Quantum entanglement

EPR paradox in high energy colliders

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BNL

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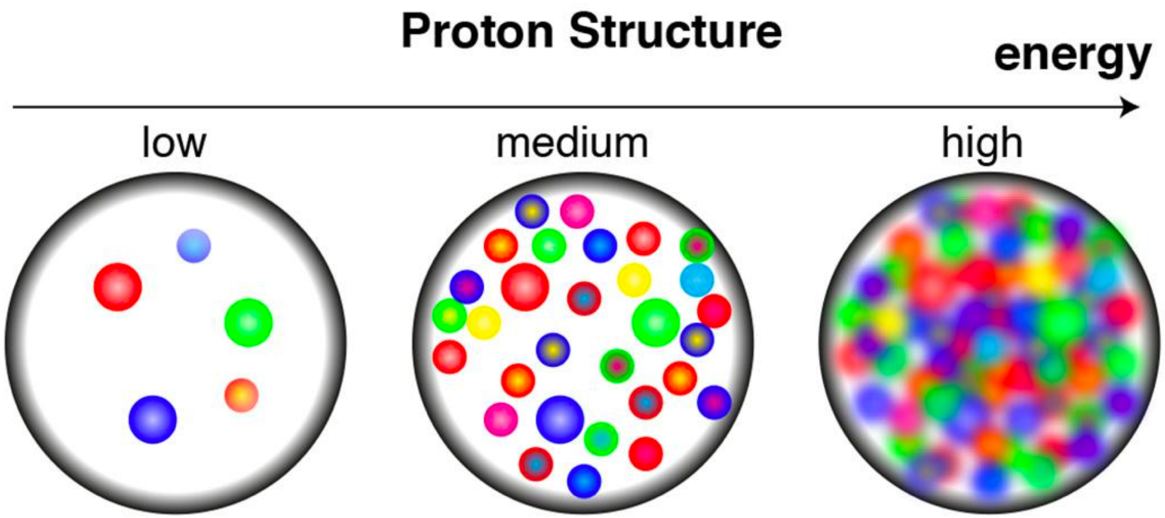
Manuscript will be submitted after DIS

Building block - Nucleon



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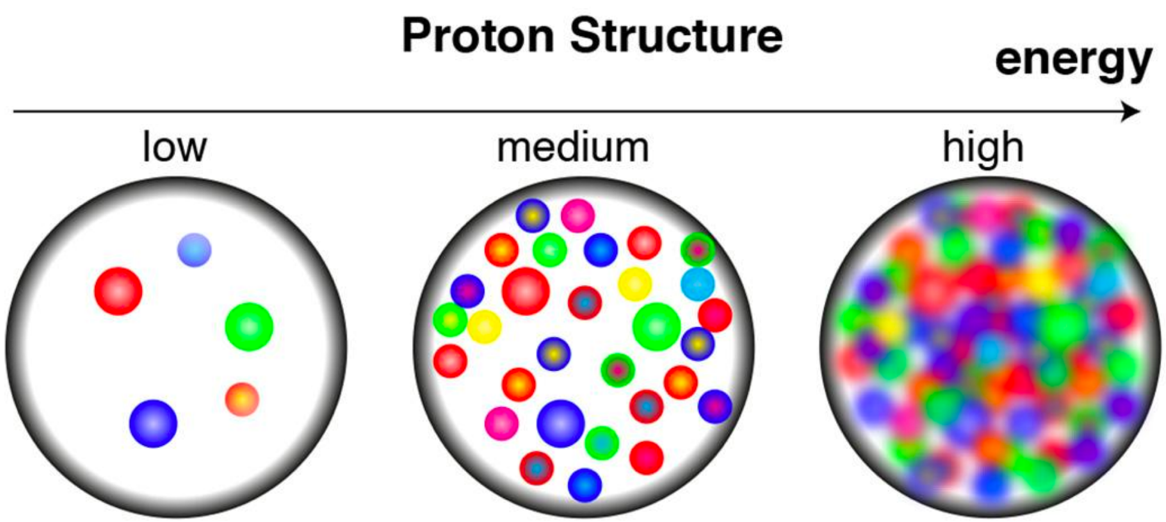
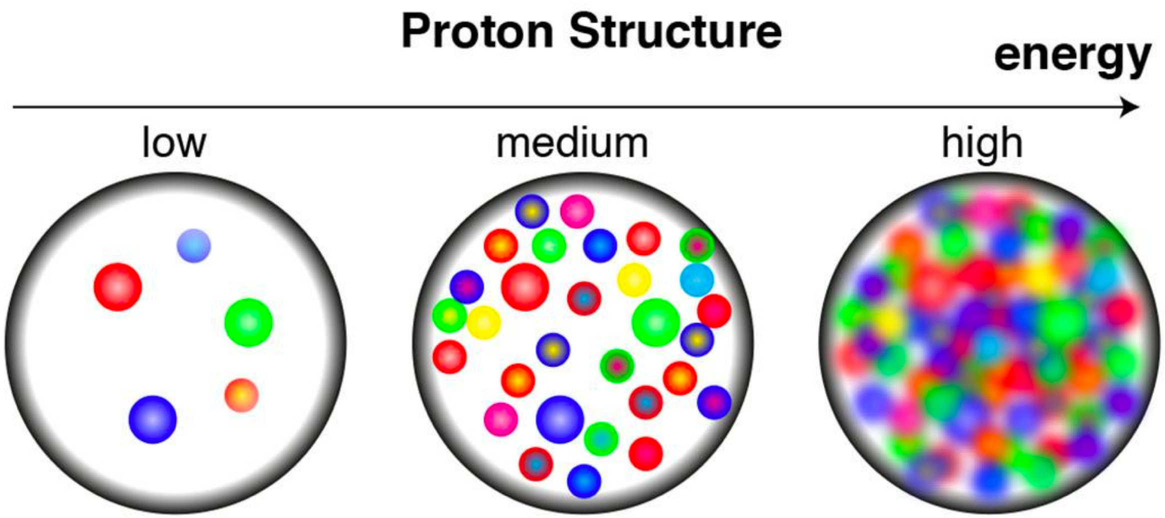


