Parton Branching TMDs with angular ordering condition and their application to Z boson $q_\perp$ spectrum

DIS 2019

• Ola Lelek [1] Francesco Hautmann [1,2] Hannes Jung [1,3]
Mees van Kampen [1] Lissa Keersmaekers [1]

[1]University of Antwerp (UAntwerp)

[2]University of Oxford

[3]Deutsches Elektronen-Synchrotron (DESY)

Motivation

Motivation: why do we need TMDs?

Collinear PDFs:

$\widetilde{f}_a\left(x, \mu^2\right)$ - proton structure in longitudinal direction

3D mapping:

Transverse Momentum Dependent (TMD) PDFs

TMD $\left(x, k_\perp, \mu^2\right)$

Motivation: why do we need TMDs?

Collinear PDFs:

$\widetilde{f}_a\left(x, \mu^2\right)$ - proton structure in longitudinal direction

3D mapping:

Transverse Momentum Dependent (TMD) PDFs

TMD $\left(x, k_\perp, \mu^2\right)$

Some observables problematic for collinear factorization and pQCD.

$\rightarrow$ TMD factorization theorems (Collins-Soper-Sterman, High energy ($k_\perp$-) factorization)

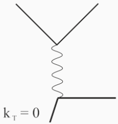
Wide area of applications

Motivation

Motivation:
We want to develop an approach in which transverse momentum kinematics will be treated without any mismatch between matrix element (ME) and PS

Standard MC predictions

Eur. Phys. J. C19, 351 (2001)



Alternative approach:

Eur. Phys. J. C19, 351 (2001)

Mismatch between PDF used by $\hat{\sigma}$ and PS

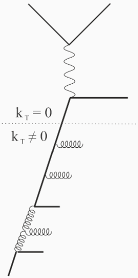
No mismatch: $\hat{\sigma}$ and PS follow the same TMD

Motivation

Motivation:
We want to develop an approach in which transverse momentum kinematics will be treated without any mismatch between matrix element (ME) and PS

Standard MC predictions

Eur. Phys. J. C19, 351 (2001)



Mismatch between PDF used by $\hat{\sigma}$ and PS

Alternative approach:

Eur. Phys. J. C19, 351 (2001)

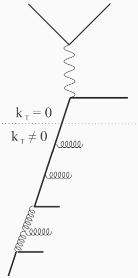
No mismatch: $\hat{\sigma}$ and PS follow the same TMD

Goal: to construct TMDs in a wide range of x, $k_\perp$ and μ^2

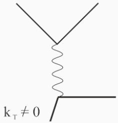
Motivation

Motivation:
We want to develop an approach in which transverse momentum kinematics will be treated without any mismatch between matrix element (ME) and PS

Standard MC predictions



Alternative approach:
Eur. Phys. J. C19, 351 (2001)



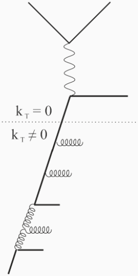
Mismatch between PDF used by $\hat{\sigma}$ and PS

Motivation

Motivation:
We want to develop an approach in which transverse momentum kinematics will be treated without any mismatch between matrix element (ME) and PS

Standard MC predictions

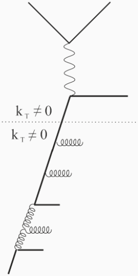
Eur. Phys. J. C19, 351 (2001)



Mismatch between PDF used by $\hat{\sigma}$ and PS

Alternative approach:

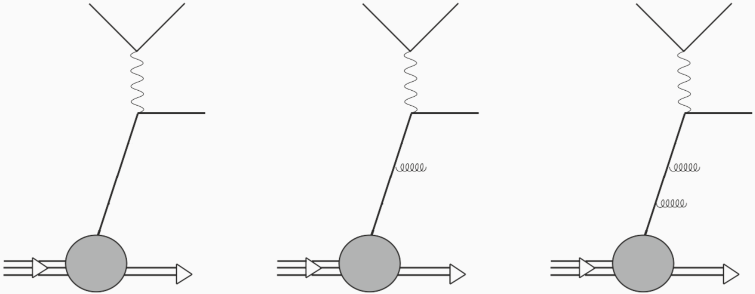
Eur. Phys. J. C19, 351 (2001)



No mismatch: $\hat{\sigma}$ and PS follow the same TMD

Goal: to construct TMDs in a wide range of x, $k_\perp$ and μ^2

Parton Branching (PB) method



We construct (and solve using MC solution) an iterative equation for a parton density which takes into account also the transverse momentum at each branching

Plan for today

Plan for today:

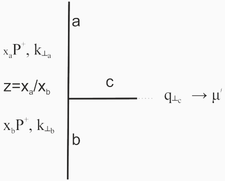
- Parton Branching (PB) TMDs
- comparison of PB with other existing approaches

Transverse momentum in PB

How to connect branching scale μ'^2 and $q^2_{\perp,c}$?

resolvable & non-resolvable $\Rightarrow$ condition on min $q^2_{\perp,c} \Rightarrow z_M$

The argument of α_s should be $q^2_{\perp,c}$

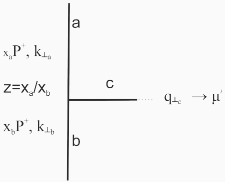


$$\begin{array}{rccc}
p_\perp\text{-ordering:} & q^2_{\perp,c} = \mu'^2 & z_M = \text{fixed} & \alpha_s\left(\mu'^2\right) \\
\text{virtuality ordering:} & q^2_{\perp,c} = (1-z)\mu'^2 & z_M = 1 - \left(\frac{q_0}{\mu'}\right)^2 & \alpha_s\left((1-z)\mu'^2\right) \\
\text{angular ordering:} & q^2_{\perp,c} = (1-z)^2\mu'^2 & z_M = 1 - \left(\frac{q_0}{\mu'}\right) & \alpha_s\left((1-z)^2\mu'^2\right)
\end{array}$$

$$\vec{k}_{\perp,a} = \vec{k}_{\perp,b} - \vec{q}_{\perp,c}$$

Transverse momentum in PB

How to connect branching scale μ'^2 and $q^2_{\perp,c}$?
resolvable & non-resolvable $\Rightarrow$ condition on min $q^2_{\perp,c}$ $\Rightarrow$ z_M
The argument of α_s should be $q^2_{\perp,c}$

