The picture is from M.Shifman’s live journal.
On correlators of Reggeon fields in high energy QCD

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Based on:
S. Bondarenko, L. Lipatov and A. Prygarin,
S. Bondarenko, L. Lipatov, S. Pozdnyakov and A. Prygarin,
S. Bondarenko, M. Zubkov,
S. Bondarenko, S. Pozdnyakov,
S. Bondarenko, S. Pozdnyakov,
Effective action setup

- Consider QCD action with added sources of longitudinal gluons:

\[
S_{\text{eff}} = - \int d^4 x \left( \frac{1}{4} F_{\mu \nu}^{a} F_{a}^{\mu \nu} + tr \left[ v_+ J^+(v_+) - A_+ j_{\text{reg}}^+ + v_- J^-(v_-) - A_- j_{\text{reg}}^- \right] \right)
\]

The real currents \( J_a^{\pm} (v_{\pm}) \) under a variation with respect to the gluon fields behave as

\[
\delta \left( v_\pm J^{\pm}(v_{\pm}) \right) = (\delta v_{\pm}) j_{\mp}^{\text{ind}}(v_{\pm}) = (\delta v_{\pm}) j^{\pm}(v_{\pm})
\]

where induced currents possess the covariant conservation property:

\[
\left( D_{\pm} j_{\mp}^{\text{ind}}(v_{\pm}) \right)^a = (D_{\pm} j^{\pm}(v_{\pm}))^a = 0
\]

Additionally we require:

\[
v_{+ \text{cl}} = A_+ , \quad v_{- \text{cl}} = A_- , \quad \partial_- A_+ = \partial_+ A_- = 0
\]

and that is enough (almost) for the determination of the form of \( J_a^{\pm} (v_{\pm}) \) currents.
The construction above is equivalent to the construction of the Lipatov’s gauge-invariant action local in rapidity interval \((y_0 - \eta, y_0 + \eta)\) for the gluon-reggeon interactions with \(A_{\pm}\) as reggeon fields similar to the Gribov’s RFT:

\[
S_{\text{eff}} = - \int d^4x \left( \frac{1}{4} G_{\mu\nu}^a G_{a}^{\mu\nu} + \text{tr} \left[ \left( \mathcal{T}_+(v_+) - A_+ \right) j_{\text{reg}}^+ + \left( \mathcal{T}_-(v-) - A_- \right) j_{\text{reg}}^- \right] \right)
\]

\[
\mathcal{T}_\pm(v_\pm) = \frac{1}{g} \partial_\pm O(x_\pm, v_\pm) = v_\pm O(x_\pm, v_\pm),
\]

\[
j_{\text{reg}a}^\pm = \frac{1}{C(R)} \partial_i^2 A_\pm^a, \quad \partial_+ A_+ = \partial_- A_- = 0,
\]

where \(C(R)\) is the eigenvalue of Casimir operator in the representation \(R\), \(C(R) = N\) in the case of adjoint representation and \(O\) as some operators of Wilson lines. See in:


Effective action setup

• In the simplest case:

\[ O_x = P e^{g \int_{-\infty}^{x^+} dx' + v_+(x')}, \quad O_x^T = P e^{g \int_{x^+}^{\infty} dx' + v_+(x')} \]

are usual ordered exponentials and

\[ \delta (v_+ J^+) = \delta \text{tr}[(v_+ x O_x \partial_i^2 A^+)] = -\delta v^a_+ \text{tr}[T_a O T_b O^T] \left( \partial_i^2 A_b^+ \right) \]

• Correspondingly, the new gluon-reggeon vertices are arising in the action; it can be used for the diagrammatic construction of the amplitudes with gluon and reggeon fields:

