsymbol{x}\bar{\boldsymbol{x}}\boldsymbol{z}}(\phi^n_{\boldsymbol{x}\bar{\boldsymbol{x}}} - \phi^n_{\boldsymbol{x}\boldsymbol{z}} - \phi^n_{\boldsymbol{z}\bar{\boldsymbol{x}}} + \phi^n_{\boldsymbol{z}\boldsymbol{z}})] + \mathcal{O}(\epsilon^{3/2}, \phi^{3/2})$$

- Fourier transform

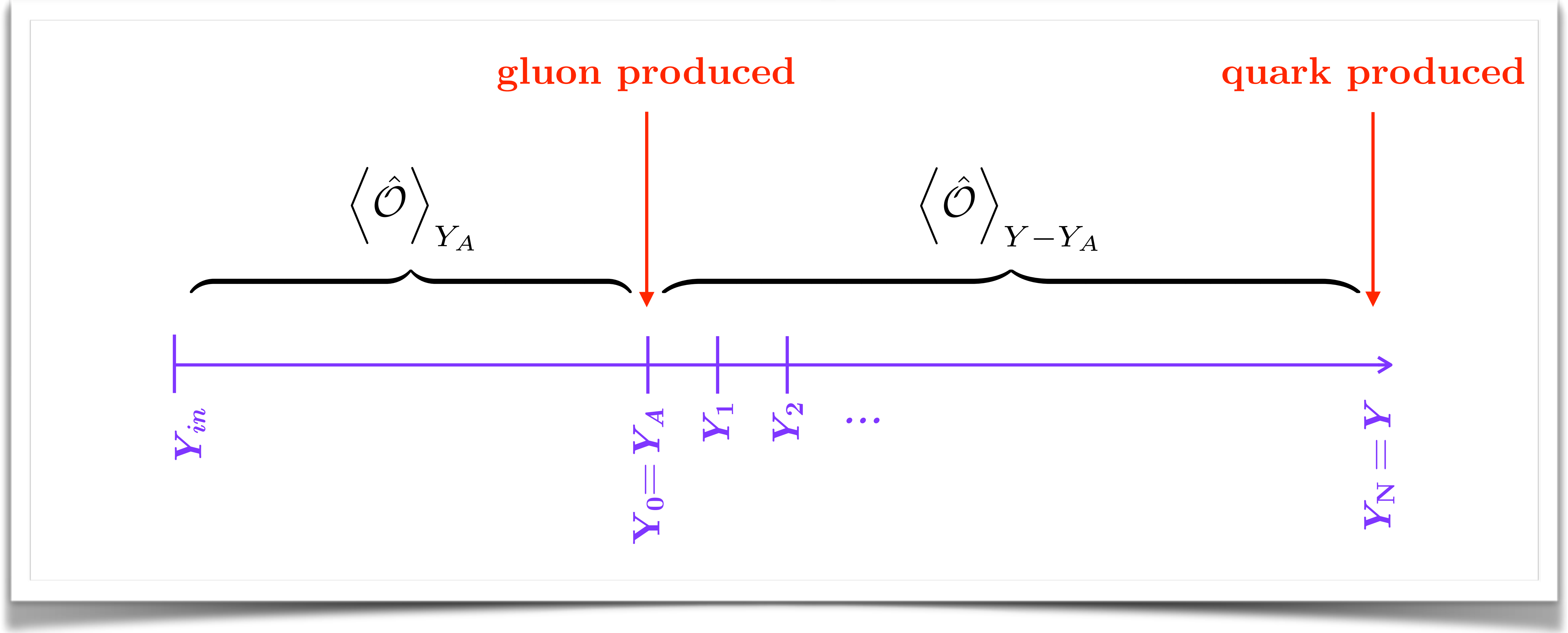
$$\phi^{n+1}(\boldsymbol{q}) - \phi^n(\boldsymbol{q}) = +4N_c\epsilon\alpha_s \int_{\boldsymbol{p}} \frac{1}{(\boldsymbol{q}-\boldsymbol{p})^2}\left(\frac{\phi^n(\boldsymbol{p})\boldsymbol{p}^2}{\boldsymbol{q}^2} - \frac{1}{2}\frac{\phi^n(\boldsymbol{q})\boldsymbol{q}^2}{\boldsymbol{p}^2}\right) + \mathcal{O}(\epsilon^{3/2}, \phi^{3/2})$$

