# One-loop corrections to multiscale effective vertices in the EFT for Multi-Regge processes in QCD

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DIS-2019,
Torino,
8-12 April 2019.

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# Outline.

The talk is based on [hep-ph/1902.11030].

1. Motivation
2. Introduction to Lipatov's EFT
3. One-loop rapidity-divergent integrals
4. Computation of $\gamma^\star Qq$ and $gRg$ vertices
5. Comparison with QCD

# Motivation

- The gauge-invariant EFT for Multi-Regge processes in QCD, which includes *Reggeized gluons* [Lipatov; 1995] and *Reggeized quarks* [Lipatov, Vyazovsky; 2001] has been introduced as a systematic tool to *compute and resum* the higher-order corrections in QCD, enhanced by $\log(s/(-t))$, with the arbitrary $N^k LL$ accuracy.
- Another motivation is the *unitarization program* for high-energy scattering. The BFKL equation at the fixed logarithmic accuracy predicts *power-like* growth of the cross-section with $s$, which violates Froissart bound ($\Leftarrow$ Unitarity). The basic idea is to write-down the *Hermitian* effective Lagrangian for QCD at high energies, so that Unitarity will hold automatically.

# Motivation

- Currently, a number of approaches is developed with the aim of taking into account both DGLAP and BFKL effects. Many of them try to generalize the amplitudes from the Lipatov's EFT to the Soft and Collinear regions (e.g. PRA [M.N., V.A.S., *et. al.*] or HEJ [J. Andersen, *et. al.*] approaches, `KaTie` [A. van Hameren, *et. al.*] Monte-Carlo code) or incorporate BFKL effects into the framework of SCET (e.g. [I. Stewart, I. Rothstein, 2016]). Going beyond tree level is an important part of this activity.
- In the talk I would like to describe the one-loop structiure of Lipatov's EFT. The complete picture, similar to one in ordinary QCD, emerges.

# Introduction to Lipatov's EFT

# Sudakov (light-cone) decomposition of momenta.

It is convenient to relate the basis vectors of Sudakov decomposition with (almost) light-like momenta of colliding highly energetic particles ($P_{1,2}^2 = 0$):

$$n_-^\mu = \frac{2P_1^\mu}{\sqrt{S}},\ n_+^\mu = \frac{2P_2^\mu}{\sqrt{S}},\ S = 2P_1P_2 \Rightarrow n_+n_- = 2.$$

Then for any four-vector $k^\mu$ one has:

$$\boxed{k^\mu = \frac{1}{2}\left(k_+n_-^\mu + k_-n_+^\mu\right) + k_T^\mu,}$$

where $k_\pm = k^\pm = n_\pm k$, $n_\pm k_T = 0$. For the dot-product one has:

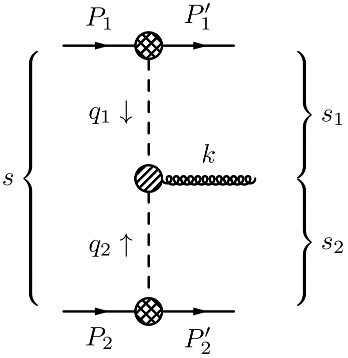
$$kq = \frac{1}{2}\left(k_+q_- + k_-q_+\right) - \mathbf{k}_T\mathbf{q}_T,\ \boxed{k^2 = k_+k_- - \mathbf{k}_T^2.}$$

Rapidity:

$$\boxed{\boxed{y = \frac{1}{2}\log\left(\frac{q^+}{q^-}\right).}}$$

# Multi-Regge Kinematics.

At high energies, $t$-channel exchange diagrams with Multi-Regge(MRK) or Quasi-Multi Regge(QMRK) Kinematics of the final-state dominate in the $2 \to 2 + n$ amplitude.




Double Regge limit (MRK):

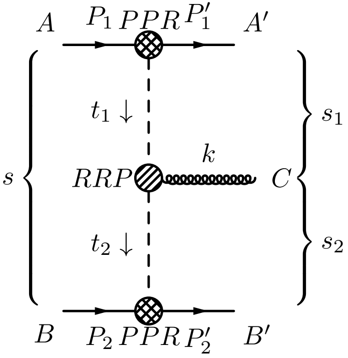
$$s_1 \gg -q_1^2, \; s_2 \gg -q_2^2,$$

momentum fractions $z_1 = q_1^+ / P_1^+$, $z_2 = q_2^- / P_2^-$.

**Properties of MRK**:

- $y(P_1') \to +\infty$, $y(P_2') \to -\infty$, $y(k)$ – finite,
- $\boxed{z_1 \sim z_2 \sim z \ll 1}$, $|\mathbf{k}_T| \ll \sqrt{s}$ ,
- $q_1^+ \sim |\mathbf{q}_{T1}| \sim O(z) \gg q_1^- \sim O(z^2)$, $q_2^- \sim |\mathbf{q}_{T2}| \sim O(z) \gg q_2^+ \sim O(z^2)$.