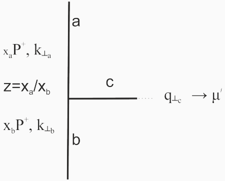


$p_\perp$-ordering:	$q^2_{\perp,c} = \mu'^2$	$z_M = \text{fixed}$	$\alpha_s(\mu'^2)$
virtuality ordering:	$q^2_{\perp,c} = (1-z)\mu'^2$	$z_M = 1 - \left(\frac{q_0}{\mu'}\right)^2$	$\alpha_s((1-z)\mu'^2)$
angular ordering:	$q^2_{\perp,c} = (1-z)^2\mu'^2$	$z_M = 1 - \left(\frac{q_0}{\mu'}\right)$	$\alpha_s((1-z)^2\mu'^2)$

$$\vec{k}_{\perp,a} = \vec{k}_{\perp,b} - \vec{q}_{\perp,c}$$

Transverse momentum in PB

How to connect branching scale μ'^2 and $q^2_{\perp,c}$?
resolvable & non-resolvable $\Rightarrow$ condition on min $q^2_{\perp,c}$ $\Rightarrow z_M$
The argument of α_s should be $q^2_{\perp,c}$

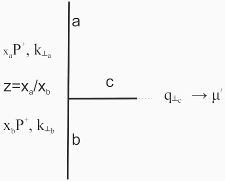


$p_\perp$-ordering:	$q^2_{\perp,c} = \mu'^2$	$z_M = \text{fixed}$	$\alpha_s\left(\mu'^2\right)$
virtuality ordering:	$q^2_{\perp,c} = (1-z)\mu'^2$	$z_M = 1 - \left(\frac{q_0}{\mu'}\right)^2$	$\alpha_s\left((1-z)\mu'^2\right)$
angular ordering:	$q^2_{\perp,c} = (1-z)^2\mu'^2$	$z_M = 1 - \left(\frac{q_0}{\mu'}\right)$	$\alpha_s\left((1-z)^2\mu'^2\right)$

$$\vec{k}_{\perp,a} = \vec{k}_{\perp,b} - \vec{q}_{\perp,c}$$

Transverse momentum in PB

How to connect branching scale μ'^2 and $q_{\perp,c}^2$?
resolvable & non-resolvable $\Rightarrow$ condition on min $q_{\perp,c}^2 \Rightarrow z_M$
The argument of α_s should be $q_{\perp,c}^2$

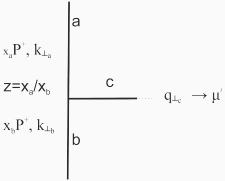


$p_\perp$-ordering:	$q_{\perp,c}^2 = \mu'^2$	$z_M = \text{fixed}$	$\alpha_s\left(\mu'^2\right)$
virtuality ordering:	$q_{\perp,c}^2 = (1-z)\mu'^2$	$z_M = 1 - \left(\frac{q_0}{\mu'}\right)^2$	$\alpha_s\left((1-z)\mu'^2\right)$
angular ordering:	$q_{\perp,c}^2 = (1-z)^2\mu'^2$	$z_M = 1 - \left(\frac{q_0}{\mu'}\right)$	$\alpha_s\left((1-z)^2\mu'^2\right)$

$$\vec{k}_{\perp,a} = \vec{k}_{\perp,b} - \vec{q}_{\perp,c}$$

Transverse momentum in PB

How to connect branching scale μ'^2 and $q^2_{\perp,c}$?
resolvable & non-resolvable $\Rightarrow$ condition on min $q^2_{\perp,c} \Rightarrow z_M$
The argument of α_s should be $q^2_{\perp,c}$



$p_\perp$-ordering:	$q^2_{\perp,c} = \mu'^2$	$z_M = \text{fixed}$	$\alpha_s\left(\mu'^2\right)$
virtuality ordering:	$q^2_{\perp,c} = (1-z)\mu'^2$	$z_M = 1 - \left(\frac{q_0}{\mu'}\right)^2$	$\alpha_s\left((1-z)\mu'^2\right)$
angular ordering:	$q^2_{\perp,c} = (1-z)^2\mu'^2$	$z_M = 1 - \left(\frac{q_0}{\mu'}\right)$	$\alpha_s\left((1-z)^2\mu'^2\right)$

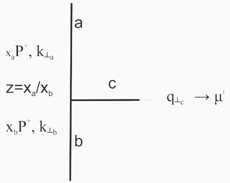
$$\vec{k}_{\perp,a} = \vec{k}_{\perp,b} - \vec{q}_{\perp,c}$$

JHEP 1801 (2018) 070

$$\widetilde{A}_a\left(x, k_\perp, \mu^2\right) = \Delta_a\left(\mu^2\right)\widetilde{A}_a\left(x, k_\perp, \mu_0^2\right) +$$
$$\sum_b \int_{\mu_0^2}^{\mu^2} \frac{\mathrm{d}^2\mu'_\perp}{\pi\mu'^2}\frac{\Delta_a\left(\mu^2\right)}{\Delta_a\left(\mu'^2\right)}\int_x^{z_M} \mathrm{d}z P^R_{ab}\left(z, \mu'^2, \alpha_s(a(z)^2\mu'^2)\right)\widetilde{A}_b\left(\frac{x}{z}, k_\perp + a(z)\mu_\perp, \mu'^2\right)$$

