

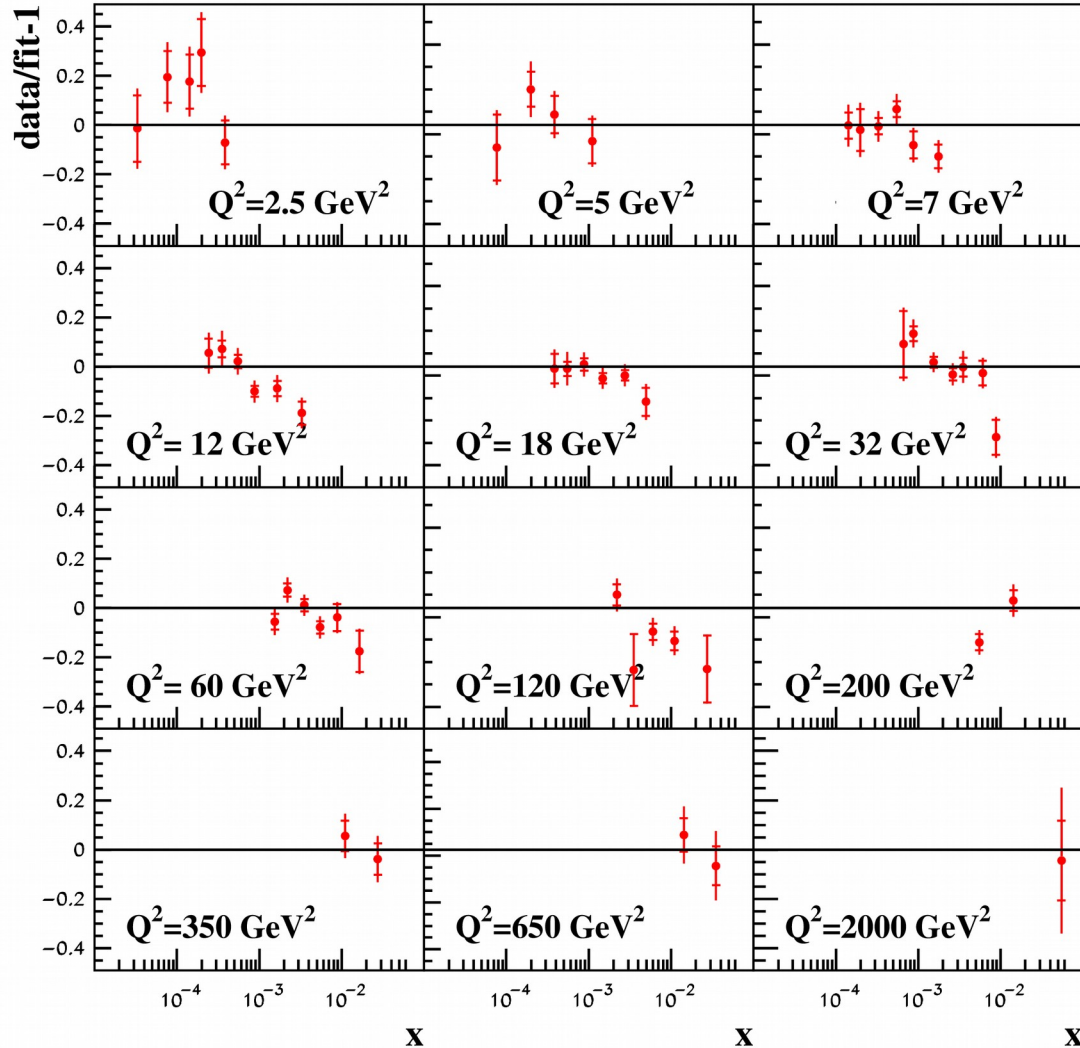
Large-log resummation in the VFN scheme of the DIS heavy-quark production

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(*in collaboration with J.Blümlein and S.Moch*)

HERA charm data and m_c

H1, ZEUS EPJC 78, 473 (2018)

HERA I+II (ep --> e charm X)



$\chi^2/NDP=86/52$

Theory: FFN scheme, running mass definition

$$m_c(m_c)=1.245\pm 0.019(\text{exp.}) \text{ GeV}$$

present analysis

$$m_c(m_c)=1.252\pm 0.018(\text{exp.}) \text{ GeV}$$

ABMP16

$$m_c(\text{pole})\sim 1.9 \text{ GeV (NNLO)}$$

Marquard et al. PRL 114, 142002 (2015)

$$m_c(m_c)=1.246\pm 0.023 \text{ (h.o.) GeV NNLO}$$

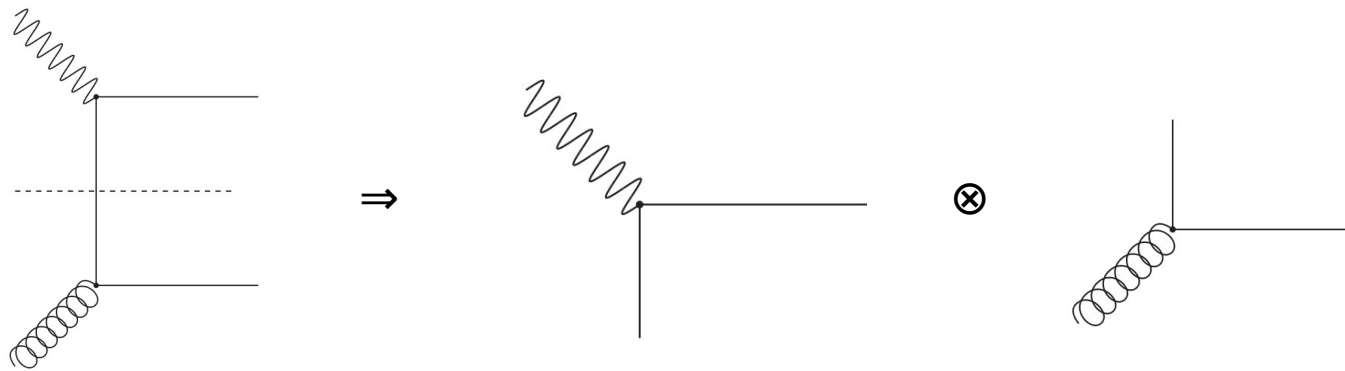
Kiyo, Mishima, Sumino PLB 752, 122 (2016)

$$m_c(m_c)=1.279\pm 0.008 \text{ GeV}$$

Kühn, LoopsLegs2018

Good consistency with the earlier results and other determinations → further confirmation of the FFN scheme relevance for the HERA kinematics

