Shear forces and tensor polarization

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Main Topics

- Gravitational formfactors and (Extended) Equivalence Principle: spin ½ and GPDs
- Spin 1 : INCLUSIVE processes with tensor polarizartion
- HERMES data and (Ex)EP
- New possibilities: DY and hadronic hard processes
- NICA@JINR

Recent development: pressure

- Publihsed in Nature, already used for EoS (talk of A. Bacchetta)

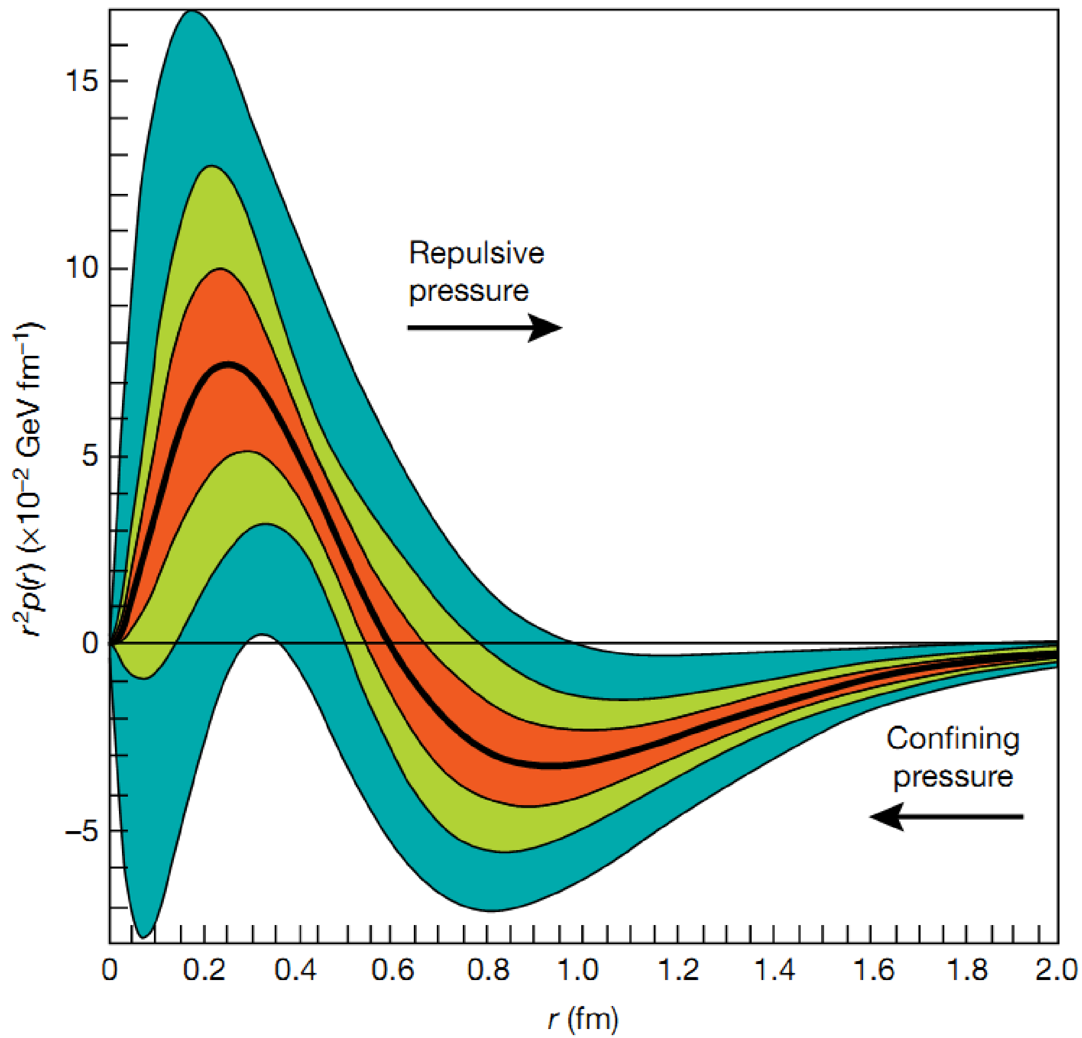
LETTER

NATURE | www.nature.com/nature

https://doi.org/10.1038/s41586-018-0060-z

The pressure distribution inside the proton

V. D. Burkert[1]*, L. Elouadrhiri[1] & F. X. Girod[1]



Gravitational Formfactors (spin ½)

$$\langle p'|T^{\mu\nu}_{q,g}|p\rangle \;=\; \bar{u}(p')\Big[A_{q,g}(\Delta^2)\gamma^{(\mu}p^{\nu)} + B_{q,g}(\Delta^2)P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M]u(p)$$

- Conservation laws - zero Anomalous Gravitomagnetic Moment : $\mu_G = J$ (g=2)

$$P_{q,g} = A_{q,g}(0) \qquad A_q(0) + A_g(0) = 1$$

$$J_{q,g} = \frac{1}{2}\left[A_{q,g}(0) + B_{q,g}(0)\right] \qquad A_q(0) + B_q(0) + A_g(0) + B_g(0) = 1$$

- May be extracted from high-energy experiments/NPQCD calculations
- Describe the partition of angular momentum between quarks and gluons
- Describe interaction with both classical and TeV gravity

Electromagnetism vs Gravity (OT'99)

- Interaction – field vs metric deviation

$$M = \langle P' | J_q^\mu | P \rangle A_\mu(q)$$

$$M = \frac{1}{2} \sum_{q,G} \langle P' | T_{q,G}^{\mu\nu} | P \rangle h_{\mu\nu}(q)$$

