

# *A new simple PDF parametrization: improved description of the HERA data*

---

M. Bonvini, F. Giuli

27<sup>th</sup> International Workshop on Deep Inelastic Scattering  
and Related Topics (Turin, Italy)

10/04/2019



# Motivations

- Default HERAPDF (xFitter) parametrisation:

$$xf(x, \mu_0^2) = Ax^B(1-x)^C[1 + Dx + Ex^2] - \underbrace{A'x^{B'}(1-x)^{C'}}_{\text{Negative term (only for gluon)}}$$

- Asymptotic behaviours:

$$x^B(1-x)^C$$

- To model large-x region → Polynomial in x

$$(1 + Dx + Ex^2 + \dots)$$

- **More flexibility in the small-x regime is needed**

- Other groups use polynomial in  $\sqrt{x}$  → Some gain is obtained

- Very important in the light of future higher-energy colliders:

- Large Hadron-electron Collider (LHeC)
- Future Circular electron-hadron or hadron-hadron Colliders (FCC-eh and FCC-hh)

# New proposed parametrization

- To model small-x region we proposed polynomial in  $\log(x)$

$$(1 + F \log(x) + G \log^2(x) + H \log^3(x) + \dots)$$

- Considered both a multiplicative and an additive option, and we chose the latter:

[arXiv:1902.11125 \[Bonvini,FG\]](https://arxiv.org/abs/1902.11125)

$$xf(x, \mu_0^2) = Ax^B(1-x)^C[1 + Dx + Ex^2 + F \log(x) + G \log^2(x) + H \log^3(x)]$$

- Minimal parametrization:

$$xg(x, \mu_0^2) = A_g x^{B_g} (1-x)^{C_g} \left[ 1 + F_g \log x + G_g \log^2 x \right]$$

$$xu_v(x, \mu_0^2) = A_{u_v} x^{B_{u_v}} (1-x)^{C_{u_v}} \left[ 1 + E_{u_v} x^2 + F_{u_v} \log x + G_{u_v} \log^2 x \right]$$

$$xd_v(x, \mu_0^2) = A_{d_v} x^{B_{d_v}} (1-x)^{C_{d_v}}$$

$$x\bar{u}(x, \mu_0^2) = A_{\bar{u}} x^{B_{\bar{u}}} (1-x)^{C_{\bar{u}}} \left[ 1 + D_{\bar{u}} x + F_{\bar{u}} \log x \right]$$

$$x\bar{d}(x, \mu_0^2) = A_{\bar{d}} x^{B_{\bar{d}}} (1-x)^{C_{\bar{d}}} \left[ 1 + D_{\bar{d}} x + F_{\bar{d}} \log x \right],$$

$$xs(x, \mu_0^2) = x\bar{s}(x, \mu_0^2) = r_s xd(x, \mu_0^2) \quad r_s = \frac{f_s}{1-f_s} \quad \text{with } f_s = 0.4 \text{ fixed}$$

# New proposed parametrization

- Our new PDF parametrization:
  - Depends on 18 free parameters that must be fitted
  - This is to be compared with HERAPDF2.0 (14 free parameters)
  - Two extra parameters for  $u_V$ ,  $\bar{u}$  and  $\bar{d}$
  - Major improvement comes from the gluon PDF (same number of free parameters)

$$xg(x, \mu_0^2) = A_g x^{B_g} (1-x)^{C_g} \left[ 1 + F_g \log x + G_g \log^2 x \right]$$

$$xu_v(x, \mu_0^2) = A_{u_v} x^{B_{u_v}} (1-x)^{C_{u_v}} \left[ 1 + E_{u_v} x^2 + F_{u_v} \log x + G_{u_v} \log^2 x \right]$$

$$xd_v(x, \mu_0^2) = A_{d_v} x^{B_{d_v}} (1-x)^{C_{d_v}}$$

$$x\bar{u}(x, \mu_0^2) = A_{\bar{u}} x^{B_{\bar{u}}} (1-x)^{C_{\bar{u}}} \left[ 1 + D_{\bar{u}} x + F_{\bar{u}} \log x \right]$$

$$x\bar{d}(x, \mu_0^2) = A_{\bar{d}} x^{B_{\bar{d}}} (1-x)^{C_{\bar{d}}} \left[ 1 + D_{\bar{d}} x + F_{\bar{d}} \log x \right],$$

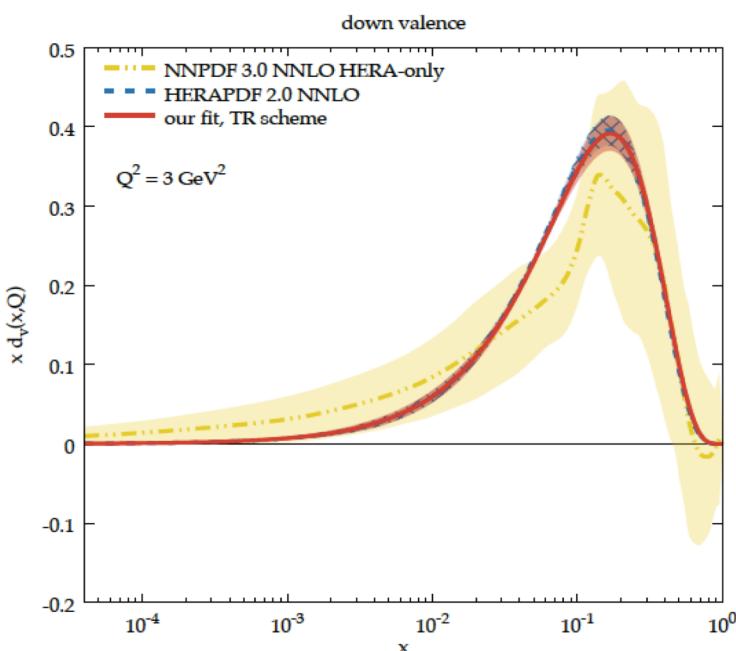
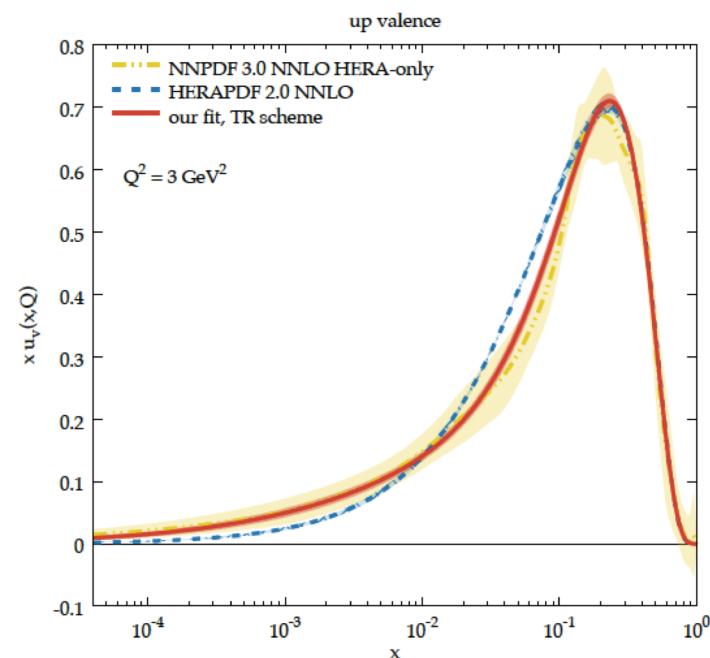
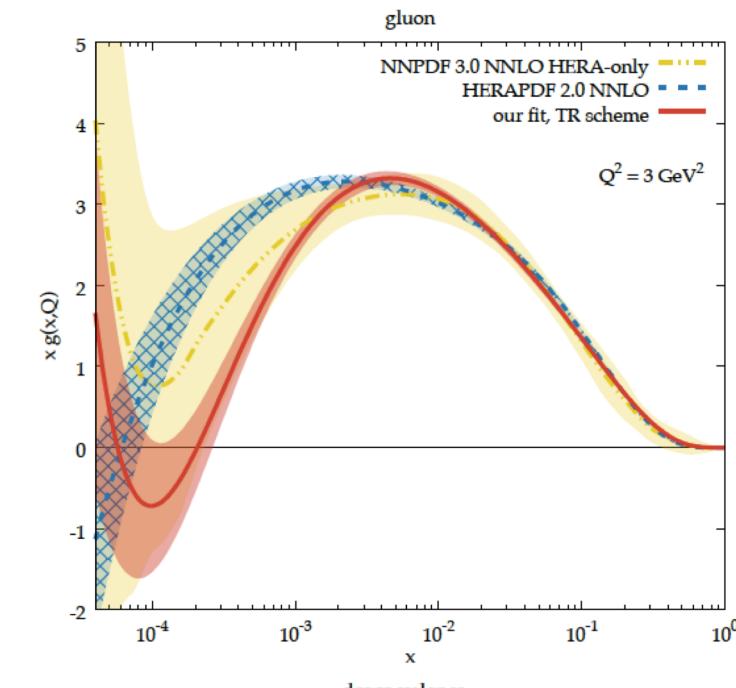
$$xs(x, \mu_0^2) = x\bar{s}(x, \mu_0^2) = r_s xd(x, \mu_0^2) \quad r_s = \frac{f_s}{1-f_s} \quad \text{with } \underline{f_s = 0.4 \text{ fixed}}$$

