Next-to-leading logarithmic processes in High Energy Jets

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Table of Contents

An Introduction to HEJ

High Energy Jets

- A Partonic Monte Carlo Generator which aims to describe **high multiplicity events**.

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- Hard corrections are α_s suppressed but phase space enhanced in the **large invariant mass limit**.

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- Provides perturbative predictions at LL accuracy ($\log(\hat{s}/|\hat{t}|)$) with **resummation of hard corrections to all orders**.
- Hard corrections are α_s suppressed but phase space enhanced in the **large invariant mass limit**.
- but we need a formalism...

Multi Regge Kinematic (MRK) Limit

The MRK Limit:

large $\hat{s}$; small P_T ; **strongly ordered jet rapidities** $(\mathbf{y_j})$:

$$y_1 \ll y_2 \ll ... \ll y_i \ll .. \ll y_{n-1} \ll y_n$$

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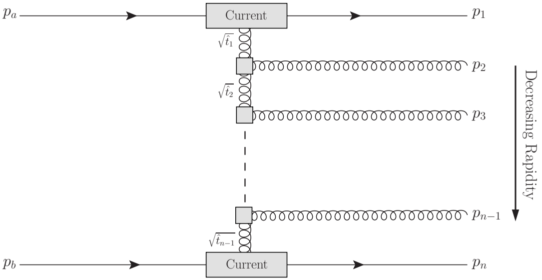
Some nice relations:

$$\begin{aligned} \hat{s}^2 &\sim -\hat{u}^2 \to large \\ \hat{t}_i &\sim -p_{\perp j_i}^2 \sim -p_\perp^2 \\ \log\left(\frac{\hat{s}_{ij}}{|\hat{t}_{ij}|}\right) &\approx |y_j - y_i| \end{aligned}$$

FKL Contributions

FKL configurations are the leading contributions in the MRK limit.

- $(2 \rightarrow n)$ amplitudes with strong rapidity ordering in final state



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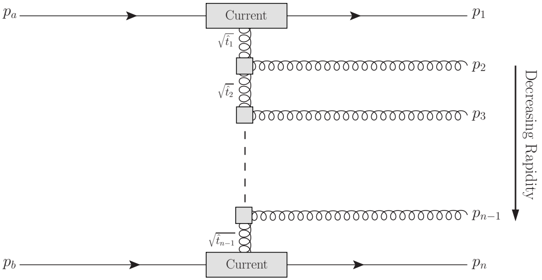

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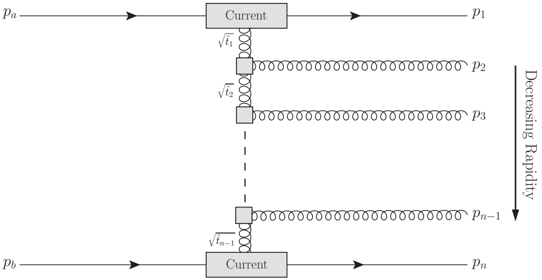

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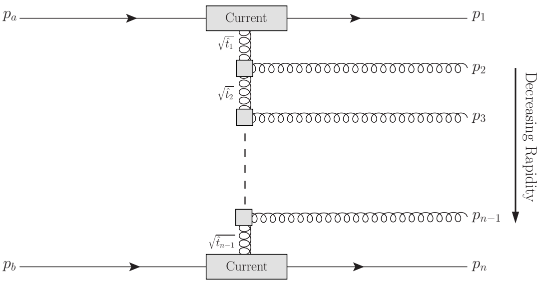

Resum via effective **Lipatov Vertices** and the **Lipatov Ansatz**.

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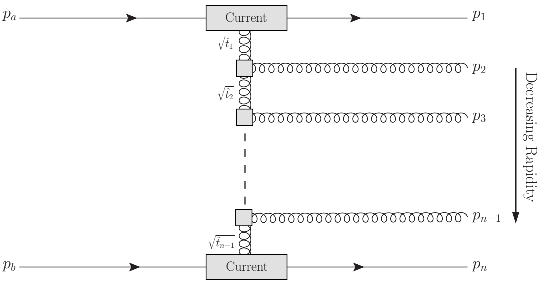
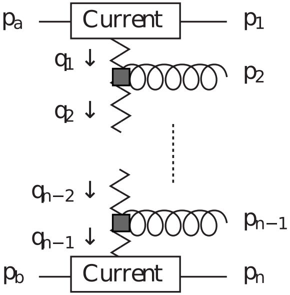


Resum via effective **Lipatov Vertices** and the **Lipatov Ansatz**.

For MRK kinematics, these are leading power in power expansion. After integration, gives **leading logarithmic** contribution.