Parton model

- Based on “quasi-free” partons that are frozen in the Infinite momentum frame.

Building block - Nucleon



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Color confinement

- Partons are not just correlated, they cannot exist as free particles in nature

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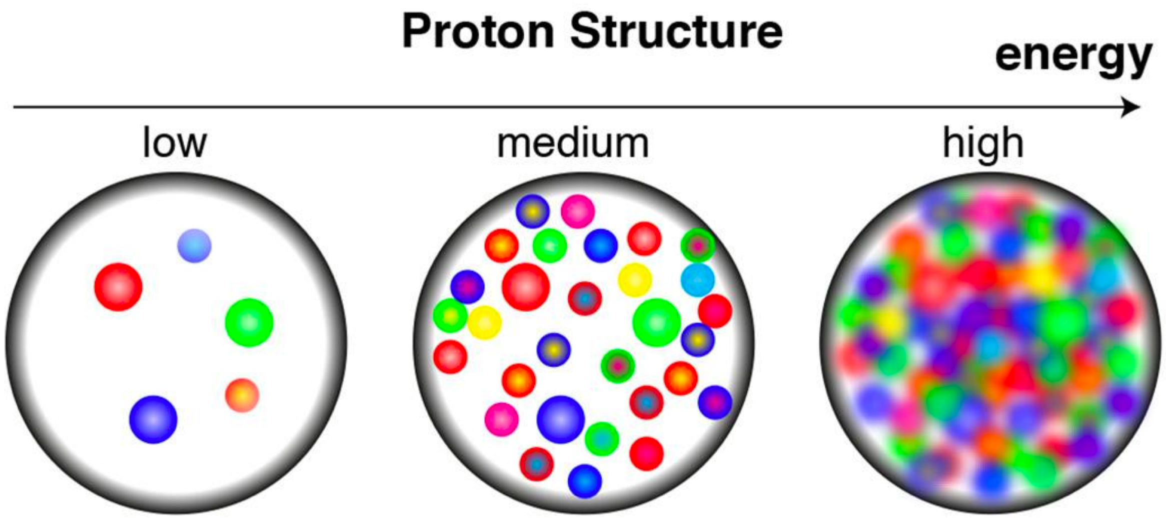

Parton model

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Color confinement

- Partons are not just correlated, they cannot exist as free particles in nature

One conceptual question arises:

- One set of incoherent partons corresponds to a non-zero von Neumann entropy $S \neq 0$

How to understand?

- Proton is a pure quantum mechanical state, its entropy is zero $S = 0$

Entanglement

Entanglement is the natural “picture” in addressing this conceptual question

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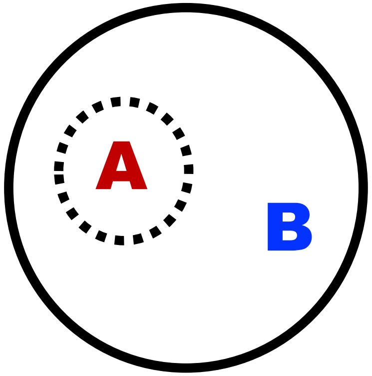
$|\Psi\rangle$ is a pure quantum state, density matrix is therefore $\rho_{tot} = |\Psi\rangle\langle\Psi|$

Entanglement Entropy (EE) is defined:

$$S_A = -\mathrm{Tr}\rho_{\mathrm{A}} \ln \rho_{\mathrm{A}}$$

, where $\rho_A \equiv \mathrm{Tr}_{\mathrm{B}}(\rho_{\mathrm{tot}})$, A and B are two complementary parts of $|\Psi\rangle$

pure quantum state



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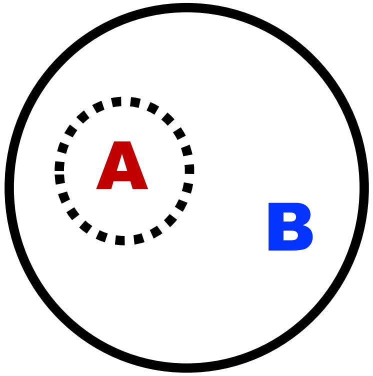
2. Take-home messages:

pure quantum state



1) For the whole system ρ_{tot}, von Neumann entropy is zero by definition (i.e., proton)

2) **When measuring A only:**
   i. $S_{EE} > 0$ if A and B are entangled.
   ii. $S_{EE} = 0$ if A and B are independent.