# Reggeization of amplitudes in QCD.




In MRK asymptotics, $2 \to 3$-amplitude factorizes (up to $O(z^{\#}_{1,2})$):

$$\mathcal{A}_{AB}^{A'B'C} = \gamma^{R_1}_{A'A} \cdot \left(\frac{s_1}{s_0}\right)^{\omega(t_1)} \frac{-i}{2t_1} \times$$

$$\Gamma^{C}_{R_1 R_2}(q_1, q_2) \cdot \frac{-i}{2t_2} \left(\frac{s_2}{s_0}\right)^{\omega(t_2)} \cdot \gamma^{R_2}_{B'B}$$

$\Gamma^{C}_{R_1 R_2}(q_1, q_2)$ – $RRP$ production vertex,

$\gamma^{R}_{A'A}$ – $PPR$-scattering vertex,

$\omega(t)$ - Regge trajectory.

Two ways to obtain this asymptotics:

- BFKL-approach (Unitarity, renormalizability and gauge invariance), see. [Ioffe, Fadin, Lipatov, 2010].
- Effective action approach [Lipatov, 1995; Lipatov, Vyazovsky, 2001].

# Structure of the EFT.

Light-cone derivatives:

$$\partial_\pm = n_\pm^\mu \partial_\mu = 2\frac{\partial}{\partial x^\mp}$$

EFT Lagrangian [Lipatov, 1995]:

$$L = L_{\rm kin} + \sum_i \left[ L_{QCD}^{(y_i \leq y \leq y_{i+1})} + L_R^{(y_i \leq y \leq y_{i+1})} \right],$$

the separate copy of $L_{QCD}^{(y_i \leq y \leq y_{i+1})}$ lives in each interval in rapidity $y_i \leq y \leq y_{i+1}$. Different intervals interact via Reggeon exchanges ($R_\pm^a = R_\pm^a T_a$):

$$L_{\rm kin} = 2\partial_\mu R_+^a \partial^\mu R_-^a,$$

kinematic constraints on Reggeon-fields ($\Leftrightarrow$ QMRK):

$$\partial_- R_+ = \partial_+ R_- = 0 \Rightarrow$$

$R_+$ carries $(k_+, \mathbf{k}_T)$ and $R_-$ carries $(k_-, \mathbf{k}_T)$.

# (Semi-)Infinite light-like Wilson lines

Particles highly separated in rapidity "perceive" each-other as light-like Wilson lines.

$$W_{x^\mp}[A_\pm] = \quad P\exp\left[\frac{-ig_s}{2}\int_{-\infty}^{x_\mp} dx'_\mp A_\pm(x_\pm, x'_\mp, \mathbf{x}_T)\right] = \left(1 + ig_s\partial_\pm^{-1}A_\pm\right)^{-1},$$

$$W^\dagger_{x^\mp}[A_\pm] = \quad \overline{P}\exp\left[\frac{ig_s}{2}\int_{-\infty}^{x_\mp} dx'_\mp A_\pm(x_\pm, x'_\mp, \mathbf{x}_T)\right] = \overline{P}\left(1 - ig_s\partial_\pm^{-1}A_\pm\right)^{-1},$$

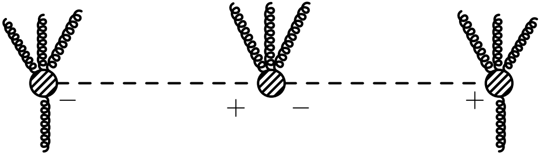
Notation for ordered integrals:

$$\frac{1}{2^n}\int_{-\infty}^{x^\mp} dx_1^\mp f_1(x_1^\mp)\int_{-\infty}^{x_1^\mp} dx_2^\mp f_2(x_2^\mp)\ldots\int_{-\infty}^{x_{n-1}^\mp} dx_n^\mp f_n(x_n^\mp) = \underbrace{\partial_\pm^{-1}f\ldots\partial_\pm^{-1}f}_{n}.$$

In the Feynman rules:

$$\boxed{\partial_\pm^{-1} \to \frac{-i}{k^\pm + i\varepsilon}.}$$

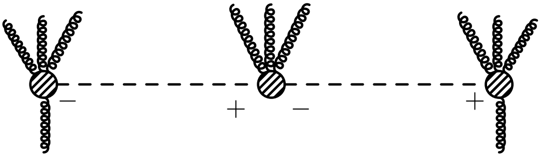
# Basic structure of Induced interactions.




**Induced interactions** of particles and Reggeons [Lipatov, 1995]:

$$
\begin{aligned}
L_R^{(y_1<y<y_2)}(x) \quad &\supset \quad \frac{i}{g_s}\,\mathrm{tr}\Big(R_+(x)\partial_\rho^2\partial_- W_x\left[A_-^{(y_1<y<y_2)}\right] \\
&+ \quad R_-(x)\partial_\rho^2\partial_+ W_x\left[A_+^{(y_1<y<y_2)}\right]\Big)\,.
\end{aligned}
$$

# Basic structure of Induced interactions.




**Induced interactions** of particles and Reggeons:

$$L_R^{(y_1<y<y_2)} \supset \frac{i}{g_s} \mathrm{tr} \left[ R_+ \partial_\rho^2 \partial_- W \left[ A_-^{(y_1<y<y_2)} \right] + R_- \partial_\rho^2 \partial_+ W \left[ A_+^{(y_1<y<y_2)} \right] \right],$$

expansion of $P$-exponent generaties induced vertices:

$$\begin{aligned} L_R \quad \supset \quad & \mathrm{tr} \left[ \left( R_+ \partial_\sigma^2 A_- + R_- \partial_\sigma^2 A_+ \right) + \right. \\ & (-ig_s)(\partial_\sigma^2 R_+)(A_- \partial_-^{-1} A_-) + (-ig_s)^2 (\partial_\sigma^2 R_+)(A_- \partial_-^{-1} A_- \partial_-^{-1} A_-) + \\ & (-ig_s)(\partial_\sigma^2 R_-)(A_+ \partial_+^{-1} A_+) + (-ig_s)^2 (\partial_\sigma^2 R_-)(A_+ \partial_+^{-1} A_+ \partial_+^{-1} A_+) \\ & \left. + O(g_s^3) \right], \end{aligned}$$

but this structure is non-Hermitian: $R_+$ – Hermitian, $W$ – Unitary!