→ well-known color singlet, zero momentum transfer BFKL equation (not "Mueller's BFKL" - backup slide)

Two-Particle Production

Iancu & Triantafyllopoulos [JHEP 1311 (2013) 067]

- Double inclusive quark-gluon production at unequal rapidities $Y_N - Y_0 \geq 1/\alpha_s$



- Modified expectation value $\left\langle \hat{\mathcal{O}} \right\rangle_{Y-Y_A} \equiv \int [DU D\bar{U}] W_{Y-Y_A}[U,\bar{U}|U_A,\bar{U}_A]\hat{\mathcal{O}}$

- Modified evolution equation $\dfrac{\partial}{\partial Y} W_{Y-Y_A}[U,\bar{U}|U_A,\bar{U}_A] = H_{\text{evol}} W_{Y-Y_A}[U,\bar{U}|U_A,\bar{U}_A]$

Two-Particle Cross Section

Iancu & Triantafyllopoulos
[JHEP 1311 (2013) 067]

- Cross section

$$\left\langle \hat{\mathcal{O}} \right\rangle_{Y_A} \equiv \int [DU] W_{Y_A}[U] \hat{\mathcal{O}}$$

$$\frac{d\sigma_{qg}}{dY\, d^2\boldsymbol{p}\, dY_A d^2\boldsymbol{k}_A} = \frac{1}{(2\pi)^4} \int_{\boldsymbol{x}\bar{\boldsymbol{x}}} e^{-i\boldsymbol{p}\cdot(\boldsymbol{x}-\bar{\boldsymbol{x}})} \left\langle H_{\text{prod}}(\boldsymbol{k}_A) \left\langle \hat{S}_{\boldsymbol{x}\bar{\boldsymbol{x}}} \right\rangle_{Y-Y_A} \Big|_{\bar{U}_A = U_A} \right\rangle_{Y_A}$$

$$\left\langle \hat{\mathcal{O}} \right\rangle_{Y-Y_A} \equiv \int [DU\, D\bar{U}] W_{Y-Y_A}[U, \bar{U} | U_A, \bar{U}_A] \hat{\mathcal{O}}$$

- Hamiltonian produces gluon (Wilson lines and derivatives do not evolve, but remain at $Y_A = Y_0$)

$$H_{\text{prod}}(\boldsymbol{k}) = \frac{1}{4\pi^3} \int_{\boldsymbol{y}\bar{\boldsymbol{y}}} e^{-i\boldsymbol{k}\cdot(\boldsymbol{y}-\bar{\boldsymbol{y}})} \int_{\boldsymbol{u}\bar{\boldsymbol{u}}} \mathcal{K}^i_{\boldsymbol{y}\boldsymbol{u}} \mathcal{K}^i_{\bar{\boldsymbol{y}}\bar{\boldsymbol{u}}} (L^a_{\boldsymbol{u},0} - U^{\dagger ab}_{\boldsymbol{y},0} R^b_{\boldsymbol{u},0})(\bar{L}^a_{\bar{\boldsymbol{u}},0} - \bar{U}^{\dagger ac}_{\bar{\boldsymbol{y}},0} \bar{R}^c_{\bar{\boldsymbol{u}},0})$$

$\rightarrow$ need Lie differentiated Langevin equations

Bilocal Langevin Equation

- Full (unexpanded) bilocal evolution equation for Wilson line

$$R^a_{\boldsymbol{u},0}U^\dagger_{\boldsymbol{x},n+1} = \exp\left\{i\epsilon g\alpha^L_{\boldsymbol{x},n}\right\} R^a_{\boldsymbol{u},0}U^\dagger_{\boldsymbol{x},n} \exp\left\{-i\epsilon g\alpha^R_{\boldsymbol{x},n}\right\} - \frac{i\epsilon g}{\sqrt{4\pi^3}} \exp\left\{i\epsilon g\alpha^L_{\boldsymbol{x},n}\right\} U^\dagger_{\boldsymbol{x},n} \int_{\boldsymbol{z}} \mathcal{K}^i_{\boldsymbol{xz}} \times [U_{\boldsymbol{z},n}\nu^i_{\boldsymbol{z},n}U^\dagger_{\boldsymbol{z},n}, U_{\boldsymbol{z},n}R^a_{\boldsymbol{u},0}U^\dagger_{\boldsymbol{z},n}]$$

