The angular momentum decomposition in the scalar diquark model

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Outline

Proton Spin Crisis

For a proton with helicity $+1/2$

$$\frac{1}{2} = \langle\langle S_z^q \rangle\rangle + \langle\langle S_z^G \rangle\rangle + \langle\langle L_z \rangle\rangle$$
$$\langle\langle S_z^q \rangle\rangle = \frac{1}{2}\int_0^1 dx \Delta\Sigma(x)$$
$$\langle\langle S_z^G \rangle\rangle = \int_0^1 dx \Delta G(x)$$

EMC experiment $\Rightarrow \int_0^1 dx \Delta\Sigma(x) \approx 0.06$

[E. Leader and M. Anselmino, Z. Phys. C 41 , 239 (1988)]

COMPASS, HERMES $\Rightarrow \int_0^1 dx \Delta\Sigma(x) \approx 0.3$

[V. Y. Alexakhin et al. (COMPASS Collaboration), Phys.Lett. B 647, 8 (2007)]
[A. Airapetian et al. (HERMES Collaboration), Phys.Rev. D 75, 012007 (2007)]

PHENIX, STAR, COMPASS $\Rightarrow \int_{0.05}^{0.2} dx \Delta G(x) \approx 0.2$

[D. de Florian et al (DSSV Collaboration). Phys Rev. Lett. 113, 012001 (2014)]
[E. R. Nocera et al. (NNPDF Collaboration), Nuc. Phys. B 887, 276 (2014)]

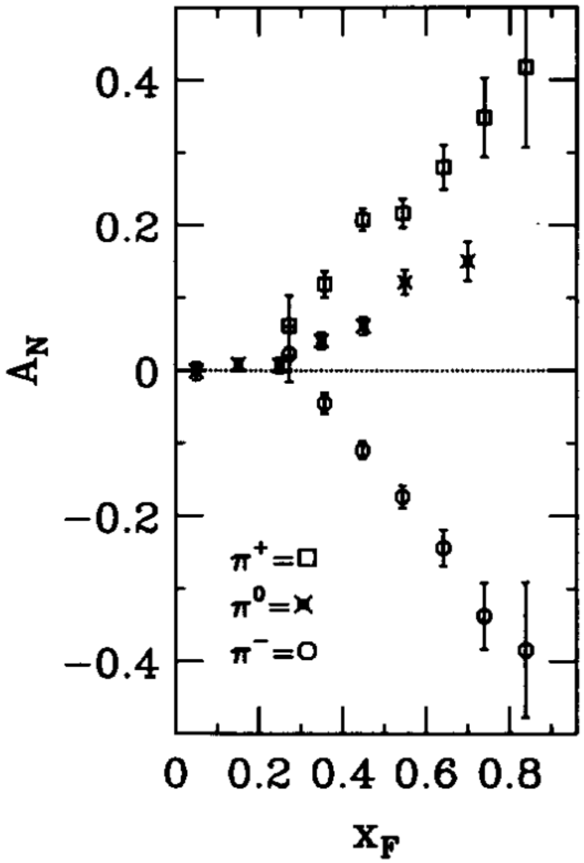
The Three Graces from Botticelli's Primavera

MAIN GOAL:

- To understand $\langle\langle L_z \rangle\rangle$ and how it can be described (decomposed).

Single Spin Asymmetries in DIS

- Single Spin Asymmetries (SSA) are sensitive to the orbital momentum of quarks.
- Provide insights in the physics of partonic initial and final state interactions.
- Observable over a wide range of experiments and in several facilities.
- $A_\pi = \frac{\sigma_{\pi\uparrow} - \sigma_{\pi\downarrow}}{\sigma_{\pi\uparrow} + \sigma_{\pi\downarrow}}$
- First measurements of SSA in DIS with longitudinally polarized targets by HERMES, closely followed by COMPASS.
- Nonzero SSA was observed for the first time with CLAS for transversely polarized quarks in a longitudinally polarized proton.