FFN and VFN schemes



Collins, Tung NPB 278, 934 (1986)

$$H_{g,2}^{\text{asympt}} = a_s(N_f) A_{hg}^{(1)} \quad \text{Asymptotic 3-flavor coefficient function}$$

LO: $A_{ij}^{(1)}\left(z, \frac{m_h^2}{\mu^2}\right) = a_{ij}^{(1,1)}(z) \ln\left(\frac{\mu^2}{m_h^2}\right)$ Massive operator matrix elements (OMEs)

$$h^{(1)}(x, \mu^2) = a_s(N_f + 1, \mu^2) \left[A_{hg}^{(1)}\left(\frac{m_h^2}{\mu^2}\right) \otimes G^{(2)}(N_f, \mu^2) \right](x) \quad \text{Matching condition for the heavy-quark PDFs}$$

$$F_{g,2}^{\text{asympt}} = e_h^2 x h^{(1)}(x, \mu^2)$$

NLO: ...

Buza, Matiounine, Smith, van Neerven EPJC 1, 301 (1998)

$$A_{ij}^{(2)}\left(z, \frac{m_h^2}{\mu^2}\right) = a_{ij}^{(2,2)}(z) \ln^2\left(\frac{\mu^2}{m_h^2}\right) + a_{ij}^{(2,1)}(z) \ln\left(\frac{\mu^2}{m_h^2}\right) + a_{ij}^{(2,0)}(z)$$

2-mass contributions in NLO and NNLO

Blümlein et al. PLB 782, 362 (2018)

NNLO: log-terms; constant terms up to the gluonic one

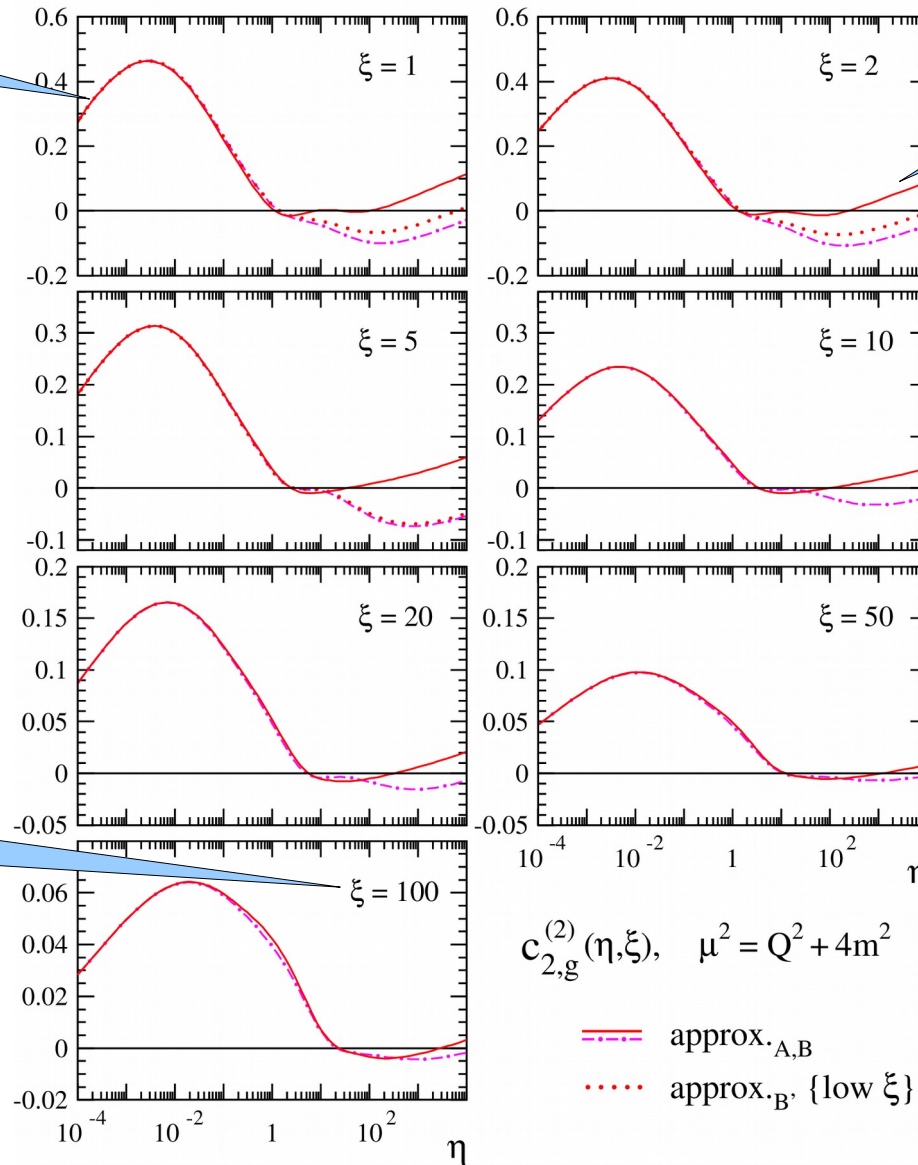
Blümlein, et al., work in progress

- The VFN scheme works well at $\mu \gg m_h$ (W,Z,t-quark production,....)
- Problematic for DIS \Rightarrow additional modeling of power-like terms required at small scales (ACOT, BMSN, FONLL, RT....)

