Shear forces and tensor polarization

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Main Topics

- Gravitational formfactors and (Extended) Equivalence Principle: spin $\frac{1}{2}$ and GPDs
- Spin 1: INCLUSIVE processes with tensor polarization
- HERMES data and (Ex)EP
- New possibilities: DY and hadronic hard processes
- NICA@JINR
Recent development: pressure

- Published in Nature, already used for EoS (talk of A. Bacchetta)
The pressure distribution inside the proton

V.D. Burkert, L. Elouadrhiri & F. X. Girod

\[ r^2 p(r) \times 10^{-2} \text{ GeV fm}^{-1} \]

- Repulsive pressure
- Confining pressure

\[ r \text{ (fm)} \]

\[ r^2 p(r) \times 10^{-2} \text{ GeV fm}^{-1} \]
Gravitational Formfactors

\( \text{(spin } \frac{1}{2}) \)

- Conservation laws - zero Anomalous Gravitomagnetic Moment: \( \mu_G = J \) (g=2)
  
  \[
  P_{q,g} = A_{q,g}(0) \quad A_q(0) + A_g(0) = 1
  \]
  
  \[
  J_{q,g} = \frac{1}{2} [A_{q,g}(0) + B_{q,g}(0)] \quad A_q(0) + B_q(0) + A_g(0) + B_g(0) = 1
  \]

- May be extracted from high-energy experiments/NPQCD calculations

- Describe the partition of angular momentum between quarks and gluons

- Describe interaction with both classical and TeV gravity
Electromagnetism vs Gravity (OT’99)

- Interaction – field vs metric deviation

\[ M = \langle P' | J_q^\mu | P \rangle A_\mu(q) \]

- Static limit

\[ \langle P | J_q^\mu | P \rangle = 2e_q P^\mu \]

\[ M_0 = \langle P | J_q^\mu | P \rangle A_\mu = 2e_q M \phi(q) \]

\[ \sum_{q,G} \langle P | T_{i,\mu\nu} | P \rangle = 2P^\mu P^{\nu} \]

\[ h_{00} = 2\phi(x) \]

\[ M_0 = \frac{1}{2} \sum_{q,G} \langle P | T_{i,\mu\nu} | P \rangle h_{\mu\nu} = 2M \cdot M \phi(q) \]

- Mass as charge – equivalence principle
Gravitomagnetism

- Gravitomagnetic field (weak, except in gravity waves) – action on spin from

\[ M = \frac{1}{2} \sum_{q,G} \langle P' | T_{q,G}^{\mu\nu} | P \rangle h_{\mu\nu}(q) \]

spin dragging twice smaller than EM

- Lorentz force – similar to EM case: factor ½ cancelled with 2 from Larmor frequency same as EM

\[ h_{00} = 2\phi(x) \]

\[ \omega_J = \frac{\mu_G}{J} H_J = \frac{H_L}{2} = \omega_L \]

- Orbital and Spin momenta dragging – the same - Equivalence principle

\[ \vec{H}_L = \text{rot} \vec{g} \]
Equivalence principle

- Newtonian – "Falling elevator" – well known and checked (also for elementary particles)
- Post-Newtonian – gravity action on SPIN – known since 1962 (Kobzarev and Okun’; rederived from conservation laws - Kobzarev and V.I. Zakharov
- Anomalous gravitomagnetic (and electric-CP-odd) moment is ZERO or
- Classical and QUANTUM rotators behave in the SAME way
- Gravitational analog of Ji’s SR $\int dx \times (\Sigma E_q + E_G) = 0!$
Generalization of Equivalence principle

- Various arguments: $\text{AGM} \approx 0$ separately for quarks and gluons – most clear from the lattice (LHPC/SESAM)
Recent lattice study (M. Deka et al. Phys.Rev. D91 (2015) no.1, 014505)

- Sum of u and d for Dirac (T1) and Pauli (T2) FFs
Extended Equivalence Principle = Exact EquiPartition

- In pQCD – violated
- Reason – in the case of ExEP - no smooth transition for zero fermion mass limit (Milton, 73)
- Conjecture (O.T., 2001 – prior to lattice data) – valid in NP QCD – zero quark mass limit is safe due to chiral symmetry breaking
- Gravityproof confinement? Nucleons do not break even by black holes?
- Support by recent observation of smalness of Cbar
Spin-1 hadrons