HEJ Matrix element



$$= \frac{1}{4\,(N_C^2-1)}\,\left\|S_{f_a f_b \to f_1 f_n}\right\|^2 \cdot \left(g_s^2\,K_{f_1}\,\frac{1}{t_1}\right) \cdot \left(g_s^2\,K_{f_n}\,\frac{1}{t_{n-1}}\right) \cdot \prod_{i=1}^{n-2}\left(\frac{-\,g_s^2 C_A}{t_i t_{i+1}}\,V^{\mu}(q_i,q_{i+1})V_{\mu}(q_i,q_{i+1})\right) \cdot \prod_{j=1}^{n-1}\exp\left[\omega^0(q_{j\perp})(y_{j+1}-y_j)\right]$$

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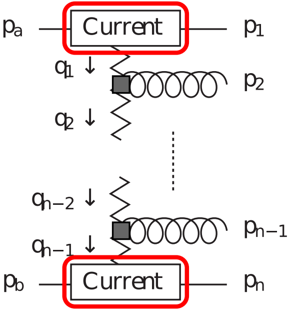

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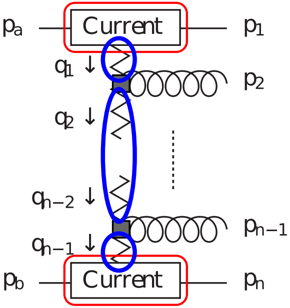

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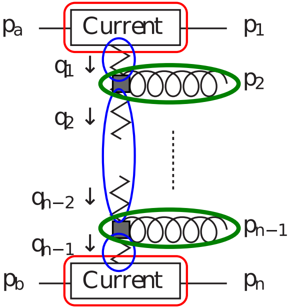

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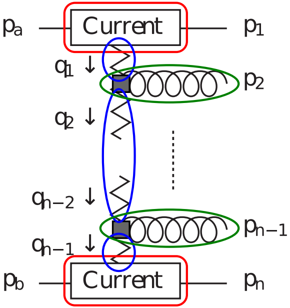

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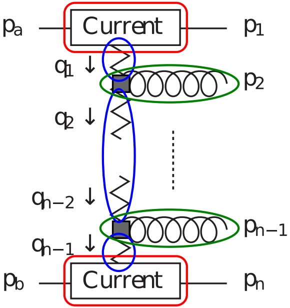
Processes $\Leftrightarrow$ currents, e.g. $\mathcal{S}_{f_1 f_2 \to f_1 H f_2} = j_\mu(p_1,p_a) V_H^{\mu\nu}(q_j,q_{j+1}) j_\nu(p_b,p_n)$.



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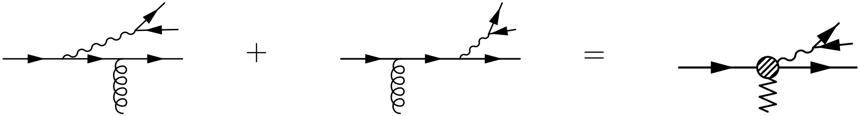
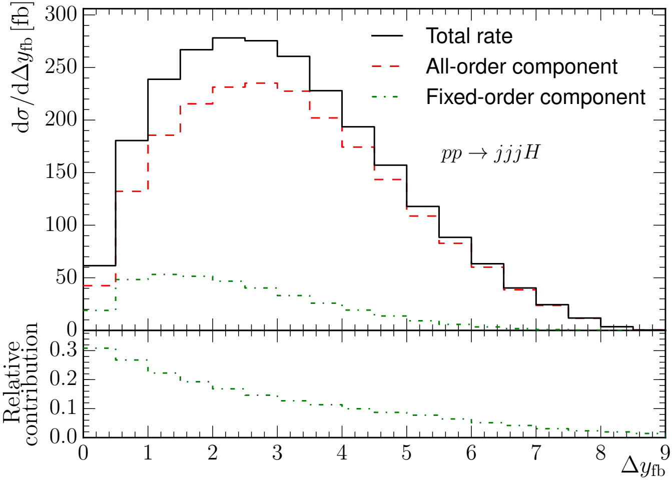


Motivation: H+Jets FKL Only



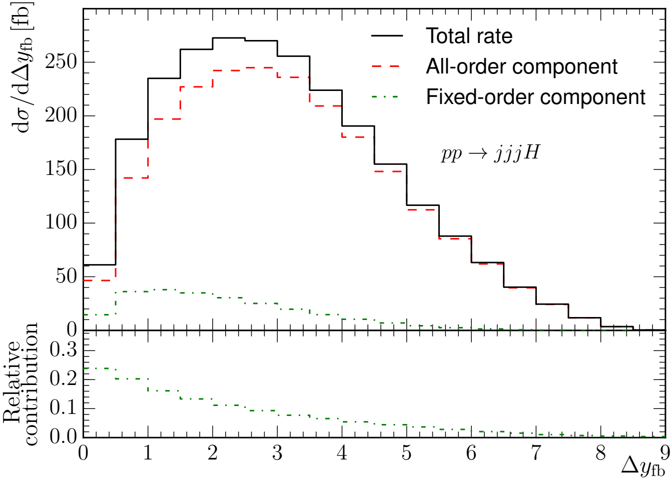
Resumed

- FKL

Processes at FO

- Unordered
- Extremal $q\bar{q}$
- Other

Motivation: H+Jets Including Unordered



Resummed

- FKL
- Unordered

Processes at FO

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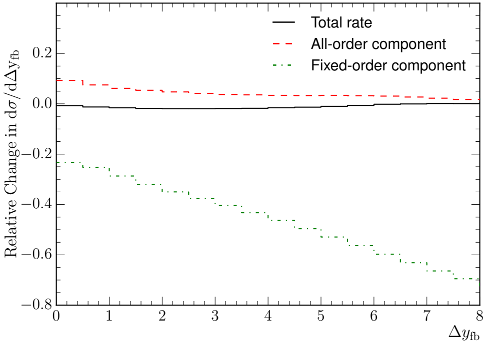
Change due to Unordered



Points of Interest:

- All-order component increased.
- FO component decreases linearly with y_{fb}
- Total rate mostly unchanged.