# Comparison to HERAPDF2.0

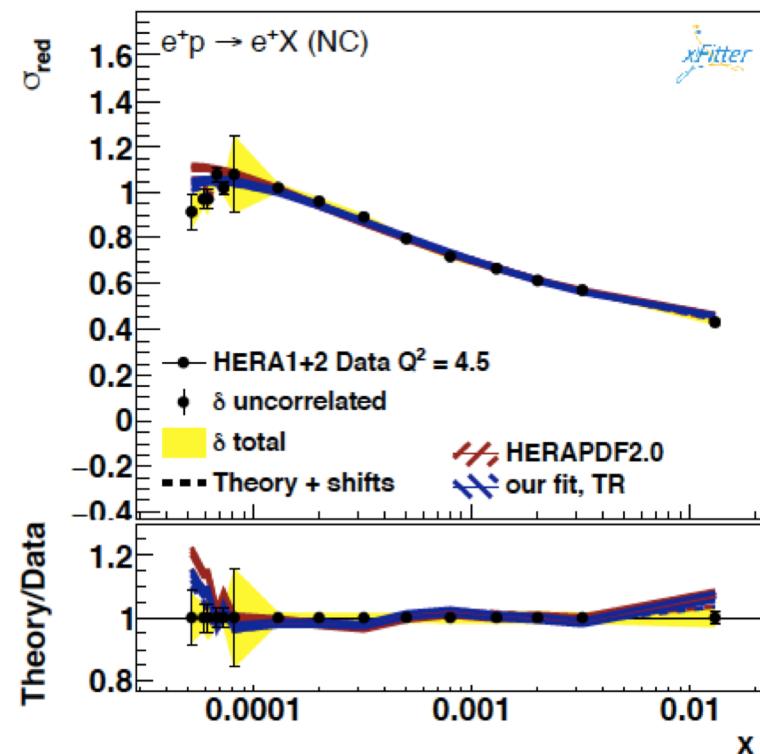
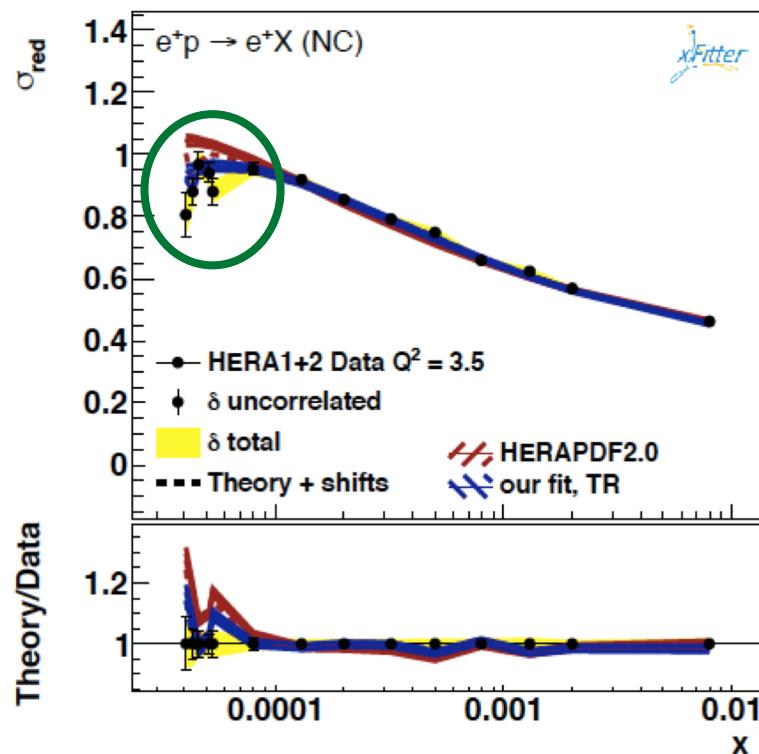
Contribution to  $\chi^2$       HERAPDF2.0      Our fit (new parametrization)

subset NC $e^+$ 920 $\tilde{\chi}^2/\text{n.d.p.}$	444/377	403/377 
subset NC $e^+$ 820 $\tilde{\chi}^2/\text{n.d.p.}$	66/70	74/70
subset NC $e^+$ 575 $\tilde{\chi}^2/\text{n.d.p.}$	219/254	221/254
subset NC $e^+$ 460 $\tilde{\chi}^2/\text{n.d.p.}$	217/204	222/204
subset NC $e^-$ $\tilde{\chi}^2/\text{n.d.p.}$	219/159	220/159
subset CC $e^+$ $\tilde{\chi}^2/\text{n.d.p.}$	45/39	38/39
subset CC $e^-$ $\tilde{\chi}^2/\text{n.d.p.}$	56/42	50/42
correlation term + log term	$91 + 5$	$75 - 3$
<b>Total <math>\chi^2/\text{d.o.f.}</math></b>	<b>1363/1131</b>	<b>1301/1127</b>

$$\chi^2 = \underbrace{\sum_i \frac{\left[ D_i - T_i \left( 1 - \sum_j \gamma_{ij} b_j \right) \right]^2}{\delta_{i,\text{uncor}}^2 T_i^2 + \delta_{i,\text{stat}}^2 D_i T_i}}_{\text{Exp. term}} + \underbrace{\sum_j b_j^2}_{\text{Corr. term}} + \underbrace{\sum_i \log \frac{\delta_{i,\text{uncor}}^2 T_i^2 + \delta_{i,\text{stat}}^2 D_i T_i}{\delta_{i,\text{uncor}}^2 D_i^2 + \delta_{i,\text{stat}}^2 D_i^2}}_{\text{Log term}}$$



- Richer structure at medium-/high- $x$  than HERAPDF2.0
- Gluon decreases more rapidly for  $x \sim 10^{-2}$  and starts rising again for  $x < 10^{-4}$
- Up-valence rather different
- Down-valence is identical (same parametrization as in HERAPDF2.0)
- If compared to NNPDF3.0 (HERA data only), qualitatively same behavior



- How could the fit quality improve so much?
- In most of the cases the agreement is at the same level
- Exception for low- $Q^2$  and low- $x$  data, where a **clear improvement of the theoretical description** is manifest
- This region is where the impact of  $\log(1/x)$  terms is expected to be largest
- $\chi^2$  improvement of the same size as the one found in xFitter small- $x$  resummation paper  
[Eur. Phys. J. C78 \(2018\) 621](#) – **is resummation really needed?**

# From TR to FONLL

- Various variations studied
- First of all, migration from TR scheme to FONLL (to include small-x resummation in a later stage) – as done in [Eur. Phys. J. C78 \(2018\) 621](#)
- The charm PDF must be generated perturbatively at a matching scale  $\mu_c > \mu_0 > m_c$

## Differences in the fit setup

---

heavy flavour scheme	TR	FONLL
initial scale $\mu_0$	1.38 GeV	1.6 GeV
charm matching scale $\mu_c$	$m_c$	$1.12m_c$
charm mass $m_c$	1.43 GeV	1.46 GeV

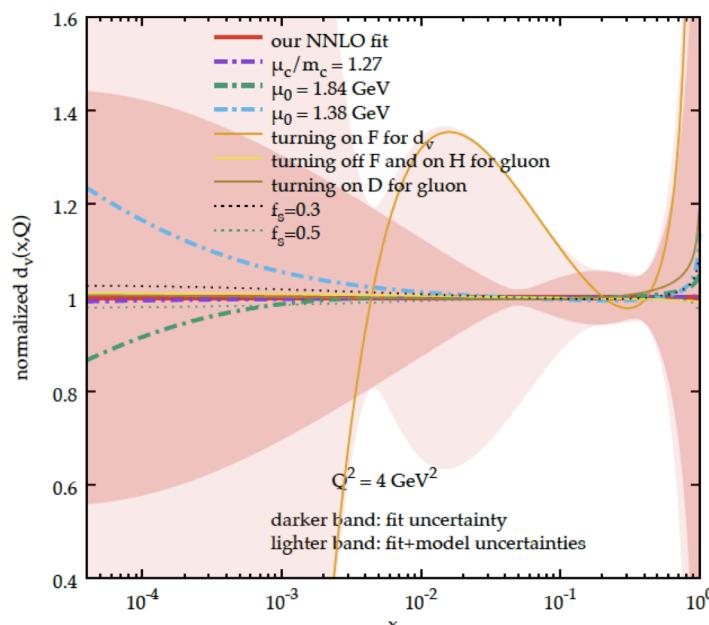
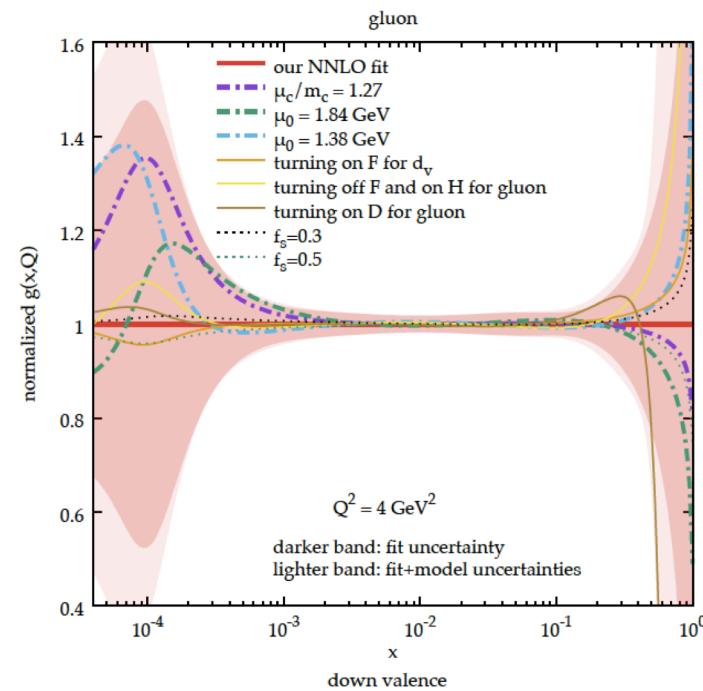
# From TR to FONLL

Contribution to $\chi^2$	Old parametrization	New parametrization
subset NC $e^+$ 920 $\tilde{\chi}^2/\text{n.d.p.}$	451/377	406/377
subset NC $e^+$ 820 $\tilde{\chi}^2/\text{n.d.p.}$	68/70	74/70
subset NC $e^+$ 575 $\tilde{\chi}^2/\text{n.d.p.}$	220/254	222/254
subset NC $e^+$ 460 $\tilde{\chi}^2/\text{n.d.p.}$	218/204	225/204
subset NC $e^-$ $\tilde{\chi}^2/\text{n.d.p.}$	215/159	217/159
subset CC $e^+$ $\tilde{\chi}^2/\text{n.d.p.}$	44/39	37/39
subset CC $e^-$ $\tilde{\chi}^2/\text{n.d.p.}$	57/42	50/42
correlation term + log term	$100 + 15$	$79 + 2$
<b>Total <math>\chi^2/\text{d.o.f.}</math></b>	<b>1388/1131</b>	<b>1312/1127</b>