[HERMES Collaboration (A. Airapetian et al.), Phys. Rev. Lett. 94, 012002 (2005)]
[COMPASS Collaboration (V.Y. Alexakhin et al.), Phys. Rev. Lett. 94, 202002 (2005)]
[CLAS Collaboration (H. Avakian et al.), Phys. Rev. Lett. 105, 262002 (2010)]
[FNAL E704 Collaboration (D. L. Adams et al.), Phys. Lett. B 264, 3, (1991)]

Transverse Momentum Distributions

- SIDIS probes orbital motion of quarks through TMDs.
- Access to PDFs not accessible in inclusive DIS.
- Factorization and Universality has been proven.

nucleon pol. \ quark pol.	U	L	T	
U	$\boldsymbol{f_1}$		$h_1^{\perp}$	
L		$\boldsymbol{g_1}$	$h_{1L}^{\perp}$	
T	$f_{1T}^{\perp}$	g_{1T}	$\boldsymbol{h_1}$	$h_{1T}^{\perp}$

Twist-2 Transverse-Momentum–dependent Distribution functions (TMDs). The U,L, T correspond to unpolarized, longitudinally polarized and transversely polarized nucleons (rows) and quarks (columns).

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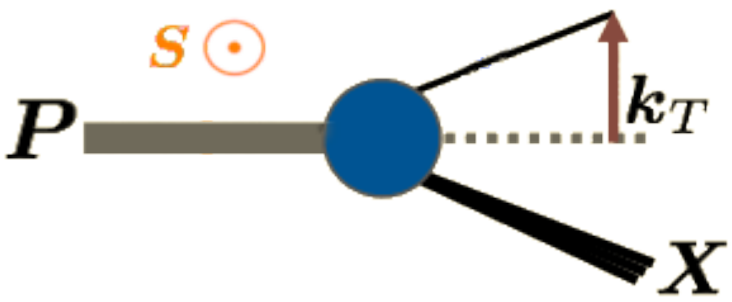
Sivers Function and torque

- The Sivers function $f_{1T}^{\perp}$ describes unpolarized quarks in a transversely polarized nucleon
- Can be related to average transverse momentum via

$$\langle k_{\perp}^{j} \rangle = \epsilon^{ij} S_{\perp}^{i} \int dx\, d^2k_{\perp} \frac{k_{\perp}^2}{2M^2} f_{1T}^{\perp}(x, k_{\perp}^2)$$

- In SIDIS, $f_{1T}^{\perp}$ is associated with final-state interactions (FSI) through gluon exchange
- FSI are responsible for a “chromomagnetic lensing” effect that provides a torque that acts on the struck quark.
- This mechanism is responsible for changing the orbital angular momentum!



[M. Burkardt, Phys. Rev. D 88, 014014 (2013)]

Jaffe-Manohar and Ji Decompositions

- Differ in their definition of Orbital Angular Momentum

$$\vec{L}_{Ji} \sim \vec{r} \times i\vec{D} \qquad \vec{\mathcal{L}}_{JM} \sim \vec{r} \times i\vec{D}^{pure} \longrightarrow \vec{r} \times i\vec{\partial}$$

- Ji:

$$\tfrac{1}{2}\Delta\Sigma + L_q + J_g = \tfrac{1}{2}$$

- Jaffe-Manohar:

$$\tfrac{1}{2}\Delta\Sigma + \Delta G + \mathcal{L}_q + \mathcal{L}_g = \tfrac{1}{2}$$

- At the moment, the only practical way to extract partonic total angular momentum is to use Ji's relation

$$J_g = \tfrac{1}{2}\int dx\, x[H(x,0,0) + E(x,0,0)]$$

- An exploratory lattice calculation of both decompositions of quark OAM was given in 2017 and demonstrates that their difference can be clearly resolved.