Regge Field Theory (RFT) construction

- Expanding around the classical solutions

\[ v_i^a \rightarrow v_{i\text{cl}}^a + \varepsilon_i^a, \quad v_+^a \rightarrow v_{+\text{cl}}^a + \varepsilon_+^a \]

and integrating over the fluctuations we obtain Regge Field Theory (RFT) effective action:

\[ \Gamma = - \int d^4 x \ (s_1[g, A_+, A_-] + g_2 s_2[g, A_+, A_-] + \cdots) \]

which can be considered as generating functional of the Reggeons interaction vertices:

\[ \Gamma = \sum_{n,m=1} A_+^{a_1} \cdots A_+^{a_n} K_+^{b_1 \cdots b_m} A_-^{b_1} \cdots A_-^{b_m} \]

- The perturbative order and precision of the effective vertices (kernels) \( K \) in \( \Gamma \) are determined by QCD degrees of freedom only. It is possible to fix the precision and consider the corrections from the RFT sector exclusively.
Lipatov’s effective action from another angle of view

- Consider two Lipatov’s operators interacting in eikonal approximation and averaged over the gluon fields:

\[
Z[J] = \frac{1}{Z'} \int Dv \exp \left( i S^0[v] + \frac{i}{2 C(R)} \int d^4 x \mathcal{T}_+ \partial^2 \mathcal{T}_- + \right.
\]

\[
+ \frac{i}{2 C(R)} \int d^4 x J_-(x^-, x_\perp) \mathcal{T}_+ + \left. \frac{i}{2 C(R)} \int d^4 x J_+(x^+, x_\perp) \mathcal{T}_- \right)
\]

with

\[
\mathcal{T}_\pm(v_\pm) = \frac{1}{g} \partial_\pm O(v_\pm) = v_\pm O(v_\pm),
\]


This reformulation of the Lipatov’s effective action be used for establishing of a connection between the correlators of the Reggeon fields and Wilson lines (Balitsky-JIMWLK approach).
Schwinger-Dyson equations for the correlators

• Lipatov’s effective action generating functional variation with respect to Reggeon fields:

\[ \delta Z[J] = \int DA \delta A_\pm \left( \frac{\delta \Gamma[A]}{\delta A_\pm} - \int dx^{\mp} J_\pm^a(x^{\mp}, x_\perp) \right) \exp(\imath \Gamma[A] - \imath \int d^4x J_\pm^a A_\pm^a - \imath \int d^4x J_\pm^a A_\pm^a) = 0 \]

• One-field correlator:

\[ \langle \frac{\delta \Gamma[A]}{\delta A_\pm^a} \rangle = 0 \]

provides

\[ \langle A_\pm^a(x, \eta) \rangle = \sum_{n=1} \left( \tilde{K}(\eta; x, x_1 \cdots, x_n) \right)^{a_a_1 \cdots a_n} \langle A_\pm^{a_1}(x_1) \cdots A_\pm^{a_n}(x_n) \rangle, \]

The BFKL like evolution is obtained by taking the derivative with respect to the \( \eta \):

\[ \frac{\partial}{\partial \eta} \langle A_\pm^a \rangle = \frac{\partial}{\partial \eta} \left( \sum_{n=1} \left( \tilde{K}(\eta) \right)^{a_a_1 \cdots a_n} \langle A_\pm^{a_1} \cdots A_\pm^{a_n} \rangle \right) \]
Schwinger-Dyson equations for correlators of Reggeons

• Taking derivative with respect to currents:

\[
< \frac{\delta \Gamma[A]}{\delta A_{\pm}^{a_1}} A_{\pm}^{a_2} - \imath \delta^{a_1 a_2} \delta_{\pm 1} \pm 2 \delta(x_{\perp 1} - x_{\perp 2}) >= 0.
\]

that provides

\[
\partial_{\perp 1}^2 < A_{\pm}^{a_1} A_{\mp}^{a_2} > = - \imath \delta^{a_1 a_2} \delta_{\pm 1} \pm 2 \delta(x_{\perp 1} - x_{\perp 2}) + \left( K_{b_1}^{a_1} \right)^{\pm} < A_{\pm}^{b_1} A_{\mp}^{a_2} > + \ldots
\]