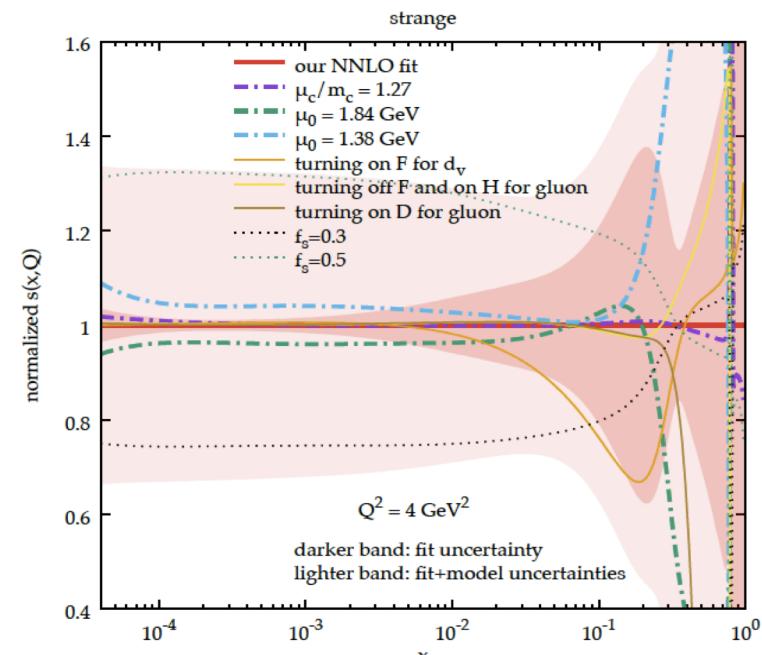
- Worse than the  $\chi^2$  presented in Slide 4 but:
  - Old parameterisation → deterioration of **25 units** wrt TR
  - New parameterisation → deterioration of **just 11 units** wrt TR
- **PDFs largely compatible** (backup)

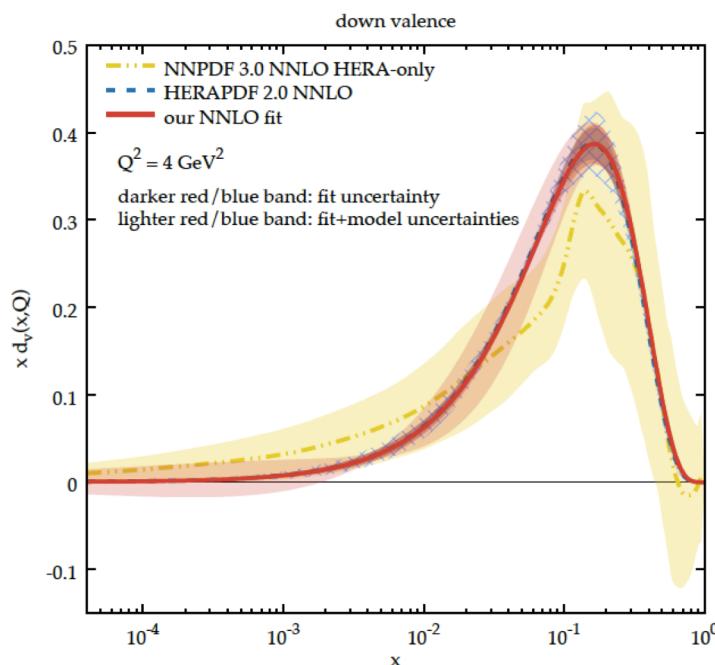
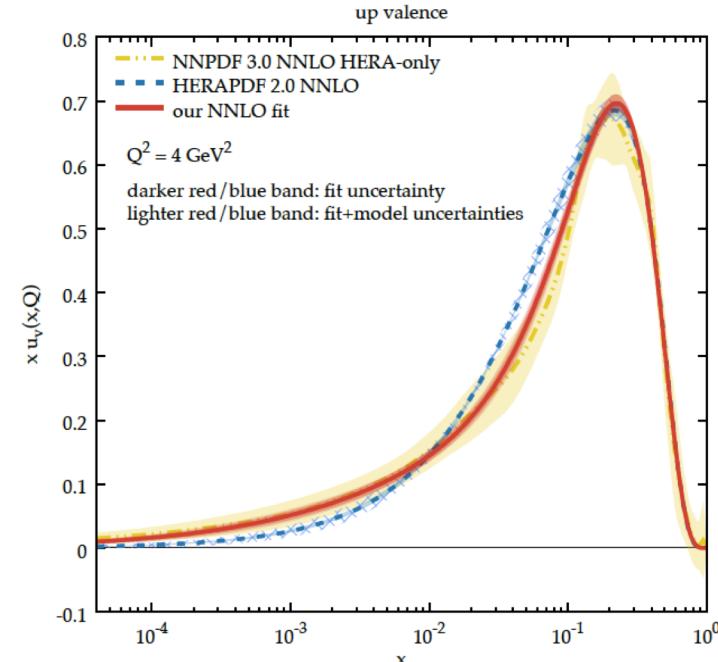
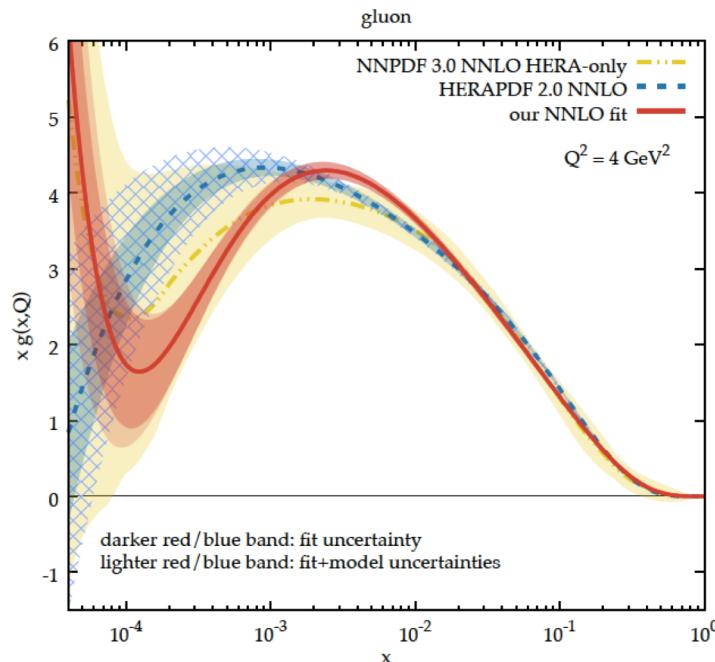
# Stability of our fit

- First, we consider variations of the fit scale:
  - $\mu_0 = 1.38 \text{ GeV}$  and  $\mu_c/m_c = 1.12$  ( $\mu_c = 1.46 \text{ GeV}$ ) – **Down variation**
  - $\mu_0 = 1.60 \text{ GeV}$  and  $\mu_c/m_c = 1.27$  ( $\mu_c = 1.85 \text{ GeV}$ ) – Intermediate step
  - $\mu_0 = 1.84 \text{ GeV}$  and  $\mu_c/m_c = 1.27$  ( $\mu_c = 1.85 \text{ GeV}$ ) – **Up variation**
- Strange fraction variations:
  - $f_s = 0.5$  (up variation) and  $f_s = 0.3$  (down variation) – same as HERAPDF2.0
- Parametrization uncertainties addressed adding or removing parameters that do not change the fit quality. The ones giving the largest effect are:
  - Adding  $F_{d_\nu}$
  - Adding  $D_g$  (more flexibility at large-x)
  - Adding  $H_g$  and removing  $F_g$  (possible effect at small-x)



- The addition of the log term to  $d_v$  has the largest effect (negative for  $x \lesssim 10^{-3}$ )
- When  $D_g$  is activated, large- $x$  shape changes substantially, but in a region where the gluon PDF is very small and largely unconstrained
- Effect of  $H_g$  (without  $F_g$ ) very mild
- Up/down variations of  $f_s$  have a larger effect on the strange PDF (as expected)
- $\mu_0$  variations have small effects





- We combined the uncertainties (**exp+th+model**) into a **symmetric uncertainty band**
- Our final PDF set including the full uncertainty is largely compatible with NNPDF30

# Including small-x resummation

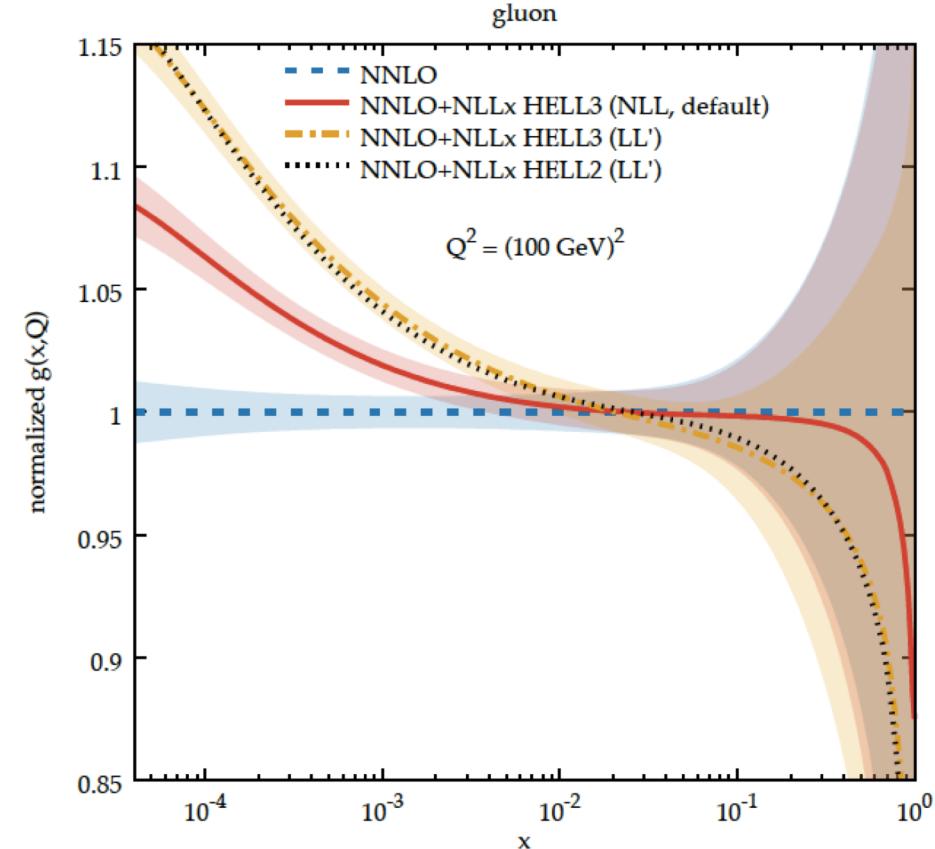
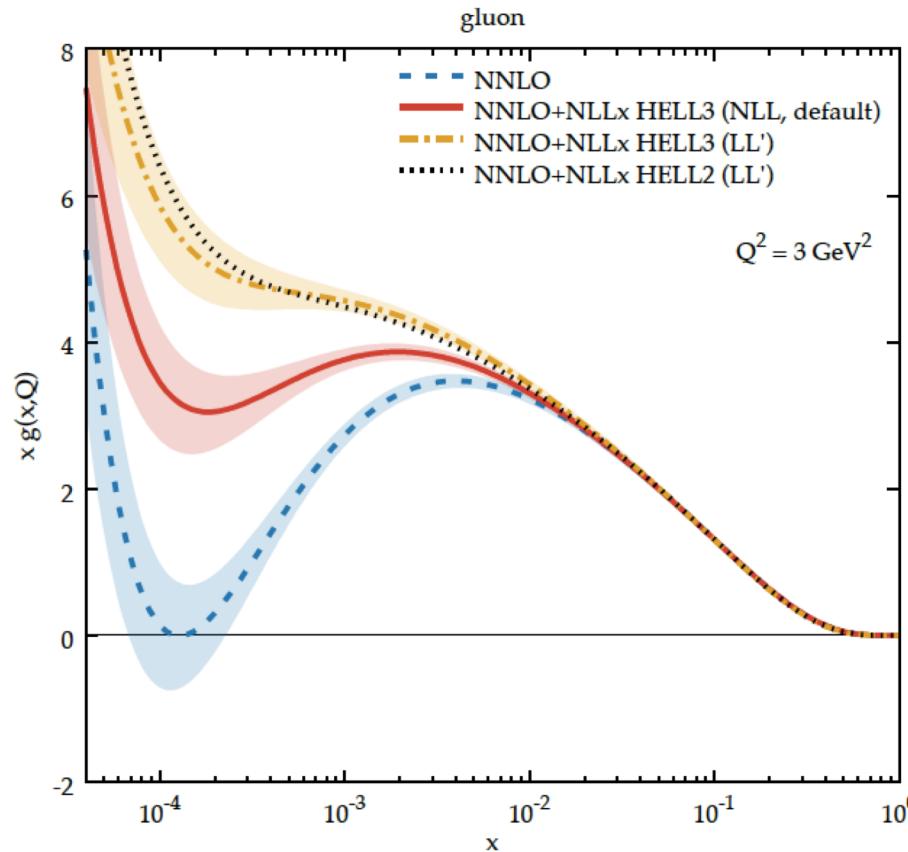
- Done using HELL (v3.0 for **the first time**) – [1805.08785](#), [1805.06460](#), [1708.07510](#)
- It provides resummed contributions to the DGLAP splitting functions, the heavy quark matching conditions and the DIS coefficient functions at NLLx

