

# Fracture Functions in Different Kinematic Regions and Their Factorizations

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*Based on: X.P. Chai, K.B. Chen, J.P. Ma and X.B. Tong, arXiv:1903.00809 [hep-ph]*



# Contents

- Fracture functions and  $pp \rightarrow \gamma^*\gamma X$  at forward Rapidity
- Factorization of TMD fracture function
- Factorization of integrated fracture function
- Summary



# Contents

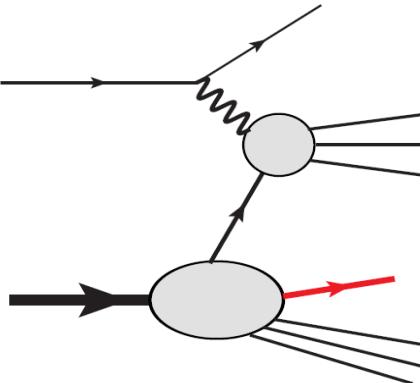
➤ Fracture functions and  $pp \rightarrow \gamma^*\gamma X$  at forward Rapidity

➤ Factorization of TMD fracture function

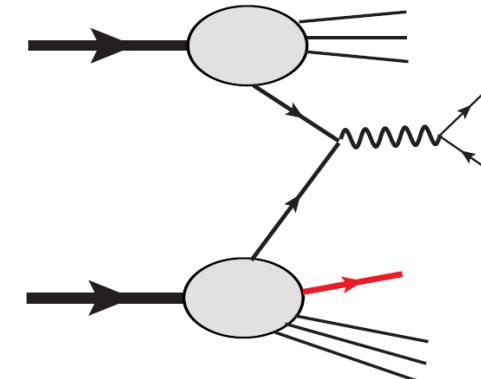
➤ Factorization of integrated fracture function

➤ Summary

## ■ Fracture Functions



SIDIS in the target fragmentation region



Diffractive Drell-Yan + hadron at forward rapidity

- Parton distributions of a hadron in the presence of an almost collinear particle observed in the final state.
- Contain information of target fragmentation.
- Important ingredients in QCD factorization for semi-inclusive processes where a particle is produced diffractively

See e.g., L. Trentadue and G. Veneziano, *Phys. Lett. B*323 (1994) 201

A. Berera and D.E. Soper, *Phys. Rev. D*53 (1996) 6162

J.C. Collins, *Phys. Rev. D*57 (1998) 3051

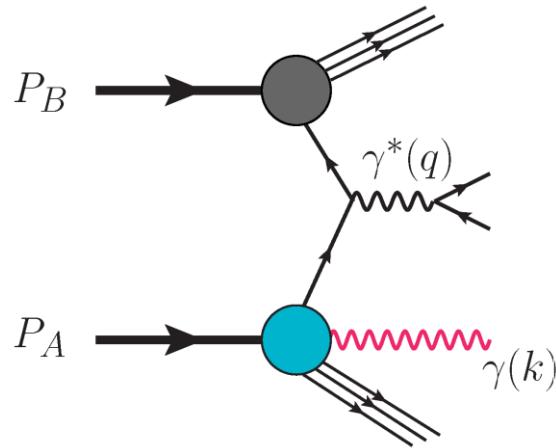
M. Anselmino, V. Barone and A. Kotzinian, *Phys. Lett. B*699 (2011) 108

F.A. Ceccopieri and L. Trentadue, *Phys. Lett. B*668 (2008) 319

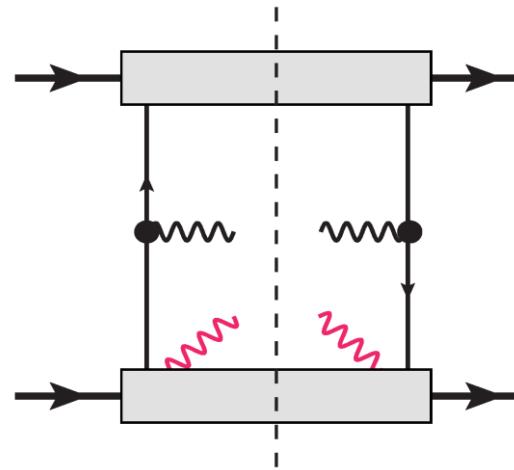
F.A. Ceccopieri, *Phys. Lett. B*703 (2011) 491