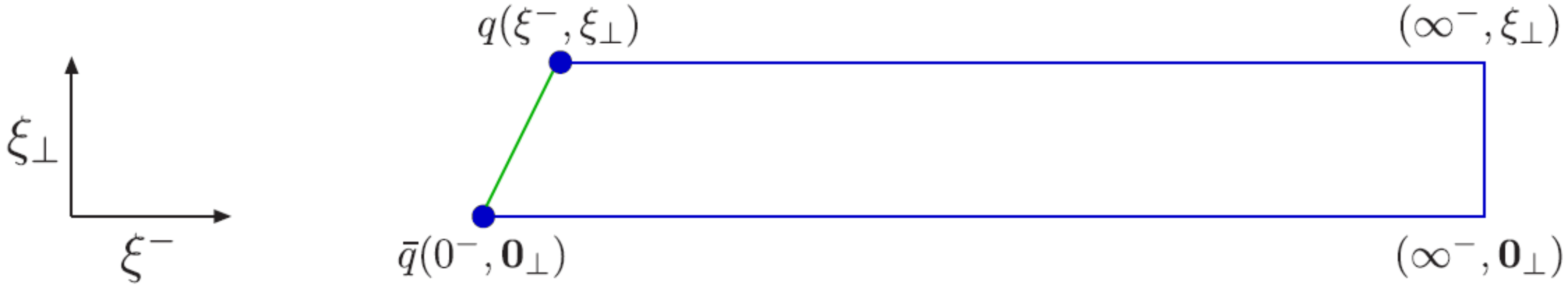
[X. D. Ji, Phys. Rev. Lett. 78, 610 (1997)]
[R. L. Jaffe, A. Manohar, Nucl. Phys. B 337, 509 (1990)]
[M. Engelhardt, Phys. Rev. D 95, 094505 (2017)]

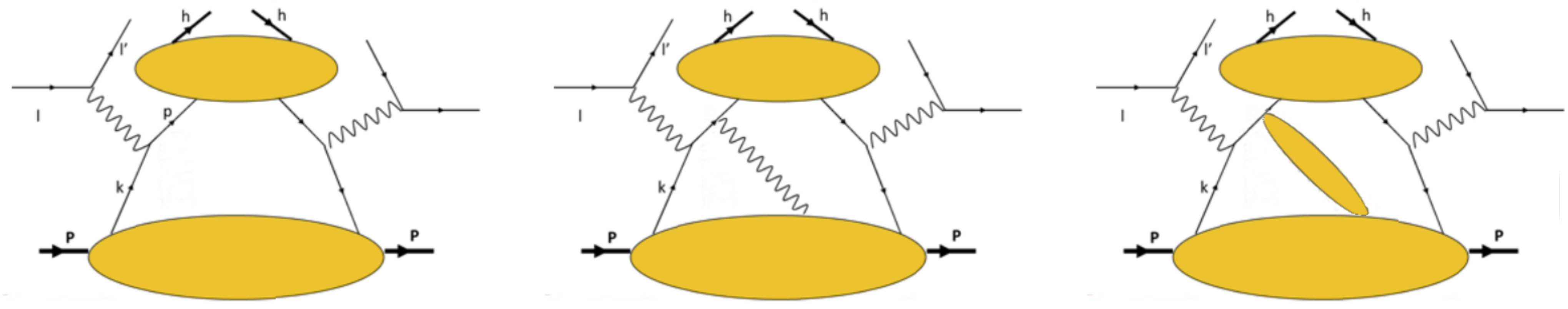
Jaffe-Manohar and Ji Decompositions

- In terms of the Covariant derivative:

$$\vec{D}_\perp = \vec{D}_\perp^{pure} + ie\vec{A}_\perp^{phys}$$

$$\langle \bar{\Psi}\gamma^+\vec{D}_\perp\Psi\rangle \; - \; \langle \bar{\Psi}\gamma^+\vec{D}_\perp^{pure}\Psi\rangle \; = \; \langle \bar{\Psi}\gamma^+ ie\vec{A}_\perp^{phys}\Psi\rangle$$

