Large-log resummation in the VFN scheme of the DIS heavy-quark production

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HERA charm data and $m_c$

H1, ZEUS EPJC 78, 473 (2018)

**Theory:** FFN scheme, running mass definition

- $m_c(m_c) = 1.245 \pm 0.019$ (exp.) GeV
- $m_c(m_c) = 1.252 \pm 0.018$ (exp.) GeV

**Present analysis**

- $m_c(m_c) = 1.246 \pm 0.023$ (h.o.) GeV NNLO

**ABMP16**

- $m_c$ (pole) $\sim 1.9$ GeV (NNLO)

Marquard et al. PRL 114, 142002 (2015)

- $m_c(m_c) = 1.246 \pm 0.023$ (h.o.) GeV NNLO

Kiyo, Mishima, Sumino PLB 752, 122 (2016)

- $m_c(m_c) = 1.279 \pm 0.008$ GeV

Kühn, LoopsLegs2018

*Good consistency with the earlier results and other determinations → further confirmation of the FFN scheme relevance for the HERA kinematics*
**FFN and VFN schemes**

\[ H_{g,2}^{\text{asympt}} = a_s(N_f)A_{h_g}^{(1)} \]  

Asymptotic 3-flavor coefficient function

**LO:**

\[ A_{ij}^{(1)} \left( z, \frac{m_h^2}{\mu^2} \right) = a_{ij}^{(1,1)}(z) \ln \left( \frac{\mu^2}{m_h^2} \right) \]  

Massive operator matrix elements (OMEs)

\[ h^{(1)}(x, \mu^2) = a_s(N_f + 1, \mu^2) \left[ A_{h_g}^{(1)} \left( \frac{m_h^2}{\mu^2} \right) \otimes G^{(2)} \left( N_f, \mu^2 \right) \right](x) \]  

Matching condition for the heavy-quark PDFs

\[ F_{g,2}^{\text{asympt}} = e_{h}^{2} x h^{(1)}(x, \mu^2) \]

**NLO:**

\[ A_{ij}^{(2)} \left( z, \frac{m_h^2}{\mu^2} \right) = a_{ij}^{(2,2)}(z) \ln^2 \left( \frac{\mu^2}{m_h^2} \right) + a_{ij}^{(2,1)}(z) \ln \left( \frac{\mu^2}{m_h^2} \right) + a_{ij}^{(2,0)}(z) \]

2-mass contributions in NLO and NNLO

Blümlein et al. PLB 782, 362 (2018)

**NNLO:** log-terms; constant terms up to the gluonic one

- The VFN scheme works well at \( \mu \gg m_h \) (W,Z,t-quark production,....)
- Problematic for DIS ⇒ additional modeling of power-like terms required at small scales (ACOT, BMSN, FONLL, RT....)
NNLO massive Wilson coefficients

Combination of the threshold corrections (small $s$), high-energy limit (small $x$), and the NNLO massive OMEs (large $Q^2$)


\[ \xi = \frac{Q^2}{m^2} \]

\[ \eta = \frac{s}{4m^2} - 1 \]
Recent progress in NNLO FFN scheme

Update with the pure singlet massive OMEs $\rightarrow$ improved theoretical uncertainties

Ablinger et al. NPB 890, 48 (2014)

sa, Blümlein, Moch, Plačakytė PRD 96, 014011 (2017)
BMSN prescription of GMVFN scheme
Buza, Matiounine, Smith, van Neerven EPJC 1, 301 (1998)

\[
F_{2}^{h,\text{BMSN}}(N_f + 1, x, Q^2) = F_{2}^{h,\text{exact}}(N_f, x, Q^2) + F_{2}^{h,\text{GMVFN}}(N_f + 1, x, Q^2) - F_{2}^{h,\text{asymp}}(N_f, x, Q^2).
\]

Cacciari, Greco, Nason JHEP 9805, 007 (1998)

In the \(O(\alpha_s^2)\) the FFNS and GMVFNS are comparable at large scales since the big logs appear in the high order corrections to the massive coefficient functions

Glück, Reya, Stratmann NPB 422, 37 (1994)

- Smooth matching with the FFNS at \(Q \to m_h\) without additional damping or re-scaling factors
- FOPT heavy-quark PDFs \(\to\) large logs missing?
Comparison of the FOPT and evolved c-quark PDFs

**LO:** The FOPT and evolved PDFs nicely match at $m_h$, at large scales they diverge due to large logs resummed by evolution.

**NLO (NLO OMES and NLO evolution):** The difference between FOPT and evolved PDFs at large scales dramatically reduces due to large log are partially included into NLO OMEs and therefore are taken into account in the FOPT as well.