NNLO massive Wilson coefficients

small s

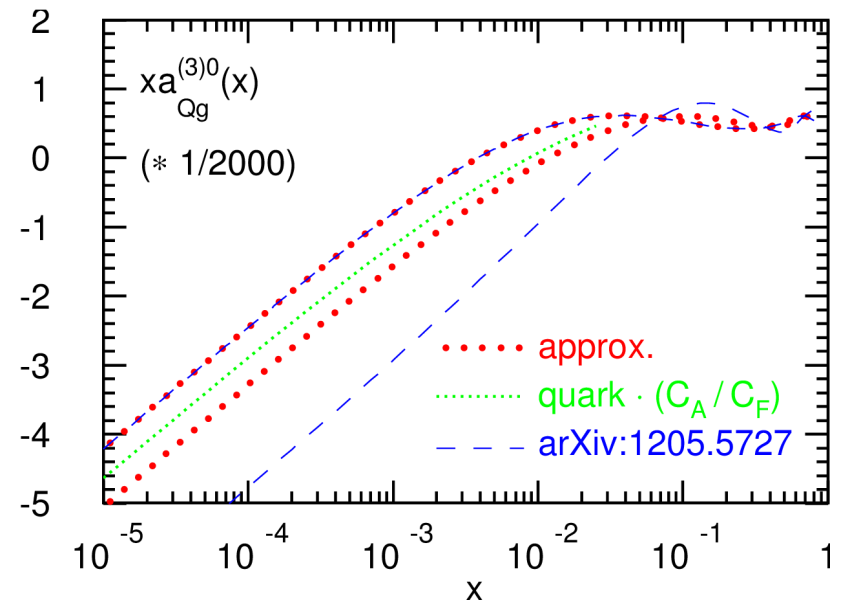
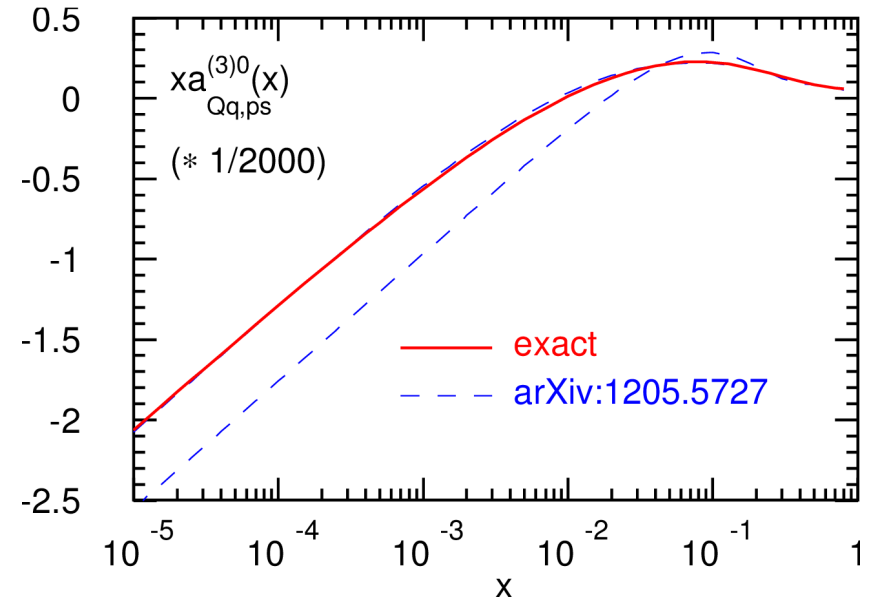
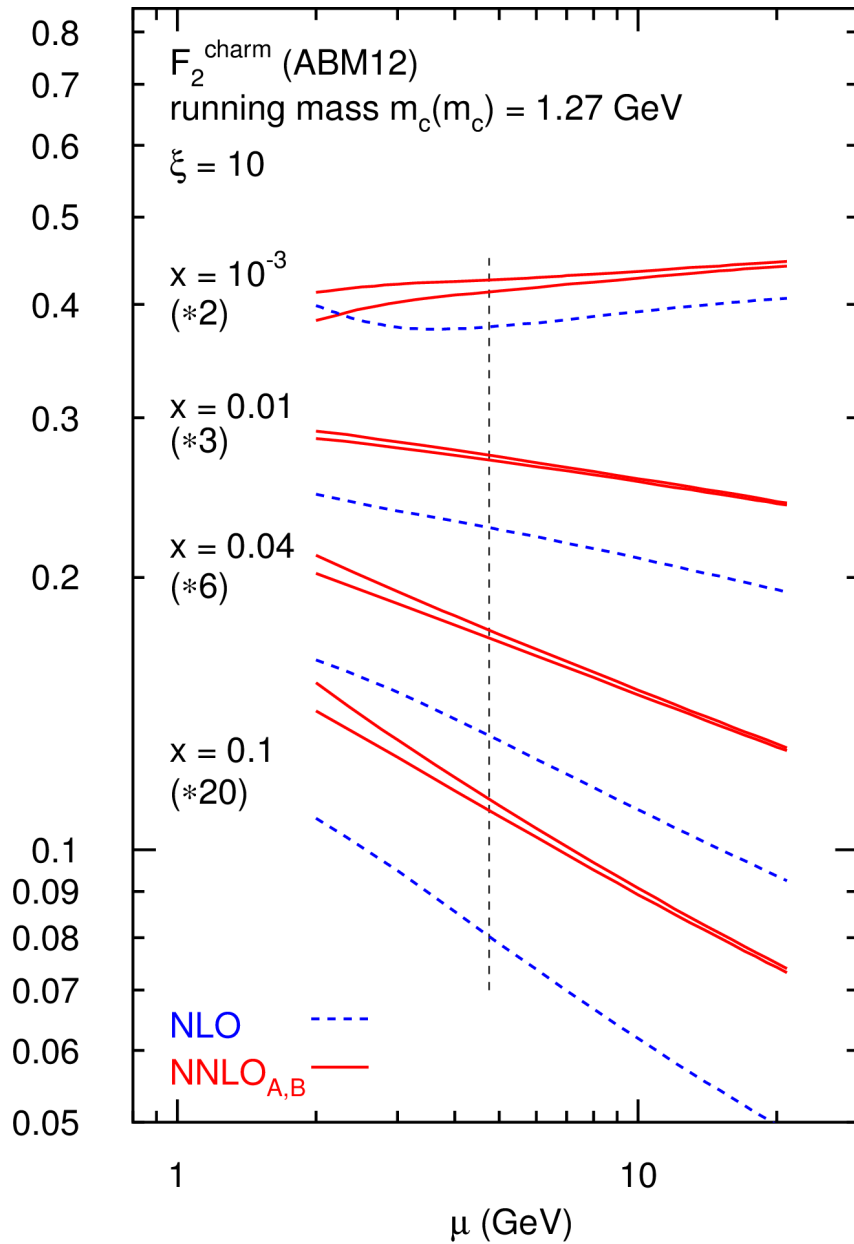
small x



large Q^2

Combination of the threshold corrections (small s), high-energy limit (small x), and the NNLO massive OMEs (large Q^2)