- Static limit

$$\langle P | J_q^\mu | P \rangle = 2 e_q P^\mu$$

$$\sum_{q,G} \langle P | T_i^{\mu\nu} | P \rangle = 2 P^\mu P^\nu$$

$$h_{00} = 2\phi(x)$$

$$M_0 = \langle P | J_q^\mu | P \rangle A_\mu = 2 e_q M \phi(q)$$

$$M_0 = \frac{1}{2} \sum_{q,G} \langle P | T_i^{\mu\nu} | P \rangle h_{\mu\nu} = 2M \cdot M \phi(q)$$

- Mass as charge – equivalence principle

Gravitomagnetism

- Gravitomagnetic field (weak, except in gravity waves) – action on spin from $M = \frac{1}{2}\sum_{q,G}\langle P'|T^{\mu\nu}_{q,G}|P\rangle h_{\mu\nu}(q)$

$$\vec{H}_J = \frac{1}{2} rot\vec{g};\ \vec{g}_i \equiv g_{0i}$$

spin dragging twice smaller than EM

- Lorentz force – similar to EM case: factor ½ cancelled with 2 from $h_{00} = 2\phi(x)$ Larmor frequency same as EM

$$\omega_J = \frac{\mu_G}{J} H_J = \frac{H_L}{2} = \omega_L\ \vec{H}_L = rot\vec{g}$$

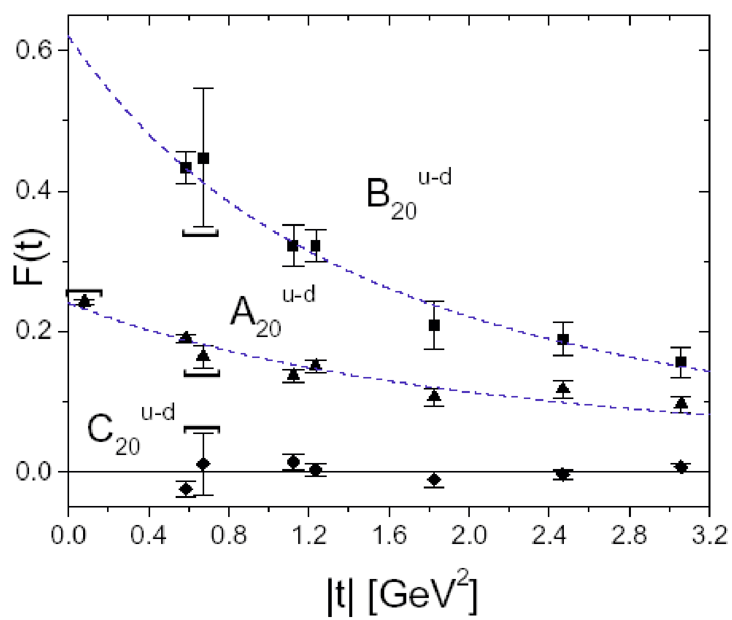
- Orbital and Spin momenta dragging – the same - Equivalence principle

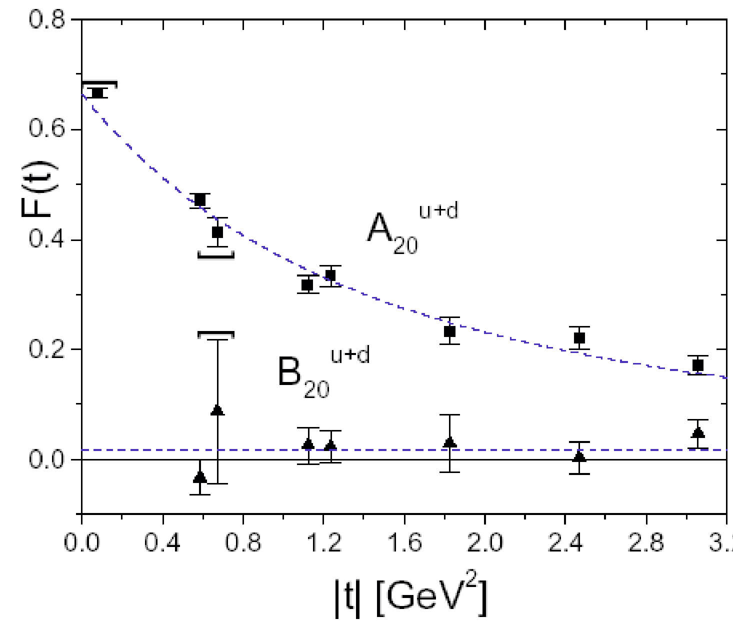
Equivalence principle

- Newtonian – "Falling elevator" – well known and checked (also for elementary particles)
- Post-Newtonian – gravity action on SPIN – known since 1962 (Kobzarev and Okun'; rederived from conservarion laws - Kobzarev and V.I. Zakharov
- Anomalous gravitomagnetic (and electric-CP-odd) moment iz ZERO or
- Classical and QUANTUM rotators behave in the SAME way
- Gravitational analog of Ji's SR $\int dx\, x\ (\Sigma\, E_q+E_G) = 0$!