- MANY new FFs!
- Recent extensive analysis
- Cosyn, Cotogno, Freese, Lorce: 1903.00408
- Polyakov, Sun: 1903.02738
- A lot of integral relations between GPDs and FFs
Spin 1 EMT and inclusive processes

- Forward matrix element -> density matrix
- Contains \textit{P-even} term: tensor polarization
- Symmetric and \textit{traceless}: correspond to (average) \textit{shear} forces
- For spin \( \frac{1}{2} \): \textit{P-odd} vector polarization requires another vector \((q)\) to form vector product
Spin 1 in QCD

- Tensor polarization in QCD: Frankfurt, Strikman (81), Efremov, OT (81)
- Spin $\frac{1}{2}$: kinematically enhanced longitudinal polarization transverse-twist 3
- Spin 1: LL/TT related by tracelessness
SUM RULEs

- We (A.V. Efremov, OT’81) derived zero sum rules:
  - 1\textsuperscript{st} moment: also in parton model by Close and Kumano (90)
  - 2\textsuperscript{nd} moment (forward analog of Ji’s SR)
  - Average shear force (compensated between quarks and gluons)
  - Gravity and (Ex)EP (zero average shear separately for quarks and gluons) – OT’09
Manifestation of post-Newtonian (Ex)EP for spin 1 hadrons

- Tensor polarization - coupling of EMT to spin in forward matrix elements - inclusive processes

\[ A_T = \frac{\sigma_+ + \sigma_- - 2\sigma_0}{3\bar{\sigma}} \]

\[ \int_0^1 C_i^T(x)dx = 0 \] (AVE.OT'91.93)

\[ \sum_q \langle P, S|\bar{\psi}(0)\gamma^\nu D^{\mu_1}...D^{\mu_n}\psi(0)|P, S\rangle_{\mu^2} = i^{-n}M^2S^{\nu\nu_1}P^{\mu_2}...P\nu_n\int_0^1 C_q^T(x)x^n\,dx \]

\[ \sum_q \langle P, S|T_i^{\mu\nu}|P, S\rangle_{\mu^2} = 2P^\mu P^\nu(1 - \delta(\mu^2)) + 2M^2S^{\mu\nu}\delta_1(\mu^2) \]

\[ \sum_q \int_0^1 C_i^T(x)x\,dx = \delta_1(\mu^2) = 0 \] for ExEP
HERMES – data on tensor spin structure function

- Isoscalar target – proportional to the sum of u and d quarks – combination required by (Ex)EP
- Second moments – compatible to zero better than the first one (collective glue << sea)
Where else to test?

- COMPASS
- EIC
- DY@J-PARC: (Song, Kumano: 1902.04712)

However: ET’81 - any hard process

\[ f_{Al} \sim b_1 \]

Suggestion: hadronic tensor SSA
Vector vs Tensor SSA

- **Vector**: \( A = (\sigma(+) - \sigma(-))/(\sigma(+) + \sigma(-)) \)

- **Tensor**: \( A = (\sigma(+) + \sigma(-))/(\sigma(+) + \sigma(-) + \sigma(0)) \)

- Inclusive pion production: (T-odd) vector SSA may be also excluded by summing \( \sigma(L) + \sigma(R) \)
Tensor polarized beams

- Opportunity: NICA@JINR with polarized hadronic beams

- Polarized deuterons is easier to accelerate: no depolarizing resonances

- DY, J/Ψ (+hadronic SSA)
NICA: heavy ions and hadrons
Conclusions

- Tensor polarization: way to test of gravitational coupling of quarks and gluons in inclusive processes
- HERMES data are compatible with validity of EP separately for quarks and gluons
- Tests in DY and hadronic TSSA at NICA are possible
One more gravitational formfactor

- Quadrupole
  \[ \langle P + q/2 | T^{\mu\nu} | P - q/2 \rangle = C(q^2)(g^{\mu\nu} q^2 - q^\mu q^\nu) + ... \]

- Cf vacuum matrix element – cosmological constant (vacuum pressure)
  \[ \langle 0 | T^{\mu\nu} | 0 \rangle = \Lambda g^{\mu\nu} \]
  \[ \Lambda = C(q^2)q^2 \]

- Inflation ~ annihilation \((q^2 > 0)\) OT’15

- How to measure experimentally – Deeply Virtual Compton Scattering
D-term interpretation: Inflation and annihilation