- Define $R^a_{\boldsymbol{ux},n} \equiv U_{\boldsymbol{x},n}R^a_{\boldsymbol{u},0}U^\dagger_{\boldsymbol{x},n}$ to write linear equation with no explicit Wilson lines

$$\frac{1}{\epsilon}(R^a_{\boldsymbol{ux},n+1} - R^a_{\boldsymbol{ux},n}) = \frac{ig}{\sqrt{4\pi^3}} \int_{\boldsymbol{z}} \mathcal{K}^i_{\boldsymbol{xz}}[\tilde{\nu}^i_{\boldsymbol{z},n}, R^a_{\boldsymbol{ux},n} - R^a_{\boldsymbol{uz},n}] - \frac{N_c}{2}\frac{g^2}{4\pi^3} \int_{\boldsymbol{z}} \mathcal{K}_{\boldsymbol{xxz}}(R^a_{\boldsymbol{ux},n} - R^a_{\boldsymbol{uz},n}) + \mathcal{O}(\epsilon^{3/2})$$

$\rightarrow$ no need for full nonlinear numerics

- But cross section still needs full nonlinear evolution for explicit Wilson lines

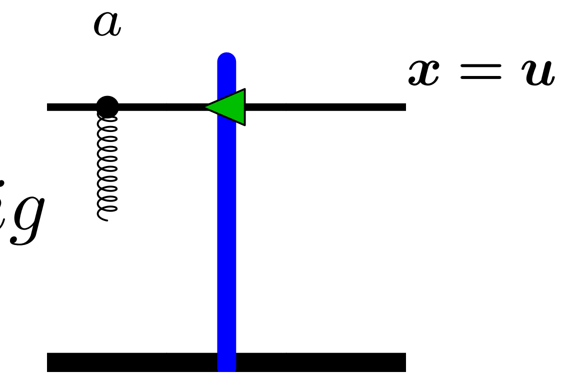
$$\sigma \sim \mathrm{tr}\left\{\bar{L}^a_{\boldsymbol{v\bar{x}},N}U_{\boldsymbol{\bar{x}},N}U^\dagger_{\boldsymbol{x},N}L^a_{\boldsymbol{ux},N}\right\} - \bar{\tilde{U}}^{\dagger ac}_{\boldsymbol{\bar{y}},0}\mathrm{tr}\left\{\bar{R}^c_{\boldsymbol{v\bar{x}},N}U_{\boldsymbol{\bar{x}},N}U^\dagger_{\boldsymbol{x},N}L^a_{\boldsymbol{ux},N}\right\} - U^{\dagger ab}_{\boldsymbol{y},0}\mathrm{tr}\left\{\bar{L}^a_{\boldsymbol{v\bar{x}},N}U_{\boldsymbol{\bar{x}},N}U^\dagger_{\boldsymbol{x},N}R^b_{\boldsymbol{ux},N}\right\} + U^{\dagger ab}_{\boldsymbol{y},0}\bar{\tilde{U}}^{\dagger ac}_{\boldsymbol{\bar{y}},0}\mathrm{tr}\left\{\bar{R}^c_{\boldsymbol{v\bar{x}},N}U_{\boldsymbol{\bar{x}},N}U^\dagger_{\boldsymbol{x},N}R^b_{\boldsymbol{ux},N}\right\}$$

Bilocal Equation - One Step

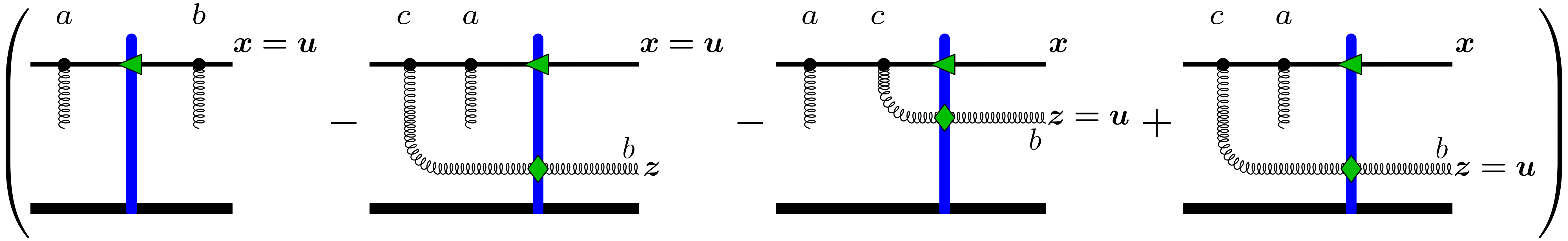
- Epsilon-expanded Lie differentiated evolution equation for Wilson line