Recent progress in NNLO FFN scheme



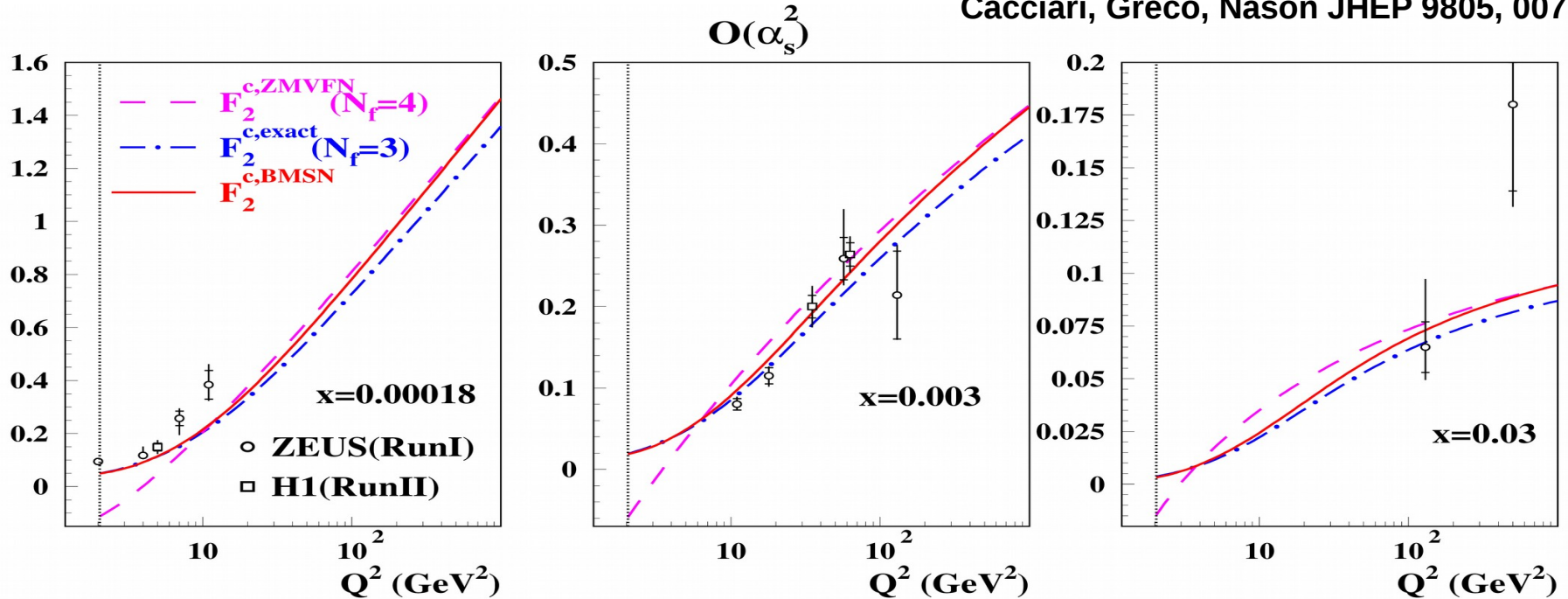
Update with the pure singlet massive OMEs \rightarrow improved theoretical uncertainties

BMSN prescription of GMVFN scheme

Buza, Matiounine, Smith, van Neerven EPJC 1, 301 (1998)

$$F_2^{h,BMSN}(N_f + 1, x, Q^2) = F_2^{h,exact}(N_f, x, Q^2) + F_2^{h,ZMVFN}(N_f + 1, x, Q^2) - F_2^{h,asymp}(N_f, x, Q^2),$$

Cacciari, Greco, Nason JHEP 9805, 007 (1998)



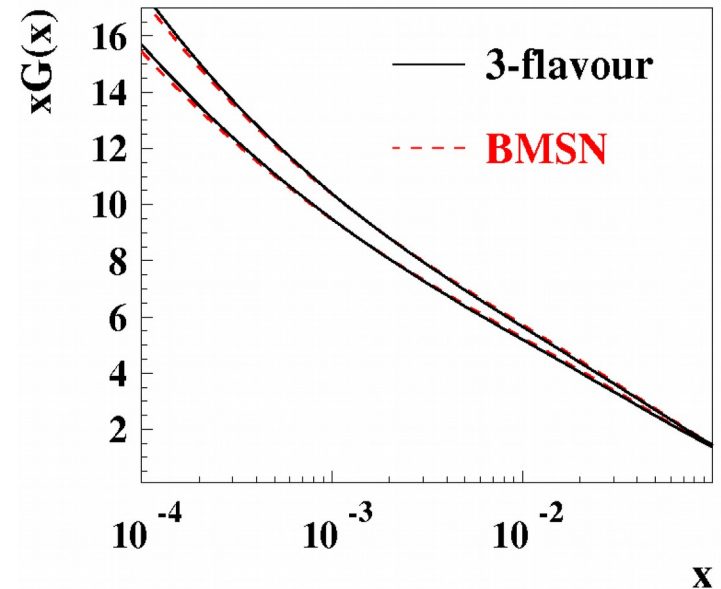
sa, Blümlein, Klein, Moch PRD 81, 014032 (2010)

In the $O(\alpha_s^2)$ the FFNS and GMVFNs are comparable at large scales since the big logs appear in the high order corrections to the massive coefficient functions