Transverse momentum in PB

How to connect branching scale μ'^2 and $q^2_{\perp,c}$?
resolvable & non-resolvable $\Rightarrow$ condition on min $q^2_{\perp,c} \Rightarrow z_M$
The argument of α_s should be $q^2_{\perp,c}$



$$
\begin{array}{llll}
p_\perp\text{-ordering:} & q^2_{\perp,c} = 1\mu'^2 & z_M = \text{fixed} & \alpha_s\left(1\mu'^2\right) \\
\text{virtuality ordering:} & q^2_{\perp,c} = (1-z)\mu'^2 & z_M = 1 - \left(\frac{q_0}{\mu'}\right)^2 & \alpha_s\left((1-z)\mu'^2\right) \\
\text{angular ordering:} & q^2_{\perp,c} = \underbrace{(1-z)^2}_{a^2(z)}\mu'^2 & z_M = 1 - \left(\frac{q_0}{\mu'}\right) & \alpha_s(\underbrace{(1-z)^2}_{a^2(z)}\mu'^2)
\end{array}
$$

$$\vec{k}_{\perp,a} = \vec{k}_{\perp,b} - \vec{q}_{\perp,c}$$

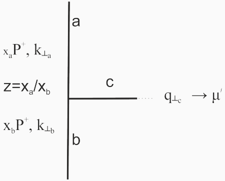
JHEP 1801 (2018) 070

$$
\widetilde{A}_a\left(x, k_\perp, \mu^2\right) = \Delta_a\left(\mu^2\right)\widetilde{A}_a\left(x, k_\perp, \mu_0^2\right) +
$$
$$
\sum_b \int_{\mu_0^2}^{\mu^2} \frac{\mathrm{d}^2\mu'_\perp}{\pi\mu'^2}\frac{\Delta_a\left(\mu^2\right)}{\Delta_a\left(\mu'^2\right)}\int_x^{z_M}\mathrm{d}z P^R_{ab}\left(z, \mu'^2, \alpha_s(a(z)^2\mu'^2)\right)\widetilde{A}_b\left(\frac{x}{z}, k_\perp + a(z)\mu_\perp, \mu'^2\right)
$$

- PB method: effect of every individual part of the ordering definition can be studied separately

Transverse momentum in PB

How to connect branching scale μ'^2 and $q^2_{\perp,c}$?
resolvable & non-resolvable $\Rightarrow$ condition on min $q^2_{\perp,c}$ $\Rightarrow z_M$
The argument of α_s should be $q^2_{\perp,c}$



$p_\perp$-ordering:	$q^2_{\perp,c} = \mu'^2$	$z_M = \text{fixed}$	$\alpha_s\left(\mu'^2\right)$
virtuality ordering:	$q^2_{\perp,c} = (1-z)\mu'^2$	$z_M = 1 - \left(\frac{q_0}{\mu'}\right)^2$	$\alpha_s\left((1-z)\mu'^2\right)$
angular ordering:	$q^2_{\perp,c} = (1-z)^2\mu'^2$	$z_M = 1 - \left(\frac{q_0}{\mu'}\right)$	$\alpha_s\left((1-z)^2\mu'^2\right)$

$$\vec{k}_{\perp,a} = \vec{k}_{\perp,b} - \vec{q}_{\perp,c}$$

JHEP 1801 (2018) 070

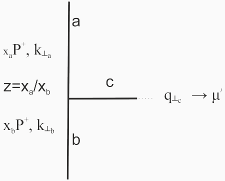
$$\widetilde{A}_a\left(x,k_\perp,\mu^2\right) = \Delta_a\left(\mu^2\right)\widetilde{A}_a\left(x,k_\perp,\mu_0^2\right) +$$

$$\sum_b \int_{\mu_0^2}^{\mu^2} \frac{\mathrm{d}^2\mu'_\perp}{\pi\mu'^2}\frac{\Delta_a\left(\mu^2\right)}{\Delta_a\left(\mu'^2\right)}\int_x^{z_M\approx 1}\mathrm{d}z P^R_{ab}\left(z,\mu'^2,\alpha_s(1\mu'^2)\right)\widetilde{A}_b\left(\frac{x}{z},k_\perp + a(z)\mu_\perp,\mu'^2\right)\Bigg| \int \mathrm{d}k^2_\perp$$

- PB method: effect of every individual part of the ordering definition can be studied separately.

Transverse momentum in PB

How to connect branching scale μ'^2 and $q^2_{\perp,c}$?
resolvable & non-resolvable $\Rightarrow$ condition on min $q^2_{\perp,c}$ $\Rightarrow z_M$
The argument of α_s should be $q^2_{\perp,c}$



$p_\perp$-ordering:	$q^2_{\perp,c} = \mu'^2$	$z_M = \text{fixed}$	$\alpha_s\left(\mu'^2\right)$
virtuality ordering:	$q^2_{\perp,c} = (1-z)\mu'^2$	$z_M = 1 - \left(\frac{q_0}{\mu'}\right)^2$	$\alpha_s\left((1-z)\mu'^2\right)$
angular ordering:	$q^2_{\perp,c} = (1-z)^2\mu'^2$	$z_M = 1 - \left(\frac{q_0}{\mu'}\right)$	$\alpha_s\left((1-z)^2\mu'^2\right)$

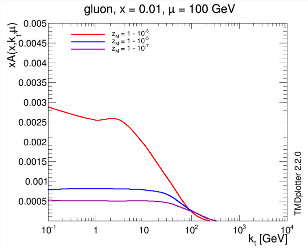
$$\vec{k}_{\perp,a} = \vec{k}_{\perp,b} - \vec{q}_{\perp,c}$$

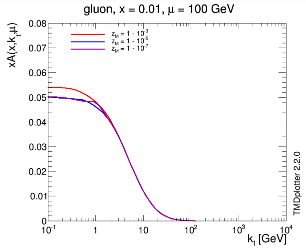
JHEP 1801 (2018) 070

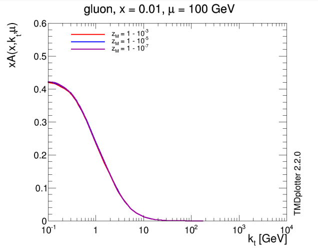
$$\widetilde{f}_a(x,\mu^2) = \widetilde{f}_a(x,\mu_0^2)\Delta_a(\mu^2)$$
$$+ \quad \int_{\ln \mu_0^2}^{\ln \mu^2} d\ln \mu'^2 \frac{\Delta_a(\mu^2)}{\Delta_a(\mu'^2)} \sum_b \int_x^{z_M} dz P^R_{ab}\left(\mu'^2, z\right) \widetilde{f}_b\left(\frac{x}{z}, \mu'^2\right)$$