$$R^a_{\boldsymbol{u},0} U^\dagger_{\boldsymbol{x},1}$$

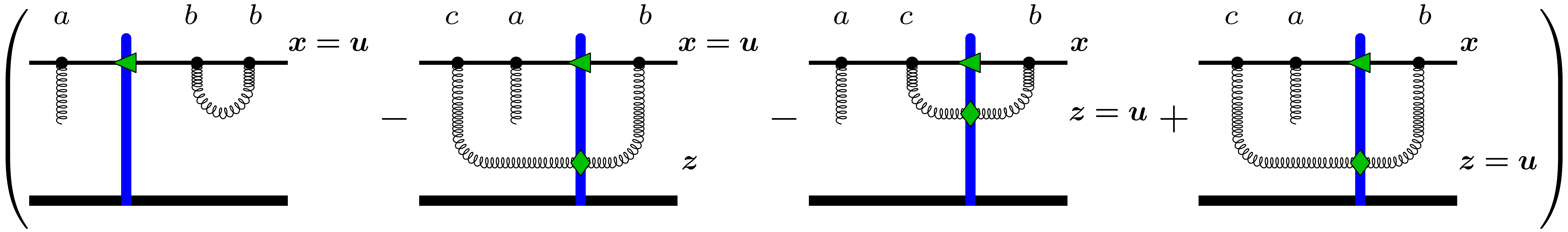
$$= ig$$



$$+ ig \frac{i\epsilon g}{\sqrt{4\pi^3}} \int_{\boldsymbol{z}} \mathcal{K}^i_{\boldsymbol{xz}} \nu^{i,b}_{\boldsymbol{z},0}$$



$$- ig \frac{\epsilon g^2}{4\pi^3} \int_{\boldsymbol{z}} \mathcal{K}_{\boldsymbol{xxz}}$$



$$+ \mathcal{O}(\epsilon^{3/2})$$

Bilocal Equation - Dilute Limit

- Expand in lambda

$$L^a_{\boldsymbol{u},0} = g\left(\delta^{ac} - \frac{1}{2} f^{abc}\lambda^b_{\boldsymbol{u},0} + \mathcal{O}(\lambda^2)\right)\frac{\delta}{\delta\lambda^c_{\boldsymbol{u},0}} \qquad R^a_{\boldsymbol{u},0} = g\left(\delta^{ac} + \frac{1}{2} f^{abc}\lambda^b_{\boldsymbol{u},0} + \mathcal{O}(\lambda^2)\right)\frac{\delta}{\delta\lambda^c_{\boldsymbol{u},0}}$$

- Evolution equation

$$R^a_{\boldsymbol{u},0}\lambda_{\boldsymbol{x},n+1} = R^a_{\boldsymbol{u},0}\lambda_{\boldsymbol{x},n} + \int_{\boldsymbol{z}}\left(\frac{i\epsilon g}{\sqrt{4\pi^3}}\mathcal{K}^i_{\boldsymbol{xz}}\nu^{i,d}_{\boldsymbol{z},n} - \frac{\epsilon g^2}{4\pi^3}\mathcal{K}_{\boldsymbol{xxz}}t^d\right) i f^{dbc} t^c R^a_{\boldsymbol{u},0}(\lambda^b_{\boldsymbol{x},n} - \lambda^b_{\boldsymbol{z},n}) + \mathcal{O}(\epsilon^{3/2}, \lambda^2)$$