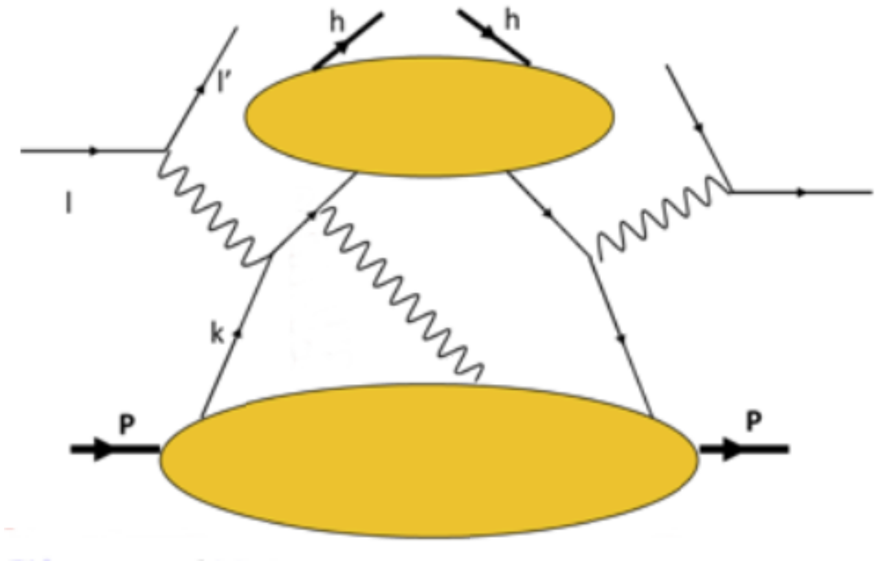


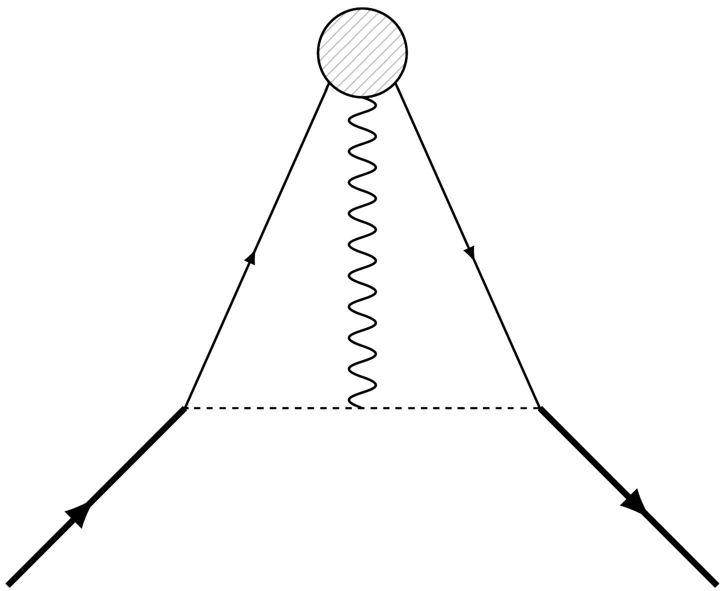


[X. Ji, F. Yuan, Phys.Lett. B 543 (2002)]

Scalar Diquark Model

- The proton splits into a quark and a diquark structure. While the active quark interacts with the photon, the diquark acts as the ’spectator’ and vice versa.
- Only QED is evaluated for Initial/Final State Interactions.
- A simple model that provides analytic results while giving an estimate of the magnitude of the observables.
- Explicit Lorentz covariance is maintained.