# Hermitian effective action and pole prescription

Recently the new derivation of effective action has been proposed [Bondarenko, Zubkov, 2018] which fixes the **Hermitian** form of Reggeon-gluon interaction:

$$\frac{i}{g_s}\,\mathrm{tr}\left[R_+\partial_\rho^2\partial_-\left(W\left[A_-\right]-W^\dagger\left[A_-\right]\right)\right],$$

E.g. $Rgg$-vertex:

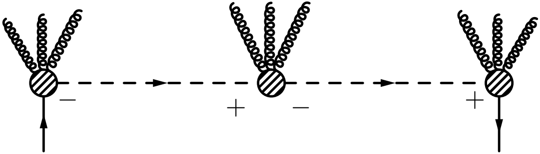
$$\frac{-ig_s}{2}\left(\partial_\sigma^2 R_+^a(x)\right)\left(A_-^{b_1}(x)\int\limits_{-\infty}^{x_-}dx_1^- A_-^{b_2}(x_1)\right)\mathrm{tr}\left[T^a\left[T^{b_1},T^{b_2}\right]\right],$$

$\Rightarrow$ Feynman rule:

$$g_s(-q^2)f^{ab_1b_2}(n_-^{\mu_1}n_-^{\mu_2})\frac{1}{2}\left[\frac{1}{k_1^-+i\varepsilon}+\frac{1}{k_1^--i\varepsilon}\right]=g_s(-q^2)(n_-^{\mu_1}n_-^{\mu_2})\frac{f^{ab_1b_2}}{[k_1^-]},$$

i.e. the PV-prescription for the $1/k^\pm$ poles for simplest induced vertices. For more complicated induced vertices, Hermitian $Rg$-interaction leads to pole prescriptions generalizing that of [Hentschinski, 2013].

# EFT for QMRK-processes with quark exchange.



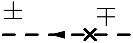

EFT for Reggeized quarks [Lipatov, Vyazovsky, 2001]:

$$L_Q = \bar{Q}_- i\hat{\partial}\left(Q_+ - W^\dagger\left[A_+\right]\psi\right) + \bar{Q}_+ i\hat{\partial}\left(Q_- - W^\dagger\left[A_-\right]\psi\right) + \text{h.c.},$$

where $\hat{p} = p_\mu \gamma^\mu$, QMRK kinematic constraints:

$$\partial_\pm Q_\mp = \partial_\pm \bar{Q}_\mp = 0,$$
$$\hat{n}^\pm Q_\mp = 0,\ \bar{Q}_\mp \hat{n}^\pm = 0. \Rightarrow$$

Reggeized quark propagator ($\hat{P}_\pm = \hat{n}_\mp \hat{n}_\pm / 4$):

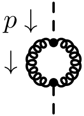
$= \hat{P}_\pm \dfrac{i\hat{q}}{q^2},$


$= \dfrac{i\hat{q}}{q^2}\hat{P}_\pm.$

# Rapidity divergences and regularization.

Due to the presence of the $1/q^{\pm}$-factors in the induced vertices, loop integrals in EFT contain the light-cone (Rapidity) divergences:

$$\Sigma^{(1)}_{ab} = q \downarrow$$




$$= g_s^2 C_A \delta_{ab} \int \frac{d^d q}{(2\pi)^D} \frac{\left(\mathbf{p}_T^2 (n_+ n_-)\right)^2}{q^2 (p-q)^2 q^+ q^-}$$

The regularization by explicit cutoff in rapidity was proposed by Lipatov [Lipatov, 1995] ($q^{\pm} = \sqrt{q^2 + \mathbf{q}_T^2} e^{\pm y}$):

$$\int \frac{dq^+ dq^-}{q^+ q^-} = \int_{y_1}^{y_2} dy \int \frac{dq^2}{q^2 + \mathbf{q}_T^2},$$

then

$$\Sigma^{(1)}_{ab} \sim \delta_{ab} \mathbf{p}_T^2 \times \underbrace{C_A g_s^2 \int \frac{\mathbf{p}_T^2 d^{D-2} \mathbf{q}_T}{\mathbf{q}_T^2 (\mathbf{p}_T - \mathbf{q}_T)^2}}_{\omega^{(1)}(\mathbf{p}_T^2)} \times (y_2 - y_1) + \text{finite terms}$$

# Rapidity divergent one-loop integrals

# Covariant regularization.

The regularization and **pole prescription** was introduced in a series of papers [Hentschinski, Sabio Vera, Chachamis *et. al.*, 2012-2013], also known in TMD factorization as "tilted Wilson lines" [Collins, 2011].