A two-body example

(i) $|\Psi\rangle = \frac{1}{2}\left[|\uparrow\rangle_A + |\downarrow\rangle_A\right] \otimes \left[|\uparrow\rangle_B + |\downarrow\rangle_B\right]$

$$\Rightarrow \rho_A = \mathrm{Tr_B}\left[|\Psi\rangle\langle\Psi|\right] = \frac{1}{2}\left[|\uparrow\rangle_A + |\downarrow\rangle_A\right]\cdot\left[\langle\uparrow|_A + \langle\downarrow|_A\right].$$

Not Entangled

$$S_A = 0$$

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$$S_{\mathrm{A}} = 0$$

$$\text{(ii)} \quad |\Psi\rangle = \left[|\uparrow\rangle_A \otimes |\downarrow\rangle_B + |\downarrow\rangle_A \otimes |\uparrow\rangle_{\mathrm{B}}\ \right] / \sqrt{2}$$

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$$S_{\mathrm{A}} = \log 2$$

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Entangled

$$S_{\mathrm{A}} = \log 2$$

"EE is a measure of how much a given state is quantum mechanically entangled"

Taken from a lecture note of Tadashi Takayanagi, http://www2.yukawa.kyoto-u.ac.jp/~tadashi.takayanagi/CERNEE.pdf

EPR paradox

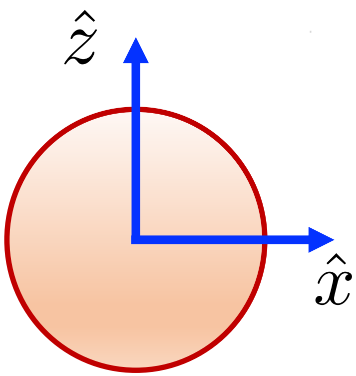
MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. Einstein, B. Podolsky and N. Rosen, *Institute for Advanced Study, Princeton, New Jersey*

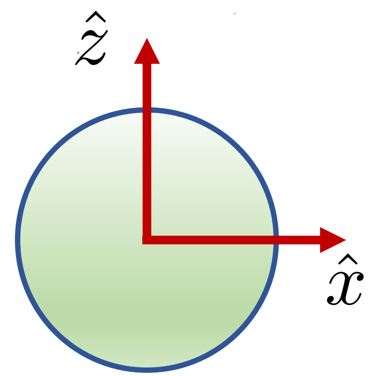
(Received March 25, 1935)

Alice



Entanglement ?

"spooky action at a distance"



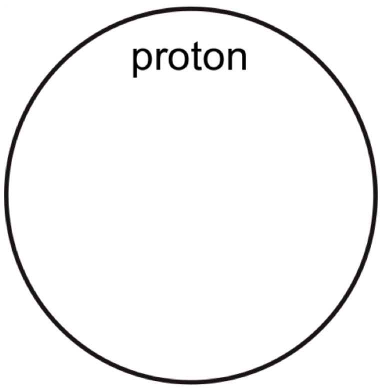
Bob

- Many modern experiments have seen evidence of EPR paradox (e.g., in cold atom experiments)

Experiments at Colliders

(a)

before collision



Proton: a pure quantum state (by definition)

$$S_{\mathrm{EE}} = 0$$

Experiments at Colliders

(a)

before collision

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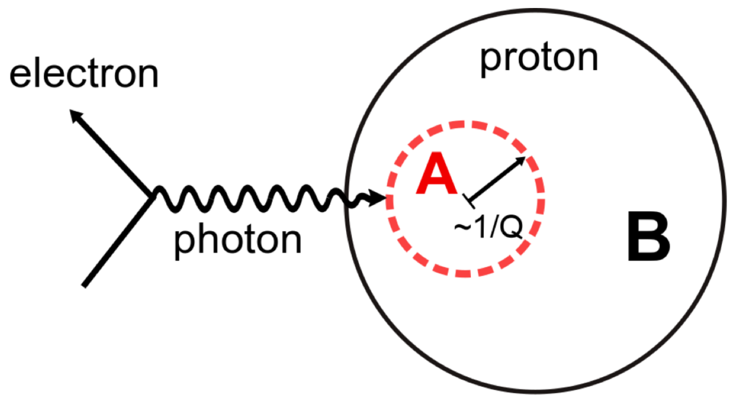
(b)