Generalization of Equivalence principle

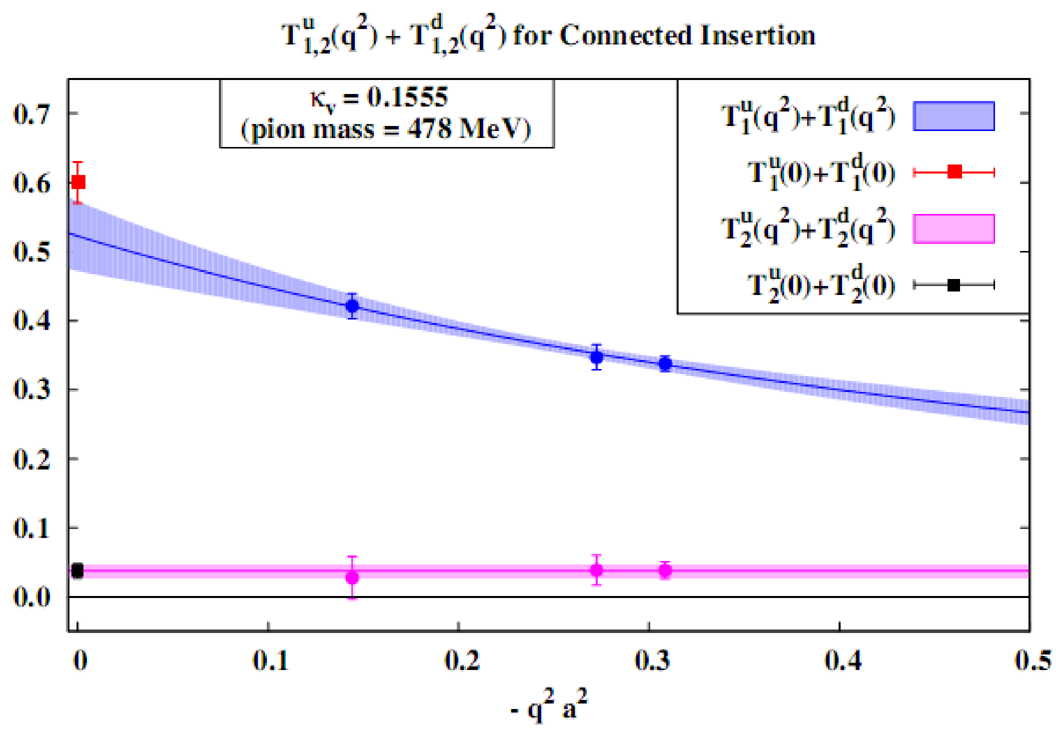
- Various arguments: AGM ≈ 0 separately for quarks and gluons – most clear from the lattice (LHPC/SESAM)





Recent lattice study (M. Deka et al. Phys.Rev. D91 (2015) no.1, 014505)

- Sum of u and d for Dirac (T1) and Pauli (T2) FFs



Extended Equivalence Principle=Exact EquiPartition

- In pQCD – violated
- Reason – in the case of ExEP- no smooth transition for zero fermion mass limit (Milton, 73)
- Conjecture (O.T., 2001 – prior to lattice data) – valid in NP QCD – zero quark mass limit is safe due to chiral symmetry breaking
- Gravityproof confinement? Nucleons do not break even by black holes?
- Support by recent observation of smalness of Cbar

Spin-1 hadrons

- MANY new FFs!
- Recent extensive analysis
- Cosyn,Cotogno, Freese,Lorce: 1903.00408
- Polyakov, Sun: 1903.02738
- A lot of integral relations between GPDs and FFs

Spin 1 EMT and inclusive processes

- Forward matrix element ->density matrix
- Contains P-even term: tensor polarization
- Symmetric and traceless: correspond to (average) shear forces
- For spin ½: P-odd vector polarization requires another vector (q) to form vector product

Spin 1 in QCD

- Tensor polarization in QCD: Frankfurt, Strikman (81), Efremov,OT (81)

- Spin ½: kinematically enhanced longitudinal polarization transverse-twist 3
- Spin 1: LL/TT related by tracelessness

SUM RULEs

- We (A.V. Efremov,OT'81) derived zero sum rules:
- 1st moment: also in parton model by Close and Kumano (90)
- 2nd moment (forward analog of Ji's SR)
- Average shear force (compensated between quarks and gluons)
- Gravity and (Ex)EP (zero average shear separately for quarks and gluons) – OT'09

Manifestation of post-Newtonian (Ex)EP for spin 1 hadrons

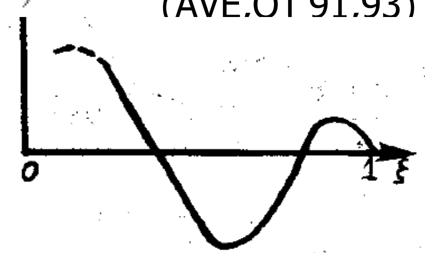
- Tensor polarization - coupling of EMT to spin in forward matrix elements - inclusive processes
- Second moments of tensor distributions should sum to zero

$$A_T = \frac{\sigma_+ + \sigma_- - 2\sigma_0}{3\bar{\sigma}}$$

$$\int_0^1 C_i^T(x)dx = 0$$

$$\langle P,S|\bar{\psi}(0)\gamma^\nu D^{\nu_1}...D^{\nu_n}\psi(0)|P,S\rangle_{\mu^2} = i^{-n}M^2 S^{\nu\nu_1}P^{\nu_2}...P\nu_n \int_0^1 C_q^T(x)x^n dx$$

(AVE,OT'91,93)



$$\sum_q \langle P,S|T_i^{\mu\nu}|P,S\rangle_{\mu^2} = 2P^\mu P^\nu(1-\delta(\mu^2)) + 2M^2 S^{\mu\nu}\delta_1(\mu^2)$$