- DGLAP reproduced for $z_M \to 1$ and $\alpha_s(\mu'^2)$

Effect of ordering choice and z_M on TMDs







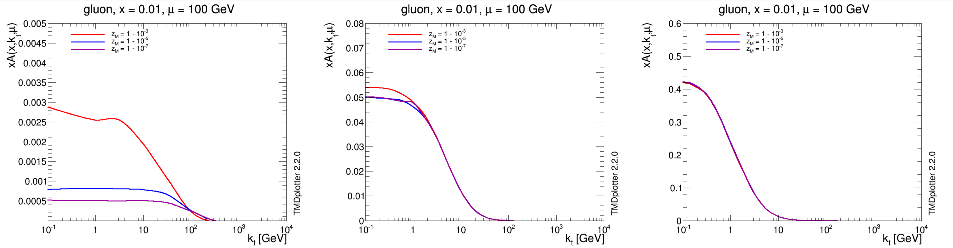
$p_\perp$- ordering
$q_\perp^2 = 1\mu'^2$
NOT stable TMDs

virtuality ordering
$q_\perp^2 = (1-z)\mu'^2$

angular ordering
$q_\perp^2 = (1-z)^2\mu'^2$
stable TMDs

Note1: Everywhere $\alpha_s\left(\mu'^2\right)$

Effect of ordering choice and z_M on TMDs



$p_\perp$- ordering
$q_\perp^2 = 1\mu'^2$
NOT stable TMDs

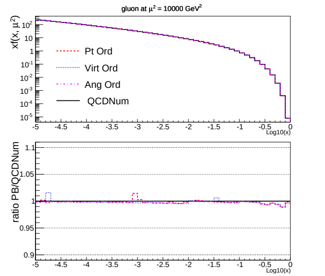
virtuality ordering
$q_\perp^2 = (1-z)\mu'^2$

angular ordering
$q_\perp^2 = (1-z)^2\mu'^2$
stable TMDs

Note1: Everywhere $\alpha_s\left(\mu'^2\right)$

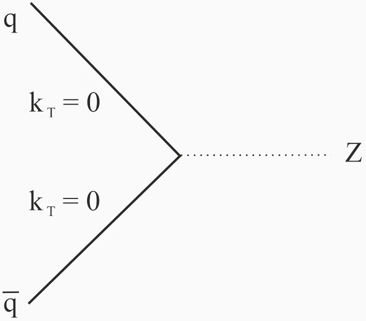
Note2: All these TMDs after integration over $k_\perp$ give the same collinear PDF ✓

Phys.Lett. B772 (2017) 446-451
JHEP 1801 (2018) 070



Prediction for Z boson $p_\perp$ spectrum using TMDs

Z. Phys. C32, 67 (1986)

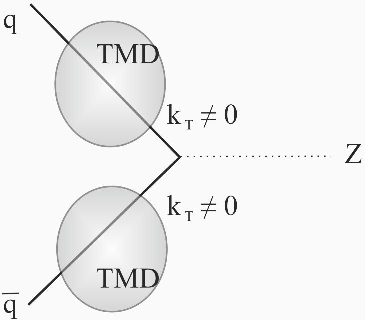


Procedure:

- DY collinear ME
- Generate $k_\perp$ of $q\overline{q}$ according to TMDs (m_{DY} fixed, x_1, x_2 change)
- compare with the 8 TeV ATLAS measurement

Prediction for Z boson $p_\perp$ spectrum using TMDs

Z. Phys. C32, 67 (1986)

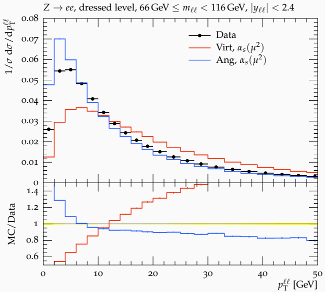


Procedure:

- DY collinear ME
- Generate $k_\perp$ of $q\overline{q}$ according to TMDs (m_{DY} fixed, x_1, x_2 change)
- compare with the 8 TeV ATLAS measurement

Prediction for Z boson $p_\perp$ spectrum using TMDs

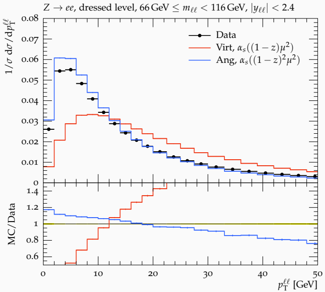
here: DY LO matrix element from Pythia: $q\bar{q} \to Z$
data from ATLAS measurement Eur. Phys. J. C76, 291 (2016)



- difference between angular and virtuality ordering visible
- angular ordering: the shape of Z boson $p_\perp$ spectrum reproduced

Prediction for Z boson $p_\perp$ spectrum using TMDs

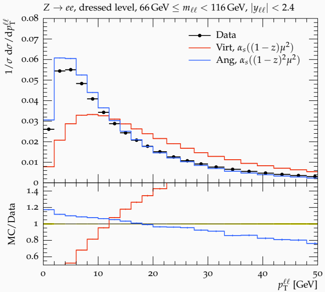
here: DY LO matrix element from Pythia: $q\bar{q} \rightarrow Z$
data from ATLAS measurement Eur. Phys. J. C76, 291 (2016)



- difference between angular and virtuality ordering visible
- angular ordering: the shape of Z boson $p_\perp$ spectrum reproduced
- with $\alpha_s\left((1-z)^2\mu'^2\right)$ agreement within the data much better than for $\alpha_s(\mu'^2)$

Prediction for Z boson $p_\perp$ spectrum using TMDs

here: DY LO matrix element from Pythia: $q\bar{q} \rightarrow Z$
data from ATLAS measurement Eur. Phys. J. C76, 291 (2016)