• In general we have:

\[
< A_{\pm}^{a_1} A_{\mp}^{a_2} \cdots A_{\pm}^{a_m} > = \sum_{n=1} \left( \hat{K}(\eta) \right)^{a_{b_1} \cdots b_n} < A_{\pm}^{b_1} \cdots A_{\pm}^{b_n} A_{\pm}^{a_1} \cdots A_{\pm}^{a_m} >
\]

or in the form of evolution equation:

\[
\frac{\partial}{\partial \eta} < A_{\pm}^{a_1} A_{\mp}^{a_2} \cdots A_{\pm}^{a_m} > = \frac{\delta}{\delta \eta} \sum_{n=1} \left( \hat{K}(\eta) \right)^{a_{b_1} \cdots b_n} < A_{\pm}^{b_1} \cdots A_{\pm}^{b_n} A_{\pm}^{a_1} \cdots A_{\pm}^{a_m} >
\]
Schwinger-Dyson equations for correlators of two Reggeons

For two Reggeon fields correlator we have:

\[ \partial_\perp^2 < B^a_+ B^{a_1}_- > = -i \delta^{aa_1} + (K^{a_2}_a)_+ < B^a_+ B^{a_1}_- > + 2 (K^{a_2}_a)_- < B^{a_2}_- B^{a_1}_- > + \]
\[ + (K^{a_2 a_3}_a)^{++} < B^a_+ B^{a_2}_+ B^{a_1}_- > + 2 (K^{a_2 a_3}_a)^{+} < B^{a_2}_+ B^{a_3}_+ B^{a_1}_- > + \]
\[ + 3 (K^{a_2 a_3 a_4}_a)^{---} < B^{a_3}_- B^{a_2}_- B^{a_1}_- > + (K^{a_2 a_3 a_4}_a)^{+++} < B^{a_2}_+ B^{a_3}_+ B^{a_4}_+ B^{a_1}_- > + \]
\[ + 2 (K^{a_3 a_4}_a)^{++} < B^a_+ B^{a_4}_+ B^{a_2}_- B^{a_1}_- > + 3 (K^{a_4}_a a_2 a_3)^{+} < B^{a_4}_+ B^{a_3}_- B^{a_2}_- B^{a_1}_- > + \]
\[ + 4 (K^{a_2 a_3 a_4}_a)^{---} < B^{a_4}_- B^{a_3}_- B^{a_2}_- B^{a_1}_- > + \cdots \]
QCD propagator of reggeized gluons

• “Usual” reggeized correlator (two first terms in r.h.s. of above expression):

\[ \partial_{\perp}^2 < A_+^a A_{-1}^a > = -i \delta^{aa_1} + (K_{a2}^{a_2})^+ < A_+^{a_2} A_{-1}^{a_1} > \]

or in terms of propagator

\[ < A_+^a (x^+, x_\perp) A_-^b (y^-, y_\perp) > = i \delta(x^+) \delta(y^-) \delta^{ab} D(x_\perp, y_\perp) \]

\[ D^{ac}_{xy} = D^{ac}_{xy0} - \int d^4z \int d^4w D^{ab}_{xz0} \left( \sum_{k=1} K^{bd}_{zwk} \right) D^{dc}_{wy} \]