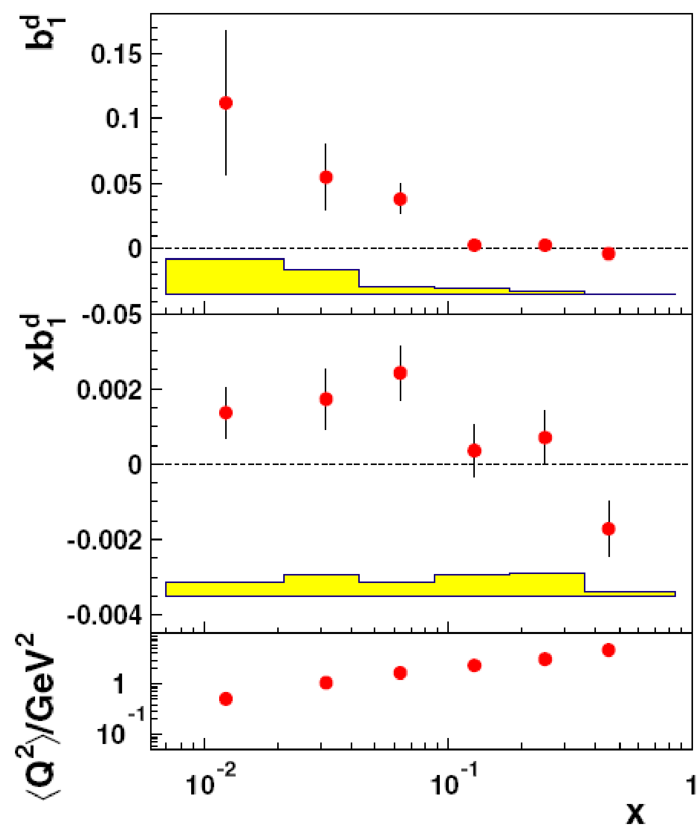
$$\langle P,S|T_g^{\mu\nu}|P,S\rangle_{\mu^2} = 2P^\mu P^\nu \delta(\mu^2) - 2M^2 S^{\mu\nu}\delta_1(\mu^2)$$

$\sum_q \int_0^1 C_i^T(x)xdx = \delta_1(\mu^2)$ =0 for ExEP

HERMES – data on tensor spin structure function

PRL **95**, 242001 (2005)

- Isoscalar target – proportional to the sum of u and d quarks – combination required by (Ex)EP
- Second moments – compatible to zero better than the first one (collective glue << sea)



Where else to test?

- COMPASS
- EIC
- DY@J-PARC: (Song,Kumano:1902.04712)
- However: ET'81-any hard process
- $f_{Al} \sim b_1$

$$P_{zz} = -2P_{yy} = -2P_{xx} = \frac{2\int_0^1 d\xi\, f_{Al}(\xi)\, \mathrm{Sp}[\hat{P} E(\xi, P)]}{3\int_0^1 d\xi\, f(\xi)\, \mathrm{Sp}[\hat{P} E(\xi, P)]} = \frac{2F_{Al}(x_1^*, x_2)}{3F(x_1, x_2)},$$