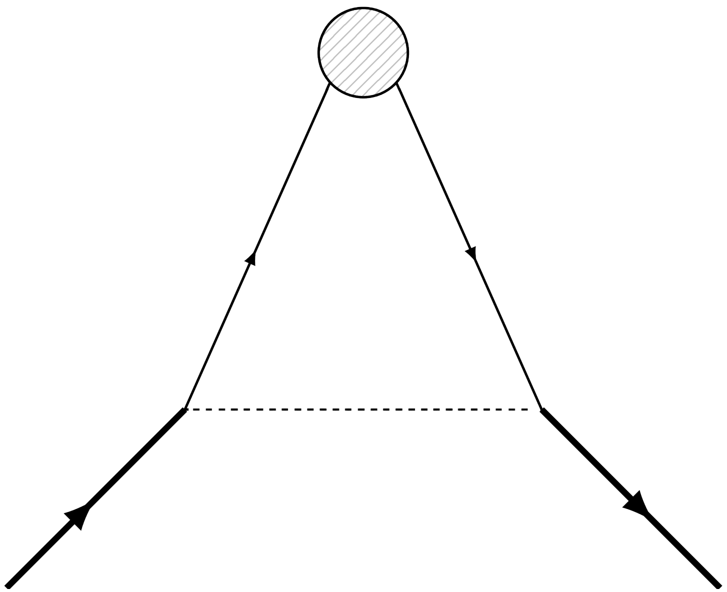




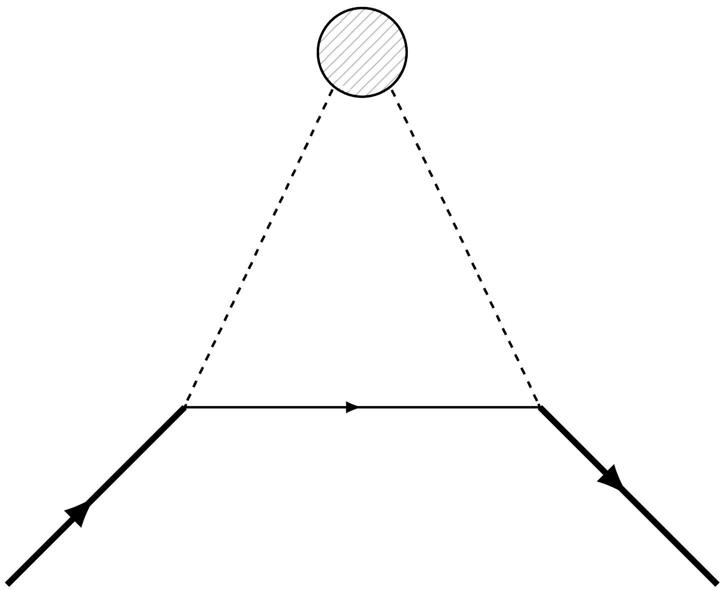
Potential Momentum

- The potential momentum corresponds to the difference between Ji and JM decompositions:

$$\langle k_\perp^i \rangle_{Ji} - \langle k_\perp^i \rangle_{JM} = \frac{1}{2} e \langle \int d^3r\, \bar{\Psi}(r) \gamma^+ \vec{A}^i_{phys}(r) \Psi(r) \rangle$$

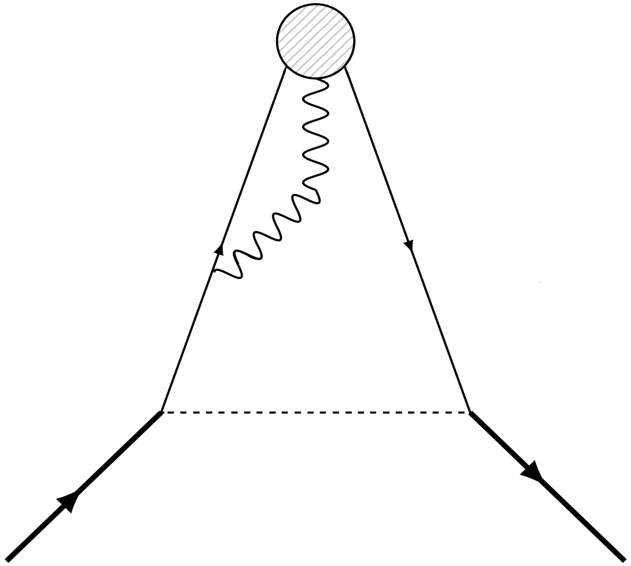


$$\langle \vec{k}_\perp^q \rangle_{Ji} = \vec{0}$$
$$\langle \vec{k}_\perp^q \rangle_{JM} = \vec{0}$$



$$\langle \vec{k}_\perp^s \rangle_{Ji} = \vec{0}$$
$$\langle \vec{k}_\perp^s \rangle_{JM} = \vec{0}$$