The equation after Fourier transform:

\[ \tilde{D}^{ab}(p_\perp, \eta) = \frac{\delta^{ab}}{p_\perp^2} + \epsilon(p_\perp^2) \int_0^\eta d\eta' \tilde{D}^{ab}(p_\perp, \eta') \]

\[ \tilde{D}^{ab}(p_\perp, \eta) = \frac{\delta^{ab}}{p_\perp^2} e \eta \epsilon(p_\perp^2) , \quad \epsilon(p_\perp^2) = -\frac{\alpha_s N}{4\pi^2} \int d^2k_\perp \frac{p_\perp^2}{k_\perp^2 (p_\perp - k_\perp)^2} \]
We can take into account LO RFT contribution to the propagator with bare QCD vertices:

\[
\begin{align*}
(K^{abc}_{xyz})^{++-}_0 &= \frac{1}{2} g f^{abc} (\tilde{G}^{+0}_{x+y} - \tilde{G}^{+0}_{y+x}) \delta^2(z_\perp - x_\perp) \delta^2(z_\perp - y_\perp) \partial^2_{iz} \\
(K^{abc}_{xyz})^{+-+}_0 &= -g f^{abc} (\tilde{G}^{-0}_{y-z} - \tilde{G}^{-0}_{z-y}) \delta^2(y_\perp - x_\perp) \delta^2(z_\perp - x_\perp) \partial^2_{ix}
\end{align*}
\]

obtaining

\[
\begin{align*}
\partial^2_{ix} < A^a_+ (x^+, x_\perp) A^{a_1}_- (y^-, y_\perp) > &= -i \delta^{aa_1} \delta(x^+) \delta(y^-) \delta^2(x_\perp - y_\perp) + \\
+ \int d^2z_\perp \int d^2z_{1 \perp} \\
K^{b_1 b_2 a}_{++-} (x^+, z_\perp; z^+_1, z_{1 \perp}; x_\perp) < A^{b_1}_+ (x^+, z_\perp) A^{b_2}_+ (z^+_1, z_{1 \perp}) A^{a_1}_- (y^-, y_\perp) > + \\
+ \int d^2z_\perp \int d^2z_{1 \perp} \\
K^{b_1 b_2 a}_{+-+} (z_\perp; z^-_1, z_{1 \perp}; x^-, x_\perp) < A^{b_1}_+ (x^+, z_\perp) A^{b_2}_- (z^-_1, z_{1 \perp}) A^{a_1}_- (y^-, y_\perp)>
\end{align*}
\]
One loop RFT (bare QCD) propagator of reggeized gluons

- Have to consider 4-d Regeon fields:

\[
A_+(x^+, x_\perp) \rightarrow B_+(x^+, x^-, x_\perp) = A_+(x^+, x_\perp) + D_+(x^+, x^-, x_\perp)
\]

\[
A_-(x^-, x_\perp) \rightarrow B_-(x^+, x^-, x_\perp) = A_-(x^-, x_\perp) + D_-(x^+, x^-, x_\perp)
\]

\[
D_+(x^+, x^- = 0, x_\perp) = D_-(x^+ = 0, x^-, x_\perp) = 0
\]

that provides:

\[
\partial_{\perp x} < D^a_+(x^+, x^-, x_\perp) A^{a_1}_-(y^-, y_\perp) > = \int d^2 z_\perp \int d^2 z_1^+ d^2 z_1^\perp \\
K^{b_1 b_2 a}_{++--}(x^+, z_\perp; z^+_1, z^-_1; x_\perp) < A^{b_1}_+(x^+, z_\perp) A^{b_2}_-(z^+_1, z^-_1) A^{a_1}_-(y^-, y_\perp) > + \\
+ \int d^2 z_\perp \int d^2 z^-_1 d^2 z^\perp_1 \\
K^{b_1 b_2 a}_{+--}(z_\perp; z^-_1, z^-_1; x^-, x_\perp) < A^{b_1}_+(x^+, z_\perp) A^{b_2}_-(z^-_1, z^-_1) A^{a_1}_-(y^-, y_\perp) >
\]
One loop RFT (bare QCD) propagator of reggeized gluons