hard collision



Hard interaction, fast enough to test entanglement, e.g.,

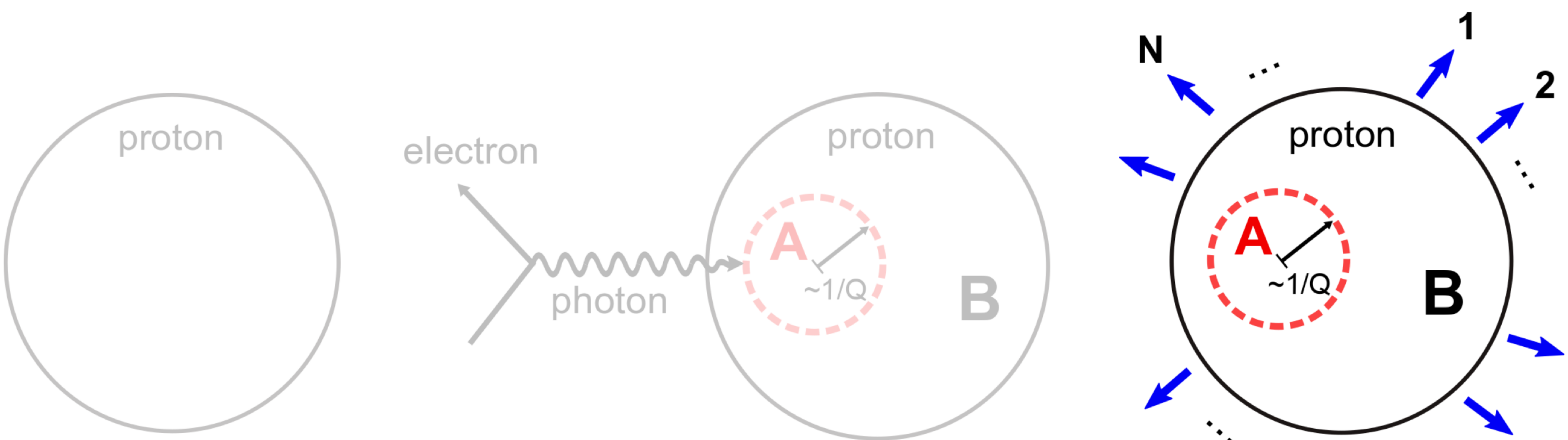
$$\frac{1}{Q} \sim 1\ \mathrm{GeV}^{-1} \sim 0.2\ \mathrm{fm}$$

Experiments at Colliders

(a) before collision

(b) hard collision

(c) after collision



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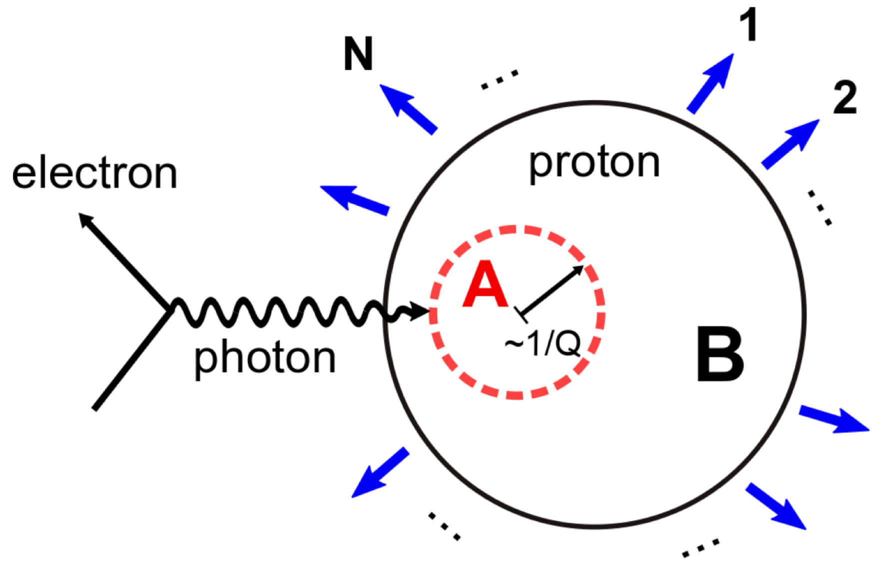
Hadronization and if A,B are entangled, entropy:

$$S^{A}_{EE} = S^{B}_{EE}$$

Principle and Practice

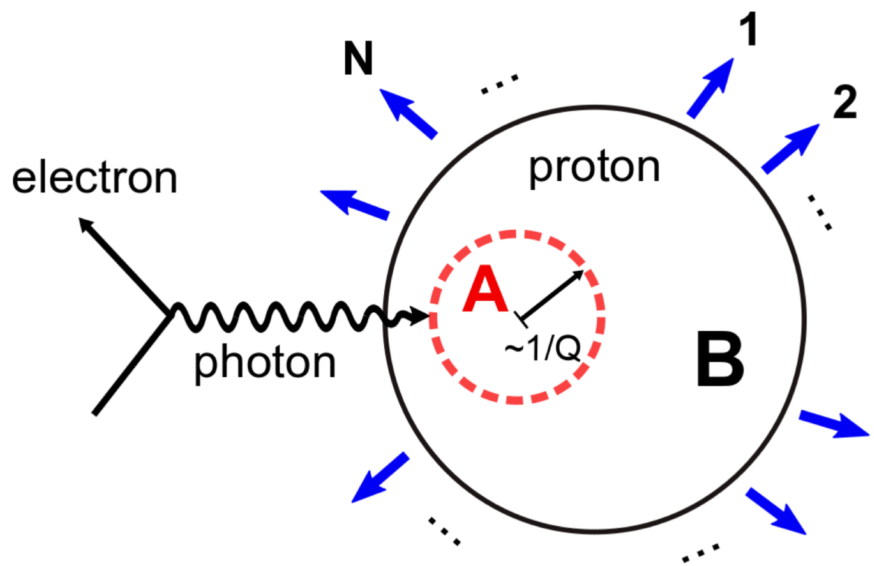


In principle

- Measure $\boldsymbol{S_A}$ **and** $\boldsymbol{S_B}$ independently, and directly test against each other.