Contribution to $\chi^2$	HELL3.0 (NLL)	HELL3.0 (LL')	HELL2.0 (LL')
subset NC $e^+$ 920 $\tilde{\chi}^2/\text{n.d.p.}$	402/377	403/377	403/377
subset NC $e^+$ 820 $\tilde{\chi}^2/\text{n.d.p.}$	70/70	69/70	69/70
subset NC $e^+$ 575 $\tilde{\chi}^2/\text{n.d.p.}$	219/254	219/254	218/254
subset NC $e^+$ 460 $\tilde{\chi}^2/\text{n.d.p.}$	223/204	224/204	224/204
subset NC $e^-$ $\tilde{\chi}^2/\text{n.d.p.}$	219/159	220/159	220/159
subset CC $e^+$ $\tilde{\chi}^2/\text{n.d.p.}$	38/39	38/39	38/39
subset CC $e^-$ $\tilde{\chi}^2/\text{n.d.p.}$	49/42	49/42	49/42
correlation term + log term	73 – 7	72 – 11	72 – 10
Total $\chi^2/\text{d.o.f.}$	1284/1127	1283/1127	1283/1127

- If compared to NNLO fit, further reduction of **~30 units in  $\chi^2$**

# Including small-x resummation

- The difference between two versions of HELL v3.0 is the introduction of a new default treatment of subleading logarithmic contributions
- NLL variant predicts a softer gluon at small- $x$ , which is still significantly harder than the NNLO one



# Conclusion and outlook

- We proposed a **new simple parametrization** for the PDFs at the initial scale that includes a low degree polynomial in  $\log(x)$  – more flexibility at low- $x$
- Improvement of the fit quality (62 units reduction in  $\chi^2$  wrt HERAPDF2.0)
- Accomplished using **18 parameters** (only 4 more than HERAPDF2.0)
- Stability of our fit tested upon several model and parametrization variations → **results very robust**
- Flexibility of our parametrization allows for a more reliable determination of the uncertainties
- The impact of small- $x$  logarithms resummation investigated → gain of ~30 units in  $\chi^2$
- Small- $x$  resummation crucial for low- $x$  (HERA/LHC/FCC) phenomenology

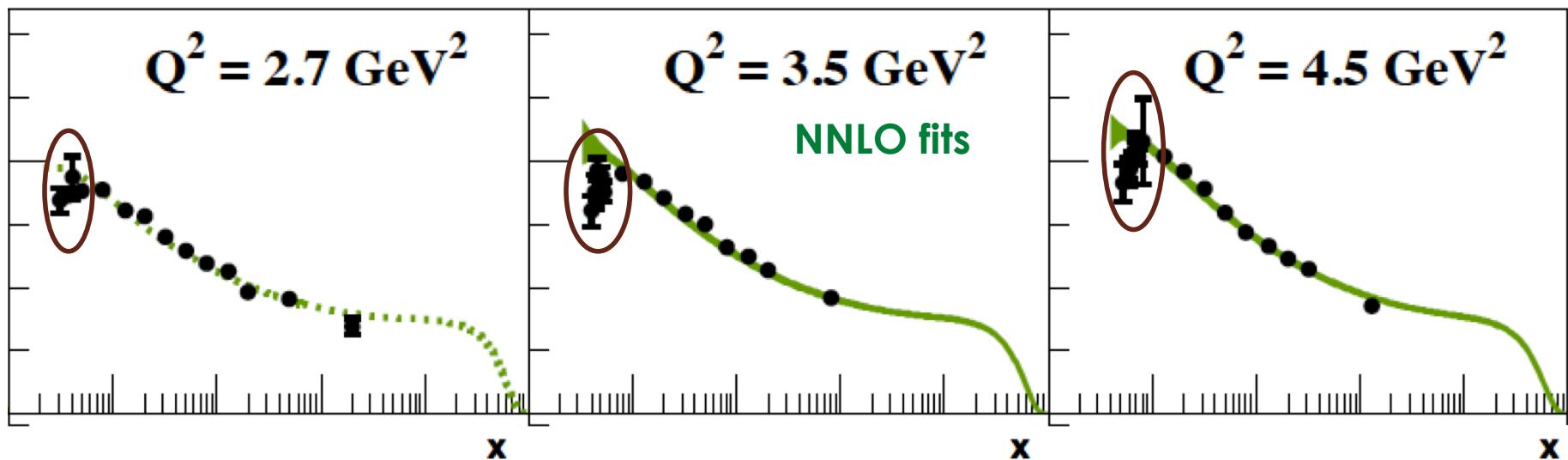
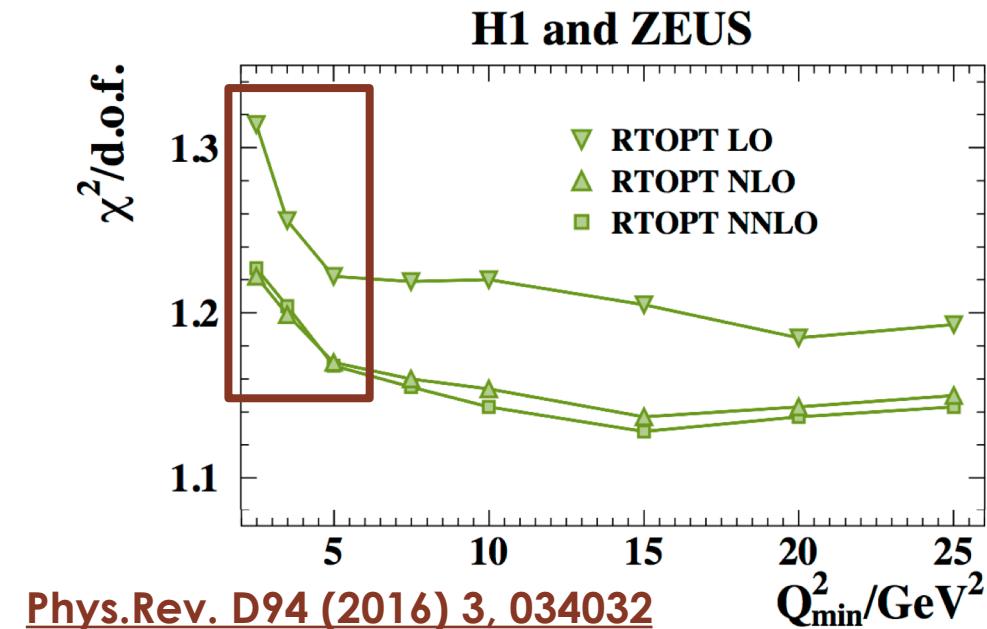
**LHAPDF grids available upon request**

# *Backup Slides*

---

# Why are we interested in small- $x$ resummation?

- **Crucial observation:** low- $x$  and low- $Q^2$  HERA data are not well described by FO pQCD
- Deterioration of  $\chi^2/\text{ndf}$  when including data at low- $Q^2$  at all orders in perturbation theory
- Data turnover at small- $x$  not described by **pQCD fits**



# Small-x logarithmic enhancement

$$\sigma_{DIS} = \textcolor{red}{C_i} \otimes f_i$$

$$\mu^2 \frac{\partial}{\partial \mu^2} f_i(\mu) = \textcolor{red}{P_{ij}} \otimes f_j(\mu)$$

**LO**       $\frac{1}{x} \alpha_S^0 [ \quad 1 \quad ]$

**NLO**       $\frac{1}{x} \alpha_S [ \# \log\left(\frac{1}{x}\right) + 1 ]$

**NNLO**       $\frac{1}{x} \alpha_S^2 [ \# \log^2\left(\frac{1}{x}\right) + \# \log\left(\frac{1}{x}\right) + 1 ]$

**N<sup>3</sup>LO**       $\frac{1}{x} \alpha_S^3 [ \# \log^3\left(\frac{1}{x}\right) + \# \log^2\left(\frac{1}{x}\right) + \# \log\left(\frac{1}{x}\right) + 1 ]$

**LL****NLL****NNLL**

If  $\alpha_S \log\left(\frac{1}{x}\right) \sim 1$  → all such terms in the perturbative series are equally important

Reorganisation of the expansion:

$$\frac{1}{x} \left[ 1 + \# \alpha_S \log\left(\frac{1}{x}\right) + \# \alpha_S^2 \log^2\left(\frac{1}{x}\right) + \# \alpha_S^3 \log^3\left(\frac{1}{x}\right) + \dots \right] \quad (\text{LL})$$

$$\frac{\alpha_S}{x} \left[ 1 + \# \alpha_S \log\left(\frac{1}{x}\right) + \# \alpha_S^2 \log^2\left(\frac{1}{x}\right) + \# \alpha_S^3 \log^3\left(\frac{1}{x}\right) + \dots \right] \quad (\text{NLL})$$