- Suggestion: hadronic tensor SSA

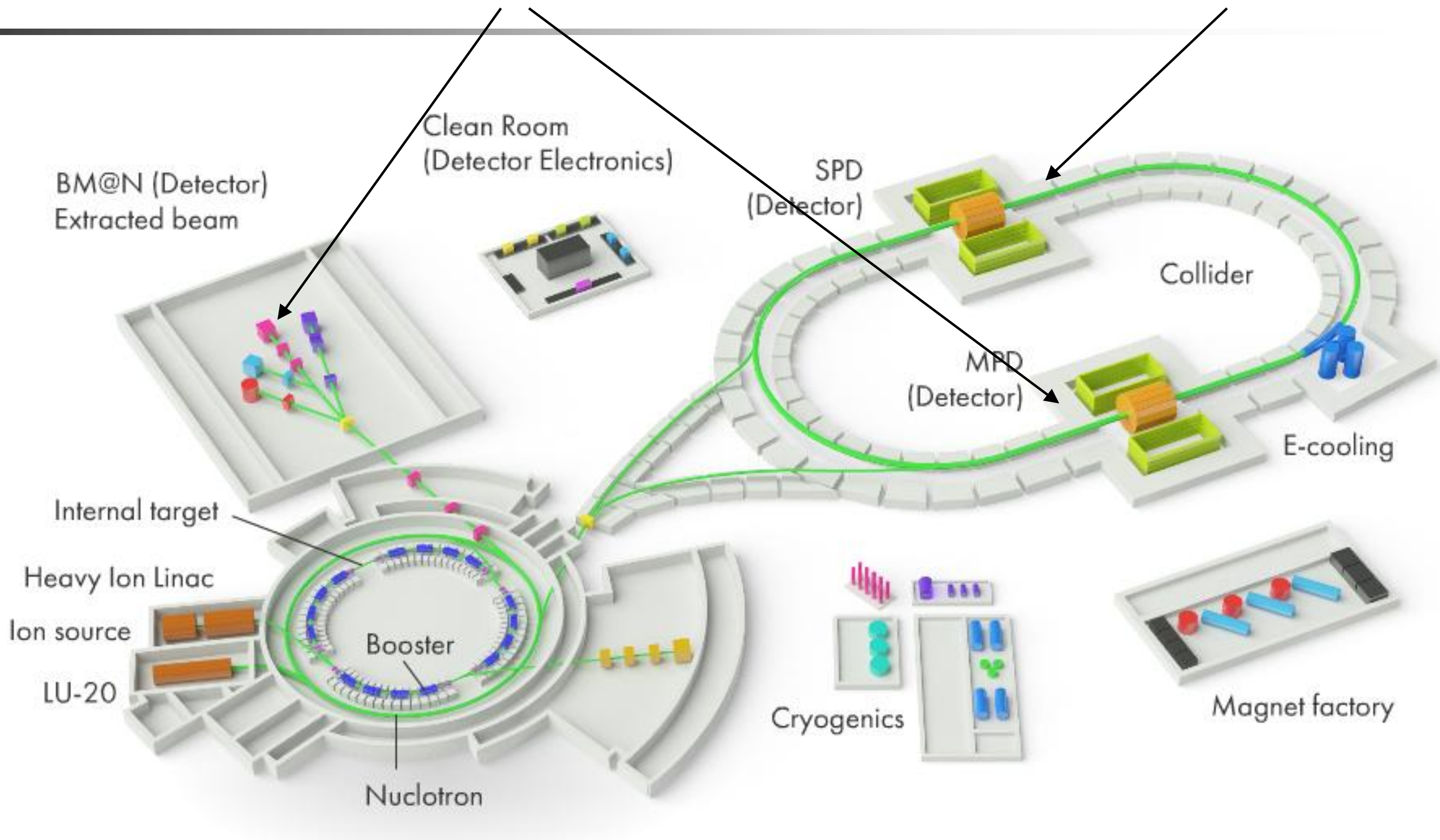
Vector vs Tensor SSA

- Vector: $A = (\sigma(+)-\sigma(-))/(\sigma(+)+\sigma(-))$
- Tensor:
  $A = (\sigma(+)+\sigma(-))/(\sigma(+)+\sigma(-)+\sigma(0))$
- Inclusive pion production: (T-odd) vector SSA may be also excluded by summing $\sigma(L)+\sigma(R)$

Tensor polarized beams

- Opportunity: NICA@JINR with polarized hadronic beams
- Polarized deuterons is easier to accelerate: no depolarizing resonances
- DY, J/Ψ (+hadronic SSA)

NICA: heavy ions and hadrons



Conclusions

- Tensor polarization: way to test of gravitational coupling of quarks and gluons in inclusive processes
- HERMES data are compatible with validity of EP separately for quarks and gluons
- Tests in DY and hadronic TSSA at NICA are possible

BACKUP

One more gravitational formfactor

- Quadrupole

$$\langle P+q/2|T^{\mu\nu}|P-q/2\rangle = C(q^2)(g^{\mu\nu}q^2 - q^\mu q^\nu) + \ldots$$

- Cf vacuum matrix element – cosmological constant (vacuum pressure)

$$\langle 0|T^{\mu\nu}|0\rangle = \Lambda g^{\mu\nu}$$

$$\Lambda = C(q^2)q^2$$

- Inflation ~ annihilation ($q^2>0$) OT'15
- How to measure experimentally – Deeply Virtual Compton Scattering

D-term interpretation: Inflation and annihilation

- Quadrupole gravitational FF

$$\langle P + q/2|T^{\mu\nu}|P - q/2\rangle = C(q^2)(g^{\mu\nu}q^2 - q^\mu q^\nu) + \ldots$$

- Moment of D-term – positive
- Vacuum – Cosmological Constant $\langle 0|T^{\mu\nu}|0\rangle = \Lambda g^{\mu\nu}$

- 2D effective CC – negative in scattering, positive in annihilation

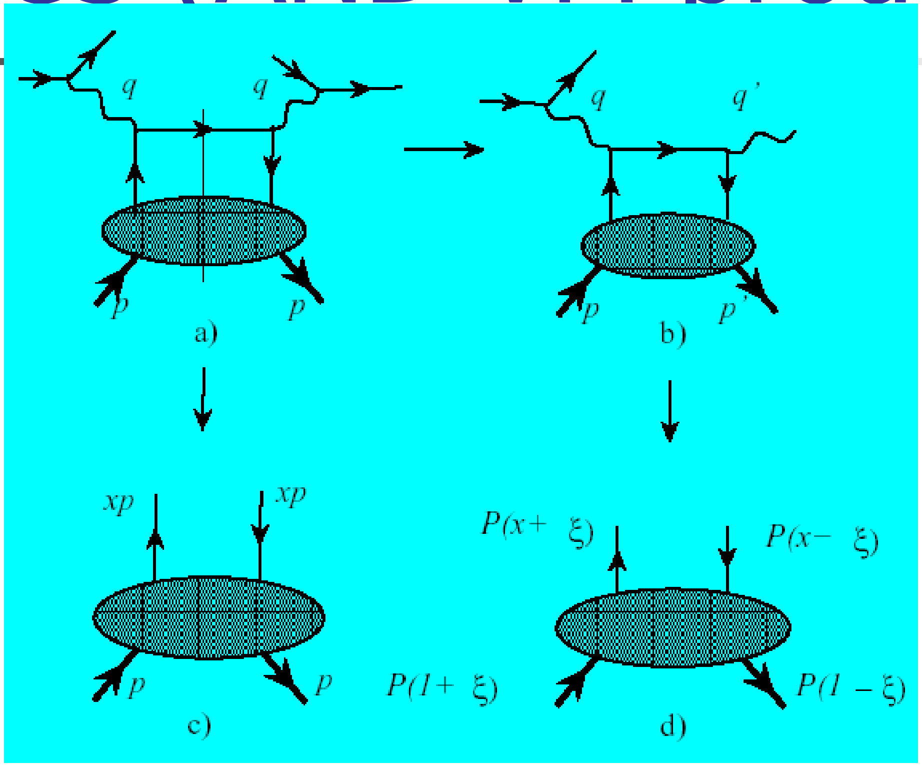
$$\Lambda = C(q^2)q^2$$

- Similarity of inflation and Schwinger pair production – Starobisnky, Zel'dovich
- Was OUR Big Bang resulting from one graviton annihilation at extra dimensions??! Version of "ekpyrotic" ("pyrotechnic") universe

C vs Cbar

- Cancellations of Cbars – negative pressure (cf Chaplygin gas)
- Cancellation in vacuum; Pauli (divergent), Zel'dovich (finite)
- Flavour structure of pressure: DVMP!