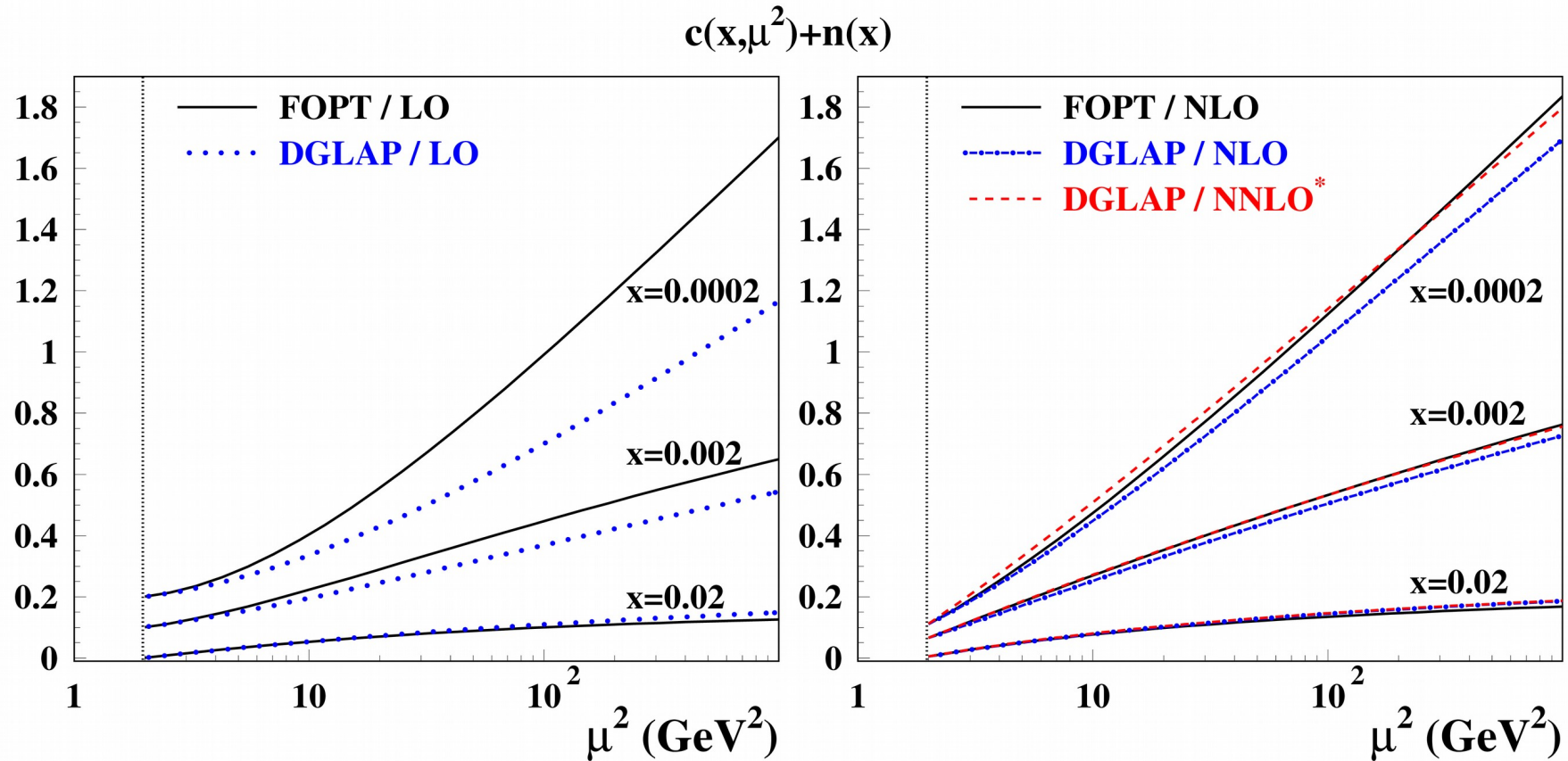
Glück, Reya, Stratmann NPB 422, 37 (1994)

- Smooth matching with the FFNS at $Q \rightarrow m_h$ without additional damping or re-scaling factors

- FOPT heavy-quark PDFs \rightarrow large logs missing?



Comparison of the FOPT and evolved c-quark PDFs



LO: The FOPT and evolved PFGs nicely match at m_h ; at large scales they diverge due to large logs resummed by evolution

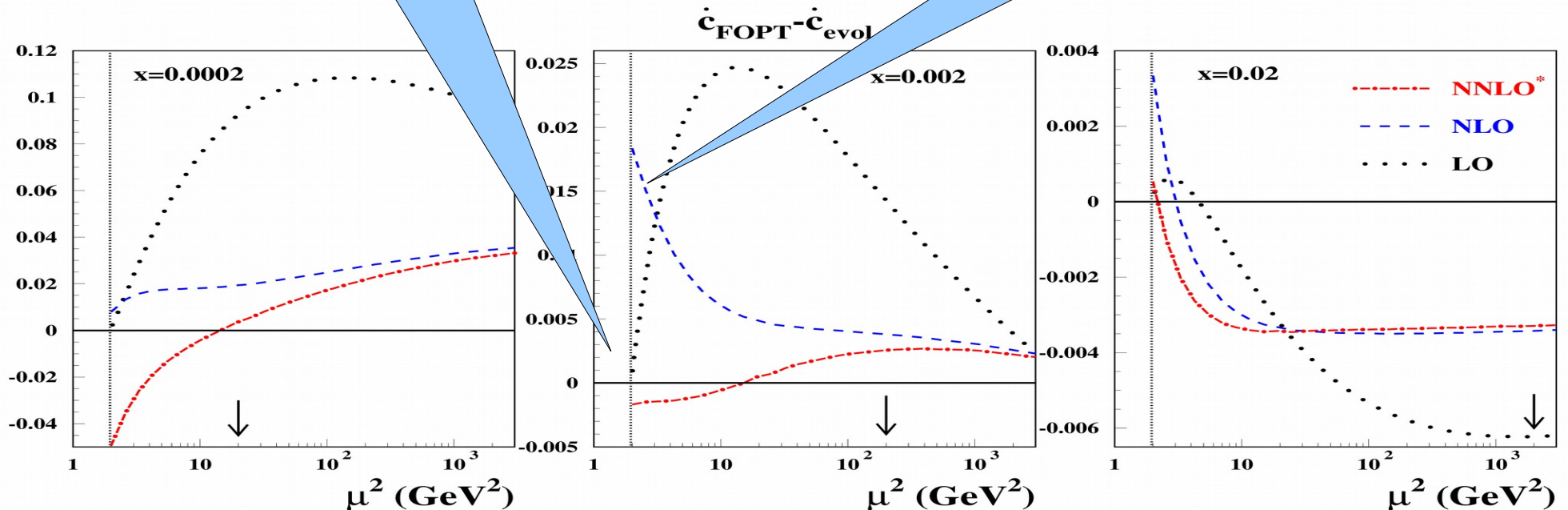
NLO (NLO OMES and NLO evolution): The difference between FOPT and evolved PDFs at large scales dramatically reduces due to large log are partially included into NLO OMES and therefore are taken into account in the FOPT as well.