## All-order resummation

# Small-x resummation

- Small-x resummation formalism based on  **$k_T$ -factorization** and **BFKL**
- Resummation affects just the singlet sector (gluon and quark singlet)
- Developed in the 90s-00s

[Catani,Ciafaloni,Colferai,Hautmann,Salam,Stasto]  
[Thorne,White][Altarelli,Ball,Forte]

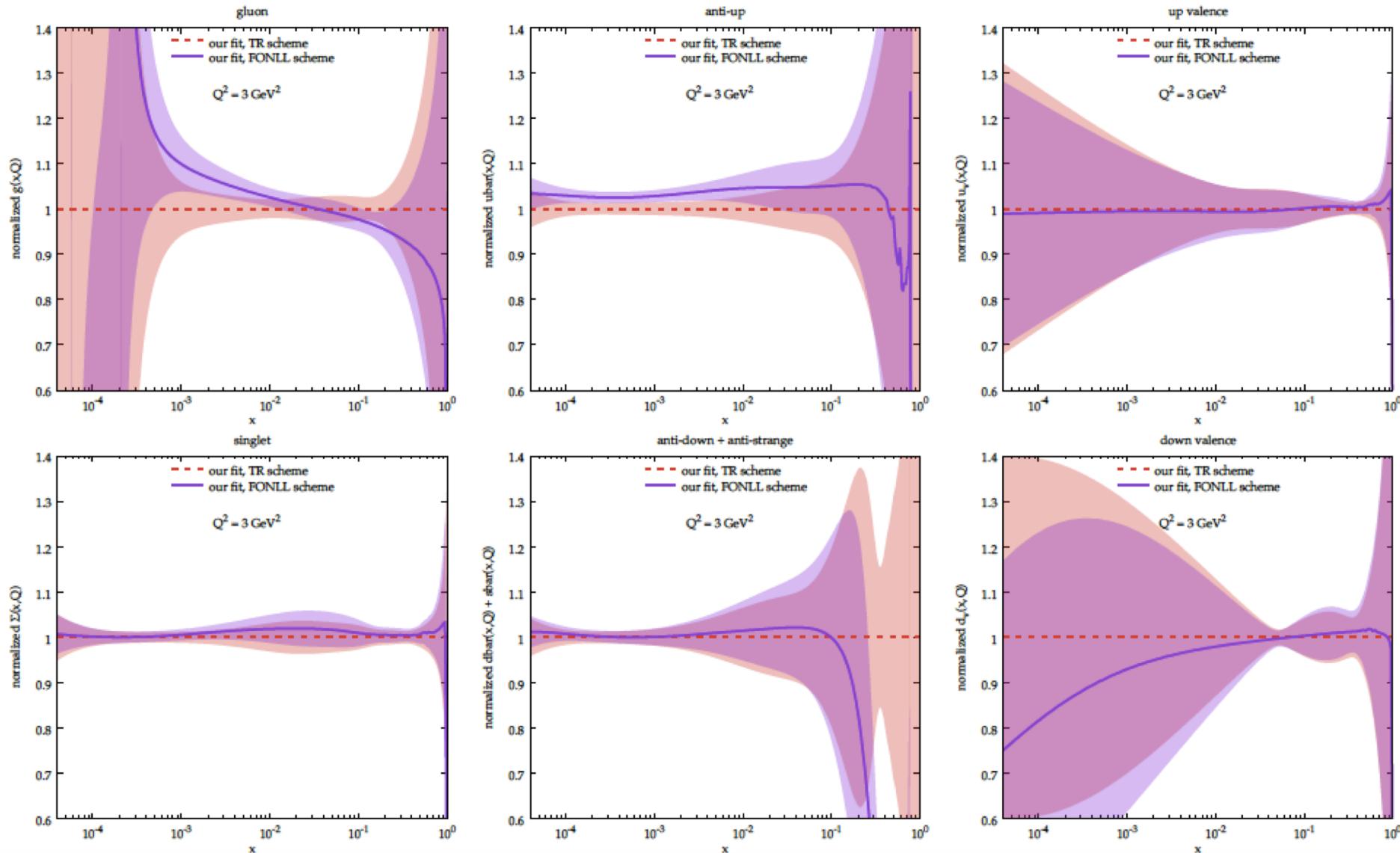
- Recent developments:
  - Improved ABF procedure to resum splitting functions and new formalism for coefficient functions
  - Resummation matched to NNLO, allowing NNLO+NLLx phenomenology
  - Public code: HELL

[Bonvini,Marzani,Peraro 1607.02153]  
[Bonvini,Marzani,Muselli 1708.07510]

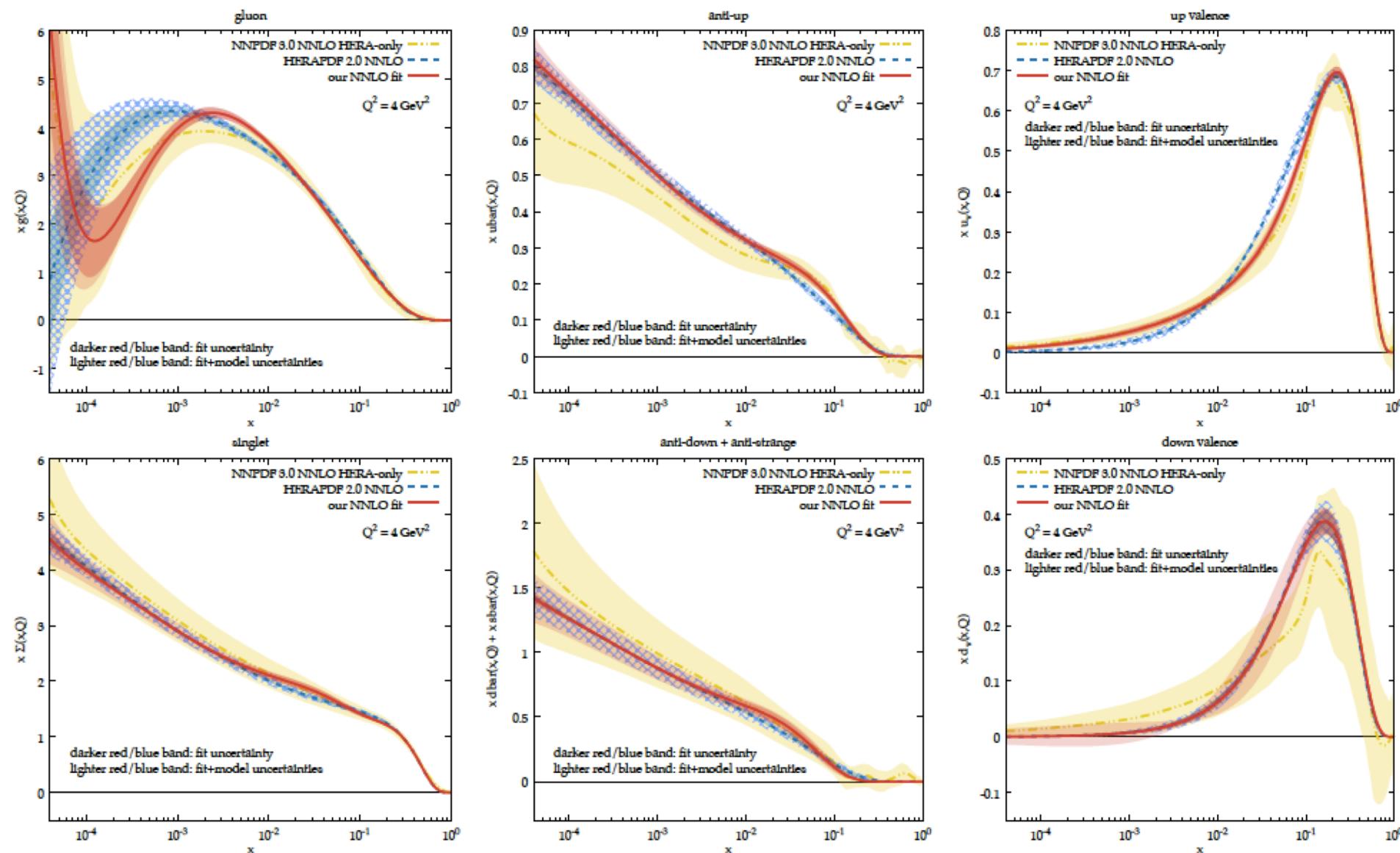
# $\chi^2$ definition

$$\chi^2 = \sum_i \underbrace{\frac{\left[ D_i - T_i \left( 1 - \sum_j \gamma_j^i b_j \right) \right]^2}{\delta_{i,\text{unc}}^2 T_i^2 + \delta_{i,\text{stat}}^2 D_i T_i}}_{\tilde{\chi}^2} + \underbrace{\sum_j b_j^2}_{\text{corr}} + \underbrace{\sum_i \ln \frac{\delta_{i,\text{unc}}^2 T_i^2 + \delta_{i,\text{stat}}^2 D_i T_i}{\delta_{i,\text{unc}}^2 D_i^2 + \delta_{i,\text{stat}}^2 D_i^2}}_{\log}$$

- First term: Data description (partial  $\chi^2$ )
  - $\gamma_j^i$  = Correlated systematic uncertainties
  - $b_j$  = Correlated systematic uncertainties shifts
- Second term: Correlated term
  - Reduction of this term indicates that the fit does not require the predictions to be shifted so far within the tolerance of the systematic uncertainties
- Third term: Log penalty term
  - Reduction of this term reflects a better agreement of the theoretical predictions ( $T_i$ ) with the data ( $D_i$ )