QCD Factorization for DIS and DVCS (AND VM production)



- Manifestly spectral

$$\mathcal{H}(x_B) = \int_{-1}^{1} dx \frac{H(x)}{x - x_B + i\epsilon}.$$

- Extra dependence on ξ

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,\xi)}{x - \xi + i\epsilon},$$

Unphysical regions

- DIS : Analytical function – polynomial in $1/x_B$ if $1 \leq |x_B|$

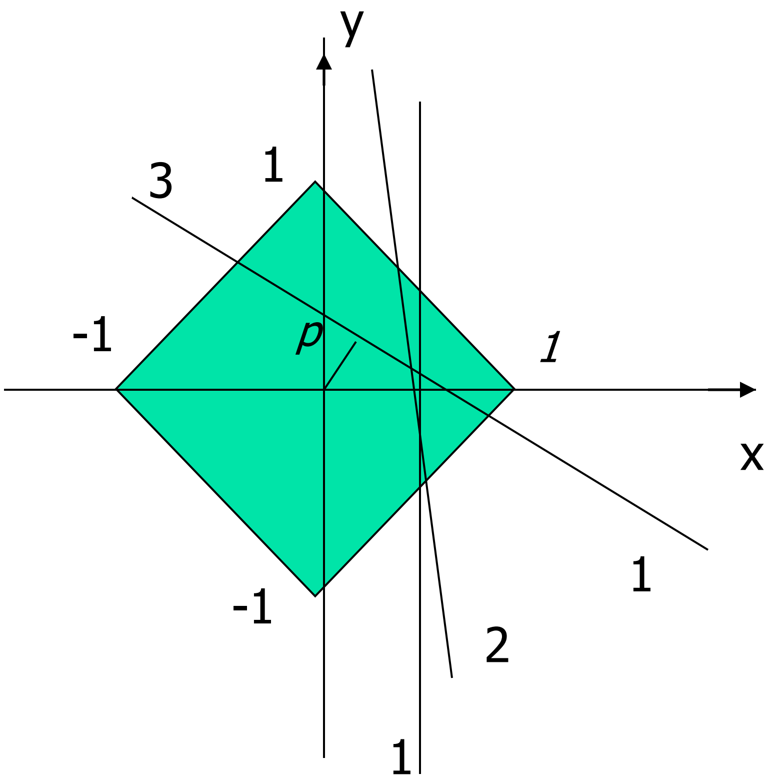
$$H(x_B) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x) \frac{x^n}{x_B^{n+1}}$$

- DVCS – additional problem of analytical continuation of $H(x,\xi)$
- Solved by using of Double Distributions Radon transform

$$H(z,\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy (F(x,y) + \xi G(x,y)) \delta(z - x - \xi y)$$

Double distributions and their integration

- Slope of the integration line- skewness
- Kinematics of DIS: $\xi = 0$
  ("forward") - vertical line (1)
- Kinematics of DVCS: $\xi < 1$
  - line 2
- Line 3: $\xi > 1$ unphysical region - required to restore DD by inverse Radon transform: tomography

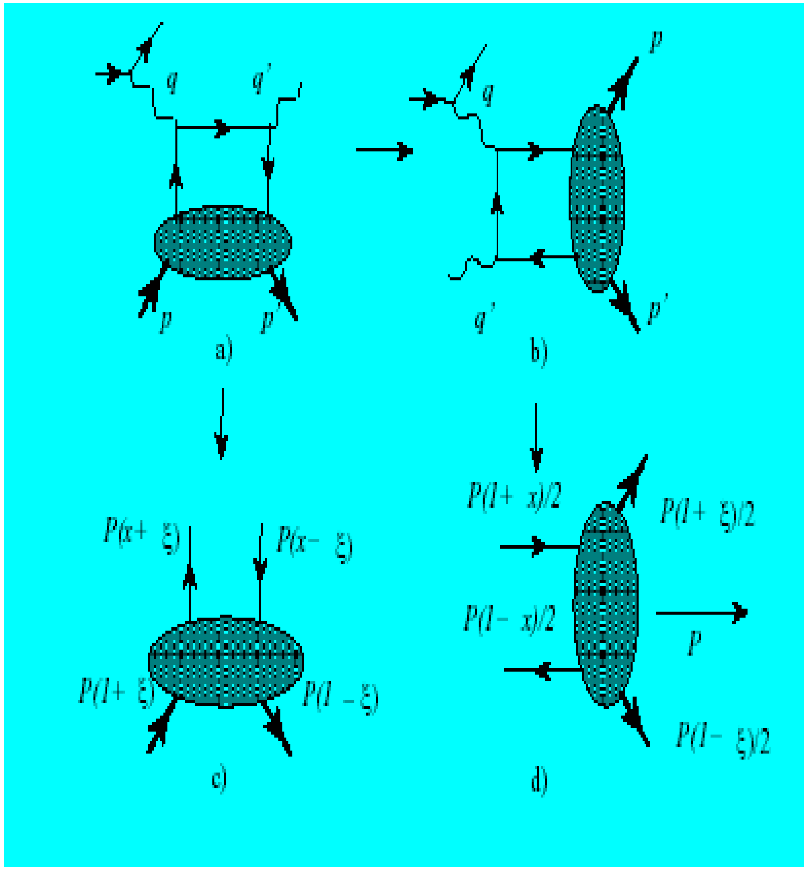


$$f(x,y) = -\frac{1}{2\pi^2}\int_0^\infty \frac{dp}{p^2}\int_0^{2\pi} d\phi |cos\phi| (H(p/cos\phi + x + ytg\phi, tg\phi) - H(x + ytg\phi, tg\phi)) =$$

$$= -\frac{1}{2\pi^2}\int_{-\infty}^{\infty} \frac{dz}{z^2}\int_{-\infty}^{\infty} d\xi (H(z + x + y\xi, \xi) - H(x + y\xi, \xi))$$