Three field correlator:

$$\partial_\perp^2 x < A_+^a(x^+, x_\perp) A_{+1}^{a1}(y^+, y_\perp) A_{+2}^{a2}(z^-, z_\perp) > =$$

$$\int d^2w_\perp dw_1^- d^2w_1 \perp K_{+ - - -}^{a4a3}(w_\perp; x^-, x_\perp; w_1^-, w_1 \perp)$$

$$< A_+^a(x^+, w_\perp) A_{+1}^{a1}(y^+, y_\perp) A_{+2}^{a2}(z^-, z_\perp) A_{+3}^{a3}(w_1^-, w_1 \perp) >$$

and four field correlator

$$< A_+^a(x^+, w_\perp) A_{+1}^{a1}(y^+, y_\perp) A_{+2}^{a2}(z^-, z_\perp) A_{+3}^{a3}(w_1^-, w_1 \perp) > =$$

$$= i\delta^{a4a3} \delta(x^+) \delta(w_1^-) D_0(w_\perp, w_1 \perp) < A_{+1}^{a1} A_{+2}^{a2} > +$$
$$+ i\delta^{a4a2} \delta(x^+) \delta(z^-) D_0(w_\perp, z_\perp) < A_{+1}^{a1} A_{+3}^{a3} >$$

we obtain

$$< A_+^a(x^+, x_\perp) A_{+1}^{a1}(y^+, y_\perp) A_{+2}^{a2}(z^-, z_\perp) > = -i \int d^2w_\perp \int dw_1^- d^2w_1 \perp \int d^2w_2 \perp$$

$$D_0(x_\perp, w_\perp) K_{+ - - -}^{a2a3}(w_\perp; x^-, w_\perp; w_1^-, w_1 \perp) D_0(w_2 \perp, z_\perp)$$

$$< A_{+1}^{a1}(y^+, y_\perp) A_{+3}^{a3}(w_1^-, w_1 \perp) > \delta(x^+) \delta(z^-),$$
**One loop RFT (bare QCD) propagator of reggeized gluons**

- In terms of propagators:

\[
< A^a_+(x^+, x_\perp) A^b_-(y^-, y_\perp) > = i \delta(x^+) \delta(y^-) \delta^{ab} D(x_\perp, y_\perp)
\]

\[
< D^a_+(x^+, x^-, x_\perp) A^{a1}_-(y^-, y_\perp) > = i G^{aa1}_+(x^+, x^-, x_\perp; y^-, y_\perp),
\]

finally we obtain

\[
G^{aa1}_+(x^+, x^-, x_\perp; y^-, y_\perp) =
\]

\[
= -\frac{i}{2} g^2 N \delta^{aa1} \left( \tilde{G}^{+0}_{x_+^0} - \tilde{G}_0^{+0} \right) \left( \tilde{G}^{-0}_{x_-^0} - \tilde{G}_0^{-0} \right) (D_0(x_\perp, y_\perp) D(x_\perp, y_\perp) -
\]

\[
- \int d^2 w_\perp \int d^2 z_\perp D_0(x_\perp, z_\perp) \left( \partial^2_{iz} \partial^2_{iw} D_0(z_\perp, w_\perp) \right) D_0(w_\perp, z_\perp) D(w_\perp, y_\perp) -
\]

\[
- \int d^2 w_\perp \int d^2 z_\perp D_0(x_\perp, z_\perp) \left( \partial^2_{iz} \partial^2_{iw} D_0(z_\perp, w_\perp) \right) D_0(w_\perp, y_\perp) D(w_\perp, z_\perp) \right) \delta(x^+) \delta(y^-)
\]

The only first term provides non-zero contribution.
One loop RFT (bare QCD) propagator of reggeized gluons