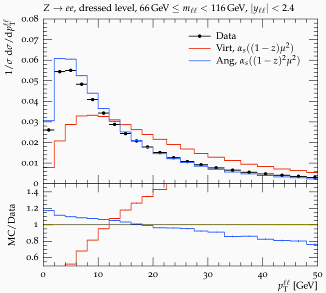


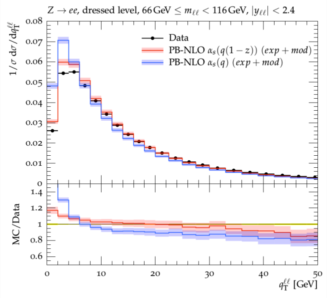
Summarizing: angular ordering allows for stable TMDs definition and Z $p_\perp$ spectrum description:
$\rightarrow$ Fit using `xFitter`: HERA H1 and ZEUS combined DIS measurement Eur.Phys.J. C75 (2015) no.12, 580
for angular ordering for two scenarios:

- Set1: $\alpha_s\left(\mu'^2\right)$, reproduces HERAPDF2.0 ✓
- Set2: $\alpha_s\left((1-z)^2\mu'^2\right)$, different HERAPDF2.0 ✓

Prediction for Z boson $p_\perp$ spectrum using TMDs

here: DY LO matrix element from Pythia: $q\bar{q} \rightarrow Z$
data from ATLAS measurement Eur. Phys. J. C76, 291 (2016)





Results after the fit. Experimental and model uncertainty arXiv:1804.11152, in Phys. Rev. D soon

Results after the fit. Experimental and model uncertainty arXiv:1804.11152, in Phys. Rev. D soon

Summarizing: angular ordering allows for stable TMDs definition and Z $p_\perp$ spectrum description:
$\rightarrow$ Fit using `xFitter`: HERA H1 and ZEUS combined DIS measurement Eur.Phys.J. C75 (2015) no.12, 580
for angular ordering for two scenarios:

- Set1: $\alpha_s\left(\mu'^2\right)$, reproduces HERAPDF2.0 ✓
- Set2: $\alpha_s\left((1-z)^2\mu'^2\right)$, different HERAPDF2.0 ✓

prediction for the whole $p_\perp$ spectrum directly from the PB method
no tuning/adjustment of free parameters

Current Activities

- PS from TMDs $\rightarrow$ see talk by Melanie Schmitz
- off-shell ME with TMDs
- PB TMDs with low-x effects $\rightarrow$ see talk by Sara Taheri Monfared
- ...
- Comparison of PB with other approaches $\rightarrow$ I concentrate on that now

PB and other approaches

PB and Marchesini, Webber

PB with angular ordering is very successful

PB and Marchesini, Webber

PB with angular ordering is very successful

PB for angular ordering:

$$\begin{aligned} \widetilde{f}_a(x,\mu^2) &= \widetilde{f}_a(x,\mu_0^2)\Delta_a(\mu^2) \\ + &\int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \frac{\Delta_a(\mu^2)}{\Delta_a(\mu'^2)} \sum_b \int_x^{1-\frac{q_0}{\mu'}} dz P_{ab}^R\left(\alpha_s\left((1-z)^2\mu'^2\right), \mu'^2, z\right) \widetilde{f}_b\left(\frac{x}{z},\mu'^2\right) \end{aligned} \tag{1}$$

where

$$q_{\perp,i}^2 = (1-z_i)^2\mu'^2$$

Eq. (1) is identical to the Marchesini and Webber (MarWeb1988) prescription
Nuclear Physics B310 (1988) 461-526

PB and Kimber- Martin- Ryskin- Watt (KMRW)

PB for angular ordering written in terms of integral over $q_\perp$ (identical to MarWeb1988):

$$
\begin{aligned}
& \widetilde{f}_a(x,\mu^2) = \widetilde{f}_a(x,\mu_0^2)\Delta_a(\mu^2) \\
+ \quad & \int_{q_0^2}^{(1-x)^2\mu^2} \frac{dq_\perp^2}{q_\perp^2} \sum_b \int_x^{1-\frac{q_\perp}{\mu}} dz \Delta_a\left(\mu^2, \frac{q_\perp^2}{(1-z)^2}\right) P_{ab}^R\left(\alpha_s\left(q_\perp^2\right), \frac{q_\perp^2}{(1-z)^2}, z\right) \widetilde{f}_b\left(\frac{x}{z}, \frac{q_\perp^2}{(1-z)^2}\right)
\end{aligned}
$$

PB and Kimber- Martin- Ryskin- Watt (KMRW)

PB for angular ordering written in terms of integral over $q_\perp$ (identical to MarWeb1988):

$$\widetilde{f}_a(x,\mu^2) = \widetilde{f}_a(x,\mu_0^2)\Delta_a(\mu^2) + \int_{q_0^2}^{(1-x)^2\mu^2} \frac{dq_\perp^2}{q_\perp^2} \sum_b \int_x^{1-\frac{q_\perp}{\mu}} dz \Delta_a\left(\mu^2, \frac{q_\perp^2}{(1-z)^2}\right) P_{ab}^R\left(\alpha_s\left(q_\perp^2\right), \frac{q_\perp^2}{(1-z)^2}, z\right) \widetilde{f}_b\left(\frac{x}{z}, \frac{q_\perp^2}{(1-z)^2}\right)$$

KMRW: TMDs (unintegrated PDFs) obtained from the integrated PDFs and the Sudakov form factors

Phys. Rev. D63 (2001) 114027

$$\widetilde{f}_a(x,\mu^2) = \widetilde{f}_a(x,\mu_0^2)\Delta_a(\mu^2) + \int_{q_0^2}^{q_M^2} \frac{dq_\perp^2}{q_\perp^2} \underbrace{\sum_b \int_x^{z_M} dz \Delta_a(\mu^2, q_\perp^2) P_{ab}^R\left(\alpha_s\left(q_\perp^2\right), z\right) \widetilde{f}_b\left(\frac{x}{z}, q_\perp^2\right)}_{\widetilde{f}(x,\mu^2,q_\perp^2)}$$

at last step of the evolution the unintegrated distribution becomes dependent on two scales: $q_\perp$ and μ