- Production Hamiltonian

$$H_{\text{prod}}(\boldsymbol{k}) = \frac{1}{4\pi^3}\int_{\boldsymbol{y}\bar{\boldsymbol{y}}} e^{-i\boldsymbol{k}\cdot(\boldsymbol{y}-\bar{\boldsymbol{y}})}\int_{\boldsymbol{u}\bar{\boldsymbol{u}}} \mathcal{K}^i_{\boldsymbol{yu}}\mathcal{K}^i_{\bar{\boldsymbol{y}}\bar{\boldsymbol{u}}}(L^a_{\boldsymbol{u},0} - U^{\dagger ab}_{\boldsymbol{y},0}R^b_{\boldsymbol{u},0})(\bar{L}^a_{\bar{\boldsymbol{u}},0} - \bar{U}^{\dagger ac}_{\bar{\boldsymbol{y}},0}\bar{R}^c_{\bar{\boldsymbol{u}},0})$$

dilute limit

$$= g^2[f^{abc}f^{ade}(\bar{\lambda}^e_{\bar{\boldsymbol{u}},0} - \bar{\lambda}^e_{\bar{\boldsymbol{y}},0})(\lambda^c_{\boldsymbol{u},0} - \lambda^c_{\boldsymbol{y},0}) + \mathcal{O}(\lambda^3)]\frac{\delta}{\delta\bar{\lambda}^d_{\bar{\boldsymbol{u}},0}}\frac{\delta}{\delta\lambda^b_{\boldsymbol{u},0}}$$

Cross Section - Dilute Limit

- Dipole $\frac{\mathrm{tr}\left\{U_{\boldsymbol{x}}^{\dagger}U_{\boldsymbol{y}}\right\}}{N_c} = 1 - \frac{1}{4N_c}(\lambda_{\boldsymbol{x}}^a - \lambda_{\boldsymbol{y}}^a)(\lambda_{\boldsymbol{x}}^a - \lambda_{\boldsymbol{y}}^a) + \mathcal{O}(\lambda^3)$

- Cross section

$$\frac{d\sigma_{qg}}{dY\,d^2\boldsymbol{p}\,dY_A d^2\boldsymbol{k}_A} = \frac{1}{(2\pi)^4}\frac{1}{2N_c}\frac{\alpha_s}{\pi^2}\int_{\boldsymbol{x}\bar{\boldsymbol{x}}\boldsymbol{y}\bar{\boldsymbol{y}}\boldsymbol{u}\bar{\boldsymbol{u}}} \mathcal{K}^i_{\boldsymbol{y}\boldsymbol{u}}\mathcal{K}^i_{\bar{\boldsymbol{y}}\bar{\boldsymbol{u}}} e^{-i\boldsymbol{p}\cdot(\boldsymbol{x}-\bar{\boldsymbol{x}})-i\boldsymbol{k}_A\cdot(\boldsymbol{y}-\bar{\boldsymbol{y}})}(\phi^0_{\bar{\boldsymbol{u}}\boldsymbol{u}} - \phi^0_{\bar{\boldsymbol{u}}\boldsymbol{y}} - \phi^0_{\bar{\boldsymbol{y}}\boldsymbol{u}} + \phi^0_{\bar{\boldsymbol{y}}\boldsymbol{y}})\mathcal{F}^N_{\boldsymbol{x},\bar{\boldsymbol{x}},\boldsymbol{u},\bar{\boldsymbol{u}}} + \mathcal{O}(\phi^{3/2})$$

$$\mathcal{F}^n_{\boldsymbol{x},\bar{\boldsymbol{x}},\boldsymbol{u},\bar{\boldsymbol{u}}} \equiv \frac{\delta}{\delta\bar{\lambda}^a_{\bar{\boldsymbol{u}},0}}\frac{\delta}{\delta\lambda^a_{\boldsymbol{u},0}}\bar{\lambda}^b_{\bar{\boldsymbol{x}},n}\lambda^b_{\boldsymbol{x},n}$$