- We have introducing rapidity variable $y = \frac{1}{2} \ln(\Lambda k_+)$

$$\tilde{G}^{a a_1}_+(p_+, p_\perp; x^-; \eta) = g^2 N \delta^{a a_1} \left( \tilde{G}^{-0}_{x^-0} - \tilde{G}^{-0}_{0x^-} \right) \int \frac{dy}{2\pi} \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{e^{(\eta-y)\epsilon(k_\perp^2)}}{k_\perp^2 (p_\perp - k_\perp)^2}$$

and taking $\eta \to Y = Y = \ln(s/s_0)$ we obtain:

$$\tilde{G}^{a a_1}_+(p_+, p_\perp; x^-; Y) = \frac{g^2 N}{4\pi^3} \delta^{a a_1} \left( \tilde{G}^{-0}_{x^-0} - \tilde{G}^{-0}_{0x^-} \right) \int d^2 k_\perp \frac{\left(e^{\epsilon(k_\perp^2)Y} - 1\right)}{k_\perp^2 (p_\perp - k_\perp)^2 \epsilon(k_\perp^2)}$$

that after the integration on $k$ provides:

$$\tilde{G}^{a a_1}_+(p_+, p_\perp; x^-; Y) = \frac{2 \delta^{a a_1}}{p_\perp^2} \left( 1 - e^{\epsilon(p_\perp^2)Y} - \int_0^{-\epsilon(p_\perp^2)Y} dy \frac{e^{-y} - 1}{Y} \right) \left( \tilde{G}^{-0}_{x^-0} - \tilde{G}^{-0}_{0x^-} \right)$$
One loop RFT propagator of reggeized gluons with one loop QCD correction

• Including one QCD loop in all the correlators:

\[ \tilde{G}^{aa_1}_+ (p_+, p_\perp; x^-; Y) = \frac{2 \delta^{aa_1}}{p_\perp^2} \left( e^{\epsilon(p_\perp^2) Y} - e^{2 \epsilon(p_\perp^2) Y} - \int_{-\epsilon(p_\perp^2) Y}^{\epsilon(p_\perp^2) Y} dy \frac{e^{-y} - 1}{y} \right) \left( \tilde{G}^{-0}_{x^- 0} - \tilde{G}^{-0}_{0 x^-} \right) \]
Conclusion:

- Lipatov’s effective action provides systematical calculations of unitary corrections to the scattering amplitudes at high energy in the framework of the QCD RFT;

- First source of the corrections are QCD loops contributions to the vertices of the effective action;

- Additional source of the unitary corrections are the Regge Field Theory (RFT) loop’s contributions. Lipatov’s effective action can be considered as QCD variant of RFT and provides the base for these RFT calculations;

- Formulated as RFT, the Lipatov’s effective action provides non-linear unitary corrections to the correlators of Reggeon fields on the base of Dyson-Schwinger hierarchy of equations for the correlators (similar to Balitsky hierarchy);

- These non-linear corrections are not small and violate the reggeization of usual QCD propagator of reggeized gluons;

- There are interesting problems concerning the structure of Pomeron and properties of QCD RFT as field theory and etc.
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One loop RFT propagator of reggeized gluons with one loop QCD correction

Part of one-loop triple Reggeon vertex:

\[-2i K^{abc}_{x,y,z,1,2} = \]

\[-\frac{i g^3 N}{2 (2\pi)^5} f_{abc} \partial^2_{i,y} \]

\[\left( \theta(z^+ - x^+) \delta^2_{x z} y \int \frac{dk^-}{k^-} \int d^2 k_\perp \int d^2 k_{1 \perp} \frac{k^2_{1 \perp}}{k^2_\perp (k_\perp - k_{1 \perp})^2} e^{-i (x^i - z^i) k_{1 \perp}} \right) + \]

\[+ \frac{i g^3 N}{2 (2\pi)^5} f_{abc} \partial^2_{i,y} \]

\[\left( \theta(x^+ - z^+) \delta^2_{x z} y \int \frac{dk^-}{k^-} \int d^2 k_\perp \int d^2 k_{1 \perp} \frac{k^2_{1 \perp}}{k^2_\perp (k_\perp - k_{1 \perp})^2} e^{-i (z^i - x^i) k_{1 \perp}} \right) \]