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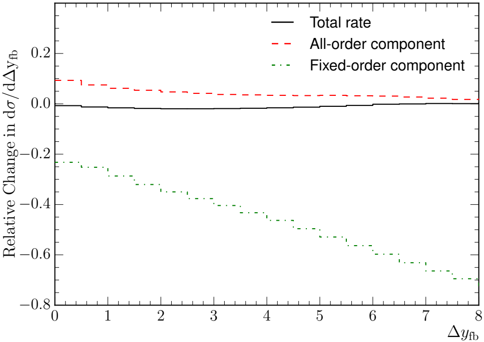

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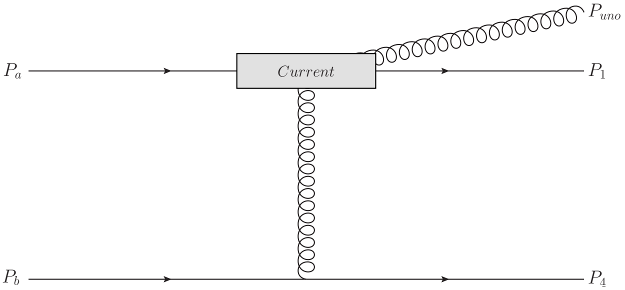
Goal: Include more subleading processes within HEJ approximation.

Unordered Contributions

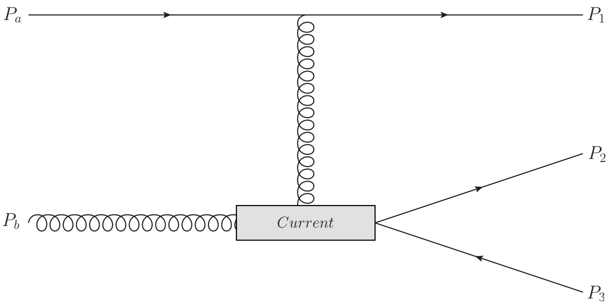


A gluon outside of FKL rapidity ordering is known as an **Unordered emission**.

In HEJ this is modelled as a modified current. Where we now allow that $y_{uno} \sim y_1$ and $y_1 >> y_2$. (QMRK Limit)

$$\mathcal{M}^{\text{uno}}_{qQ\to gqQ} \sim \frac{j^{\mu}_{\text{uno}}(p_a, p_1, p_{uno}) j_{\mu}(p_b, p_2)}{\hat{t}}$$

Extremal $q\bar{q}$



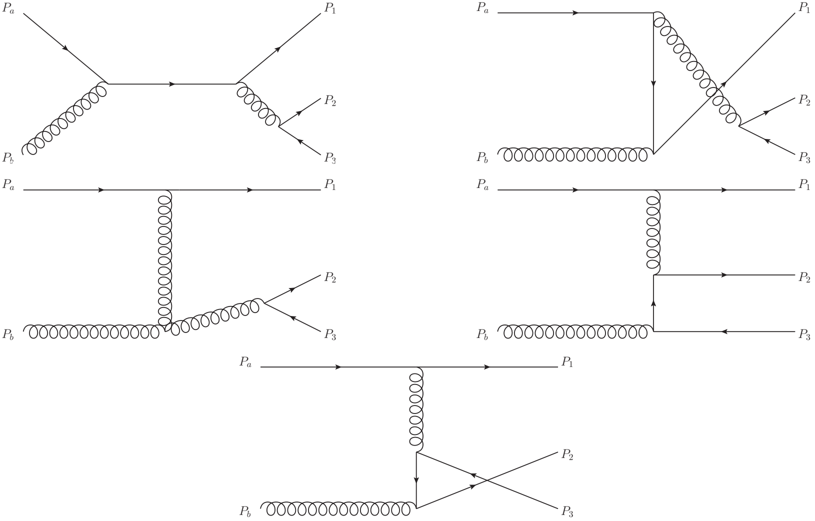
The **Extremal q$\bar{\text{q}}$** case is an incoming gluon splitting to $q\bar{q}$.

In HEJ use a a modified current (related by crossing symmetry to Uno case) in the scattering.

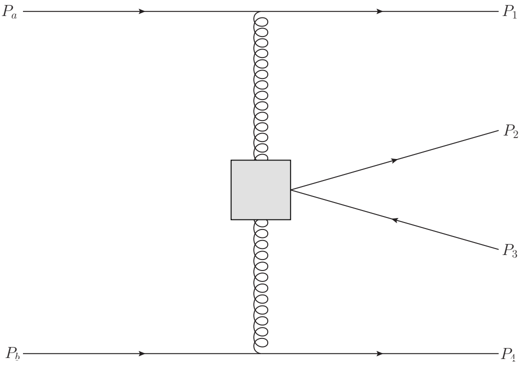
$$\mathcal{M}^{q\bar{q}}_{qg\to qQQ} \sim \frac{j^{\mu}_{q\bar{q}}(p_b, p_2, p_3) j_{\mu}(p_a, p_1)}{\hat{t}}$$

There are **5 possible diagrams** which contribute.