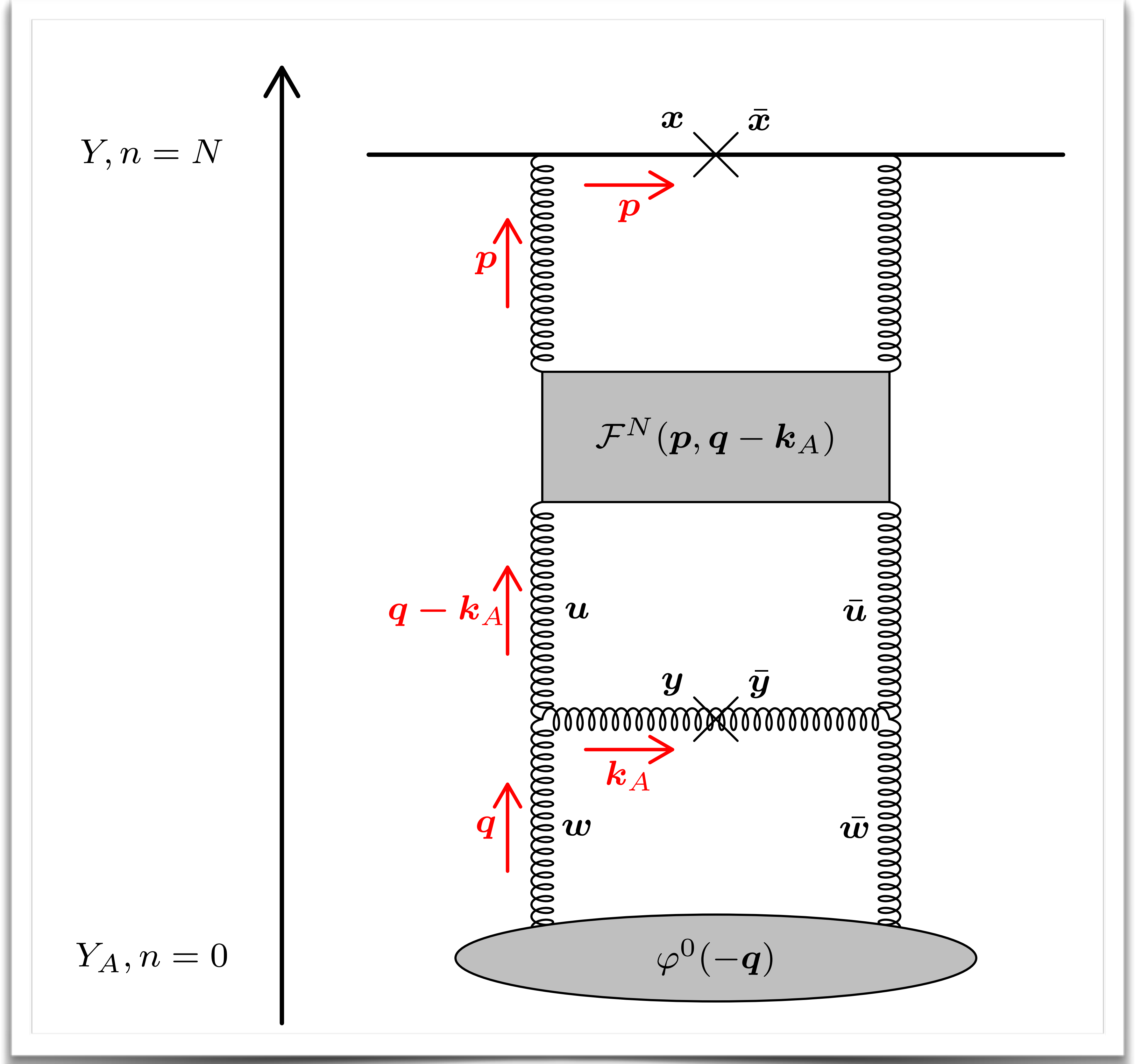
BFKL Green's function (satisfies BFKL equation)

- Final k_T-factorized cross section for quark-gluon production at unequal rapidity

$$\frac{d\sigma_{qg}}{dY\,d^2\boldsymbol{p}\,dY_A d^2\boldsymbol{k}_A} = -\frac{\alpha_s}{N_c}\int_{\boldsymbol{q}}\frac{\boldsymbol{q}^2}{(\boldsymbol{q}-\boldsymbol{k}_A)^2\boldsymbol{k}_A^2}\mathcal{F}^N(-\boldsymbol{p},\boldsymbol{p},\boldsymbol{q}-\boldsymbol{k}_A,-\boldsymbol{q}+\boldsymbol{k}_A)\phi^0(-\boldsymbol{q}) + \mathcal{O}(\varphi^{3/2})$$

BFKL Ladders

$$\frac{d\sigma_{qg}}{dY\, d^2\boldsymbol{p}\, dY_A d^2\boldsymbol{k}_A} = -\frac{\alpha_s}{N_c}\int_{\boldsymbol{q}} \frac{\boldsymbol{q}^2}{(\boldsymbol{q}-\boldsymbol{k}_A)^2\boldsymbol{k}_A^2}\mathcal{F}^N(-\boldsymbol{p},\boldsymbol{p},\boldsymbol{q}-\boldsymbol{k_A},-\boldsymbol{q}+\boldsymbol{k_A})\phi^0(-\boldsymbol{q}) + \mathcal{O}(\varphi^{3/2})$$