Regularization of the light-cone divergences is achieved by shifting $n^{\pm}$ vectors from the light-cone:

$$\boxed{\tilde{n}^{\pm} = n^{\pm} + r \cdot n^{\mp},\ \tilde{k}^{\pm} = k^{\pm} + r \cdot k^{\mp},\ r \to 0,}$$

and for the lowest-order($Rgg$, $Qqg$) induced vertices the PV prescription is at work:

$$I^{[\pm]}:\ \frac{1}{[\tilde{k}^{\pm}]} = \frac{1}{2}\left(\frac{1}{\tilde{k}^{\pm} + i\varepsilon} + \frac{1}{\tilde{k}^{\pm} - i\varepsilon}\right),$$

# Regularization and gauge-invariance

Regularization should preserve the gauge-invariance of Reggeon-gluon interactions:

$$S_{Rg}^{(-)} = \int d^2\mathbf{x}_T \int\limits_{-\infty}^{+\infty} \frac{dx_+ dx_-}{2} \,\mathrm{tr}\left[R^- \tilde{\partial}_+ \partial_\sigma^2 W_{\tilde{x}_-}[\tilde{A}_+]\right]$$

$$= \int d^2\mathbf{x}_T \int\limits_{-\infty}^{+\infty} \frac{d\tilde{x}_+ d\tilde{x}_-}{1-r^2} \,\mathrm{tr}\left[R^- \frac{\partial}{\partial \tilde{x}_-} \partial_\sigma^2 W_{\tilde{x}_-}[\tilde{A}_+]\right] =$$

$$= \int d^2\mathbf{x}_T \int\limits_{-\infty}^{+\infty} \frac{d\tilde{x}_+ d\tilde{x}_-}{1-r^2} \left\{ \frac{\partial}{\partial \tilde{x}_-} \,\mathrm{tr}\left[R^- \partial_\sigma^2 W_{\tilde{x}_-}[\tilde{A}_+]\right] - \frac{1}{2} \,\mathrm{tr}\left[\left(\tilde{\partial}_+ R_-\right) \partial_\sigma^2 W_{\tilde{x}_-}[\tilde{A}_+]\right] \right\}.$$

First term – *infinite Wilson line* is gauge invariant (w.r.t. gauge transformations trivial at $\infty$) $\Rightarrow$ **new kinematic constraint**:

$$\boxed{\tilde{\partial}_+ R_- = \tilde{\partial}_- R_+ = 0,}$$

or $\tilde{p}^+ = 0$ for $R_-$ and $\tilde{p}^- = 0$ for $R_+$.

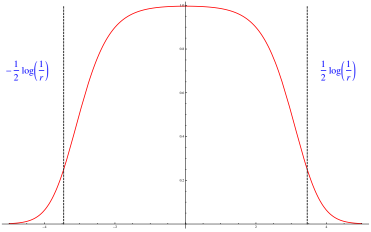
# Rapidity divergences in real corrections

New constraint allows to use same regularization for RDs in *virtual and real corrections*. Lipatov's vertex ($k = q_1 + q_2$, $k^2 = 0$):

$$\Gamma_{+\mu-} = 2\left[(q_2 - q_1)_\mu + \left(\frac{q_1^2}{\tilde{k}_-} + \tilde{q}_1^+\right)\tilde{n}_\mu^- - \left(\frac{q_2^2}{\tilde{k}_+} + \tilde{q}_2^-\right)\tilde{n}_\mu^+\right],$$

without modified constraint, the Slavnov-Taylor identity $\boxed{k^\mu \Gamma_{+\mu-} = 0}$ is broken by terms $O(r)$.

The square of regularized LV:




$$\Gamma_{+\mu-}\Gamma_{+\nu-}P^{\mu\nu} = \frac{16\mathbf{q}_{T1}^2\mathbf{q}_{T2}^2}{\mathbf{k}_T^2} f(y),$$

$$\longleftarrow f(y) = \frac{1}{(re^{-y} + e^{y})^2(re^{y} + e^{-y})^2},$$

$$\int\limits_{-\infty}^{+\infty} dy\ f(y) = -1 - \log r + O(r)$$

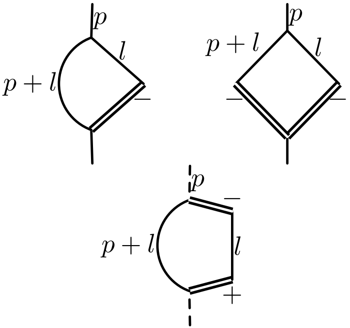
# “Tadpoles” and “Bubbles”.

“Tadpoles” (one quadratic propagator):

$$A_{[-]}(p) = \int \frac{[d^d l]}{(p+l)^2[\tilde{l}^-]}, \; A_{[--]}(p) = \int \frac{[d^d l]}{l^2[\tilde{l}^-][\tilde{l}-\tilde{p}]}$$

where $[d^D l] = \frac{(\mu^2)^\epsilon d^d l}{i\pi^{D/2} r_\Gamma}$, $r_\Gamma = \Gamma^2(1-\epsilon)\Gamma(1+\epsilon)/\Gamma(1-2\epsilon)$.