Crossing for DVCS and GPD

- DVCS -> hadron pair production in the collisions of real and virtual photons
- GPD -> Generalized Distribution Amplitudes
- Duality between s and t channels (Polyakov,Shuvaev, Guzey, Vanderhaeghen)



GDA -> back to unphysical regions for DIS and DVCS

- Recall DIS

$$H(x_B) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x) \frac{x^n}{x_B^{n+1}}$$

- Non-positive powers of x_B

- DVCS

$$H(\xi) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x,\xi) \frac{x^n}{\xi^{n+1}}$$

- Polynomiality (general property of Radon transforms): moments - integrals in x weighted with x^n - are polynomials in $1/\xi$ of power $n+1$
- As a result, analyticity is preserved: only non-positive powers of ξ appear

Holographic property (OT'05)

- Factorization Formula ->

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,\xi)}{x-\xi+i\epsilon}$$

- Analyticity -> Imaginary part -> Dispersion relation:

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,x)}{x-\xi+i\epsilon}$$

- "Holographic" equation (DVCS AND VM)

$$\Delta\mathcal{H}(\xi) \equiv \int_{-1}^{1} dx \frac{H(x,x)-H(x,\xi)}{x-\xi+i\epsilon}$$

$$= \sum_{n=1}^{\infty} \frac{1}{n!} \frac{\partial^n}{\partial \xi^n} \int_{-1}^{1} H(x,\xi) dx (x-\xi)^{n-1} = const$$

Holographic property - II

- Directly follows from double distributions

$$H(z,\xi)=\int_{-1}^{1}dx\int_{|x|-1}^{1-|x|}dy(F(x,y)+\xi G(x,y))\delta(z-x-\xi y)$$

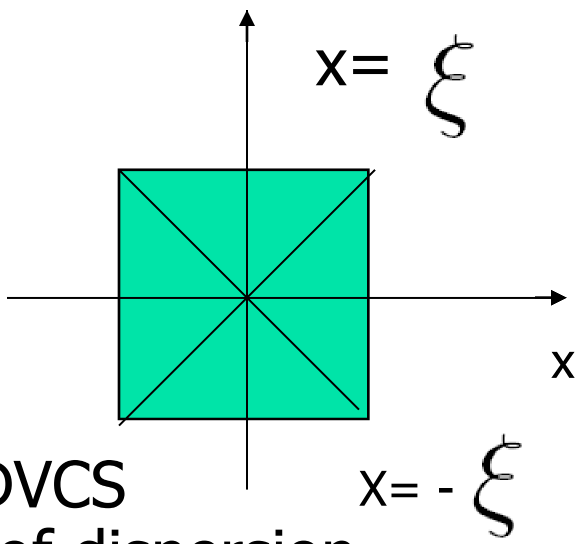
- Constant is the SUBTRACTION one - due to the (generalized) Polyakov-Weiss term G(x,y)

$$\Delta\mathcal{H}(\xi)=\int_{-1}^{1}dx\int_{|x|-1}^{1-|x|}dy\frac{G(x,y)}{1-y}$$

$$=\int_{-\xi}^{\xi}dx\frac{D(x/\xi)}{x-\xi+i\epsilon}=\int_{-1}^{1}dz\frac{D(z)}{z-1}=const$$

Holographic property - III

- 2-dimensional space -> 1-dimensional section!
- Momentum space: any relation to holography in coordinate space ?!



- Strategy (now adopted) of GPD's

studies: start at diagonals

(through SSA due to imaginary part of DVCS amplitude) and restore by making use of dispersion relations + subtraction constants

Holography vs NLO

- Depends on factorization scheme
- Special role of scheme preserving the coefficient function
- Nucleon as (scheme dependent) black hole – 3D information encoded in 2D

Pressure in hadron pairs production

- Back to GDA region
- -> moments of H(x,x) - define the coefficients of powers of cosine!– 1/ξ
- Higher powers of cosine in t-channel – threshold in s -channel
- Larger for pion than for nucleon pairs because of less fast decrease at x ->1
- Stability defines the sign of GDA and (via soft pion theorem) DA: work in progress

$$\mathscr{H}(\xi) = -\int_{-1/\xi}^{1/\xi} dx \sum_{n=0}^{\infty} H(x,\xi) \frac{x^n}{\xi^{n+1}}$$
$$= -\int_{-1/\xi}^{1/\xi} dx \sum_{n=0}^{\infty} H(x,x) \frac{x^n}{\xi^{n+1}} + \Delta\mathscr{H}.$$

Analyticity of Compton amplitudes in energy plane (Anikin,OT'07)

- Finite subtraction implied

$$\mathrm{Re}\mathcal{A}(\nu,Q^2)=\frac{\nu^2}{\pi}\mathcal{P}\int_{\nu_0}^{\infty}\frac{d\nu'^2}{\nu'^2}\frac{\mathrm{Im}\mathcal{A}(\nu',Q^2)}{(\nu'^2-\nu^2)}+\Delta \qquad \Delta=2\int_{-1}^{1}d\beta\,\frac{D(\beta)}{\beta-1}$$

$$\Delta^p_{\mathrm{CQM}}(2)=\Delta^n_{\mathrm{CQM}}(2)\approx 4.4, \qquad \Delta^p_{\mathrm{latt}}\approx\Delta^n_{\mathrm{latt}}\approx 1.1$$

- Numerically close to Thomson term for real proton (but NOT neutron) Compton Scattering!

- Duality (sum of squares vs square of sum; proton: 4/9+4/9+1/9=1)?!