NNLO* (NLO OMES and NNLO evolution): A kink w.r.t. FOPT is observed at small x

c-quark PDFs: fixed-order versus evolved

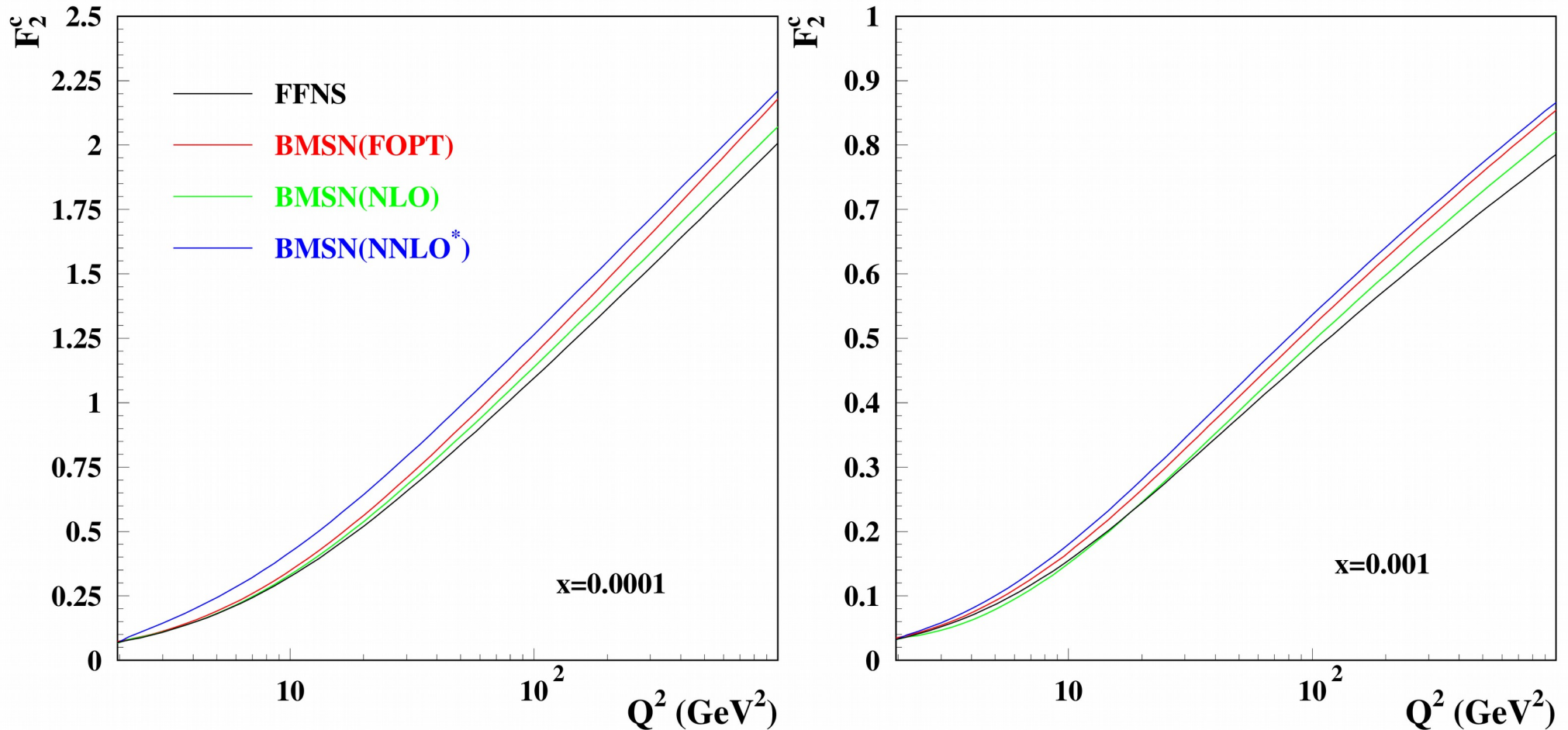
LO constant term in OME=0

NLO constant term in OME≠0



- The heavy-quark PDF evolution brings essential impact only beyond realistic kinematics of existing data (HERA)
- A consistent NNLO treatment of the VFN PDFs is problematic due to missing NNLO massive OMEs. The difference between NLO and NNLO* gives theoretical uncertainty due to missing higher orders