- But partons don't live ☹.
- Need all hadrons from A and B

Principle and Practice



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In practice

- **Theorists[1] made a prediction**

$$S_{EE} = \ln [xG]$$

at small x, e.g., $x < 10^{-3}$

- We have well constrained PDFs

1. D. Kharzeev and E.Levin, *Phys. Rev. D 95, 114008 (2017)*

A well-defined test

- At similar kinematics in x and Q^2 (region A), the S_{EE} can be checked from the entropy of finite-state hadron around region A

prediction

experiment

$$\mathrm{S_{EE}} = \ln\left[\mathrm{xG}\right] \quad \Rightarrow \quad \mathrm{S_{hadron}} = -\sum \mathrm{P(N)} \ln\left[\mathrm{P(N)}\right]$$

Assuming entropy doesn't grow much

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Assuming entropy doesn't grow much

- The event kinematics define the region of interest, using relation between x and rapidity,

$$\ln\left(\frac{1}{\mathrm{x}}\right) \approx \mathrm{y}_{\mathrm{beam}} - \mathrm{y}_{\mathrm{hadron}}$$

(arXiv:hep-ph/9903536)

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⇨

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For example,

fixed Q^2, and x, e.g., $\mathbf{x} \in (\mathbf{x_1},\mathbf{x_2})$ ⇨ Final-state hadrons $\mathbf{y} \in (\mathbf{y_1},\mathbf{y_2})$

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prediction

$$S_{EE} = \ln[xG]$$

experiment

$$S_{hadron} = -\sum P(N)\ln[P(N)]$$

Assuming entropy doesn't grow much

- The event kinematics define the region of interest, using relation between x and rapidity,

$$\ln\left(\frac{1}{x}\right) \approx y_{beam} - y_{hadron}$$

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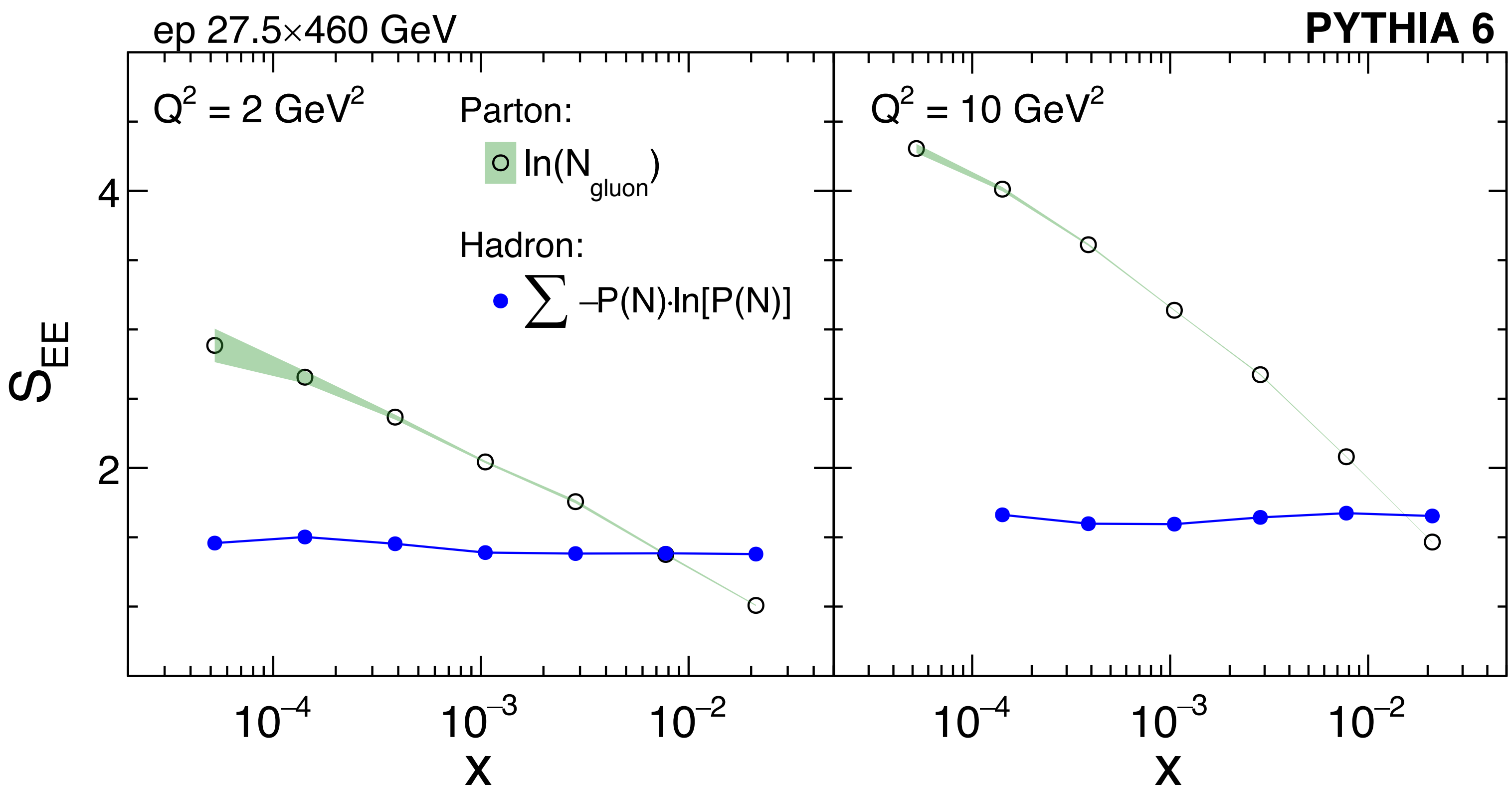
prediction