- Quadrupole gravitational FF
  \[ \langle P + q/2 | T^{\mu\nu} | P - q/2 \rangle = C(q^2)(g^{\mu\nu} q^2 - q^\mu q^\nu) + ... \]

- Moment of D-term – positive
- Vacuum – Cosmological Constant
  \[ \langle 0 | T^{\mu\nu} | 0 \rangle = \Lambda g^{\mu\nu} \]

- 2D effective CC – negative in scattering, positive in annihilation
  \[ \Lambda = C(q^2)q^2 \]

- Similarity of inflation and Schwinger pair production – Starobisnky, Zel’dovich
- Was OUR Big Bang resulting from one graviton annihilation at extra dimensions??! Version of “ekpyrotic” (“pyrotechnic”) universe
C vs Cbar

- Cancellations of Cbars – negative pressure (cf Chaplygin gas)

-Cancellation in vacuum; Pauli (divergent), Zel’dovich (finite)

- Flavour structure of pressure: DVMP!
Manifestly spectral

$$\mathcal{H}(x_B) = \int_{-1}^{1} dx \frac{H(x)}{x - x_B + i\epsilon}.$$  

Extra dependence on $\xi$

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x, \xi)}{x - \xi + i\epsilon},$$
Unphysical regions

- **DIS**: Analytical function – polynomial in $1/x_B$ if $1 \leq |X_B|$

\[ H(x_B) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x) \frac{x^n}{x_B^{n+1}} \]

- **DVCS** – additional problem of analytical continuation of $H(x, \xi)$

Solved by using of

Double Distributions

Radon transform

\[ H(z, \xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy (F(x, y) + \xi G(x, y)) \delta(z - x - \xi y) \]
Double distributions and their integration

- Slope of the integration line-skewness
- Kinematics of DIS: $\xi = 0$ ("forward") - vertical line (1)
- Kinematics of DVCS: $\xi < 1$ - line 2
- Line 3: $\xi > 1$ unphysical region - required to restore DD by inverse Radon transform: tomography

$$f(x, y) = -\frac{1}{2\pi^2} \int_0^\infty \frac{dp}{p^2} \int_0^{2\pi} d\phi |\cos\phi| (H(p/\cos\phi + x + yt\phi, t\phi) - H(x + yt\phi, t\phi)) =$$

$$= -\frac{1}{2\pi^2} \int_{-\infty}^{\infty} \frac{dz}{z^2} \int_{-\infty}^{\infty} d\xi (H(z + x + y\xi, \xi) - H(x + y\xi, \xi))$$
Crossing for DVCS and GPD

- DVCS -> hadron pair production in the collisions of real and virtual photons
- GPD -> Generalized Distribution Amplitudes
- Duality between s and t channels (Polyakov, Shuvaev, Guzey, Vanderhaeghen)
GDA -> back to unphysical regions for DIS and DVCS

- Recall DIS

\[ H(x_B) = - \int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x) \frac{x^n}{x_B^{n+1}} \]

- Non-positive powers of \( x_B \)

- DVCS

\[ H(\xi) = - \int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x, \xi) \frac{x^n}{\xi^{n+1}} \]

- Polynomiality (general property of Radon transforms): moments - integrals in \( x \) weighted with \( x^n \) - are polynomials in \( 1/\xi \) of power \( n+1 \)

- As a result, analyticity is preserved: only non-positive powers of \( \xi \) appear
Holographic property (OT’05)

Factorization Formula

\[ \mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x, \xi)}{x - \xi + i\epsilon} \]

Analyticity -> Imaginary part -> Dispersion relation:

\[ \mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x, x)}{x - \xi + i\epsilon} \]

\[ \Delta \mathcal{H}(\xi) \equiv \int_{-1}^{1} dx \frac{H(x, x) - H(x, \xi)}{x - \xi + i\epsilon} \]

“Holographic” equation (DVCS AND VM)

\[ = \sum_{n=1}^{\infty} \frac{1}{n!} \frac{\partial^n}{\partial \xi^n} \int_{-1}^{1} H(x, \xi) dx (x - \xi)^{n-1} = \text{const} \]
Holographic property - II

- Directly follows from double distributions

\[ H(z, \xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy \left( F(x, y) + \xi G(x, y) \right) \delta(z - x - \xi y) \]