PB and Kimber- Martin- Ryskin- Watt (KMRW)

PB for angular ordering written in terms of integral over $q_\perp$ (identical to MarWeb1988):

$$\widetilde{f}_a(x,\mu^2) = \widetilde{f}_a(x,\mu_0^2)\Delta_a(\mu^2)$$
$$+ \int_{q_0^2}^{(1-x)^2\mu^2} \frac{dq_\perp^2}{q_\perp^2} \sum_b \int_x^{1-\frac{q_\perp}{\mu}} dz \Delta_a\left(\mu^2, \frac{q_\perp^2}{(1-z)^2}\right) P_{ab}^R\left(\alpha_s\left(q_\perp^2\right), \frac{q_\perp^2}{(1-z)^2}, z\right) \widetilde{f}_b\left(\frac{x}{z}, \frac{q_\perp^2}{(1-z)^2}\right)$$

KMRW: TMDs (unintegrated PDFs) obtained from the integrated PDFs and the Sudakov form factors

Phys. Rev. D63 (2001) 114027

$$\widetilde{f}_a(x,\mu^2) = \widetilde{f}_a(x,\mu_0^2)\Delta_a(\mu^2)$$
$$+ \int_{q_0^2}^{q_M^2} \frac{dq_\perp^2}{q_\perp^2} \underbrace{\sum_b \int_x^{z_M} dz \Delta_a(\mu^2, q_\perp^2) P_{ab}^R\left(\alpha_s\left(q_\perp^2\right), z\right) \widetilde{f}_b\left(\frac{x}{z}, q_\perp^2\right)}_{\widetilde{f}(x,\mu^2,q_\perp^2)}$$

at last step of the evolution the unintegrated distribution becomes dependent on two scales: $q_\perp$ and μ

In KMRW:

- "Strong ordering": $q_M^2 = (1-x)^2\mu^2$ and $z_M = 1 - \frac{q_\perp}{\mu}$
- "Angular ordering" $q_M^2 = \left(\frac{1-x}{x}\right)^2 \mu^2$ and $z_M = 1 - \frac{q_\perp}{q_\perp + \mu}$

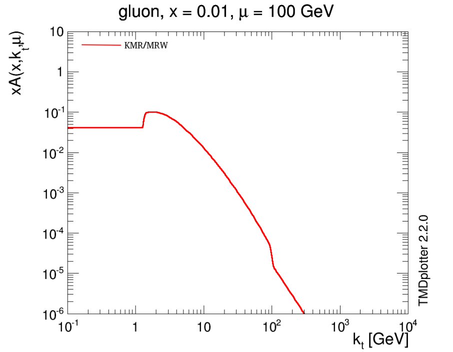
PB and KMRW: distributions

PB: intrinsic $k_\perp$ is a Gauss distribution with width=0.5 GeV

KMRW parametrization for $k_\perp < k_0 = 1\mathrm{GeV}$:

$$\frac{\widetilde{f}_a(x, k_\perp, \mu^2)}{k_\perp^2} = \frac{1}{\mu_0^2} \widetilde{f}_a(x, k_\perp, \mu_0^2) \Delta_a(\mu^2, \mu_0^2) = \mathrm{const}$$

MRW-ct10nlo: TMD sets obtained according to KMRW formalism with angular ordering included in TMDlib Eur.Phys.J.C78(2018)no.2,137



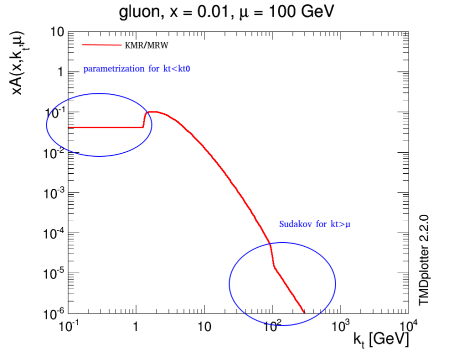
PB and KMRW: distributions

PB: intrinsic $k_\perp$ is a Gauss distribution with width=0.5 GeV

KMRW parametrization for $k_\perp < k_0 = 1\mathrm{GeV}$:

$$\frac{\widetilde{f}_a(x,k_\perp,\mu^2)}{k_\perp^2} = \frac{1}{\mu_0^2}\widetilde{f}_a(x,k_\perp,\mu_0^2)\Delta_a(\mu^2,\mu_0^2) = \mathrm{const}$$

MRW-ct10nlo: TMD sets obtained according to KMRW formalism with angular ordering included in TMDlib Eur.Phys.J.C78(2018)no.2,137

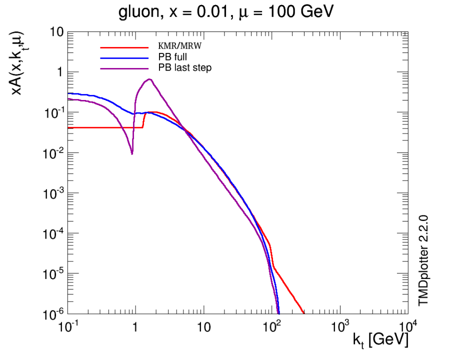


PB and KMRW: distributions

PB: intrinsic $k_\perp$ is a Gauss distribution with width=0.5 GeV
KMRW parametrization for $k_\perp < k_0 = 1\mathrm{GeV}$:

$$\frac{\widetilde{f}_a(x, k_\perp, \mu^2)}{k_\perp^2} = \frac{1}{\mu_0^2}\widetilde{f}_a(x, k_\perp, \mu_0^2)\Delta_a(\mu^2, \mu_0^2) = \mathrm{const}$$

MRW-ct10nlo: TMD sets obtained according to KMRW formalism with angular ordering included in TMDlib Eur.Phys.J.C78(2018)no.2,137



exercise:

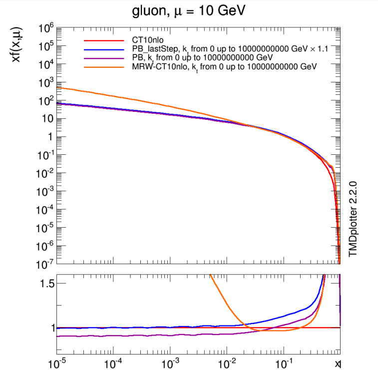
PB last Step: try to obtain KMRW from PB: take PB with angular ordering but take $k_\perp$ only from the last emission
do $\vec{k}_{\perp,a} = -\vec{q}_{\perp,c}$ instead $\vec{k}_{\perp,a} = \vec{k}_{\perp,b} - \vec{q}_{\perp,c}$ (PB full)

$k_t < 1\mathrm{GeV}$:

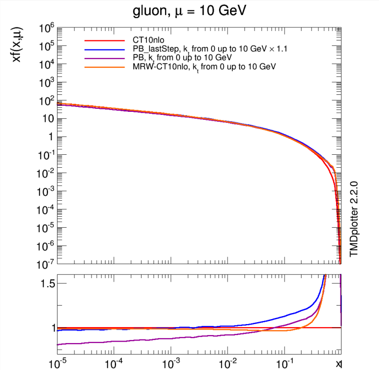
- KMRW: initial parametrization
- PB last Step: matching of intrinsic $k_\perp$ and evolution clearly visible
- PB full: matching of intrinsic $k_\perp$ and evolution smeared during evolution

For $k_t \in (\approx 10\mathrm{GeV}, \approx \mu)$:
PB full and KMRW very similar!

iTMDs



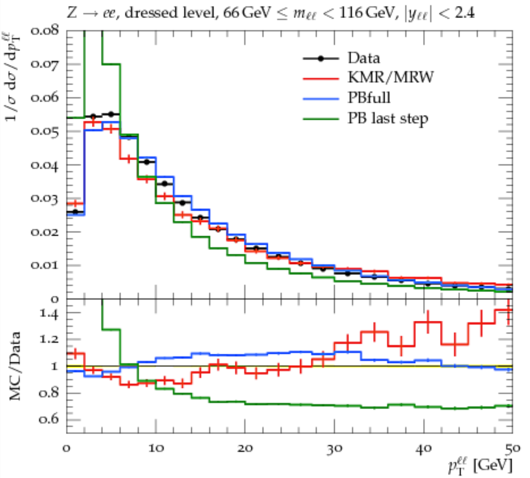
$\int^{\infty} dk_{\perp}^2\, A(x, k_{\perp}, \mu)$



$\int^{\mu} dk_{\perp}^2\, A(x, k_{\perp}, \mu)$

- PB, PB last Step and KMRW do not integrate back to ct10nlo (as expected, z_M far from 1, $\alpha_s(q_\perp)$)
- KMRW integrated up to ∞ much higher than integrated up to μ (large $k_\perp$ tail has significant contribution)

Z boson $p_\perp$ spectrum



- PB with angular ordering and full evolution works very well
- KMRW works well for small and middle-range $k_\perp$ but for higher $k_\perp$ it overestimates the data
- PB with last step evolution not sufficient

PB and Collins, Soper and Sterman

WORK IN PROGRESS

CSS: TMD factorization formula for the DY cross section:
Nuclear Physics B250 (1985) 199-224

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^2 q_\perp \mathrm{d}M^2 \mathrm{d}y} = \sum_{q,\bar{q}} \frac{\sigma^0}{s} H(\alpha_s) \int \frac{\mathrm{d}^2 b}{(2\pi)^2} e^{iq_\perp \cdot b} \mathcal{F}_q(x_1, b, M) \mathcal{F}_{\bar{q}}(x_2, b, M) + Y$$

where

$$\mathcal{F}_i(x, b, M) = \exp\left(-\frac{1}{2}\int_{c_0/b^2}^{M^2} \frac{\mathrm{d}\mu'^2}{\mu'^2}\left[A_i\left(\alpha_s(\mu'^2)\right)\ln\left(\frac{M^2}{\mu'^2}\right) + B_i\left(\alpha_s(\mu'^2)\right)\right]\right)\sqrt{G_{\mathrm{NP}(b)}}$$
$$\times \sum_j \int_x^1 \frac{\mathrm{d}z}{z} C_{ij}\left(z, \alpha_s\left(\frac{c_0}{b^2}\right)\right) f_j\left(\frac{x}{z}, \frac{c_0}{b^2}\right)$$

and A, B, C, H - have perturbative expansion

PB and Collins, Soper and Sterman

WORK IN PROGRESS

CSS: TMD factorization formula for the DY cross section:

Nuclear Physics B250 (1985) 199-224

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^2 q_\perp \mathrm{d}M^2 \mathrm{d}y} = \sum_{q,\bar{q}} \frac{\sigma^0}{s} H(\alpha_s) \int \frac{\mathrm{d}^2 b}{(2\pi)^2} e^{iq_\perp \cdot b} \mathcal{F}_q(x_1, b, M) \mathcal{F}_{\bar{q}}(x_2, b, M) + Y$$

where

$$\mathcal{F}_i(x, b, M) = \exp\left(-\frac{1}{2}\int_{c_0/b^2}^{M^2} \frac{\mathrm{d}\mu'^2}{\mu'^2}\left[A_i\left(\alpha_s(\mu'^2)\right)\ln\left(\frac{M^2}{\mu'^2}\right) + B_i\left(\alpha_s(\mu'^2)\right)\right]\right)\sqrt{G_{\mathrm{NP}(b)}}$$
$$\times \sum_j \int_x^1 \frac{\mathrm{d}z}{z} C_{ij}\left(z, \alpha_s\left(\frac{c_0}{b^2}\right)\right) f_j\left(\frac{x}{z}, \frac{c_0}{b^2}\right)$$

and A, B, C, H - have perturbative expansion

- one scale evolution up to a scale c_0/b
- in the last step of the evolution the dependence on the second scale enters