experiment

$$S_{EE}^{(x_1<x<x_2)} = \ln[xG] \stackrel{?}{=} S_{hadron}^{(y_1<y<y_2)} = -\sum P(N)\ln[P(N)]$$

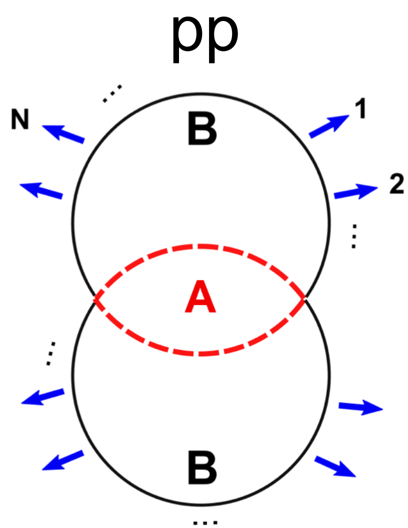
ep



No indication of entanglement in simulation

- $xG(x)$ is from LO *MSTW,* no substantial difference from using other PDFs
- Other models, DJANGO, PYTHIA6, and PYTHIA8, same conclusion

High energy pp collisions

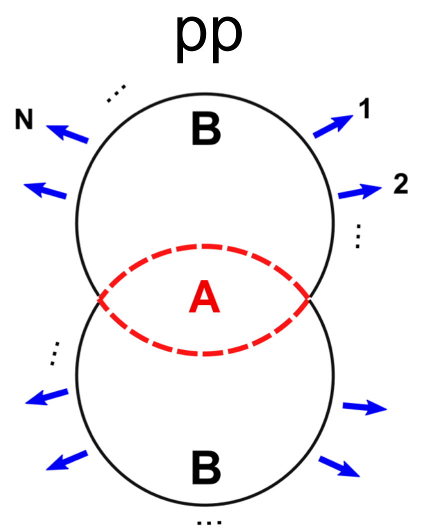


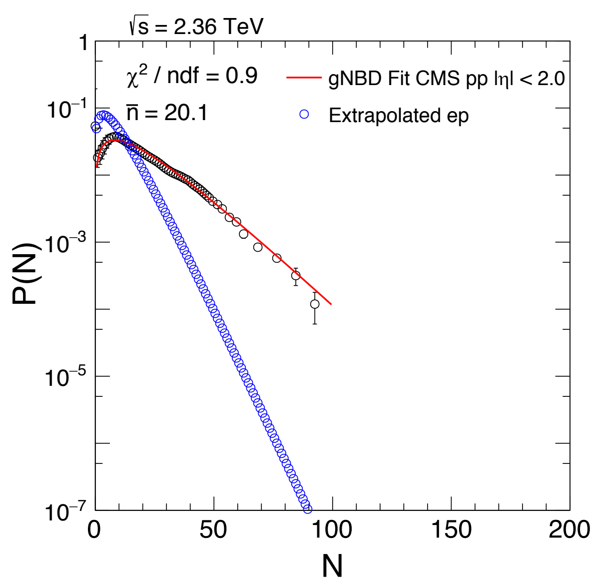
- At high energy, dominated by gluon-gluon interactions, pp collisions could be tested using similar idea.
  - Get the x value from y_{beam} and y_{hadron},

$$\ln\left(\frac{1}{x}\right) \approx y_{\mathrm{beam}} - y_{\mathrm{hadron}}$$

  - Saturation scale Q_s is used from NLO BK model [*see backup for other models*]