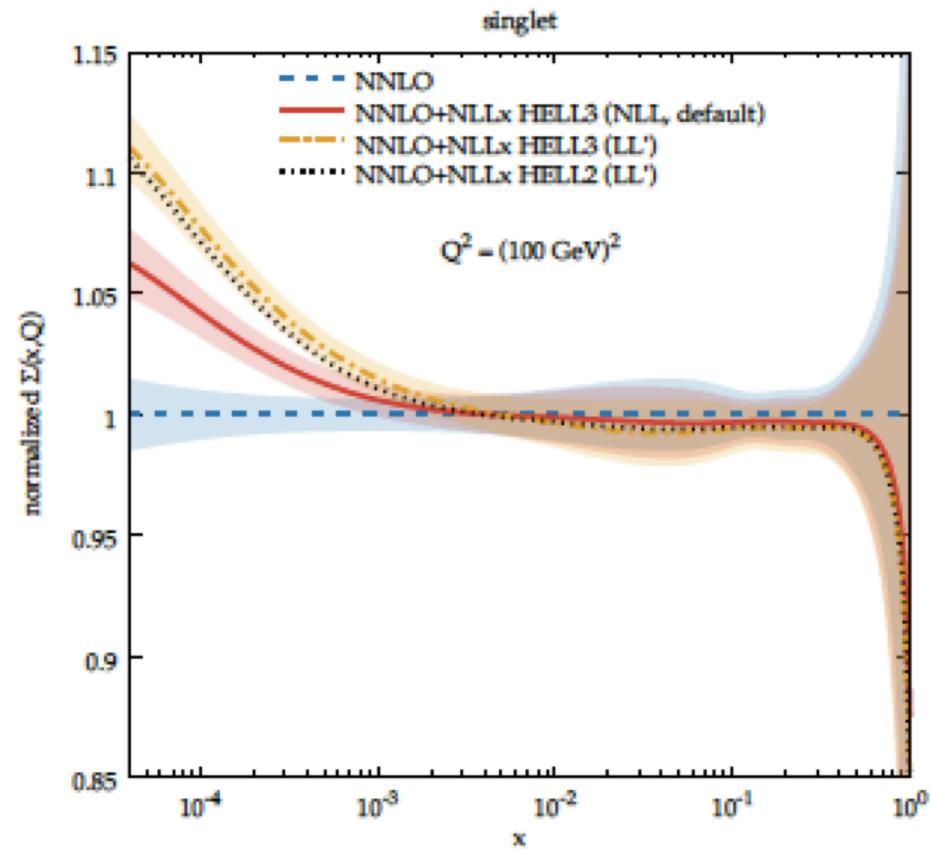
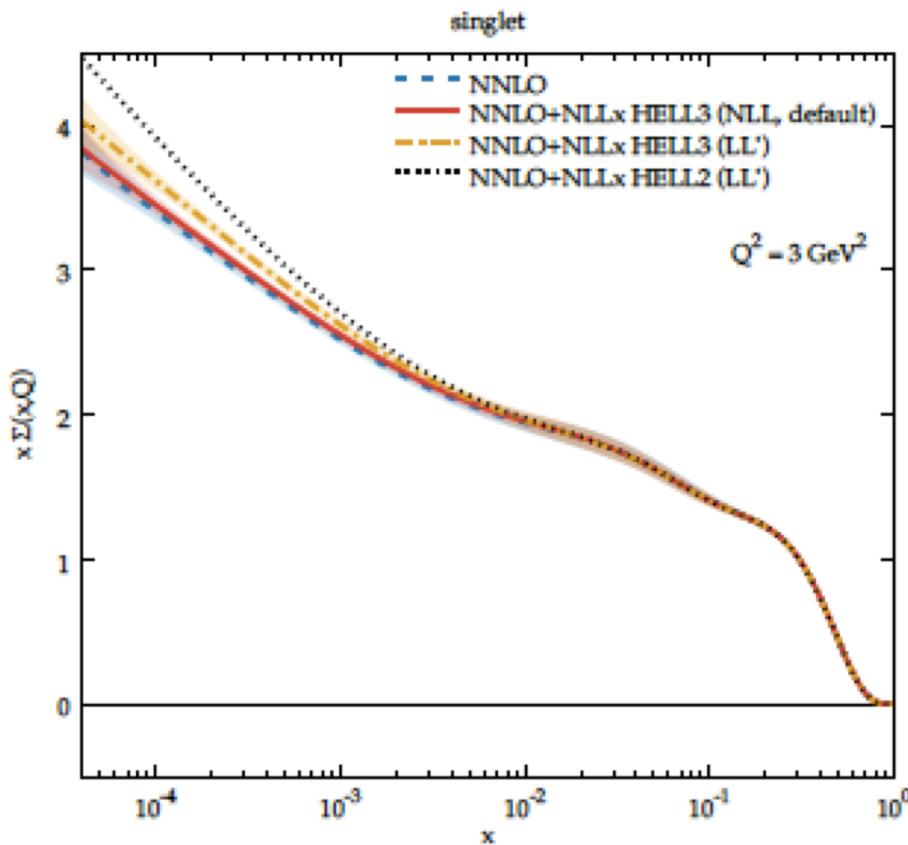
- Some differences are manifest (gluon/sea quarks)
- $1\sigma$  bands overlap or are very close to each other (apart from  $\bar{d} + \bar{s}$ )



- We combined all the uncertainties (**exp+th+model**) into a **(symmetric) uncertainty band**
- Our final PDF set including the full uncertainty is largely compatible with NNPDF30

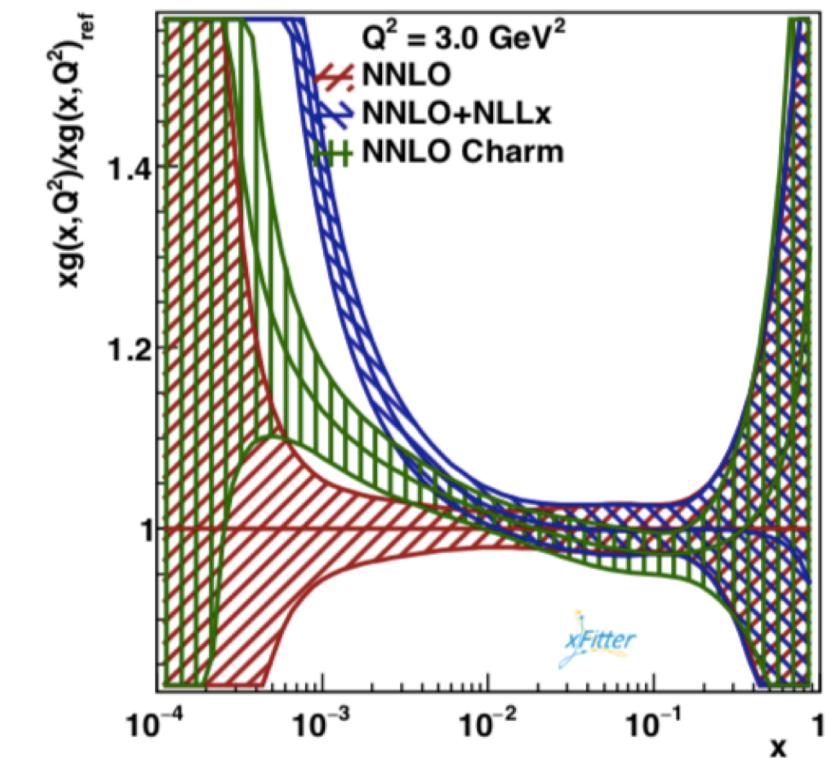
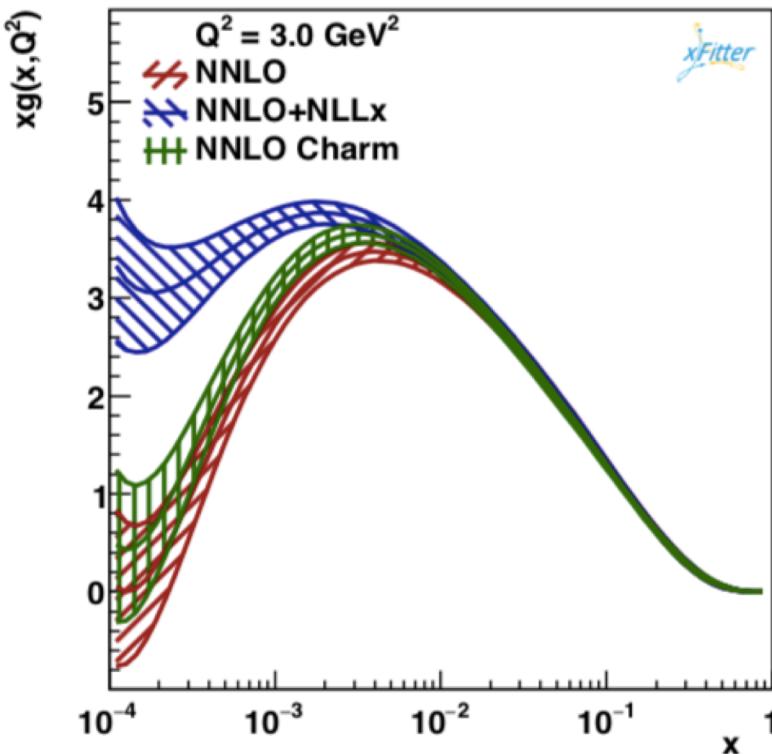
# Including small-x resummation

- The difference between two versions of HELL v3.0 is the introduction of a new default treatment of subleading logarithmic contributions
- HELL v2.0 is the previous version of the HELL code



# More sensitivity to the gluon PDF

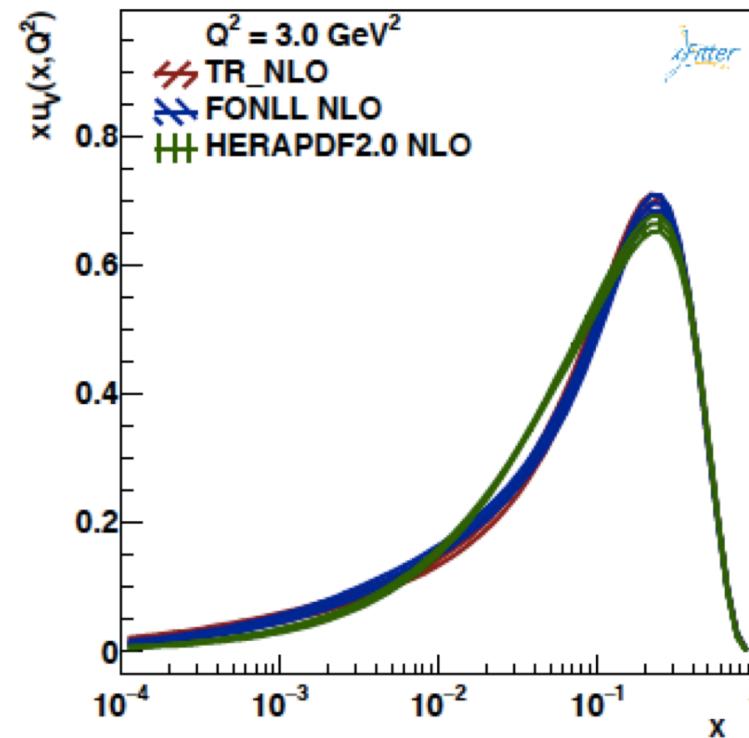
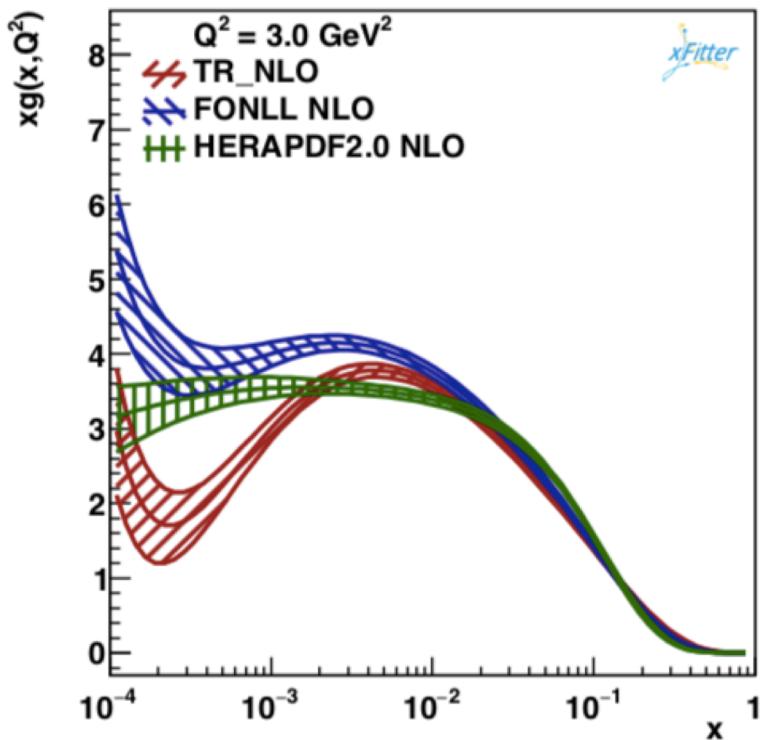
- We also studied the inclusion of HERA Charm combined data  
([Eur.Phys.J. C78 \(2018\) no.6, 473](#))
- These data are directly sensitive to  $xg(x, Q^2)$