- Constant is the SUBTRACTION one - due to the (generalized) Polyakov-Weiss term \( G(x,y) \)

\[ \Delta \mathcal{H}(\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy \frac{G(x, y)}{1 - y} \]

\[ = \int_{-\xi}^{\xi} dx \frac{D(x/\xi)}{x - \xi + i\epsilon} = \int_{-1}^{1} dz \frac{D(z)}{z - 1} = \text{const} \]
- 2-dimensional space -> 1-dimensional section!
- Momentum space: any relation to holography in coordinate space ?!
- Strategy (now adopted) of GPD’s studies: start at diagonals (through SSA due to imaginary part of DVCS amplitude) and restore by making use of dispersion relations + subtraction constants.
Holography vs NLO

- Depends on factorization scheme

- Special role of scheme preserving the coefficient function

- Nucleon as (scheme dependent) black hole – 3D information encoded in 2D
Pressure in hadron pairs production

- Back to GDA region
- \( \rightarrow \) moments of \( H(x,x) \) - define the coefficients of powers of cosine! – 1/
- Higher powers of cosine \( \xi \) in t-channel – threshold in s-channel
- Larger for pion than for nucleon pairs because of less fast decrease at \( x \rightarrow 1 \)
- Stability defines the sign of GDA and (via soft pion theorem) DA: work in progress

\[
\mathcal{H}(\xi) = -\int_{-1/\xi}^{1/\xi} dx \sum_{n=0}^{\infty} H(x, \xi) \frac{x^n}{\xi^{n+1}} \\
= -\int_{-1/\xi}^{1/\xi} dx \sum_{n=0}^{\infty} H(x, x) \frac{x^n}{\xi^{n+1}} + \Delta \mathcal{H}.
\]
Analyticity of Compton amplitudes in energy plane (Anikin, OT’07)

- Finite subtraction implied

\[ \text{Re} \mathcal{A}(\nu, Q^2) = \frac{\nu^2}{\pi} \mathcal{P} \int_{\nu_0}^{\infty} \frac{d\nu'^2}{\nu'^2} \frac{\text{Im} \mathcal{A}(\nu', Q^2)}{(\nu'^2 - \nu^2)} + \Delta \]

\[ \Delta = 2 \int_{-1}^{1} d\beta \frac{D(\beta)}{\beta - 1} \]

\[ \Delta_{\text{CQM}}^p(2) = \Delta_{\text{CQM}}^n(2) \approx 4.4, \quad \Delta_{\text{latt}}^p \approx \Delta_{\text{latt}}^n \approx 1.1 \]

- Numerically close to Thomson term for real proton (but NOT neutron) Compton Scattering!

- Duality (sum of squares vs square of sum; proton: \(4/9 + 4/9 + 1/9 = 1\))?!
From D-term to pressure

- **Inverse -> 1st moment (model)**
- **Kinematical factor – moment of pressure** $C \sim <p r^4>$ ($<p r^2> = 0$)
  
  \[ T^{\mu \nu}_{\mu \nu}(r, \vec{s}) = \frac{1}{2E} \int \frac{d^3\Delta}{(2\pi)^3} e^{ip\Delta} \langle p', S'|\hat{T}^\mu_{\mu}(0)|p, S \rangle \]

  \[ T_{ij}(r) = s(r) \left( \frac{r_i r_j}{r^2} - \frac{1}{3} \delta_{ij} \right) + p(r) \delta_{ij} \]

- **Stable equilibrium** $C>0$:
Loss of stability?

- D=0 -> extra node required (cf tensor distribution - Efremov, OT- mechanical analogy – c.m. and c.i.)
- Smooth decrease – two extra nodes
- J=2 (Talk of Barbara Pasquini, comment by Maxim – zeros of Bessel functs?!)}
CONCLUSIONS/OUTLOOK

- “Macroscopical” aspects of GPDs
- Pressure of quark flavours/gluons – DVMP
- Pressure from TMDs (TMD/GPD relations)?
- Comparison to QCD matter (HIC)
BACKUP
Is D-term independent?