“Bubbles” (two quadratic propagators):




$$B_{[-]}(p) = \int \frac{[d^d l]}{l^2(p+l)^2[\tilde{l}^-]},$$

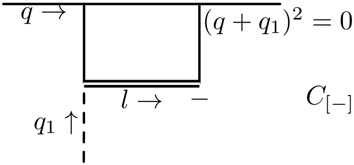
$$B_{[--]}(p) = \int \frac{[d^d l]}{l^2(p+l)^2[\tilde{l}^-][\tilde{l}^- + \tilde{p}^-]}$$

$$B_{[+-]}(p) = \int \frac{[d^d l]}{l^2(p+l)^2[\tilde{l}^+][\tilde{l}^-]},$$

where $p^+ = p^- = 0$ for the last integral.

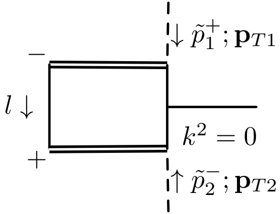
## “Triangle” integrals

One light-cone propagator:




$$C_{[-]}(-q_1^2, q^2, q^-) = \int \frac{[d^d l]}{l^2 (q_1+l)^2 (q_1+q+l)^2 [\tilde{l}^-]}.$$

Two light-cone propagators:




$$C_0^{[+-]} = \int \frac{[d^D l]}{q^2 (p_1-l)^2 (p_2+l)^2 [\tilde{l}^+][\tilde{l}^-]}.$$

# Rapidity divergences at one loop

Only log-divergence $\sim \log r$ (Blue cells in the table) is related with Reggeization of particles in $t$-channel.
Integrals which **do not** have log-divergence may still contain the power-dependence on $r$:

- $r^{-\epsilon} \to 0$ for $r \to 0$ and $\epsilon < 0$.
- $r^{+\epsilon} \to \infty$ for $r \to 0$ and $\epsilon < 0$ – **weak-power divergence** (Pink cells in the table)
- $r^{-1+\epsilon} \to \infty$ – **power divergence.** (Red)

| (# LC prop.) \ (# quadr. prop.) | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | $A_{[-]}$ | $B_{[-]}$ | $C_{[-]}$ | … |
| 2 | $A_{[+-]}$ | $B_{[+-]}$ | $C_{[+-]}$ | … |
| 3 | … | … | … | … |

The **weak-power** and **power-divergences** cancel between Feynman diagrams describing one region in rapidity, so only log-divergences are left.

## Scalar integrals with power RDs.

Notation: $\left\{\frac{\mu}{k}\right\}^{2\epsilon} = \frac{1}{2}\left[\left(\frac{\mu}{k-i\varepsilon}\right)^{2\epsilon} + \left(\frac{\mu}{-k-i\varepsilon}\right)^{2\epsilon}\right].$

Tadpoles:

$$
\begin{aligned}
A_{[-]}(p) &= -\frac{\tilde{p}^{-}\ r^{-1+\epsilon}}{\cos(\pi\epsilon)} \frac{1}{2\epsilon(1-2\epsilon)} \left\{\frac{\mu}{\tilde{p}^{-}}\right\}^{2\epsilon}, \\
A_{[--]}(p) &= \frac{1}{\tilde{p}_{-}} A_{[-]}(p).
\end{aligned}
$$

Bubbles:

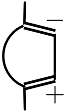
$$
\begin{aligned}
B_{[-]}(p) &= \frac{1}{p^{-}\epsilon^{2}}\left(\frac{\mu^{2}}{-p^{2}}\right)^{\epsilon} + \frac{1-2\epsilon}{\epsilon}\frac{r\cdot A_{[-]}(p)}{\tilde{p}_{-}^{2}} + \Delta B_{[-]}(-p^{2}, p_{-}) + O(r), \\
B_{[--]}(p) &= \frac{2}{\tilde{p}_{-}} B_{[-]}(p),
\end{aligned}
$$

where:

$$
\Delta B_{[-]}(-p^{2}, p_{-}) = -\frac{1}{p_{-}}\left(\frac{p_{-}^{2}\mu^{2}}{(-p^{2})^{2}}\right)^{\epsilon} \frac{\Gamma^{2}(1-2\epsilon)\Gamma(1+2\epsilon)\cdot r^{-\epsilon}}{2\epsilon^{2}\Gamma^{2}(1-\epsilon)}.
$$

# Logarithmic RDs

- $[+-]$-bubble in transverse kinematics $p^- = p^+ = 0$:




$$B_{[+-]}(\mathbf{p}_T) = \frac{1}{\mathbf{p}_T^2}\left(\frac{\mu^2}{\mathbf{p}_T^2}\right)^{\epsilon}\frac{i\pi + 2\log r}{\epsilon},$$

- $[+-]$-bubble in $p^- = 0$ kinematics:

$$\begin{aligned} B_{[+-]}(\mathbf{p}_T, p^+) &= \frac{1}{\mathbf{p}_T^2}\left(\frac{\mu^2}{\mathbf{p}_T^2}\right)^{\epsilon}\frac{\Gamma^2(1+\epsilon)\Gamma(2+\epsilon)\sin(\pi\epsilon)}{\pi\epsilon^2} \\ &\times \left[i\pi + \log r - \log\frac{p_+^2}{\mathbf{p}_T^2} - \psi(1+\epsilon) + \psi(1)\right] + O(r^{1/2}) \end{aligned}$$