- It is remarkable that the two FO fits are in agreement within uncertainties

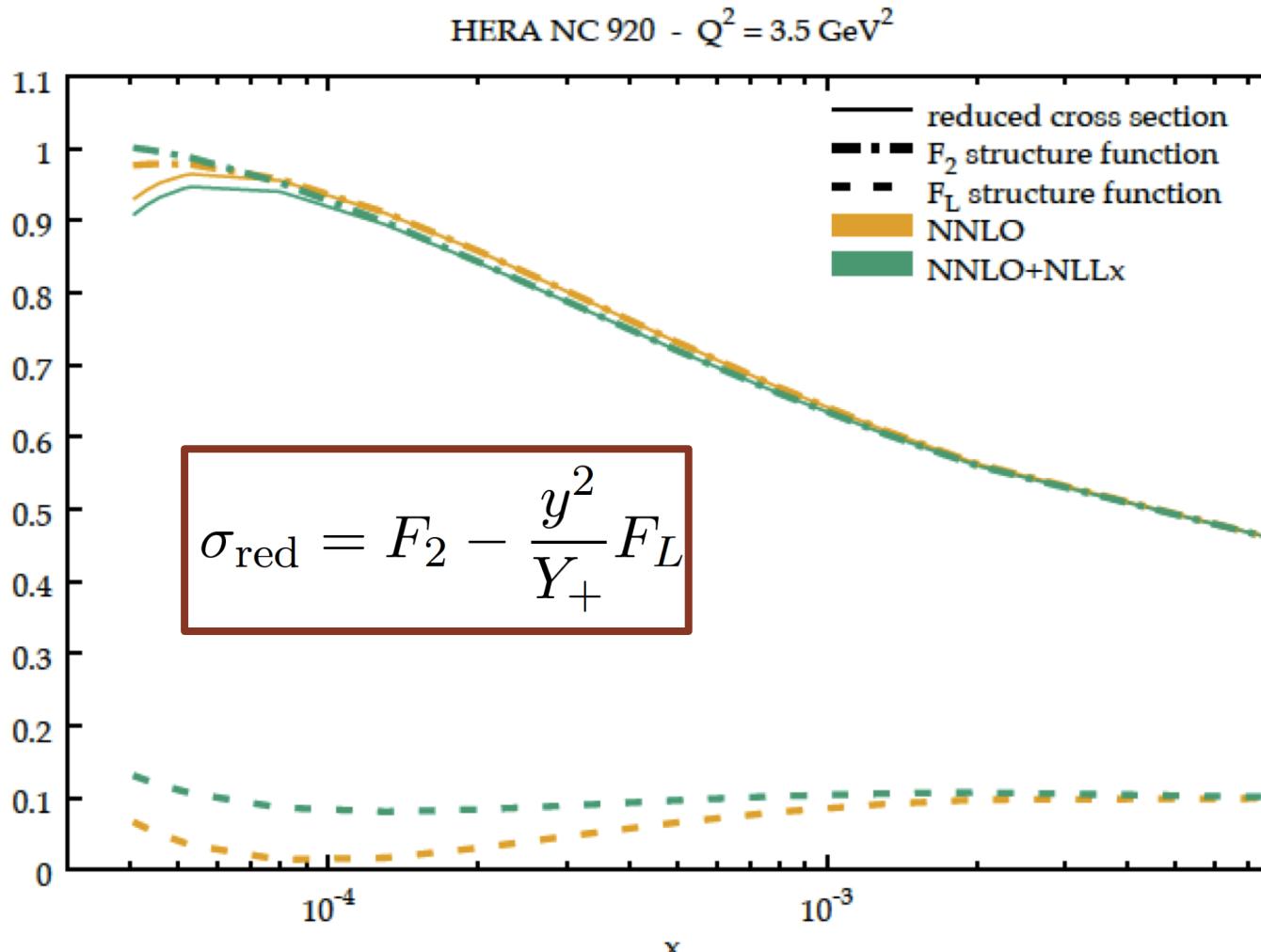
# A NLO fit

- We also tried a NLO fit (using both TR and FONLL-B) – **preliminary**
- FONLL-B provides a better description than TR
- At low- $x$ , same structure in the gluon PDF



Dataset	TR NLO	FONLL NLO	HERAPDF2.0 NLO
HERA1+2 CCep	38 / 39	39 / 39	43 / 39
HERA1+2 CCem	49 / 42	48 / 42	54 / 42
HERA1+2 NCem	220 / 159	215 / 159	222 / 159
HERA1+2 NCep 820	72 / 70	68 / 70	68 / 70
HERA1+2 NCep 920	407 / 377	400 / 377	440 / 377
HERA1+2 NCep 460	223 / 204	225 / 204	217 / 204
HERA1+2 NCep 575	222 / 254	219 / 254	219 / 254
Correlated $\chi^2$	76	67	86
Log penalty $\chi^2$	-1.39	-6.17	+8.9
Total $\chi^2 / \text{dof}$	1305 / 1127	1276 / 1127	1357 / 1145

# Reduced cross section, $F_2$ and $F_L$



Where  $Y_+ = (1 + (1 - y)^2)$  and  $y = Q^2/(sx)$

- FO and resummed calculations very similar for the reduced xsec
- NNLO prediction for  $F_2$  decreases at small  $x$  (softer gluon and quark singlet), while it rises steadily at resummed level (larger singlet)
- Resummed  $F_L$  is quite flat in  $x$  and much larger than the NNLO one ( $x \lesssim 10^{-3}$ )
- Rise of  $F_L$  due to the gluon PDF shape (rising for  $x \sim 10^{-4}$ )