# Fracture functions and $pp \rightarrow \gamma^* + \gamma + X$ at forward Rapidity

## $pp \rightarrow \gamma^* + \gamma + X$ at forward Rapidity



$$h_A(P_A) + h_B(P_B) \rightarrow \gamma^*(q) + \gamma(k) + X$$



Kinematics:

$$P_A^\mu = (P_A^+, 0, 0, 0), \quad P_B^\mu = (0, P_B^-, 0, 0)$$

$$q^\mu = (q^+, q^-, \vec{q}_\perp), \quad q^2 = Q^2 \gg \Lambda_{QCD}^2$$

$$k^\mu = (k^+, k^-, \vec{k}_\perp) \sim Q(1, \lambda^2, \lambda, \lambda), \quad \lambda \ll 1$$

$$a^\mu = ((a^0 + a^3)/\sqrt{2}, (a^0 - a^3)/\sqrt{2}, \vec{a}_\perp)$$

$$\frac{(2\pi)^3 2k^0 d\sigma}{d^4 q d^3 k} = \frac{e^4}{12\pi s Q^2} W^{\mu\nu} \left( \frac{q_\mu q_\nu}{q^2} - g_{\mu\nu} \right)$$

$$W^{\mu\nu} = \sum_X \int \frac{d^4 x}{(2\pi)^4} e^{iq \cdot x} \langle P_A, P_B | \bar{q}(0) \gamma^\nu q(0) | X, \gamma(k) \rangle \langle \gamma(k), X | \bar{q}(x) \gamma^\mu q(x) | P_A, P_B \rangle$$

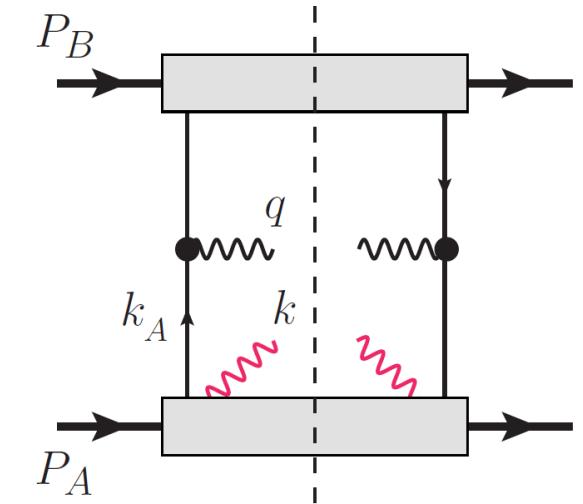
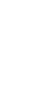
# Fracture functions and $pp \rightarrow \gamma^*\gamma X$ at forward Rapidity

## ■ Collinear and TMD factorizations for the hadronic tensor

When  $Q \gg q_\perp \gg \Lambda_{QCD}$ , the transverse momenta of partons can be neglected and integrated out. The *collinear factorization* applies:

$$W^{\mu\nu} = \int dx_A dx_B \sum_{a,b} H_{ab}^{\mu\nu}(x_A P_A, x_B P_B, q) \cdot F_a(x_A, \xi, k_\perp) \cdot f_{b/B}(x_B)$$

*Integrated fracture function*



$$F_q(x, \xi, k_\perp) = \sum_X \int \frac{dz}{4\pi} e^{ixP_A^+ z} \langle P_A | \bar{q}(0) \mathcal{L}_n(0) \gamma^+ | X, \gamma(k) \rangle \langle \gamma(k), X | \mathcal{L}_n^\dagger(zn) q(zn) | P_A \rangle$$