Potential Momentum



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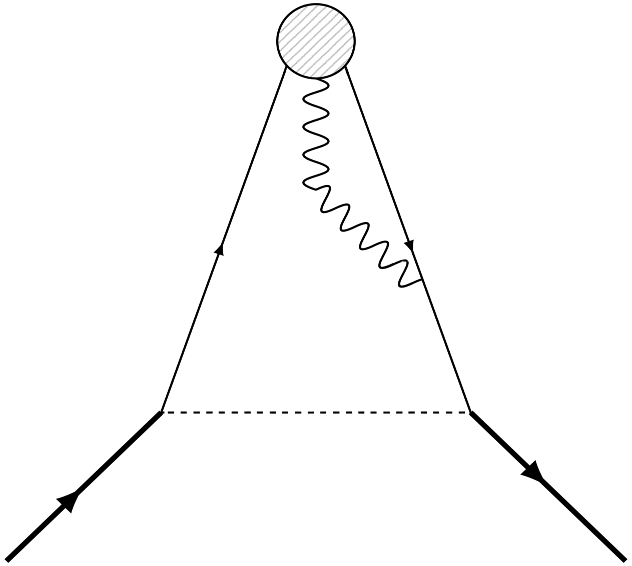
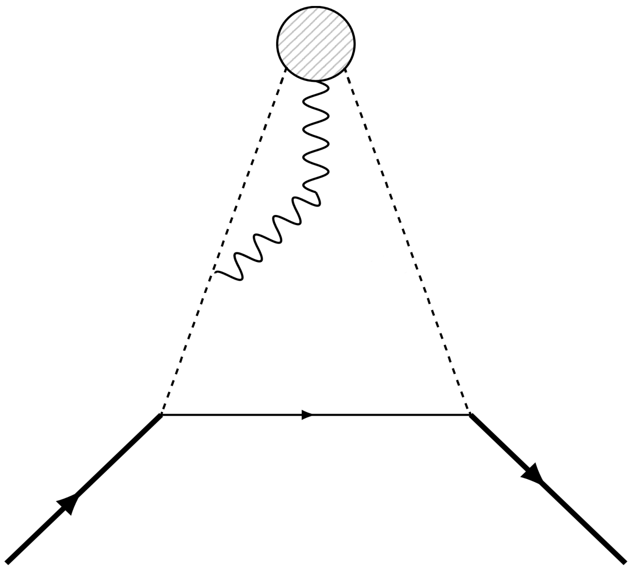


$$\langle \vec{k}_\perp^{\,q} \rangle_{Ji} = \vec{0}$$
$$\langle \vec{k}_\perp^{\,q} \rangle_{JM} = \vec{0}$$



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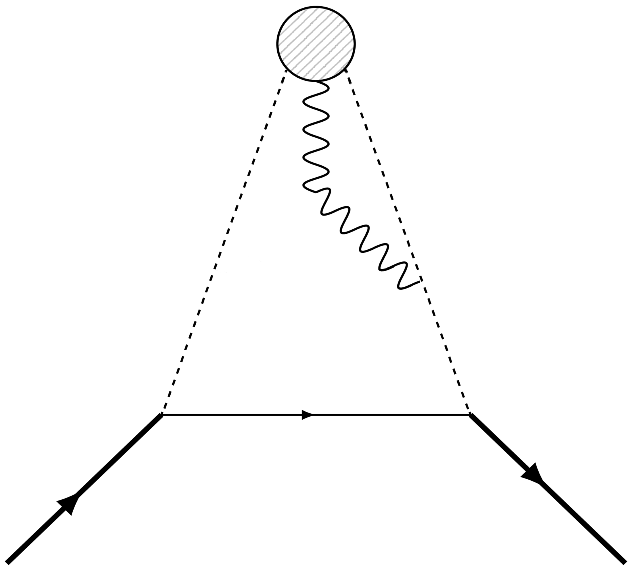


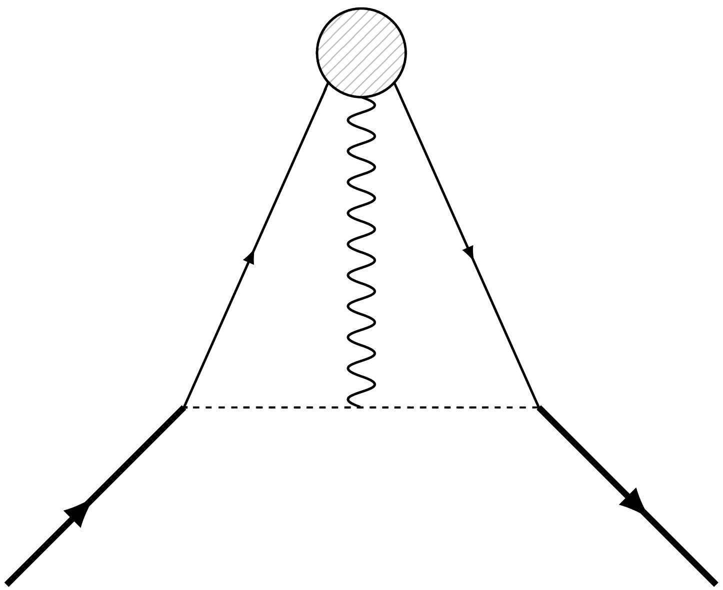
$$\langle \vec{k}_\perp^{\,s} \rangle_{Ji} = \vec{0}$$
$$\langle \vec{k}_\perp^{\,s} \rangle_{JM} = \vec{0}$$