Extremal $q\bar{q}$: Possibilities



Central $q\bar{q}$

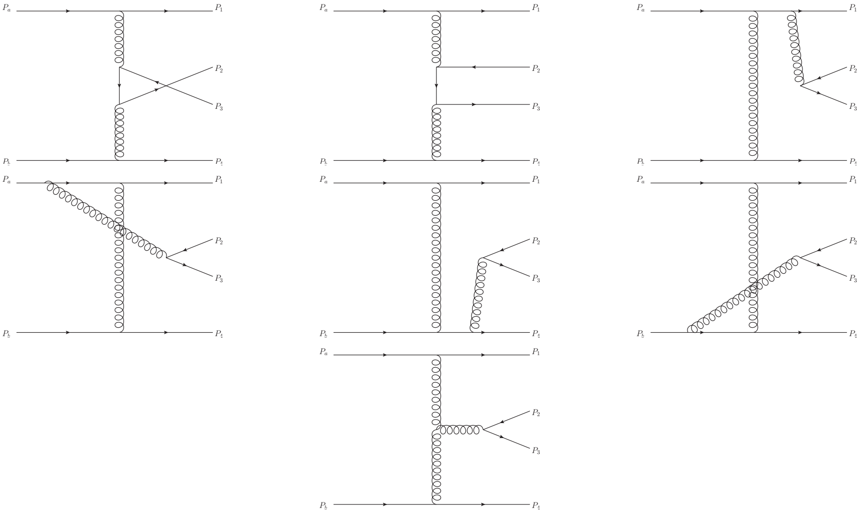


In the case a **Central q$\bar{\mathbf{q}}$** pair is produced, we use an effective vertex which fits the form:

$$\mathcal{M}_{qq \to qQ\overline{Q}q} \sim \frac{\langle 1|\mu|a\rangle \, X^{\mu\nu} \, \langle 4|\nu|b\rangle}{\hat{t}_1 \, \hat{t}_3}$$

There are **7 possible diagrams** which contribute.

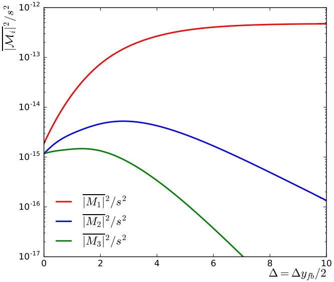
Central $q\bar{q}$

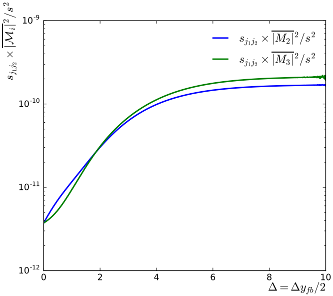


Scaling of the Matrix Elements

Higgs+3j: $qQ \to qgHQ$

Gluon Exchange (FKL) | Quark Exchange (Unordered) | Higgs Outside (Unordered)





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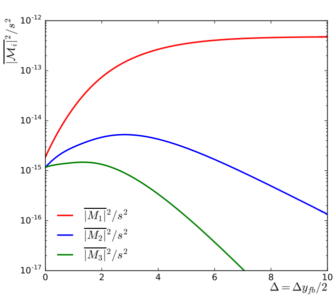
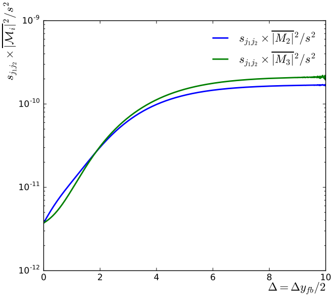


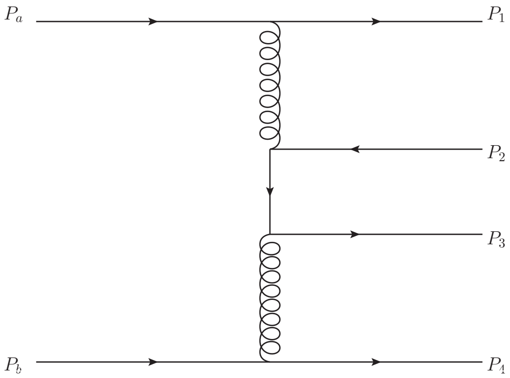


In Multi Regge Theory:

$$|\mathcal{M}| \sim (\hat{s}_{j_i j_j})^{spin}$$

Swapping propagator (gluon $\to$ quark) suppresses ME by $(\hat{s}_{j_i j_j})^{1/2}$.

Reducing Dependence on Matching



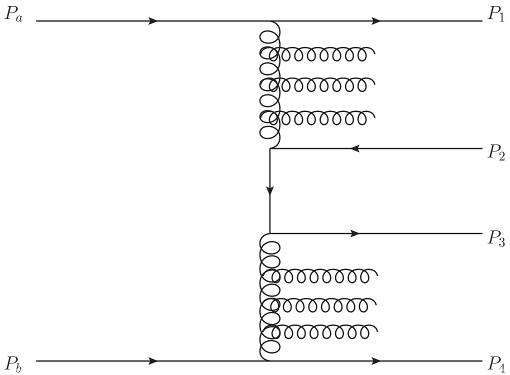
Fixed Order

α_s^4

Inclusion within HEJ:

Previously: • 4j + 5j Fixed order result used directly in HEJ

Reducing Dependence on Matching



Fixed Order

α_s^4

Add Resummation

$(\alpha_s \Delta_y)^N$

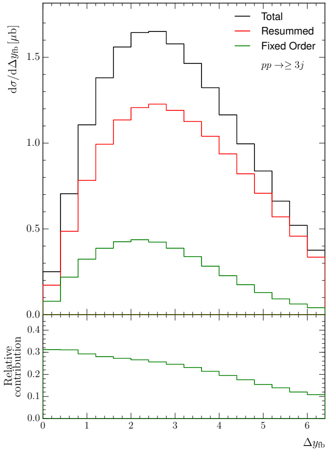
Inclusion within HEJ:

Previously: • 4j + 5j Fixed order result used directly in HEJ

Now: • 4j result reduced by virtual corrections. 5j result increased.