$$x = \frac{k_A^+}{p_A^+}, \quad \xi = \frac{k^+}{p_A^+}$$

$$n^\mu = (0, 1, 0, 0)$$

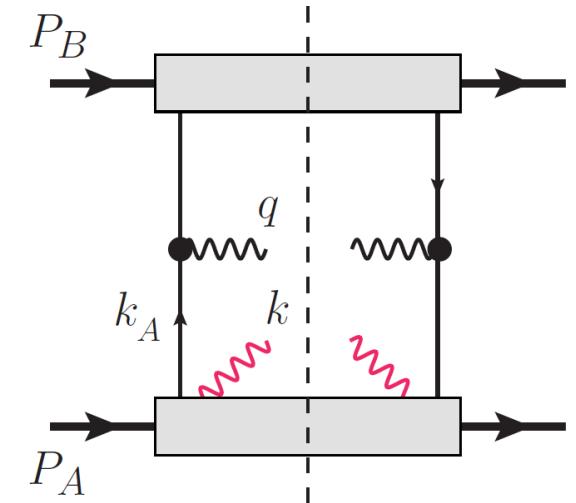
# Fracture functions and $pp \rightarrow \gamma^*\gamma X$ at forward Rapidity

## ■ Collinear and TMD factorizations for the hadronic tensor

When  $q_\perp \sim \Lambda_{QCD}$ , the transverse momenta of partons cannot be neglected, and the photon is produced nonperturbatively. We need *transverse momentum dependent (TMD) factorization*.

$$W^{\mu\nu} = -g_\perp^{\mu\nu} \frac{1}{N_c} H(Q, \zeta_u^2, \zeta_v^2) \int d^2 k_{A\perp} d^2 k_{B\perp} d^2 \ell_\perp \delta^2(k_{A\perp} + k_{B\perp} + \ell_\perp - q_\perp) \\ \cdot f_{\bar{q}}(y, k_{B\perp}, \zeta_v^2) \times \mathcal{F}_q(x, k_{A\perp}, \xi, k_\perp, \zeta_u^2) \times \tilde{S}(\ell_\perp, \rho^2) + \dots$$

*TMD PDF*      *TMD fracture function*      *Soft factor*



$$\mathcal{F}_q(x, k_{A\perp}, \xi, k_\perp) = \sum_X \int \frac{dz^- d^2 z_\perp}{2(2\pi)^3} e^{ik_A^+ z^- - k_{A\perp} \cdot z_\perp} \langle P_A | \bar{q}(0) \mathcal{L}_u(0) \gamma^+ | X, \gamma(k) \rangle \langle \gamma(k), X | \mathcal{L}_u^\dagger(z) q(z) | P_A \rangle \Big|_{z^+=0}$$

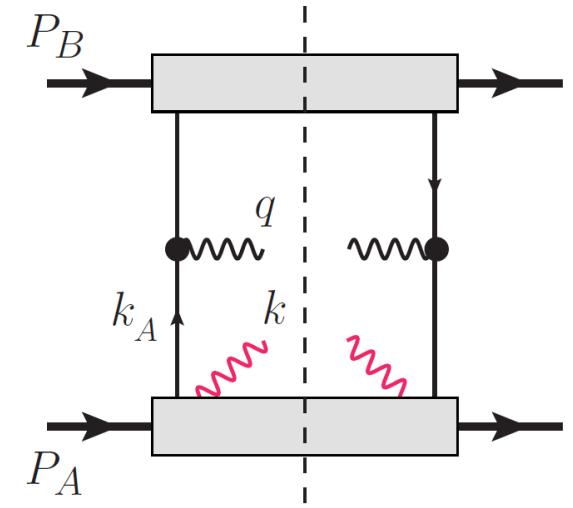
$$x = \frac{k_A^+}{p_A^+}, \quad \xi = \frac{k^+}{p_A^+} \quad \mathcal{L}_u(z) = P e^{-i g_s \int_{-\infty}^0 dy u \cdot G(yu+z)} \quad u = (u^+, u^-, 0, 0), u^- \gg u^+ \\ v = (v^+, v^-, 0, 0), v^+ \gg v^- \quad \zeta_u^2 = \frac{(2u \cdot P_A)^2}{u^2}, \quad \zeta_v^2 = \frac{(2v \cdot P_A)^2}{v^2}$$

$$f_{\bar{q}}(x, k_{B\perp}) = - \int \frac{dz^+ d^2 z_\perp}{2(2\pi)^3} e^{-ik_B^- z^+ + k_{B\perp} \cdot z_\perp} \langle P_B | \bar{q}(0) \mathcal{L}_v(0) \gamma^- \mathcal{L}_v^\dagger(z) q(z) | P_B \rangle \Big|_{z^-=0} \quad \tilde{S}(k_\perp) = \int \frac{d^2 y_\perp}{(2\pi)^2} e^{-iy_\perp \cdot k_\perp} \frac{N_c}{\langle 0 | \mathcal{L}_v^\dagger(y_\perp) \mathcal{L}_u(y_\perp) \mathcal{L}_u^\dagger(0) \mathcal{L}_v(0) | 0 \rangle}$$

Fracture functions are nonperturbative objects.