- $[+-]$-bubble in light-like kinematics $p^2 = 0$:

$$B_{[+-]}(\mathbf{p}_T^2, p^2 = 0) = \int \frac{[d^d l]}{l^2(l+p)^2[l^+][l^- + p^-]} = \frac{-2\Gamma(1-\epsilon)\Gamma(1+\epsilon)}{\mathbf{p}_T^2\epsilon^2}\left(\frac{\mu^2}{\mathbf{p}_T^2}\right)^{\epsilon}.$$

# Triangle integrals, logarithmic RD

Result for $Q^2 = 0$:

$$C_{[-]}(t_1, 0, q^-) = \frac{1}{q^- t_1}\left(\frac{\mu^2}{t_1}\right)^{\epsilon}\frac{1}{\epsilon}\left[\log r + i\pi - \log\frac{|q_-|^2}{t_1}\right.$$
$$\left. -\psi(1+\epsilon) - \psi(1) + 2\psi(-\epsilon)\right] + O(r^{1/2}),$$

coincides with the result of [G. Chachamis, *et. al.*, 2012].

Result for $Q^2 \neq 0$:

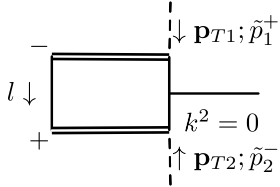
$$C_{[-]}(t_1, Q^2, q_-) = C_{[-]}(t_1, 0, q_-) + \left(\frac{\mu^2}{t_1}\right)^{\epsilon}\frac{I(Q^2/t_1)}{q_- t_1} - \frac{1}{t_1}\Delta B_{[-]}(Q^2, q_-),$$

where

$$\begin{aligned} I(X) &= -\frac{2X^{-\epsilon}}{\epsilon^2} - \frac{2}{\epsilon}\int\limits_0^X \frac{(1-x^{-\epsilon})dx}{1-x} \\ &= -\frac{2X^{-\epsilon}}{\epsilon^2} + 2\left[-\mathrm{Li}_2(1-X) + \frac{\pi^2}{6}\right] + O(\epsilon). \end{aligned}$$

# Triangle with two light-cone propagators

Usual one-loop Feynman integrals with more than 4 propagators are reducible to more simple integrals up to terms $O(\epsilon)$.




We apply method of [Bern, Dixon, Kosower, 1992]. The $O(\epsilon)$ remnant is proportional to $(d-4)I^{(d+2)}$ and integral $I^{(6)}$ is finite.
The resilt in Euclidean region ($p_1^+ > 0$, $-p_2^- > 0$, $\mathbf{p}_{T1,2}^2 > 0$):

$$C_{[+-]}(\mathbf{p}_{T1}^2, \mathbf{p}_{T2}^2, p_1^+, -p_2^-) = \frac{(-1)}{2\mathbf{p}_{T1}^2 \mathbf{p}_{T2}^2 \mathbf{k}_T^2} \times$$
$$\left\{ \mathbf{p}_{T1}^2 (\mathbf{p}_{T2}^2 - \mathbf{p}_{T1}^2 + \mathbf{k}_T^2) \left[ B_{[+-]}(\mathbf{p}_{T1}^2, p_1^+) + (-p_2^-) C_{[-]}(\mathbf{p}_{T1}^2, \mathbf{p}_{T2}^2, -p_2^-) \right] \right.$$
$$+ \mathbf{p}_{T2}^2 (\mathbf{p}_{T1}^2 - \mathbf{p}_{T2}^2 + \mathbf{k}_T^2) \left[ B_{[+-]}(\mathbf{p}_{T2}^2, -p_2^-) + p_1^+ C_{[+]}(\mathbf{p}_{T2}^2, \mathbf{p}_{T1}^2, p_1^+) \right]$$
$$\left. - \mathbf{k}_T^2 (\mathbf{p}_{T1}^2 + \mathbf{p}_{T2}^2 - \mathbf{k}_T^2) B_{[+-]}(\mathbf{k}_T^2, k^2 = 0) \right\},$$

where $\mathbf{k}_T^2 = p_1^+(-p_2^-)$.
**The $\log r$-divergence cancels within square brackets, as expected.**

# One-loop effective vertices

# Forward scattering vertices at one loop

We will consider two examples of Particle-Particle-Reggeon vertices:

$$\gamma^\star(q) + Q_+(q_1) \to q(q+q_1), \qquad (1)$$
$$g(q) + R_+(q_1) \to g(q+q_1), \qquad (2)$$

where $q^2 = Q^2 \neq 0$ in the case (1), $q^2 = 0$ in the case (2) and $\boxed{\tilde{q}_1^- = 0}$ to ensure GI at $r \neq 0$.

Example (1) is needed to compute NLO correction to the DIS SFs in *Parton Reggeization Approach* [M.N., V. Saleev, 2017]. Also the gluon-driven "DIS" process, mediated by the operator:

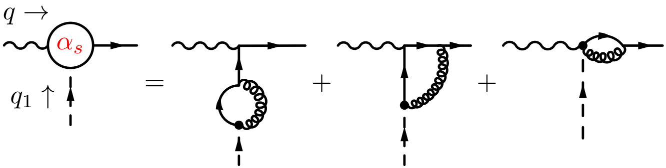
$$\mathrm{tr}[G_{\mu\nu}G^{\mu\nu}],$$

is required to construct the scheme of NLO calculations in PRA.