Summary

- Studied Langevin picture of JIMWLK evolution $\rightarrow$ alternative formulation of evolution as stochastic diffusion
- Two expansions $\rightarrow$ epsilon (rapidity step), lambda (group algebra element)
- Bilocal Langevin evolution equation is linear (full dense case)
- BFKL dynamics emerge in dilute limit
- Particle production cross section simplifies somewhat $\rightarrow$ no need for full nonlinear numerics

Backup: Reggeization

Caron-Huot [High Energ. Pays. (2015) 93]

- Evolution equation in dilute limit

$$\frac{1}{\epsilon}(\lambda_{\boldsymbol{x},n+1}-\lambda_{\boldsymbol{x},n}) = \int_{\boldsymbol{y}}\left(\frac{ig}{\sqrt{4\pi^3}}\mathcal{K}^i_{\boldsymbol{xy}}\nu^{i,a}_{\boldsymbol{y},n} - \frac{g^2}{4\pi^3}\mathcal{K}_{\boldsymbol{xxy}}t^a\right) if^{abc}t^c(\lambda^b_{\boldsymbol{x},n}-\lambda^b_{\boldsymbol{y},n}) + \mathcal{O}(\epsilon^{3/2},\lambda^2)$$

- Expectation value

$$\frac{1}{\epsilon}\langle\lambda_{\boldsymbol{x},n+1}-\lambda_{\boldsymbol{x},n}\rangle = -\frac{N_c}{2}\frac{g^2}{4\pi^3}\int_{\boldsymbol{z}}\mathcal{K}_{\boldsymbol{xxz}}\langle\lambda_{\boldsymbol{x},n}-\lambda_{\boldsymbol{z},n}\rangle + \mathcal{O}(\epsilon^2,\lambda^2)$$

- Fourier transform

$$\left\langle\frac{d}{dY}\lambda^a(\boldsymbol{p})\right\rangle = \langle\underbrace{\alpha_g(\boldsymbol{p})}\lambda^a_n(\boldsymbol{p})\rangle + \mathcal{O}(\epsilon^2,\lambda^2)$$

$$\alpha_g(\boldsymbol{p}) \equiv \frac{N_c}{2}\frac{\alpha_s}{\pi^2}\int_{\boldsymbol{z}}\frac{1}{\boldsymbol{z}^2}(e^{i\boldsymbol{p}\cdot\boldsymbol{z}}-1)$$

Regge trajectory

→ amplitude has power law behavior $\propto s^{\alpha_g}$

Backup: Mueller's BFKL

- Expand dipole in dilute limit

$$\frac{\mathrm{tr}\left\{U^\dagger_{\boldsymbol{x}} U_{\boldsymbol{y}}\right\}}{N_\mathrm{c}} = 1 - \frac{1}{4N_\mathrm{c}}(\lambda^a_{\boldsymbol{x}} - \lambda^a_{\boldsymbol{y}})(\lambda^a_{\boldsymbol{x}} - \lambda^a_{\boldsymbol{y}}) + \mathcal{O}(\lambda^3)$$

- Define "BFKL pomeron"

$$\varphi_{\boldsymbol{xy}} \equiv \left\langle(\lambda^a_{\boldsymbol{x}} - \lambda^a_{\boldsymbol{y}})(\lambda^a_{\boldsymbol{x}} - \lambda^a_{\boldsymbol{y}})\right\rangle$$

- Evolution equation for lambda gives

$$\varphi^{n+1}_{\boldsymbol{xy}} - \varphi^n_{\boldsymbol{xy}} = -\frac{N_\mathrm{c}}{2}\frac{\epsilon\alpha_s}{\pi^2}\int_{\boldsymbol{z}} \tilde{\mathcal{K}}_{\boldsymbol{xyz}}(\varphi^n_{\boldsymbol{xy}} - \varphi^n_{\boldsymbol{xz}} - \varphi^n_{\boldsymbol{zy}})$$

$\rightarrow$ "Mueller's BFKL equation"