However, when  $k_\perp, q_\perp \gg \Lambda_{QCD}$ :

There are effects which are calculable with pQCD.



Factorize the fracture functions further!



# Contents

➤ Fracture functions and  $pp \rightarrow \gamma^*\gamma X$  at forward Rapidity

➤ Factorization of TMD fracture function

➤ Factorization of integrated fracture function

➤ Summary

# Factorization of TMD fracture function

## ■ Factorization at large transverse momentum region

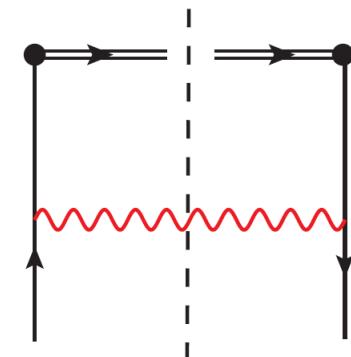
When  $k_{A\perp}, k_\perp, k_{A\perp} + k_\perp \gg \Lambda_{QCD}$ , they must be generated from hard radiation. The large transverse momentum behavior of the fracture function can be calculated using pQCD!

One expects the TMD fracture function takes the following factorization form:

$$\mathcal{F}_q(x, k_{A\perp}, \xi, k_\perp) = \int_{x+\xi}^1 \frac{dy}{y} [f_q(y)\mathcal{C}_q(x/y, \xi/y, k_{A\perp}, k_\perp) + f_g(y)\mathcal{C}_g(x/y, \xi/y, k_{A\perp}, k_\perp)]$$

- The quark TMD fracture function at  $\mathcal{O}(\alpha_s^0)$ :

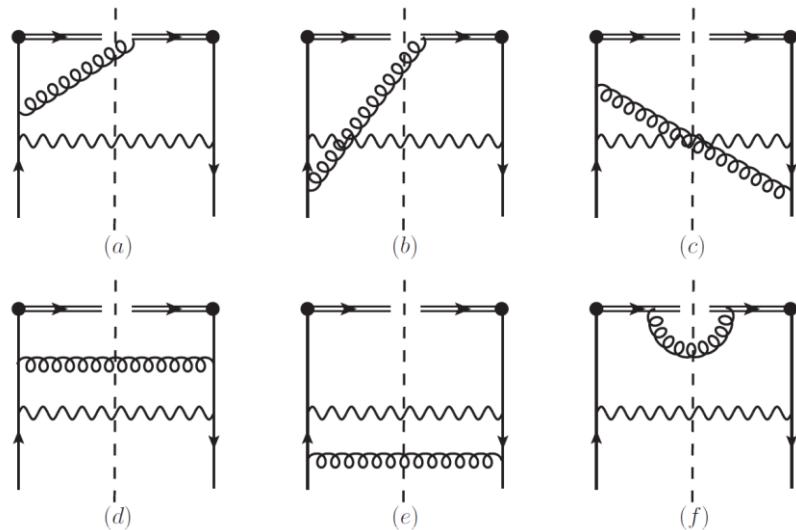
$$\begin{aligned}\mathcal{C}_q^{(0)} &= \mathcal{F}_q^{(0)}(x, k_{A\perp}, \xi, k_\perp) = 2e^2 \delta(1-x-\xi) \delta^2(k_{A\perp} + k_\perp) \frac{1}{k_\perp^2} (\xi^2 - 2\xi + 2) \\ \mathcal{C}_g^{(0)} &= 0\end{aligned}$$