• sum of 4j + 5j state is logarithmically controlled.

Pure Jets: FKL Only



Processes Resummed

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Processes only at FO

- Unordered
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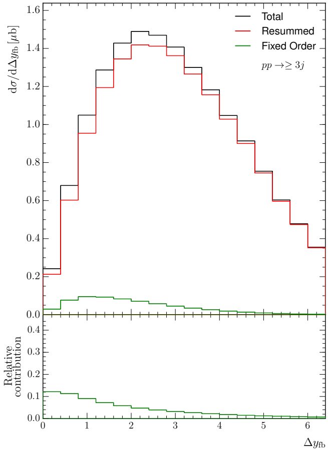
Nota Bene: we do not include Central $q\bar{q}$ since that only exists as a subleading process for 4j+

Pure Jets: All subleading processes



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- Unordered
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Processes only at FO

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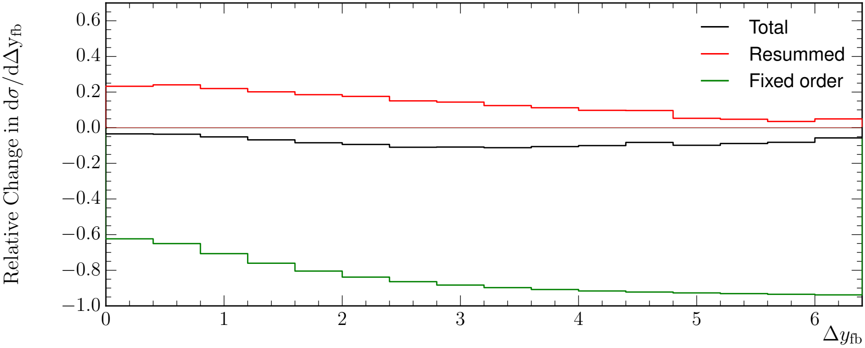
Change due to Subleading Pieces

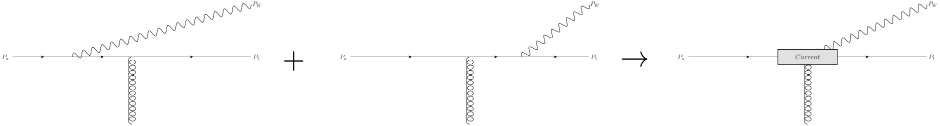


Points of interest:

- All-order component increased.
- FO component dramatically decreases with y_{fb}
- Total rate remains largely unchanged.

W+Jets at LL

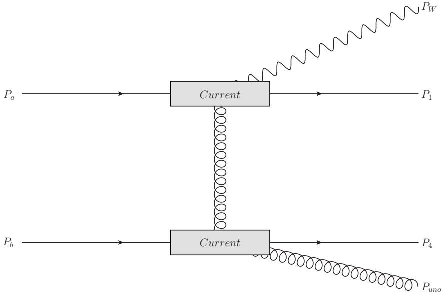
In HEJ, W+Jets are usually calculated differently from Pure Jets by the use of a **modified current**.



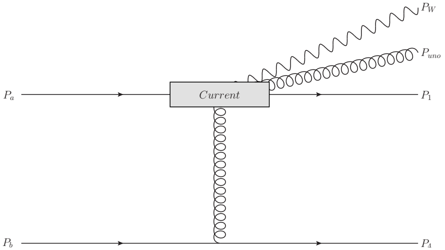
With the addition of the $q\bar{q}$ pairs we have additional places from which a W-Boson can be emitted.

Complications to Unordered

No New Objects

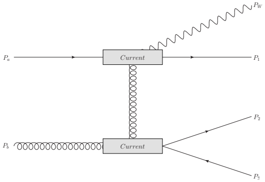


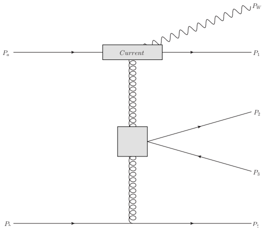
New Objects Required



Complications to $\mathbf{q\bar{q}}$

No New Objects





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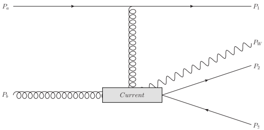
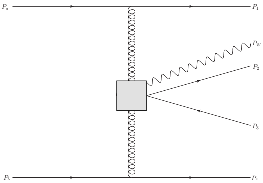




W+Jets at NLL: Extremal $q\bar{q}$

Consider Process: $qg \to qQ\overline{Q}W$

AIM:

Factorise the t channel exchanges and the current scattering, resulting in a new effective current at either end of the FKL chain.

Need to find an amplitude for the process $qg \to qQ\overline{Q}W$ of the form:

$$M_{qg\to qQ\overline{Q}W} \sim \frac{\langle 1|\mu|a\rangle \, Q^{\mu\nu\rho}\,(p_2, p_w, p_3, p_b)\, \varepsilon_\nu(p_b)\varepsilon^*_\rho(p_w)}{\hat{t}_1}$$

Where $Q^{\mu\nu\rho}$ is this effective current.