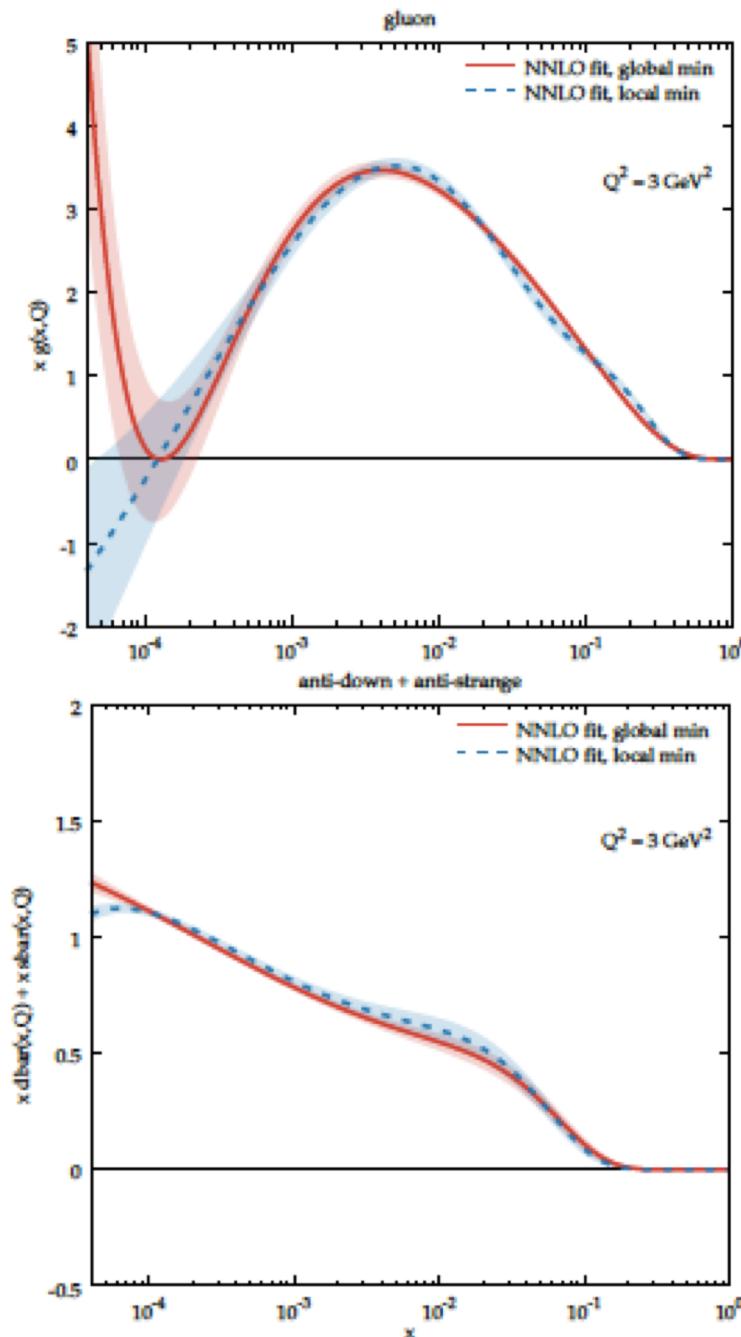
	$B_g$	$C_g$	$F_g$	$G_g$	$B_{u_v}$	$C_{u_v}$	$E_{u_v}$	$F_{u_v}$	$G_{u_v}$	$B_{d_v}$	$C_{d_v}$	$C_u$	$D_u$	$A_d$	$B_d$	$C_d$	$D_d$	$F_d$
$B_g$	1.000	0.783	-0.508	-0.465	-0.055	0.055	0.074	-0.094	-0.098	0.000	-0.058	-0.176	0.043	-0.457	-0.525	-0.285		
$C_g$	0.783	1.000	-0.093	-0.070	-0.014	0.038	0.046	-0.100	-0.091	0.061	-0.061	-0.163	0.036	-0.352	-0.383	-0.345		
$F_g$	-0.508	-0.093	1.000	0.989	-0.072	0.117	0.142	-0.150	-0.119	0.051	0.063	0.011	-0.183	0.422	0.494	0.125		
$G_g$	-0.465	-0.070	0.989	1.000	-0.075	0.124	0.149	-0.157	-0.125	0.051	0.061	0.016	-0.218	0.488	0.558	0.110		
$B_{u_v}$	-0.055	-0.014	-0.072	-0.075	1.000	-0.202	-0.598	0.485	0.897	-0.226	-0.197	-0.127	-0.244	0.126	0.050	-0.634		
$C_{u_v}$	0.055	0.038	0.117	0.124	-0.202	1.000	0.846	-0.616	-0.381	-0.042	0.030	-0.535	-0.521	0.211	0.178	0.315		
$E_{u_v}$	0.074	0.046	0.142	0.149	-0.598	0.846	1.000	-0.871	-0.777	0.184	0.248	-0.462	-0.443	0.164	0.157	0.646		
$F_{u_v}$	-0.094	-0.100	-0.150	-0.157	0.485	-0.616	-0.871	1.000	0.806	-0.409	-0.445	0.356	0.523	-0.240	-0.206	-0.673		
$G_{u_v}$	-0.098	-0.091	-0.119	-0.125	0.897	-0.381	-0.777	0.806	1.000	-0.402	-0.384	0.002	0.048	-0.031	-0.064	-0.730		
$B_{d_v}$	0.000	0.061	0.051	0.051	-0.226	-0.042	0.184	-0.409	-0.402	1.000	0.940	0.390	0.133	0.075	0.069	0.383		
$C_{d_v}$	-0.058	-0.061	0.063	0.061	-0.197	0.030	0.248	-0.445	-0.384	0.940	1.000	0.262	0.013	0.123	0.112	0.437		
$C_u$	-0.176	-0.163	0.011	0.016	-0.127	-0.535	-0.462	0.356	0.002	0.390	0.262	1.000	0.721	0.005	0.056	-0.126		
$D_u$	0.043	0.036	-0.183	-0.218	-0.244	-0.521	-0.443	0.523	0.048	0.133	0.013	0.721	1.000	-0.595	-0.517	-0.083		
$A_d$	-0.457	-0.352	0.422	0.488	0.126	0.211	0.164	-0.240	-0.031	0.075	0.123	0.005	-0.595	1.000	0.986	0.078		
$B_d$	-0.525	-0.383	0.494	0.558	0.050	0.178	0.157	-0.206	-0.064	0.069	0.112	0.056	-0.517	0.986	1.000	0.122		
$C_d$	-0.285	-0.345	0.125	0.110	-0.634	0.315	0.646	-0.673	-0.730	0.383	0.437	-0.126	-0.083	0.078	0.122	1.000		
$D_d$	-0.042	0.022	0.029	-0.010	-0.665	0.241	0.571	-0.575	-0.706	0.188	0.215	-0.317	-0.048	-0.304	-0.252	0.752	1.000	
$F_d$	0.562	0.363	-0.590	-0.652	0.013	-0.142	-0.142	0.166	0.086	-0.061	-0.095	-0.094	0.429	-0.941	-0.983	-0.147	0.200	1.000

- Correlation matrix between fit parameters
- Most of them are poorly correlated
- When present, F and G parameters strongly correlated (they probe the same x regime)
- They are also correlated to B parameters (same reason as above)
- Down-valence parameters highly correlated (same as for HERAPDF2.0)

# Local minima

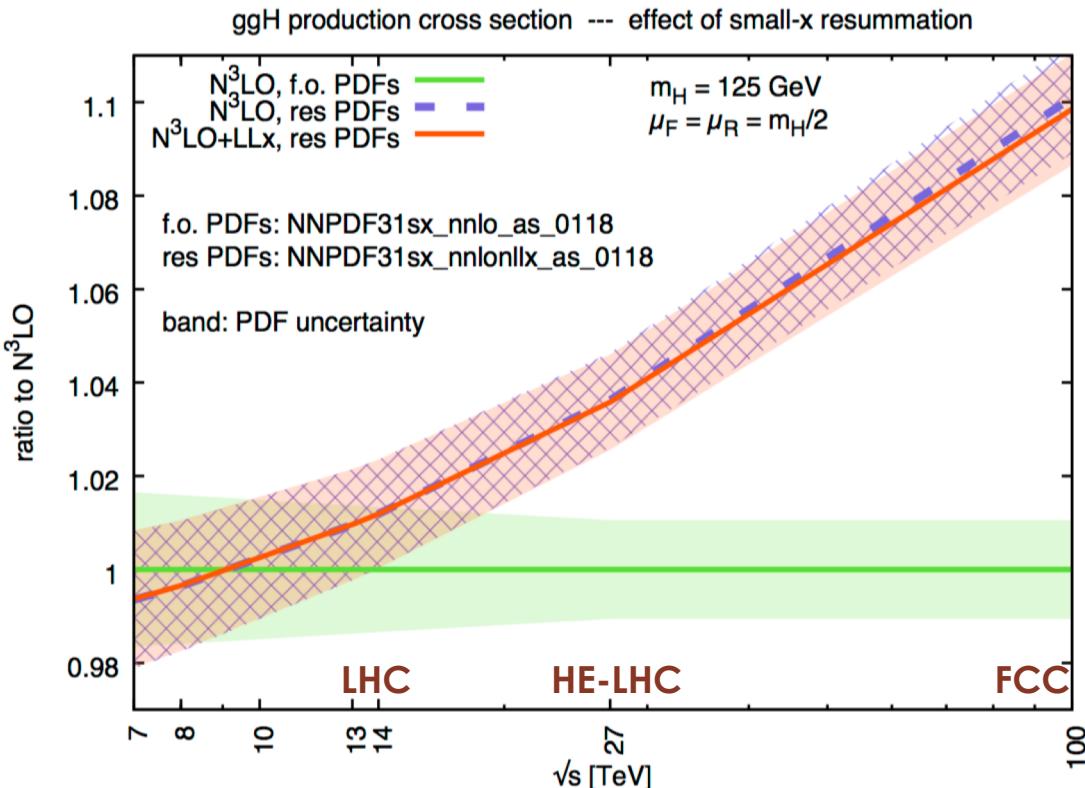
- While fitting data with fixed-order theory, we found a local minimum pretty far away from the global minimum presented in the paper
- Main difference in the gluon PDF: global minimum with  $B_g < 0$ , while local minimum with  $B_g > 0$
- The fit converged in the local minimum has an extra parameter in it: cubic logarithmic term in the gluon PDF ( $H_g$ )
- Even though very significant differences in some parameters,  $\chi^2$  really similar
- When transitioning from one minimum to the other in the parameter space, the  $\chi^2$  becomes much larger → with a standard minimization routine it is highly unlikely that once the local minimum is found, it could converge to the global minimum
- The physical expectation  $B_g < 0$  was crucial to guide us

Fitted parameter	NNLO (FONLL) local minimum	NNLO (FONLL) global minimum	NNLO+NLLx HELL 3.0 (NLL)
$B_g$	$0.34 \pm 0.07$	$-0.55 \pm 0.03$	$-0.52 \pm 0.04$
$C_g$	$8.8 \pm 1.0$	$4.5 \pm 0.5$	$4.5 \pm 0.5$
$F_g$	$0.76 \pm 0.04$	$0.230 \pm 0.003$	$0.217 \pm 0.005$
$G_g$	$0.22 \pm 0.02$	$0.0131 \pm 0.0004$	$0.0112 \pm 0.0005$
$H_g$	$0.017 \pm 0.002$		
$B_{u_v}$	$0.85 \pm 0.06$	$0.83 \pm 0.06$	$0.76 \pm 0.06$
$C_{u_v}$	$4.5 \pm 0.1$	$4.6 \pm 0.2$	$4.6 \pm 0.1$
$E_{u_v}$	$1.7 \pm 0.8$	$1.9 \pm 1.0$	$2.6 \pm 1.1$
$F_{u_v}$	$0.38 \pm 0.04$	$0.37 \pm 0.05$	$0.35 \pm 0.04$
$G_{u_v}$	$0.062 \pm 0.011$	$0.058 \pm 0.012$	$0.049 \pm 0.010$
$B_{d_v}$	$1.01 \pm 0.09$	$0.98 \pm 0.10$	$0.99 \pm 0.09$
$C_{d_v}$	$4.7 \pm 0.4$	$4.7 \pm 0.5$	$4.7 \pm 0.5$
$A_{\bar{d}}$	$0.070 \pm 0.008$	$0.13 \pm 0.02$	$0.14 \pm 0.02$
$B_{\bar{d}}$	$-0.45 \pm 0.02$	$-0.34 \pm 0.02$	$-0.33 \pm 0.02$
$C_{\bar{d}}$	$28 \pm 3$	$24 \pm 2$	$24 \pm 3$
$D_{\bar{d}}$	$76 \pm 17$	$40 \pm 12$	$38 \pm 10$
$F_{\bar{d}}$	$0.084 \pm 0.001$	$0.072 \pm 0.004$	$0.071 \pm 0.004$
$C_{\bar{u}}$	$11 \pm 1$	$11 \pm 1$	$11 \pm 1$
$D_{\bar{u}}$	$33 \pm 6$	$20 \pm 4$	$18 \pm 4$
$\chi^2/\text{d.o.f.}$	$1314/1126$	$1312/1127$	$1284/1127$



# Resummed phenomenology

- Inclusive **gluon-fusion Higgs** production process



- Resummed calculation matched to  $N^3LO$  FO calculations
- Small-x resummation has a modest impact at current LHC energies
- Its impact grows substantially with the energy, reaching 10% at 100 TeV
- **Bulk of the effect:** the resummed PDFs and their resummed evolution

[Phys.Rev.Lett. 120 \(2018\) 20 202003](#)

[Bonvini,Marzani]

[Eur.Phys.J. C78 \(2018\) no.10, 834](#)

[Bonvini]

- Here inclusive cross sections BUT a more prominent effect is expected in exclusive/differential cross section (especially in e.g. large-rapidity regions)