# Factorization of TMD fracture function

## ■ Factorization at large transverse momentum region

- At  $\mathcal{O}(\alpha_s^1)$



$$\begin{aligned} \mathcal{C}_q(x, \xi, k_{A\perp}, k_\perp) = & \frac{\alpha_s C_F e^2}{\pi^2 k_\perp^2} \left[ \frac{\delta(z)\gamma(\xi)}{\ell_\perp^2} \left( \ln \frac{\zeta_u^2 \bar{\xi}^2}{\ell_\perp^2} - 1 \right) + \frac{\gamma(\xi)/z_+ - x - \bar{\xi}}{\ell_\perp^2} - \frac{x(\xi^2 \ell_\perp^2 + z^2 k_\perp^4 / \ell_\perp^2)}{\xi \bar{\xi}^2 D_d^2} \right. \\ & \left. + \frac{\gamma(\xi) \bar{\xi}/z_+ - x - 1}{\bar{\xi} D_d} + \frac{k_\perp^2 (\gamma(\xi) - z^3 + 3z^2 + \xi z - 4z)}{\xi \bar{\xi} \ell_\perp^2 D_d} \right] + \mathcal{O}(\epsilon). \end{aligned}$$

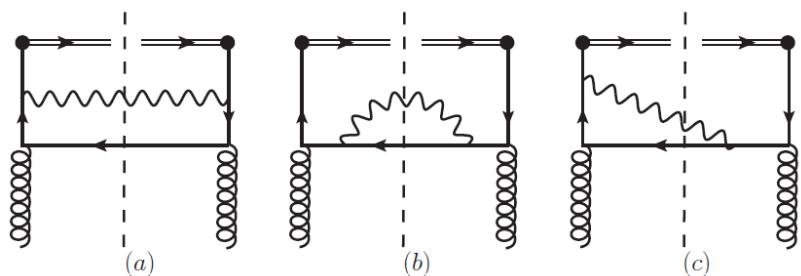
$$z = 1 - x - \xi, \quad \ell_\perp = k_{A\perp} + k_\perp,$$

$$\gamma(\xi) = 1 + \bar{\xi}^2, \quad \bar{\xi} = 1 - \xi$$

$$D_c = \left( \ell_\perp + \frac{zk_\perp}{\xi} \right)^2, \quad D_d = \left( \ell_\perp - \frac{zk_\perp}{\bar{\xi}} \right)^2 + \frac{xzk_\perp^2}{\xi \bar{\xi}^2}$$

$$\frac{\zeta_u^2 z}{(\ell_\perp^2 + z^2 \zeta_u^2)} = \frac{1}{z_+} + \frac{1}{2} \delta(z) \ln \frac{(1-\xi)^2 \zeta_u^2}{k_\perp^2} + \mathcal{O}(\zeta_u^{-2})$$

$$\text{The plus distribution: } \int_0^{1-\xi} dx \frac{T(x)}{(1-x-\xi)_+} \equiv \int_0^{1-\xi} dx \frac{T(x) - T(1-\xi)}{1-x-\xi}$$



$$\begin{aligned} \mathcal{C}_g(x, \xi, k_{A\perp}, k_\perp) = & \frac{\alpha_s e^2 z}{2\pi^2} \left[ \frac{xzk_\perp^2 - 2\xi^2 x D_c}{\xi^2 \bar{\xi}^2 D_c D_d} + \frac{2x^2(1+z) - \xi^2 x - 3x + 1}{\xi^2 \bar{\xi} D_c D_d} + \frac{z(\bar{\xi}^2 - 2xz)}{\xi^2 D_c \ell_\perp^2} \right. \\ & \left. - \frac{2x^2(z-\xi) - \xi^3 - 3\xi^2 x - x}{\xi \bar{\xi} D_d \ell_\perp^2} - \frac{xzk_\perp^2}{\xi \bar{\xi}^2 D_d^2 \ell_\perp^2} \right] + \mathcal{O}(\epsilon). \end{aligned}$$

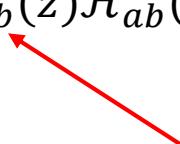
# Factorization of TMD fracture function

## ■ Factorization in impact parameter space

Transforming the fracture function into the impact parameter b-space , there is another factorization relating the fracture function to PDFs and fragmentation functions.