- Fast enough decrease at large energy -

\[
\text{Re } \mathcal{A}(\nu) = \frac{\mathcal{P}}{\pi} \int_{\nu_0}^{\infty} d\nu' \frac{\text{Im } \mathcal{A}(\nu')}{\nu'^2 - \nu^2} + C_0.
\]

\[
C_0 = \Delta - \frac{\mathcal{P}}{\pi} \int_{\nu_0}^{\infty} d\nu' \frac{\text{Im } \mathcal{A}(\nu')}{\nu'^2} = \Delta + \mathcal{P} \int_{-1}^{1} dx \frac{H^{(+)}(x, x)}{x}.
\]

- FORWARD limit of Holographic equation

\[
\Delta = \mathcal{P} \int_{-1}^{1} dx \frac{H^{(+)}(x, 0) - H^{(+)}(x, x)}{x} = 2\mathcal{P} \int_{-1}^{1} dx \frac{H(x, 0, t)}{x},
\]

\[
C_0(t) = 2\mathcal{P} \int_{-1}^{1} dx \frac{H(x, 0, t)}{x}.
\]
“D – term” 30 years before...

- Cf Brodsky, Close, Gunion’72 (seagull ~ pressure) – but NOT DVMP
- D-term – a sort of renormalization constant
- May be calculated in effective theory if we know fundamental one
- OR
- Recover through special regularization procedure (D. Mueller)?
Vector mesons and EEP

- J=1/2 -> J=1. QCD SR calculation of Rho’s AMM gives g close to 2.

- Maybe because of similarity of moments
- $g-2=<E(x)>; \ B=<xE(x)>$
- Directly for charged Rho (combinations like p+n for nucleons unnecessary!). Not reduced to non-extended EP:
EEP and AdS/QCD

- Recent development – calculation of Rho formfactors in Holographic QCD (Grigoryan, Radyushkin)
- Provides g=2 identically!
- Experimental test at time –like region possible
EEP and Sivers function

- Sivers function – process dependent (effective) one
- T-odd effect in T-conserving theory- phase
- FSI – Brodsky-Hwang-Schmidt model
- Unsuppressed by M/Q twist 3
- Process dependence- colour factors
- After Extraction of phase – relation to universal (T-even) matrix elements
EEP and Sivers function -II

- Qualitatively similar to OAM and Anomalous Magnetic Moment (talk of S. Brodsky)

\[ x \int_T f_T(x) : xE(x) \]

- Burkardt SR for Sivers functions is then related to Ji’s SR for E and, in turn, to Equivalence Principle

\[ \sum_{q,G} \int dxx f_T(x) = \sum_{q,G} \int dxxE(x) = 0 \]
EEP and Sivers function for deuteron

- EEP - smallness of deuteron Sivers function
- Cancellation of Sivers functions – separately for quarks (before inclusion gluons)
- Equipartition + small gluon spin – large longitudinal orbital momenta (BUT small transverse ones – Brodsky, Gardner)
Another relation of Gravitational FF and NP QCD (first reported at 1992: hep-ph/9303228)

- BELINFANTE (relocalization) invariance:
  decreasing in coordinate – smoothness in momentum space

- Leads to absence of massless pole in singlet channel – $U_A(1)$

- Delicate effect of NP QCD

- Equipartition – deeply related to relocalization invariance by QCD evolution

\[ M_{\mu\nu\rho} = \frac{1}{2} \epsilon_{\mu
u\rho\sigma} J_{S\sigma}^{5} + x^{\nu} T_{\mu\rho}^{B} - x^{\rho} T_{\mu\nu}^{B} \]

\[ M_{\mu\nu\rho} = x^{\nu} T_{\mu\rho}^{B} - x^{\rho} T_{\mu\nu}^{B} \]

\[ \epsilon_{\mu\nu\rho\alpha} M_{\mu\nu\rho} = 0. \]

\[ (g_{\rho\nu} g_{\alpha\mu} - g_{\rho\mu} g_{\alpha\nu}) \partial^{\theta} (J_{5S}^{\alpha} x^{\nu}) = 0 \]

\[ q^2 \frac{\partial}{\partial q^{\alpha}} \langle P | J_{5S}^{\alpha} | P + q \rangle = (q^3 \frac{\partial}{\partial q^3} - 1) q^{\nu} \langle P | J_{5S}^{\nu} | P + q \rangle \]

\[ \langle P, S | J_{5}^{\nu}(0) | P + q, S \rangle = 2M S_{\mu} G_{1} + q_{\mu} (S q) G_{2}. \]

\[ q^2 G_{2} |_{0} = 0 \]