From D-term to pressure

- Inverse -> 1st moment (model)
- Kinematical factor – moment of pressure $C \sim \langle p\, r^4 \rangle$ ($\langle p\, r^2 \rangle = 0$)

M.Polyakov'03

$$T^Q_{\mu\nu}(\vec{r},\vec{s}) = \frac{1}{2E}\int \frac{d^3\Delta}{(2\pi)^3}\, e^{i\vec{r}\cdot\vec{\Delta}}\, \langle p', S'|\hat{T}^Q_{\mu\nu}(0)|p, S\rangle$$

$$T_{ij}(\vec{r}) = s(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\,\delta_{ij}\right) + p(r)\delta_{ij}$$

- Stable equilibrium C>0:

Loss of stability?

- D=0 -> extra node required (cf tensor distribution - Efremov,OT- mechanical analogy – c.m. and c.i.)
- Smooth decrease – two extra nodes
- ++++------------------
- +++++++-------+++++--------------
- J=2 (Talk of Barbara Pasquini, comment by Maxim – zeros of Bessel functs?!)

CONCLUSIONS/OUTLOOK

- “Macroscopical” aspects of GPDs
- Pressure of quark flavours/gluons – DVMP
- Pressure from TMDs (TMD/GPD relations)?
- Comparison to QCD matter (HIC)

BACKUP

Is D-term independent?

- Fast enough decrease at large energy ->

$$\mathrm{Re}\,\mathcal{A}(\nu) = \frac{\mathcal{P}}{\pi}\int_{\nu_0}^{\infty} d\nu'^2 \frac{\mathrm{Im}\,\mathcal{A}(\nu')}{\nu'^2-\nu^2} + \mathrm{C}_0$$

$$\mathrm{C}_0 = \Delta - \frac{\mathcal{P}}{\pi}\int_{\nu_0}^{\infty} d\nu'^2 \frac{\mathrm{Im}\,\mathcal{A}(\nu')}{\nu'^2}$$

$$= \Delta + \mathcal{P}\int_{-1}^{1} dx \frac{H^{(+)}(x,x)}{x}.$$

- FORWARD limit of Holographic equation

$$\Delta = \mathcal{P}\int_{-1}^{1} dx \frac{H^{(+)}(x,0) - H^{(+)}(x,x)}{x}$$

$$= 2\mathcal{P}\int_{-1}^{1} dx \frac{H(x,0) - H(x,x)}{x},$$

$$\mathrm{C}_0(t) = 2\mathcal{P}\int_{-1}^{1} dx \frac{H(x,0,t)}{x}$$

“D – term” 30 years before…

- Cf Brodsky, Close, Gunion’72 (seagull ~ pressure) – but NOT DVMP
- D-term – a sort of renormalization constant
- May be calculated in effective theory if we know fundamental one
- OR
- Recover through special regularization procedure (D. Mueller)?

Vector mesons and EEP

- J=1/2 -> J=1. QCD SR calculation of Rho’s AMM gives g close to 2.

- Maybe because of similarity of moments
- g-2=<E(x)>; B=<xE(x)>
- Directly for charged Rho (combinations like p+n for nucleons unnecessary!). Not reduced to non-extended EP:

EEP and AdS/QCD

- Recent development – calculation of Rho formfactors in Holographic QCD (Grigoryan, Radyushkin)
- Provides g=2 identically!
- Experimental test at time –like region possible

EEP and Sivers function

- Sivers function – process dependent (effective) one
- T-odd effect in T-conserving theory- phase
- FSI – Brodsky-Hwang-Schmidt model
- Unsuppressed by M/Q twist 3
- Process dependence- colour factors
- After Extraction of phase – relation to universal (T-even) matrix elements

EEP and Sivers function -II

- Qualitatively similar to OAM and Anomalous Magnetic Moment (talk of S. Brodsky)
- Quantification : weighted TM moment of Sivers PROPORTIONAL to GPD E (OT'07, **hep-ph/0612205**): $x f_{T}(x) : xE(x)$
- Burkardt SR for Sivers functions is then related to Ji's SR for E and, in turn, to Equivalence Principle

$$\sum_{q,G} \int dx x f_{T}(x) = \sum_{q,G} \int dx x E(x) = 0$$

EEP and Sivers function for deuteron

- EEP - smallness of deuteron Sivers function
- Cancellation of Sivers functions – separately for quarks (before inclusion gluons)
- Equipartition + small gluon spin – large longitudinal orbital momenta (BUT small transverse ones –Brodsky, Gardner)

Another relation of Gravitational FF and NP QCD (first reported at 1992: **hep-ph/9303228**)

- BELINFANTE (relocalization) invariance :

decreasing in coordinate –

$$M^{\mu,\nu\rho} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} J^5_{S\sigma} + x^\nu T^{\mu\rho} - x^\rho T^{\mu\nu}$$

smoothness in momentum space

$$M^{\mu,\nu\rho} = x^\nu T^{\mu\rho}_B - x^\rho T^{\mu\nu}_B$$

- Leads to absence of massless pole in singlet channel – U_A(1)

$$\epsilon_{\mu\nu\rho\alpha} M^{\mu,\nu\rho} = 0.$$

- Delicate effect of NP QCD

$$(g_{\rho\nu} g_{\alpha\mu} - g_{\rho\mu} g_{\alpha\nu})\partial^\rho (J^\alpha_{5S} x^\nu) = 0.$$

- Equipartition – deeply related to relocalization invariance by QCD evolution

$$q^2 \frac{\partial}{\partial q^\alpha} \langle P|J^\alpha_{5S}|P+q\rangle = (q^\beta \frac{\partial}{\partial q^\beta} - 1) q_\gamma \langle P|J^\gamma_{5S}|P+q\rangle$$

$$\langle P,S|J^5_\mu(0)|P+q,S\rangle = 2MS_\mu G_1 + q_\mu (Sq) G_2,$$

$$q^2 G_2|_0 = 0$$