Potential Momentum

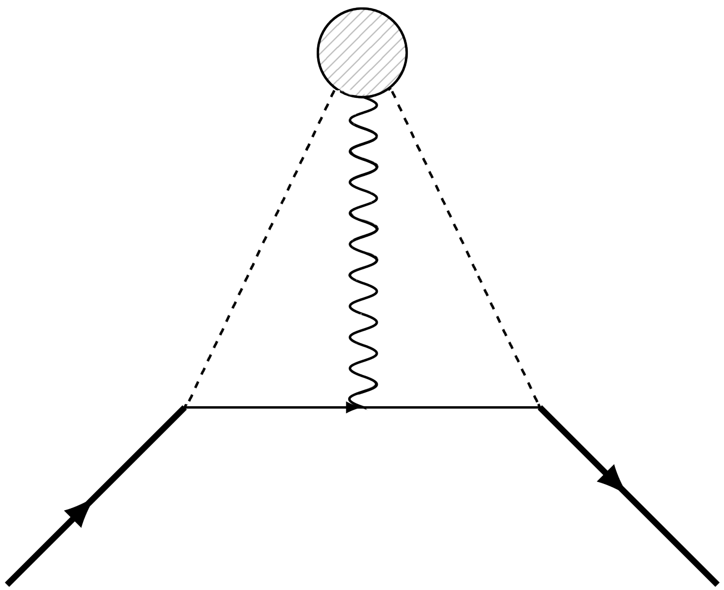
- The potential momentum corresponds to JM transverse momentum:

$$\begin{aligned}\langle k_\perp^i \rangle_{JM} &= -\tfrac{1}{2} e \langle \int d^3 r\, \bar{\Psi}(r) \gamma^+ \vec{A}^i_{phys}(r) \Psi(r) \rangle \\ &= \frac{\epsilon_T^{ij} s_\perp^j}{6(4\pi)^2} (3m_q + M) \pi e_s e_q g^2 \left(\frac{1}{4\pi\epsilon}\right)^2\end{aligned}$$

- Requires transversely polarized target



$$\langle \vec{k}_\perp^q \rangle_{Ji} = \vec{0}$$
$$\langle \vec{k}_\perp^s \rangle_{JM} \neq \vec{0}$$



$$\langle \vec{k}_\perp^s \rangle_{Ji} = \vec{0}$$
$$\langle \vec{k}_\perp^s \rangle_{JM} \neq \vec{0}$$

Physical interpretation

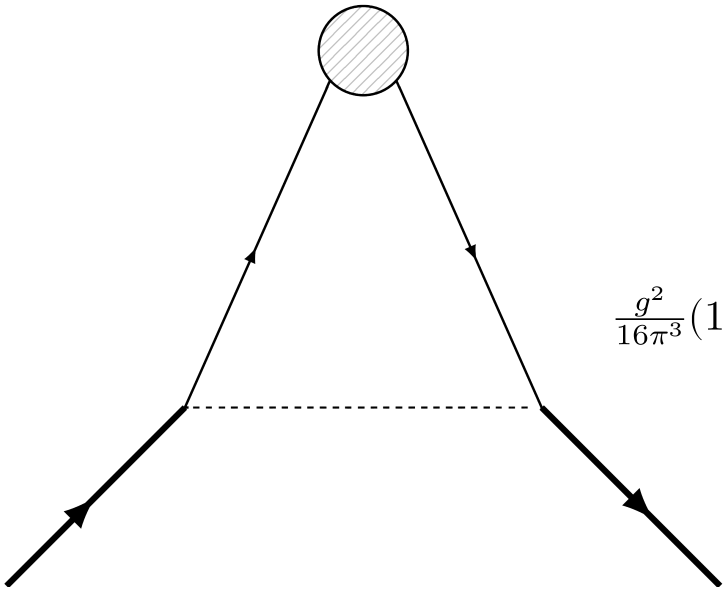
- Even though $\langle \vec{k}^a_\perp \rangle_{JM} \neq \vec{0}$, we have conservation of transverse momentum $\sum_{a=q,s} \langle \vec{k}^a_\perp \rangle_{JM} = \vec{0}$
- $\langle \vec{k}_\perp \rangle(x) \sim (\vec{P} \times \vec{S}) f^{\mathcal{W}}$ is naive T-odd
- $f^{\mathcal{W}}$ has to be naive T-odd, i.e., $T: \mathcal{W} \longmapsto \mathcal{W}'$
- $\langle \bar{\Psi}\gamma^+ \vec{D}_\perp \Psi \rangle \ - \ \langle \bar{\Psi}\gamma^+ \vec{D}^{pure}_\perp \Psi \rangle \ = \ \langle \bar{\Psi}\gamma^+ ie A^{phys}_\perp \Psi \rangle$
- Analytic expression for Sivers function $f_{1T}^\perp$
- The difference between Ji and JM decompositions appears at two-loop level
- This supports the interpretation of such a difference as originating from the torque exerted by the spectator system on the struck quark