W+Jets at NLL: Central $q\bar{q}$

Consider Process: $qq \to qQ\overline{Q}Wq$

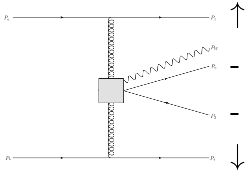
AIM:

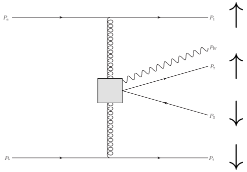
Factorise Currents and effective $q\bar{q}$ vertex. As in the extremal $q\bar{q}$

We therefore search for an expression of the form:

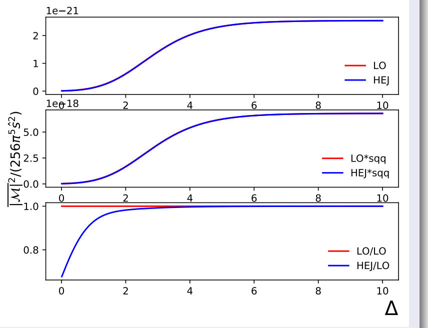
$$M_{qq \to qQ\overline{Q}qW} \sim \frac{\langle 1|\mu|a\rangle\, X^{\mu\nu}\, \langle 4|\nu|b\rangle}{\hat{t}_1 \hat{t}_3}$$

W+Jets Status Matrix Element Comparison

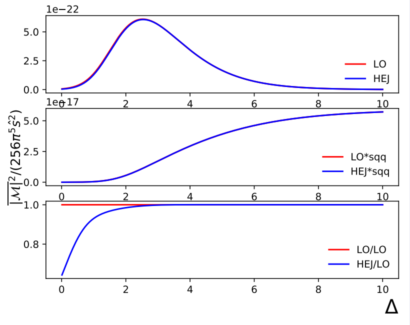




$q\bar{q}$ at fixed Δ_y



$q\bar{q}$ at increasing Δ_y



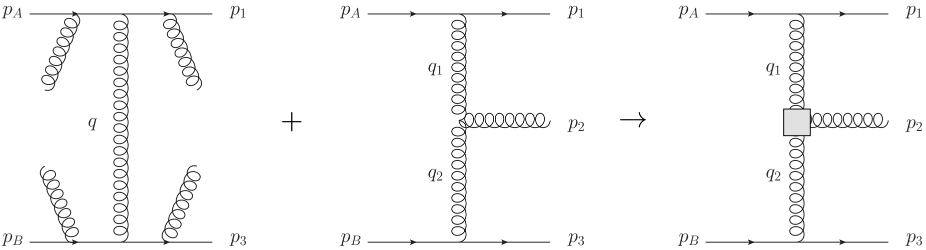
Conclusions and Further Considerations

- Added resummation for Unordered, Extremal and Central $q\bar{q}$
- HEJ is now leading log accurate for all sub-leading processes
- Verification process underway for W+Jets
- Next steps for Next-to-Leading Log:
    - Virtual Corrections
- HEJ2 recently had a public release!

https://hej.web.cern.ch/HEJ/

Backup slides

Lipatov Vertices



$$
\begin{aligned}
V^\rho(q_1, q_2) = &- (q_1 + q_2)^\rho \\
&+ \frac{p_A^\rho}{2}\left(\frac{q_1^2}{p_2 \cdot p_A} + \frac{p_2 \cdot p_B}{p_A \cdot p_B} + \frac{p_2 \cdot p_3}{p_A \cdot p_3}\right) + p_A \leftrightarrow p_1 \\
&- \frac{p_B^\rho}{2}\left(\frac{q_2^2}{p_2 \cdot p_B} + \frac{p_2 \cdot p_A}{p_B \cdot p_A} + \frac{p_2 \cdot p_1}{p_B \cdot p_1}\right) - p_B \leftrightarrow p_3 .
\end{aligned}
$$

Virtual Corrections

Lipatov Ansatz

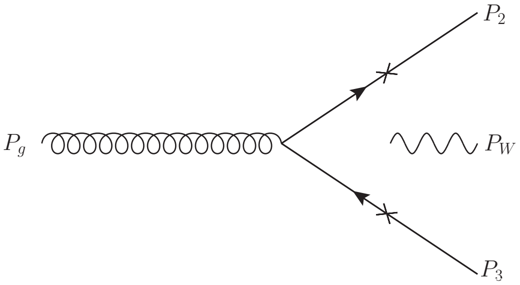
One can obtain the virtual corrections in the MRK limit with the Lipatov Ansatz, which is the following substitution within the analytic expression for the amplitudes:

$$\frac{1}{t_i} \to \frac{1}{t_i} \exp[\hat{\alpha}(q_i)(y_{i-1} - y_i)]$$

where

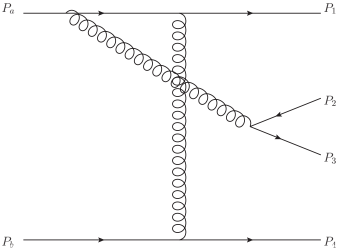
$$\hat{\alpha} = -g^2 C_A \frac{\Gamma(1-\varepsilon)}{(4\pi)^{2+\varepsilon}} \frac{2}{\varepsilon} \left(\frac{q^2}{\mu^2}\right)^{\varepsilon}$$