High energy pp collisions





CMS published data **JHEP 1101 (2011) 079**

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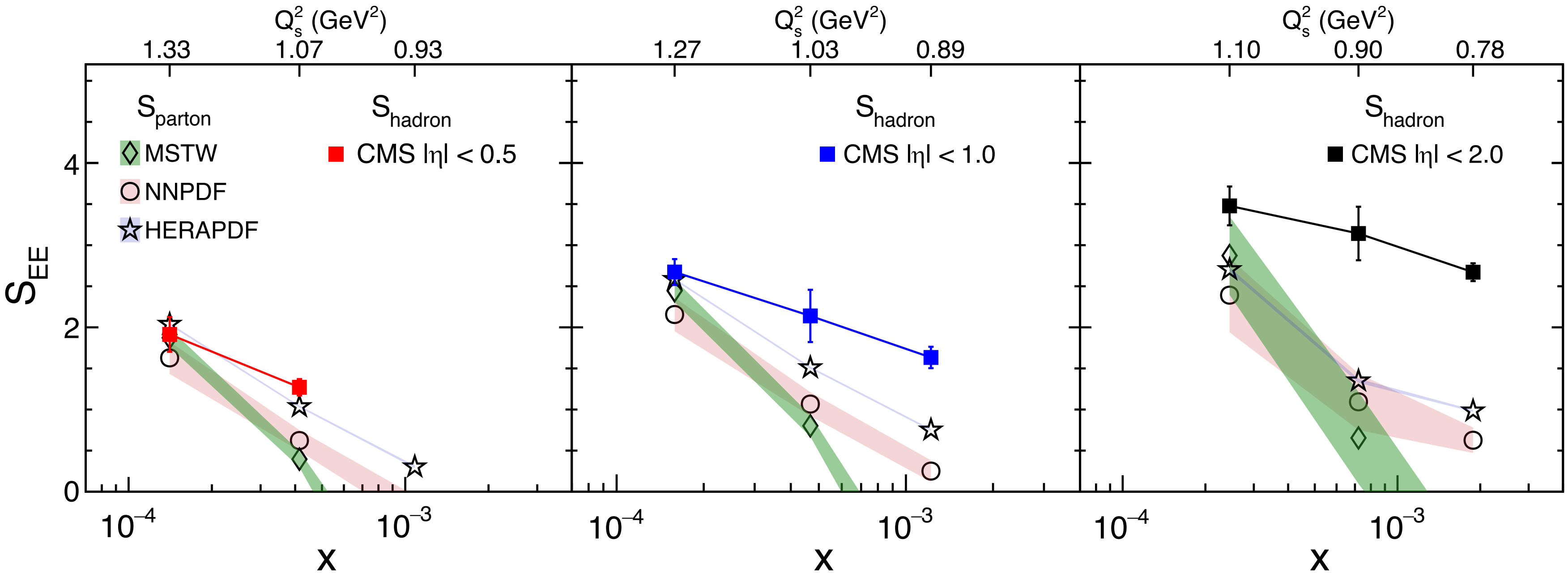
  - Saturation scale Q_s is used from NLO BK model [*see backup for other models*]
  - A negative binomial distribution (NBD) is used to extrapolate P(N) distribution per nucleon, assuming $\langle N \rangle$ is half.

(different fit ranges, double NBDs are used and included as systematics)

pp



A strong indication of quantum entanglement

- EE and its dependence on x are well predicted, e.g., expected only for $x < 10^{-3}$
- Similar at all rapidity ranges. Compatible with different PDFs.

- **Entanglement provides a new perspective on understanding the proton**

Summary and outlook

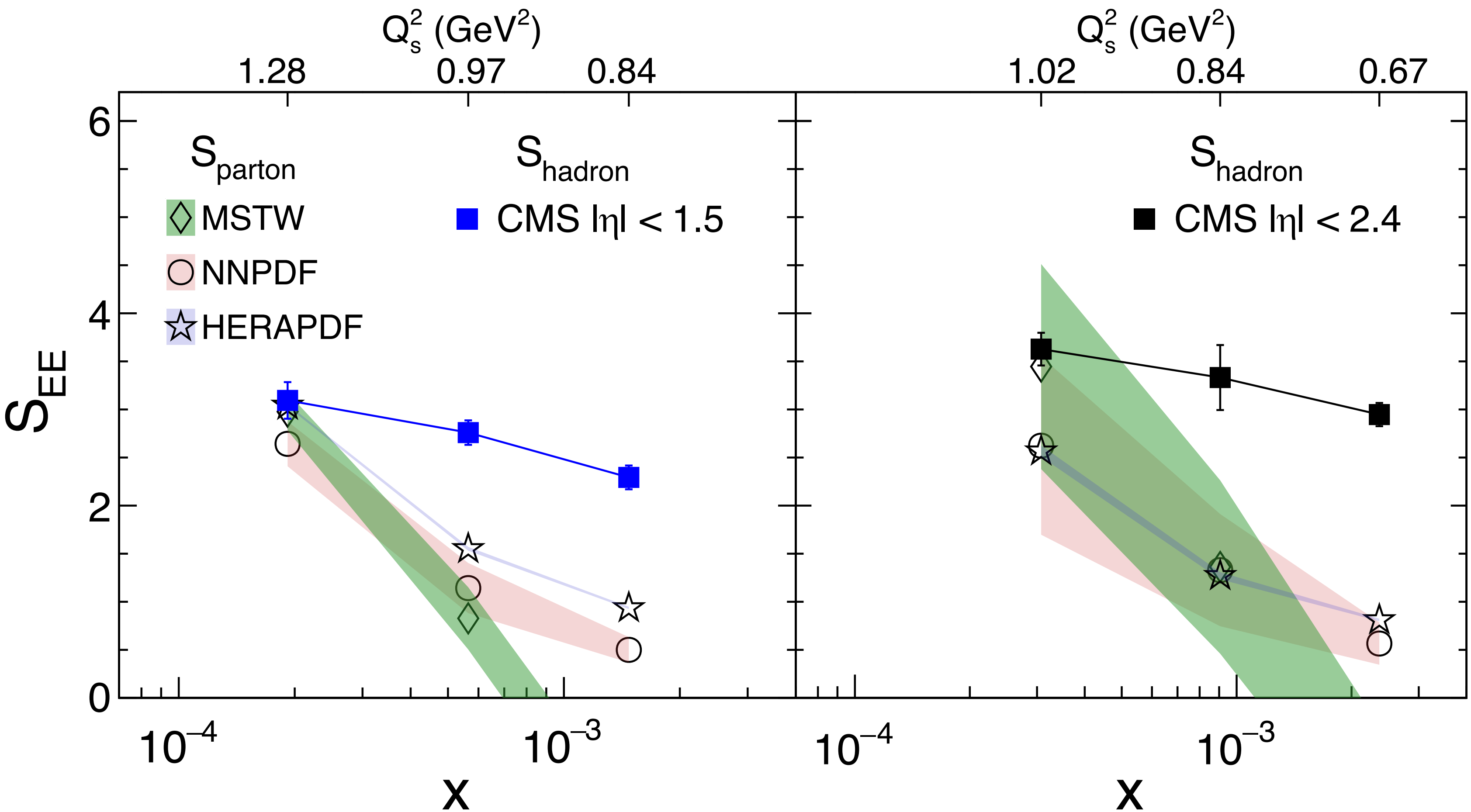
- **First indication of quantum entanglement at sub-nucleonic scales,** *encountered EPR paradox using high energy particle colliders*