Baron Münchhausen by Oskar Herrfurth

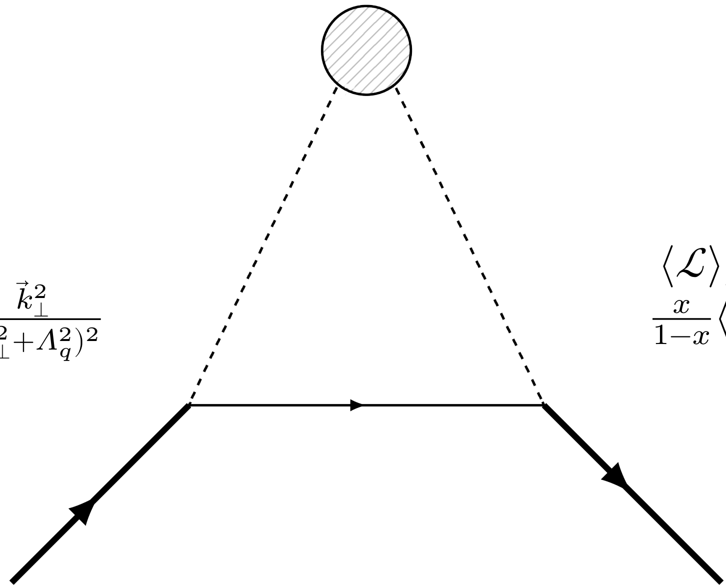
Orbital Angular Momentum

- In the Lightcone Gauge we can compute (Ji / JM) OAM as:

$$\langle L_{Ji}\rangle = \langle \mathcal{L}_{JM}\rangle = \langle \int d^2 r_\perp \, \bar{\Psi}(\vec{r})\gamma^+ \vec{r}_\perp \times i\nabla_\perp \Psi(\vec{r})\rangle$$



$$\langle \mathcal{L}\rangle_q(x) = \frac{g^2}{16\pi^3}(1-x)^2 \int d^2 k_\perp \frac{\vec{k}_\perp^2}{(\vec{k}_\perp^2 + \Lambda_q^2)^2}$$



$$\langle \mathcal{L}\rangle_s(x) = \frac{x}{1-x}\langle \mathcal{L}\rangle_q(x)$$

$$J(x) = \langle \mathcal{L}\rangle_q(x) + \langle \mathcal{L}\rangle_s(x) + \frac{1}{2} g_1^q(x) \implies J = \int dx J(x) = \frac{1}{2}$$

[C. Lorcé, L. Mantovani, B. Pasquini, Phys. Lett. B 776, 38 (2018)]

Conclusions and Outlook

Conclusions:

- The potential momentum was computed for the diquark model.
- The difference between Ji and JM decompositions appears at two-loop level.
- Ji and JM decompositions for OAM were obtained up to one-loop.
- Non-zero Sivers function at two-loops supports the possibility of non-zero Potential OAM.

Outlook: Potential OAM has to be evaluated!

- Obtain analytical expressions for the involved TMDs.
- Provide estimates of observables in leading twist.
- Crosscheck for the different sum rules.
- Address more complex/realistic models.

Questions ?