Example (2) already has been studied [G. Chachamis, *et. al.*, 2012], but *cancellation of power RDs has not been checked.*

# $\gamma^\star Qq$-vertex

$\Gamma^{(1)}_{+\mu}$ =




$$= \quad C[\Gamma]\cdot\Gamma^{(0)}_{+\mu}(q_1,q) + C[\Delta^{(1)}]\cdot\Delta^{(1)}_{+\mu}(q_1,q) + C[\Delta^{(2)}]\cdot\Delta^{(2)}_{+\mu}(q_1,q)$$

Lorentz structures:

$$\Gamma^{(0)}_{+\mu}(q_1,k,q_2) = \gamma_\mu + \frac{\hat{q}_1 n^-_\mu}{[\tilde{k}^-]}, \qquad \longleftarrow \text{[Fadin, Sherman, 1976]}$$

$$\Delta^{(1)}_{+\mu}(q_1,q) = \frac{\hat{q}}{q_-}\left(n^-_\mu - \frac{2(q_1)_\mu}{q_1^+}\right),\ \Delta^{(2)}_{+\mu}(q_1,q) = \frac{\hat{q}}{q_-}\left(n^-_\mu - \frac{q_\mu}{q^+}\right)$$

Cancellation of RDs:

- $A_{[-]} \sim r^{-1+\epsilon}$ – cancels between diagrams
- $O(r^\epsilon)$-terms cancel between $B_{[-]}(q)$ and $B_{[-]}(q+q_1)$
- $O(r^{-\epsilon})$-terms cancel between $B_{[-]}(q)$ and $C_{[-]}$.
- **only $O(\log r)$-divergence from $C_{[-]}$ is left**

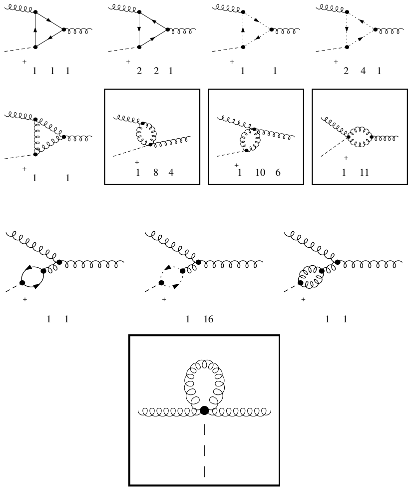
# Expressions for the coefficients

$$C[\Gamma] = -\frac{\bar{\alpha}_s C_F}{4\pi}\frac{1}{2}\left\{\frac{[(d-8)Q^2+(d-6)t_1]B(t_1)-2(d-7)Q^2B(Q^2)}{Q^2-t_1}\right.$$
$$\left.-2\left[(Q^2-t_1)C(t_1,Q^2)-q_-\left(t_1C_{[-]}(t_1,Q^2,q_-)+(B_{[-]}(q)-B_{[-]}(q+q_1))\right)\right]\right\},$$
$$C[\Delta^{(1)}] = -\frac{\bar{\alpha}_s C_F}{4\pi}\frac{(Q^2+t_1)}{2(Q^2-t_1)^2}\left[\left((d-2)Q^2-(d-4)t_1\right)B(t_1)-2Q^2B(Q^2)\right],$$
$$C[\Delta^{(2)}] = -\frac{\bar{\alpha}_s C_F}{4\pi}\frac{Q^2}{(Q^2-t_1)^2}\left[\left((d-6)t_1-(d-8)Q^2\right)B(Q^2)+2(t_1-2Q^2)B(t_1)\right],$$

were $\bar{\alpha}_s = \frac{\mu^{-2\epsilon}g_s^2}{(4\pi)^{1-\epsilon}}r_\Gamma$, $t_1 = -q_1^2$.

# $gRg$-vertex

Diagrams:




In [G. Chachamis, *et. al.*, 2012] the framed diagrams where nullified by gauge-choice for external gluons. However they participate in cancellation of power RDs:

- Contributions of $A_{[--]}$ and $A_{[-]} \sim r^{-1+\epsilon}$ diagrams cancel due to relation $A_{[--]}(q) = A_{[-]}(q)/q_-$
- Contributions of $B_{[--]}(q)$ and $B_{[-]}(q) \sim r^{\epsilon}$ cancel dur to relation $B_{[--]}(q) = 2B_{[-]}(q)/q_-$.
- **only $O(\log r)$-divergence from $C_{[-]}$ is left**

# $gRg$-vertex

Lorentz structures:

$$\delta_{\lambda_1,\lambda_2} \text{ and } \delta_{\lambda_1,-\lambda_2}.$$

One-loop amplitude:

$$\mathcal{M}_{\lambda_1,\lambda_2} = \frac{\bar{\alpha}_s}{4\pi}\left\{\frac{n_F(d-4)B(t_1)}{(d-2)(d-1)}\delta_{\lambda_1,-\lambda_2}\right.$$
$$-\frac{C_A}{2(d-1)}\left[\left((d-4)\delta_{\lambda_1,-\lambda_2}+2(d-1)\delta_{\lambda_1,\lambda_2}\right)B(t_1)\right.$$
$$\left.\left.-2(d-1)\delta_{\lambda_1,\lambda_2}q_- t_1 C_{[-]}(q_-,0,t_1)\right]\right\},$$

agrees with the result of [G. Chachamis, *et. al.*, 2012].