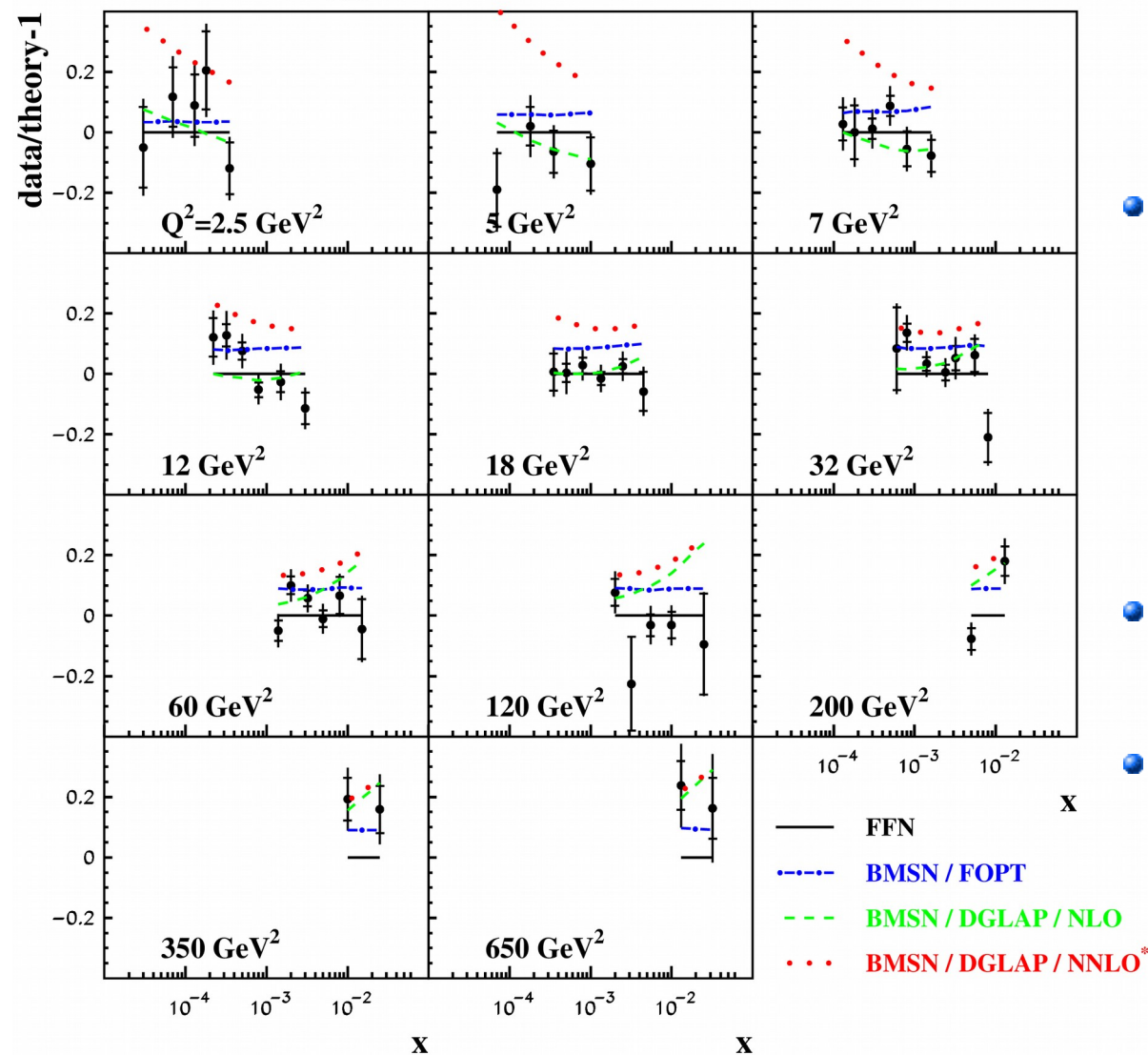
BMSN VFN scheme: FOPT versus evolved



- Impact of heavy-quark PDF evolution is marginal even at very large Q
- A significant kink w.r.t. FFN scheme appears at small Q if heavy-quark PDFs are evolved in the NNLO

VFN scheme with the evolved PDFs

$\sigma_{\text{red}}^{\text{cc}}$ (HERA RunI+II combined)



Comparison with the model FFN fit:

- NLO massive Wilson coeffs.,
- $m_c(\text{pole})=1.4 \text{ GeV}$

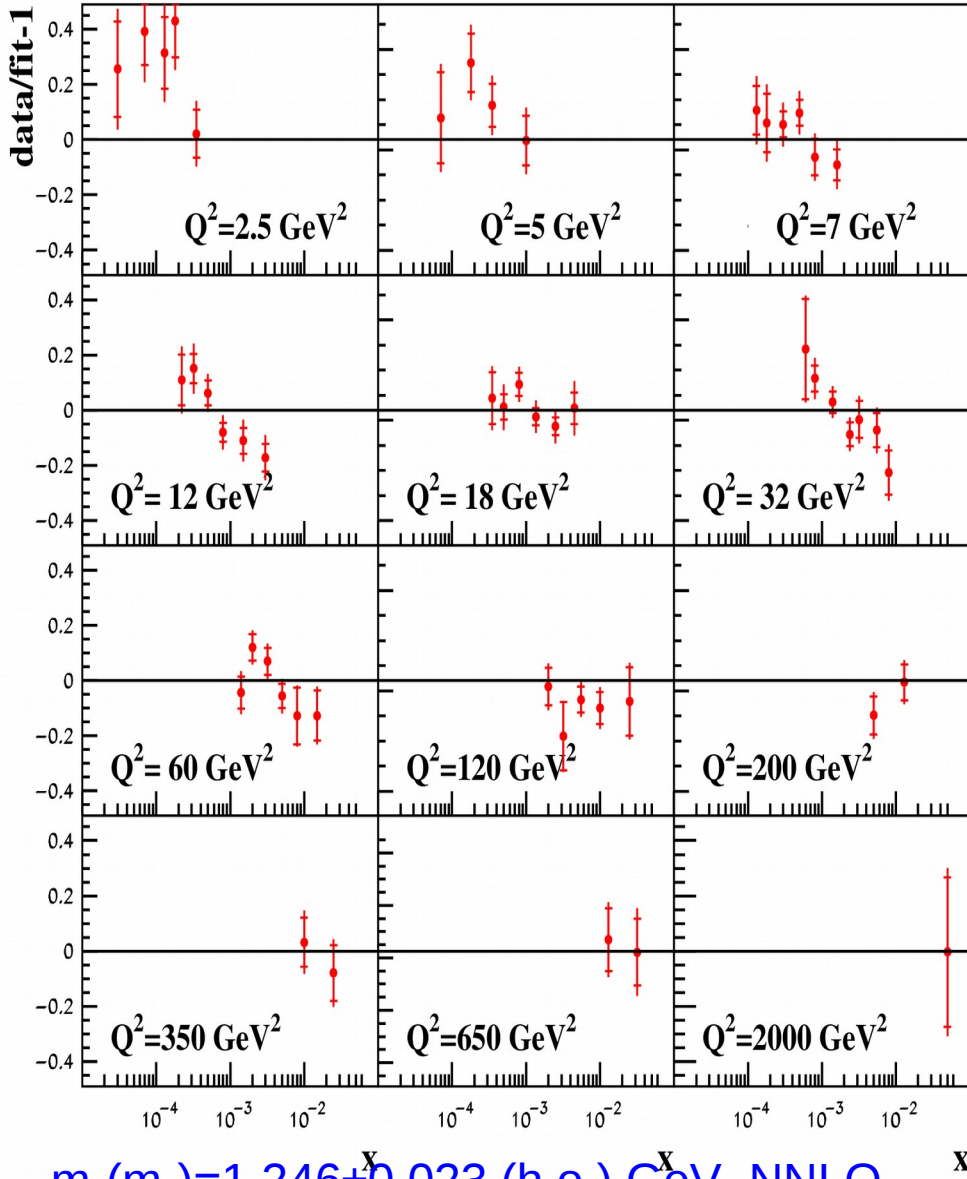
- Two variants of 4-flavor PDF evolution:
 - NNLO (commonly used in the VFN fits)
 - consistent with light PDF evolution
 - inconsistent with NLO matching
 - NLO
 - inconsistent with light PDF evolution
 - consistent with NLO matching
- Substantial difference between NLO and NNLO versions
- The evolved predictions demonstrate strong x-dependence and weak Q^2 -dependence