  - Resolved an "apparent paradox" between the Parton model and quantum mechanics.
  - Opened a new perspective on studying the proton.
  - *Entanglement as a probe of confinement* *(Nucl.Phys.B796:274-293,2008)*
  - Thermalization through entanglement in pp collisions (Phys. Rev. D 98, 054007 (2018))

- **What else can be done?**
  - DIS experiment using ep data, e.g., HERA (published data does not go down to low x)
  - LHC pp data with a different scale?
  - Electron-Ion Collider in the future

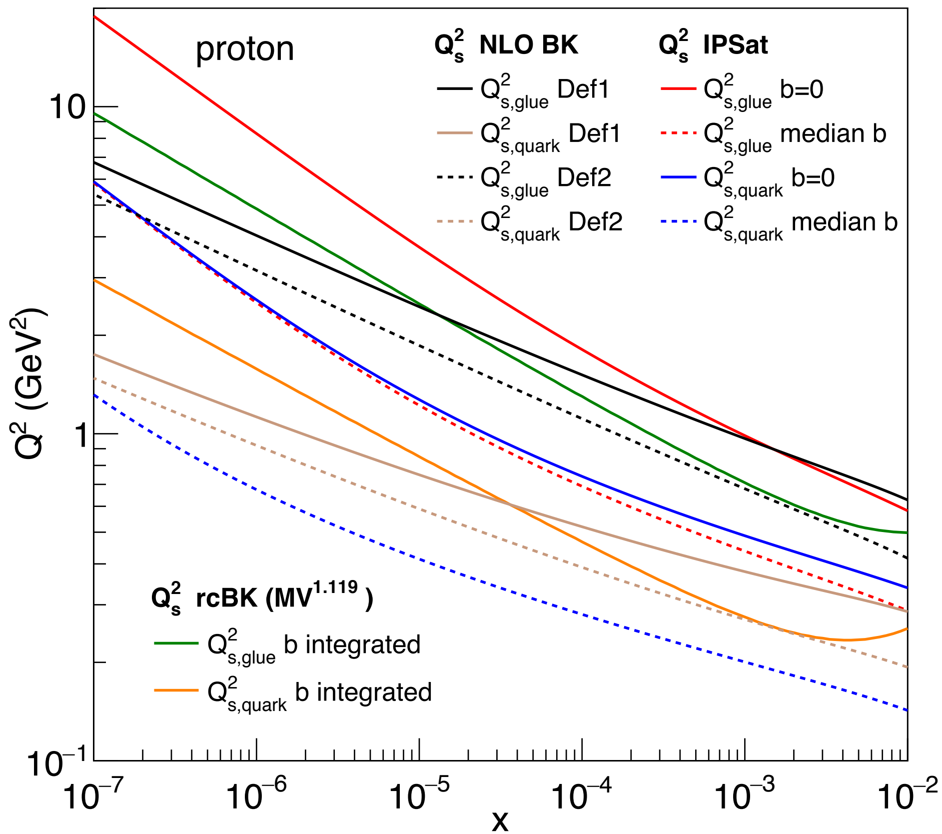
Backup

data



Q_s^2 (GeV^2)
1.28
0.97
0.84
1.02
0.84
0.67
6
4
2
0
S_EE
S_parton
MSTW
NNPDF
HERAPDF
S_hadron
CMS |η| < 1.5
CMS |η| < 2.4
10^-4
10^-3
x

Saturation scales



ee, ep, and pp multiplicities