Building Blocks



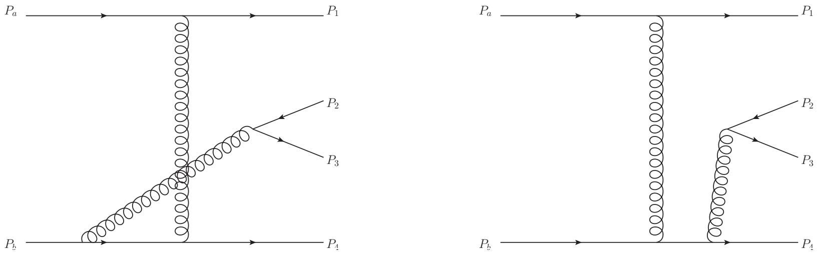
$$J_V^\mu(p_2, p_A, p_B, p_3) = \left(\frac{\bar{u}_2 \gamma^\nu (\not{p}_2 + \not{p}_A + \not{p}_B) \gamma^\mu u_3}{s_{2AB}} + \frac{\bar{u}_2 \gamma^\mu (\not{p}_3 - \not{p}_A - \not{p}_B) \gamma^\nu u_3}{s_{3AB}} \right) [\bar{u}_A \gamma_\nu u_B]$$

1a Contribution



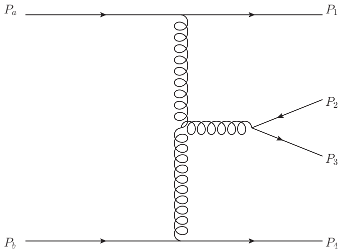
$$X_{1a}^{\mu\nu} = \frac{g^{\mu\nu} C_1 g_w g_s^4}{2\sqrt{2} s_{23AB}(s_{123AB})}(p_1^\rho) J_{V\rho}$$

4b Contribution



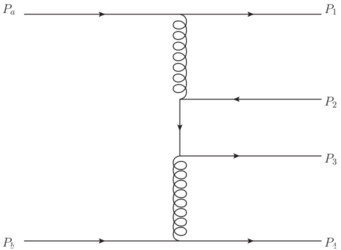
$$X_{4b}^{\mu\nu} = \frac{-g^{\mu\nu} C_1 g_w g_s^4}{2\sqrt{2} s_{23AB} (s_{234AB})} (p_4^{\rho}) J_{V\rho}$$

3 Gluon Contribution



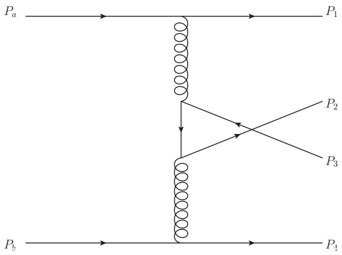
$$X^{\mu\nu}_{3g} = \frac{g_w g_s^2\, T^{geg'}\, T^e_{23}}{2\sqrt{2}\,\hat{t}_1 s_{23AB}\,\hat{t}_3}\Big[\,(q_1)^\mu\,\eta^{\nu\rho} + (q_3)^\nu\,\eta^{\mu\rho} - (q_1+q_3)^\rho\,\eta^{\mu\nu}\Big] J_{V\rho}(p_a, p_A, p_B, p_1)$$

Uncrossed Contributions



$$X^{\mu\nu}_{\text{uncross}} = \frac{\langle A|\sigma|B\rangle}{(p_A+p_B)^2}\bar{u}_2\left[\frac{\gamma^\sigma(\not{p}_2+\not{p}_A+\not{p}_B)\gamma^\nu(\not{p}_3+\not{p}_4-\not{p}_b)\gamma^\mu}{(s_{2AB})(t_{int_2})}+\right.$$
$$\frac{\gamma^\nu(\not{p}_a-\not{p}_1-\not{p}_2)\gamma^\sigma(\not{p}_3+\not{p}_4-\not{p}_b)\gamma^\mu}{(t_{int_1})(t_{int_2})}+$$
$$\left.\frac{\gamma^\nu(\not{p}_a-\not{p}_1-\not{p}_2)\gamma^\mu(\not{p}_3+\not{p}_A+\not{p}_B)\gamma^\sigma}{(t_{int_1})(s_{3AB})}\right]u_3$$

Crossed Contributions



$$X^{\mu\nu}_{cross} = \frac{\langle A|\sigma|B\rangle}{(p_A+p_B)^2}\bar{u}_2\left[\frac{\gamma^\sigma(\not{p}_2+\not{p}_A+\not{p}_B)\gamma^\nu(\not{p}_a-\not{p}_1-\not{p}_3)\gamma^\mu}{(s_{2AB})(t_{int_3})}+\frac{\gamma^\nu(\not{p}_2-\not{p}_4-\not{p}_b)\gamma^\sigma(\not{p}_a-\not{p}_1-\not{p}_3)\gamma^\mu}{(t_{int_3})(t_{int_4})}+\frac{\gamma^\nu(\not{p}_2+\not{p}_4-\not{p}_b)\gamma^\mu(\not{p}_3+\not{p}_A+\not{p}_B)\gamma^\sigma}{(t_{int_4})(s_{3AB})}\right]u_3$$