The difference with FOPT appears rather due to inconsistent evolution than due to big-logs → theoretical uncertainty in the VFN schemes

HERA charm data: FFN versus VFN

H1/ZEUS ZPC 73, 2311 (2013)

HERA I+II (ep --> e charm X)



$m_c(m_c) = 1.246 \pm 0.023$ (h.o.) GeV NNLO

Kiyo, Mishima, Sumino PLB 752, 122 (2016)

$m_c(m_c) = 1.279 \pm 0.008$ GeV

Kühn, this conference

$X^2/NDP = 66/52$

$m_c(m_c) = 1.252 \pm 0.018$ (exp.) - 0.01 (th.) GeV

ABMP16

$m_c(\text{pole}) \sim 1.9$ GeV (NNLO)

Marquard et al. PRL 114, 142002 (2015)

- RT optimal
 $X^2/NDP = 82/52$
 $m_c(\text{pole}) = 1.25$ GeV

NNLO

MMHT14 EPJC 75, 204 (2015)

- S-ACOT- χ
 $X^2/NDP = 59/47$
 $m_c(\text{pole}) = 1.3$ GeV

NNLO

CT14 PRD 93, 033006 (2016)

- F0NLL
 $X^2/NDP = 60/47$
 $m_c(\text{pole}) = 1.275$ GeV

NNLO

NNPDF3.0 JHEP 504, 040 (2015)

- F0NLL
 $X^2/NDP = 54/37$
 $m_c(\text{pole}) = 1.51$ GeV, intrinsic (fitted) charm

NNLO

NNPDF3.1 hep-ph/1706.00428

For more accurate data VFN works even worse

H1, ZEUS EPJC 78, 473 (2018)

Conclusions

- The large-log resummation provided in the VFN scheme for the heavy-quark PDFs manifests in the existing data on DIS structure functions only in the LO; in the NLO the large logs are greatly absorbed into heavy-quark matching conditions and resummations gets irrelevant for existing data kinematics.
- Limited theoretical accuracy of the matching conditions for the heavy-quark PDFs, available in the NLO only, does not allow fully consistent NNLO QCD evolution in the VFN scheme. This brings substantial theoretical uncertainty into the VFN computations at small Q^2 , which is mixed with the power corrections appearing in the VFN scheme (in practical implementations of the VFN scheme, ACOT, FONLL, RT,... all this has to be suppressed by additional parameters, like damping and rescaling factors).
- In contrast, the straightforward FFN approach is free from such ambiguities. With the QCD corrections up to NNLO available it provides good description of the existing data and consistent determination of the heavy-quark masses

$$m_c(m_c)=1.245\pm 0.019(\text{exp.}) \text{ GeV}$$

$$m_b(m_b)=3.96\pm 0.10(\text{exp.}) \text{ GeV}$$

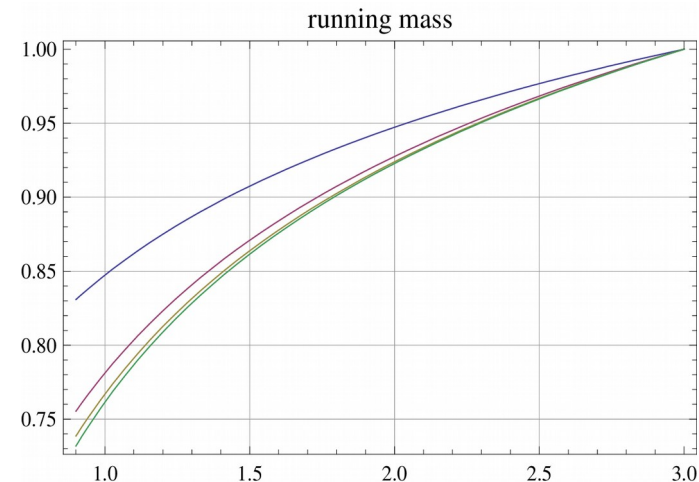
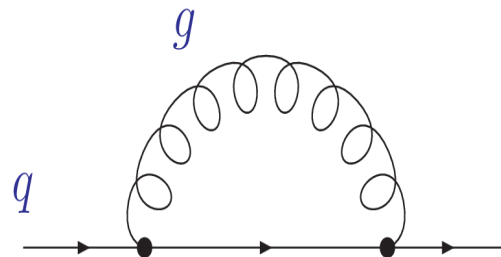
EXTRAS

Running mass in DIS

The pole mass is defined for the free (*unobserved*) quarks as a the QCD Lagrangian parameter and is commonly used in the QCD calculations

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \sum_{\text{flavors}} \bar{q} (i\not{D} - m_q) q$$

$$\not{p} - m_q - \Sigma(p, m_q) \Big|_{p^2=m_q^2}$$



The quantum corrections due to the self-energy loop Integrals receive contribution down to scale of $O(\Lambda_{\text{QCD}})$

→ sensitivity to the high order corrections, particularly at the production threshold

$$\mu^2 \frac{d}{d\mu^2} m(\mu) = \gamma(\alpha_s) m(\mu)$$