**NNLO** (NLO OMES and NNLO evolution): A kink w.r.t. FOPT is observed at small $x$. 

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**Diagram:**

- **FOPT / LO**
- **DGLAP / LO**
- **FOPT / NLO**
- **DGLAP / NLO**
- **DGLAP / NNLO***

Graphs showing the comparison of PDFs at different scales and $x$ values.
c-quark PDFs: fixed-order versus evolved

- The heavy-quark PDF evolution brings essential impact only beyond realistic kinematics of existing data (HERA)

- A consistent NNLO treatment of the VFN PDFs is problematic due to missing NNLO massive OMEs. The difference between NLO and NNLO* gives theoretical uncertainty due to missing higher orders
BMSN VFN scheme: FOPT versus evolved

- Impact of heavy-quark PDF evolution is marginal even at very large $Q$
- A significant kink w.r.t. FFN scheme appears at small $Q$ if heavy-quark PDFs are evolved in the NNLO
VFN scheme with the evolved PDFs

Comparison with the model FFN fit:
– NLO massive Wilson coeffs.,
– $m_c(pole)=1.4$ GeV

Two variants of 4-flavor PDF evolution:
- NNLO (commonly used in the VFN fits)
  – consistent with light PDF evolution
  – inconsistent with NLO matching
- NLO
  – inconsistent with light PDF evolution
  – consistent with NLO matching

Substantial difference between NLO and NNLO versions

The evolved predictions demonstrate strong $x$-dependence and weak $Q^2$-dependence

The difference with FOPT appears rather due to inconsistent evolution than due to big-logs → theoretical uncertainty in the VFN schemes
HERA charm data: FFN versus VFN

**HERA I+II (ep --→ e charm X)**

- $Q^2 = 2.5$ GeV$^2$
- $Q^2 = 5$ GeV$^2$
- $Q^2 = 7$ GeV$^2$
- $Q^2 = 12$ GeV$^2$
- $Q^2 = 18$ GeV$^2$
- $Q^2 = 32$ GeV$^2$
- $Q^2 = 60$ GeV$^2$
- $Q^2 = 120$ GeV$^2$
- $Q^2 = 200$ GeV$^2$
- $Q^2 = 350$ GeV$^2$
- $Q^2 = 650$ GeV$^2$
- $Q^2 = 2000$ GeV$^2$

**ABMP16**

$X^2/NDP=66/52$

$m_c(m_c)=1.252\pm0.018$ (exp.) - $0.01$ (th.) GeV

$m_c$ (pole) ~ 1.9 GeV (NNLO)

Marquard et al. PRL 114, 142002 (2015)

- **RT optimal**
- $X^2/NDP=82/52$
- $m_c$ (pole) = 1.25 GeV

MMHT14 EPJC 75, 204 (2015)

- **S-ACOT-χ**
- $X^2/NDP=59/47$
- $m_c$ (pole) = 1.3 GeV

CT14 PRD 93, 033006 (2016)

- **F0NLL**
- $X^2/NDP=60/47$
- $m_c$ (pole) = 1.275 GeV

NNPDF3.0 JHEP 504, 040 (2015)

- **F0NLL**
- $X^2/NDP=54/37$
- $m_c$ (pole) = 1.51 GeV, intrinsic (fitted) charm

NNPDF3.1 hep-ph/1706.00428

Kühn, this conference

**HERA charm data: FFN versus VFN**

- $X^2/NDP=66/52$
- $m_c(m_c)=1.246\pm0.023$ (h.o.) GeV

Kiyô, Mishima, Sumino PLB 752, 122 (2016)

- $X^2/NDP=54/37$
- $m_c$ (pole) = 1.279±0.008

Kühn, this conference

For more accurate data VFN works even worse

H1, ZEUS EPJC 78, 473 (2018)
Conclusions

- The large-log resummation provided in the VFN scheme for the heavy-quark PDFs manifests in the existing data on DIS structure functions only in the LO; in the NLO the large logs are greatly absorbed into heavy-quark matching conditions and resummations gets irrelevant for existing data kinematics.

- Limited theoretical accuracy of the matching conditions for the heavy-quark PDFs, available in the NLO only, does not allow fully consistent NNLO QCD evolution in the VFN scheme. This brings substantial theoretical uncertainty into the VFN computations at small $Q^2$, which is mixed with the power corrections appearing in the VFN scheme (in practical implementations of the VFN scheme, ACOT, FONLL, RT,… all this has to be suppressed by additional parameters, like damping and rescaling factors).

- In contrast, the straightforward FFN approach is free from such ambiguities. With the QCD corrections up to NNLO available it provides good description of the existing data and consistent determination of the heavy-quark masses
  
  $m_c (m_c) = 1.245 \pm 0.019 \text{(exp.) GeV}$
  $m_b (m_b) = 3.96 \pm 0.10 \text{(exp.) GeV}$


EXTRAS
Running mass in DIS

The pole mass is defined for the free \textit{(unobserved)} quarks as a the QCD Lagrangian parameter and is commonly used in the QCD calculations.

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \sum_{\text{flavors}} \bar{q} (i\gamma \sigma - m_q) q \]

\[ \phi - m_q - \Sigma(p, m_q) \bigg|_{p^2 = m_q^2} \]

The quantum corrections due to the self-energy loop integrals receive contribution down to scale of \( O(\Lambda_{QCD}) \)

\( \rightarrow \) sensitivity to the high order corrections, particularly at the production threshold.

\[ \mu^2 \frac{d}{d\mu^2} m(\mu) = \gamma(\alpha_s)m(\mu) \]

sa, Moch PLB 699, 345 (2011)