Sudakov form factor in PB and CSS

WORK IN PROGRESS

$$\mathcal{F}_i(x,b,M) = \exp\left(-\frac{1}{2}\int_{c_0/b^2}^{M^2}\frac{\mathrm{d}\mu'^2}{\mu'^2}\left[A_i\left(\alpha_s(\mu'^2)\right)\ln\left(\frac{M^2}{\mu'^2}\right)+B_i\left(\alpha_s(\mu'^2)\right)\right]\right)\sqrt{G_{\mathrm{NP}(b)}}$$

$$\times\sum_j\int_x^1\frac{\mathrm{d}z}{z}C_{ij}\left(z,\alpha_s\left(\frac{c_0}{b^2}\right)\right)f_j\left(\frac{x}{z},\frac{c_0}{b^2}\right)$$

Sudakov form factor in PB and CSS

WORK IN PROGRESS

$$\mathcal{F}_i(x,b,M) = \exp\left(-\frac{1}{2}\int_{c_0/b^2}^{M^2}\frac{\mathrm{d}\mu'^2}{\mu'^2}\left[A_i\left(\alpha_s(\mu'^2)\right)\ln\left(\frac{M^2}{\mu'^2}\right) + B_i\left(\alpha_s(\mu'^2)\right)\right]\right)\sqrt{G_{\mathrm{NP}(b)}}$$
$$\times\sum_j\int_x^1\frac{\mathrm{d}z}{z}C_{ij}\left(z,\alpha_s\left(\frac{c_0}{b^2}\right)\right)f_j\left(\frac{x}{z},\frac{c_0}{b^2}\right)$$

PB: Sudakov form factor in terms of P_a^V (momentum sum rule)
for angular ordering:

$$\Delta_a(\mu^2) = \exp\left(-\int_{q_0^2}^{\mu^2}\frac{dq_\perp^2}{q_\perp^2}\alpha_s(q_\perp)\left(\int_0^{1-\frac{q_\perp}{\mu}}dz\left(k_a\frac{1}{1-z}\right)-d\right)\right)\ .$$

notice: $\int_0^{1-\frac{q_\perp}{\mu}} dz\left(\frac{1}{1-z}\right) = \frac{1}{2}\ln\left(\frac{\mu}{q_\perp}\right)^2$

PB with angular ordering: in Sudakov the same coefficients as $\underbrace{A^1}_{\mathrm{LL}}$, $\underbrace{A^2 \text{ and } B^1}_{\mathrm{NLL}}$ in CSS

Resummation scheme dependence

WORK IN PROGRESS

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^2 q_\perp \mathrm{d}M^2 \mathrm{d}y} = \sum_{q,\bar{q}} \frac{\sigma^0}{s} H(\alpha_s) \int \frac{\mathrm{d}^2 b}{(2\pi)^2} e^{iq_\perp \cdot b} \mathcal{F}_q(x_1, b, M) \mathcal{F}_{\bar{q}}(x_2, b, M) + Y$$

NNLL: difference of CSS and PB B_2 comes from renormalization group

Sudakov form factor is process dependent Nucl.Phys. B596 (2001) 299-312

renormalization group equation: $\frac{\partial \ln H}{\partial \ln \mu^2} = \gamma(\alpha_s)$

solution: $H\left(\alpha_s(M^2)\right) = \exp\left(\int_{c_0/b^2}^{M^2} \frac{\mathrm{d}\mu'^2}{\mu'^2} \gamma\left(\alpha_s(\mu'^2)\right)\right) H\left(\alpha_s(\frac{c_0}{b^2})\right)$

This changes coefficient B in the Sudakov
$B(\alpha_s) \to B(\alpha_s) - \frac{\beta(\alpha_s)}{H(\alpha_s)} \frac{\partial H}{\partial \alpha_s}$

At $\mathcal{O}(\alpha_s^2)$: $B^2(\alpha_s) \to B^2(\alpha_s) + \pi \beta_0 H^1$
$H^1 = 16\left(\frac{\pi^2}{6} - 1\right)$

Summary and Conclusions

Summary and Conclusions

- PB: collinear PDFs and TMDs obtained
- different ordering definitions studied; visible effects on TMDs and Z boson $p_\perp$
  Angular ordering: stable (z_M-independent) TMDs and good description of Z boson $p_\perp$
- many different activities ongoing
  shown today: ongoing studies on comparison with Marchesini and Webber, KMRW and CSS
- results in:
  Phys.Lett. B772 (2017) 446-451,
  JHEP 1801 (2018) 070,
  arXiv:1804.11152 (in Phys. Rev. D soon)
  new paper in preparation!

Summary and Conclusions

- PB: collinear PDFs and TMDs obtained
- different ordering definitions studied; visible effects on TMDs and Z boson $p_\perp$
  Angular ordering: stable (z_M-independent) TMDs and good description of Z boson $p_\perp$
- many different activities ongoing
  shown today: ongoing studies on comparison with Marchesini and Webber, KMRW and CSS
- results in:
  Phys.Lett. B772 (2017) 446-451,
  JHEP 1801 (2018) 070,
  arXiv:1804.11152 (in Phys. Rev. D soon)
  new paper in preparation!

Outlook:

new level of precision in obtaining predictions for QCD observables (hard ME and PS follow the same TMD) for LHC and future colliders

Summary and Conclusions

- PB: collinear PDFs and TMDs obtained
- different ordering definitions studied; visible effects on TMDs and Z boson $p_\perp$
  Angular ordering: stable (z_M-independent) TMDs and good description of Z boson $p_\perp$
- many different activities ongoing
  shown today: ongoing studies on comparison with Marchesini and Webber, KMRW and CSS
- results in:
  Phys.Lett. B772 (2017) 446-451,
  JHEP 1801 (2018) 070,
  arXiv:1804.11152 (in Phys. Rev. D soon)
  new paper in preparation!

Outlook:

new level of precision in obtaining predictions for QCD observables (hard ME and PS follow the same TMD) for LHC and future colliders

Thank you!