# Comparison with QCD

# Test process: DIS on the on-shell photon target

To perform the comaprison with QCD we consider the process:

$$\gamma^*(q) + \gamma(P) \to X,$$

where $P$ has large $P^+$ momentum component and $P^2 = 0$. The LO subprocess is:

$$\gamma^*(q) + \gamma(P) \to q(k_1) + \bar{q}(k_2),$$

we introduce the usual variables:$Q^2 = -q^2,\ x_B = \frac{Q^2}{2(qP)}$, and work in the $(q, P)$ center of mass frame, where $q^+ = -x_B P^+$, $q^- = Q^2/(x_B P^+)$, $\mathbf{q}_T = 0$.
We parametrize final-state momenta as:

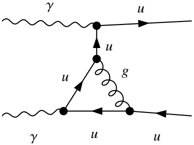
$$k_1 = q + q_1,\ k_2 = P - q_1,$$
$$t_1 = -\mathbf{q}_{T1}^2,\ x_1 = \frac{q_1^+}{P^+} = x_B \frac{Q^2 + t_1}{Q^2} \text{ for } x_B \ll 1.$$

And will study the squared amplitude projected on the $F_2$ structure function:

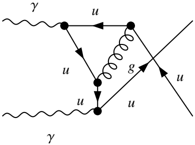
$$F_2(x_B, Q^2, t_1) \text{ in the limit } x_B \ll 1.$$

# QCD diagrams at one loop

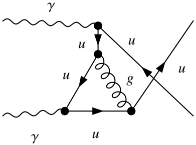
$\gamma \quad \gamma \quad \rightarrow \quad u \quad u$



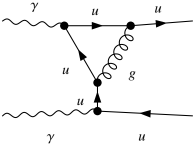
C1 N1 T1



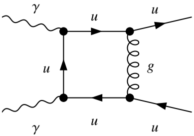
C1 N2 T2



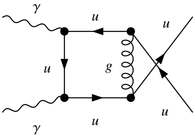
C1 N3 T3



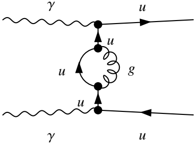
C1 N4 T4



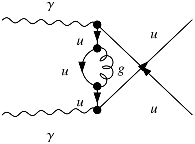
C1 N5 T5



C1 N6 T6



C1 N7 T7



C1 N8 T8

# QCD result at one loop

The QCD result (**leading power for $x_B \ll 1$, but exact in $Q^2$ and** $t_1$):

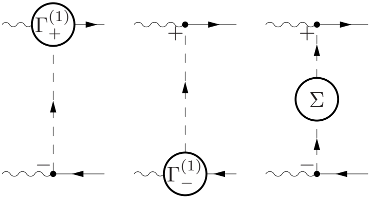
$$F_2^{(1,\mathrm{QCD})}(Q^2, t_1, x_B) = \frac{\bar{\alpha}_s C_F}{4\pi} \left\{ -\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - \frac{2L_1}{\epsilon} + \left( \frac{2\pi^2}{3} - 7 - L_1^2 - 3L_1 \right) + \right.$$
$$+ \left( \frac{1}{\epsilon} + \log\frac{\mu^2}{t_1} \right) \left( \log\frac{1}{x_B^2} - 2\pi i \right) + L_2^2 + 2\mathrm{Li}_2\left(1 - \frac{Q^2}{t_1}\right)$$
$$\left. - \frac{1}{(Q^2 - t_1)^2} \left[ Q^2(Q^2 - t_1) + (3t_1^2 - 4Q^2 t_1) L_2 \right] \right\} + O(\epsilon, x_B),$$

where $L_1 = \log(\mu^2/Q^2)$, $L_2 = \log(Q^2/t_1)$, contains:

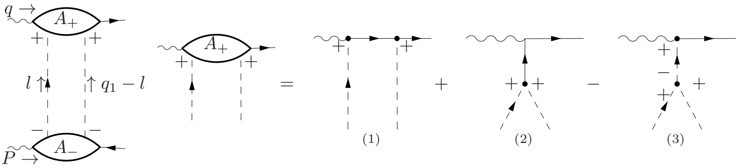
- The $1/\epsilon^2$ and $1/\epsilon$ IR-divergences,
- Single-log part in $\log x_B^{-1}$ and *imaginary part*,
- The complicated dependence on $Q^2/t_1$.

# Contributions in the EFT

One-Reggeon contribution (*positive signature*, Re+Im parts @ 1 loop):




Two-Reggeon contribution (*negative signature*, Im part @ 1 loop):




**Sum of one and two-Reggeon contributions reproduces QCD result exactly.**

# Conclusions

- The consistent procedure of rapidity regularization is proposed. One should modify not only Wilson lines, but also kinematic constraints.
- One-loop integrals with log-RDs are identified. The power-RDs are contained just in a few simplest integrals.
- Triangle integrals with one and two scales are calculated.
- Reduction of one-loop integrals with more than four propagators (quadratic or light-cone) seems to work similar to the case of ordinary loop integrals.
- The one-loop correction to the two-scale $\gamma^\star Qq$-vertex is computed. Comparison with QCD works!
- The power-RDs cancel in amplitudes with Reggeized gluons and quarks. *Is there a simple explanation?*
- Calculation of the effective vertex containing $\mathrm{tr}[G_{\mu\nu}G^{\mu\nu}]$ is in progress.

**Thank you for your attention!**