Matching with Fixed Order

$$
\begin{aligned}
\sigma_{2j}^{\mathrm{resum,match}} = \sum_{f_1,f_2} \sum_m \prod_{j=1}^{m} & \left(\int_{p_{j\perp}^B=0}^{p_{j\perp}^B=\infty} \frac{\mathrm{d}^2\mathbf{p}_{j\perp}^B}{(2\pi)^3} \int \frac{\mathrm{d}y_j^B}{2} \right) (2\pi)^4 \, \delta^{(2)}\!\left(\sum_{k=1}^{m} \mathbf{p}_{k\perp}^B \right) \\
& \cdot \; x_a^B \, f_a(x_a^B, Q_a^B) \; x_b^B \, f_b(x_b^B, Q_b^B) \, \frac{\overline{|\mathcal{M}^B|}^2}{(\hat{s}^B)^2} \\
& \cdot \; \overline{|\mathcal{M}_{\mathrm{HEJ}}^{tree}|}^{-2} \, (2\pi)^{-4+3m} \, 2^m \frac{(\hat{s}^B)^2}{x_a^B \, f_{a,f_1}(x_a^B, Q_a^B) \; x_b^B \, f_{b,f_2}(x_b^B, Q_b^B)} \\
& \cdot \; \sum_{n=2}^{\infty} \int_{p_{1\perp}=.9p_{j,\perp}}^{p_{1\perp}=\infty} \frac{\mathrm{d}^2\mathbf{p}_{1\perp}}{(2\pi)^3} \int_{p_{n\perp}=.9p_{j,\perp}}^{p_{n\perp}=\infty} \frac{\mathrm{d}^2\mathbf{p}_{n\perp}}{(2\pi)^3} \prod_{i=2}^{n-1} \int_{p_{i\perp}=\lambda}^{p_{i\perp}=\infty} \frac{\mathrm{d}^2\mathbf{p}_{i\perp}}{(2\pi)^3} (2\pi)^4 \, \delta^{(2)}\!\left(\sum_{k=1}^{n} \mathbf{p}_{k\perp} \right) \\
& \cdot \; \mathbf{T}_y \prod_{i=1}^{n} \left(\int \frac{\mathrm{d}y_i}{2} \right) \mathcal{O}_{mj}^{e} \left(\prod_{l=1}^{m-1} \delta^{(2)}(\mathbf{p}_{\mathcal{J}_l\perp}^B - \mathbf{j}_{l\perp}) \right) \left(\prod_{l=1}^{m} \delta(y_{\mathcal{J}_l}^B - y_{\mathcal{J}_l}) \right) \mathcal{O}_{2j}(\{p_i\}) \\
& \cdot \; x_a f_{a,f_1}(x_a, Q_a) \; x_b f_{b,f_2}(x_b, Q_b) \, \frac{\overline{|\mathcal{M}_{\mathrm{HEJ}}^{f_1 f_2 \to f_1 g \cdots g f_2}(\{p_i\})|}^2}{\hat{s}^2} \, .
\end{aligned}
$$

arxiv:1805.04446

Matching with Fixed Order

$$
\begin{aligned}
\sigma_{2j}^{\mathrm{resum,match}} = & \sum_{f_1,f_2} \sum_m \prod_{j=1}^{m} \left(\int_{p_{j\perp}^B=0}^{p_{j\perp}^B=\infty} \frac{\mathrm{d}^2\mathbf{p}_{j\perp}^B}{(2\pi)^3} \int \frac{\mathrm{d}y_j^B}{2} \right) (2\pi)^4 \, \delta^{(2)}\!\left(\sum_{k=1}^{m} \mathbf{p}_{k\perp}^B \right) \\
& \cdot \; x_a^B \, f_a(x_a^B, Q_a^B) \, x_b^B \, f_b(x_b^B, Q_b^B) \, \frac{\overline{|\mathcal{M}^B|}^2}{(\hat{s}^B)^2} \\
& \cdot \; \overline{|\mathcal{M}_{\mathrm{HEJ}}^{tree}|}^{-2} \, (2\pi)^{-4+3m} \, 2^m \, \frac{(\hat{s}^B)^2}{x_a^B \, f_{a,f_1}(x_a^B, Q_a^B) \, x_b^B \, f_{b,f_2}(x_b^B, Q_b^B)} \\
& \cdot \sum_{n=2}^{\infty} \int_{p_{1\perp}=.9p_{j,\perp}}^{p_{1\perp}=\infty} \frac{\mathrm{d}^2\mathbf{p}_{1\perp}}{(2\pi)^3} \int_{p_{n\perp}=.9p_{j,\perp}}^{p_{n\perp}=\infty} \frac{\mathrm{d}^2\mathbf{p}_{n\perp}}{(2\pi)^3} \prod_{i=2}^{n-1} \int_{p_{i\perp}=\lambda}^{p_{i\perp}=\infty} \frac{\mathrm{d}^2\mathbf{p}_{i\perp}}{(2\pi)^3} (2\pi)^4 \, \delta^{(2)}\!\left(\sum_{k=1}^{n} \mathbf{p}_{k\perp} \right) \\
& \cdot \; \mathbf{T}_y \prod_{i=1}^{n} \left(\int \frac{\mathrm{d}y_i}{2} \right) \mathcal{O}_{mj}^{e} \left(\prod_{l=1}^{m-1} \delta^{(2)}(\mathbf{p}_{\mathcal{J}_l\perp}^B - \mathbf{j}_{l\perp}) \right) \left(\prod_{l=1}^{m} \delta(y_{\mathcal{J}_l}^B - y_{\mathcal{J}_l}) \right) \mathcal{O}_{2j}(\{p_i\}) \\
& \cdot \; x_a f_{a,f_1}(x_a, Q_a) \, x_b f_{b,f_2}(x_b, Q_b) \, \frac{\overline{|\mathcal{M}_{\mathrm{HEJ}}^{f_1 f_2 \to f_1 g \cdots g f_2}(\{p_i\})|}^2}{\hat{s}^2} \, .
\end{aligned}
$$

Fixed Order (red box: first two lines)

Overlap (green box: third line)

HEJ (blue box: last three lines)

arxiv:1805.04446