When the impact parameter  $b$  is small, the factorization takes the form:

$$\begin{aligned}
 \hat{F}_q(x, b, \xi, k_\perp) &= \int d^{2-\epsilon} k_{A\perp} e^{-ib \cdot (k_{A\perp} + k_\perp)} \mathcal{F}_q(x, k_{A\perp}, \xi, k_\perp) \\
 &= \int \frac{dy}{y} \sum_a f_a(y) \hat{\mathcal{H}}_a(x/y, \xi/y, b, k_\perp) + \sum_{ab} \int \frac{dy}{y} \int \frac{dz}{z} f_a(y) D_b(z) \hat{\mathcal{H}}_{ab}(x, \xi, z, b, k_\perp) + \mathcal{O}(b^2)
 \end{aligned}$$


fragmentation function of  
a parton to the photon

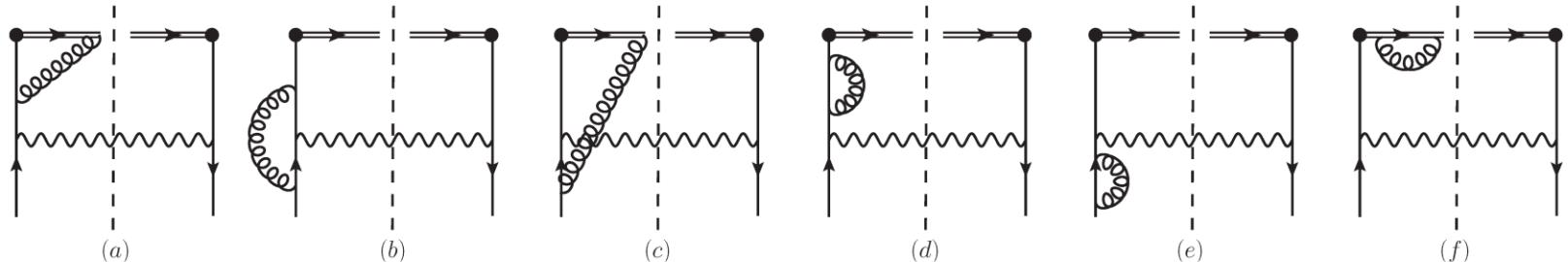
- At order of  $\alpha_s^0$ :

$$\hat{F}_q(x, b, \xi, k_\perp) = \hat{H}_q(x, \xi, b, k_\perp) = 2e^2 \delta(1 - x - \xi) \frac{1}{k_\perp^2} \left( \gamma(\xi) - \frac{\epsilon}{2} \xi^2 \right) + \mathcal{O}(\alpha_s).$$

# Factorization of TMD fracture function

## ■ Factorization in impact parameter space

- Virtual corrections of quark fracture function from initial quark



$$\hat{F}_q \Big|_a^V = -\frac{e^2 \alpha_s C_F}{2\pi} \delta(z) \gamma(\xi) \frac{1}{k_\perp^2} \left[ \ln^2 \frac{k_\perp^2}{\xi \bar{\xi}^2 \zeta_u^2} + \ln \frac{k_\perp^2}{\xi \mu^2} + \ln \frac{k_\perp^2}{\xi \bar{\xi}^2 \zeta_u^2} + \frac{4\pi^2}{3} - 2 \right]$$

$$\hat{F}_q \Big|_b^V = \frac{e^2 \alpha_s C_F}{\pi} \delta(z) \frac{1}{k_\perp^2} \left[ \frac{1}{2} \gamma(\xi) \ln \frac{\xi \mu^2}{k_\perp^2} + \xi \bar{\xi} \left( \frac{2}{\epsilon_c} + \ln \frac{\xi \tilde{\mu}_c^2}{k_\perp^2} \right) + \frac{3}{2} \xi \bar{\xi} + 1 \right]$$

$$\begin{aligned} \hat{F}_q \Big|_c^V &= \frac{e^2 \alpha_s C_F}{\pi} \delta(z) \frac{1}{k_\perp^2} \left\{ -\frac{2\gamma(\xi)}{\epsilon_c^2} + \frac{1}{\epsilon_c} \left[ \gamma(\xi) \ln \frac{\zeta_u^2}{\tilde{\mu}_c^2} + \xi^2 + 2\xi - 4 \right] + \frac{1}{4} \gamma(\xi) \left( \ln^2 \frac{\zeta_u^2}{k_\perp^2} - \frac{1}{2} \ln^2 \frac{\zeta_u^2}{\tilde{\mu}_c^2} + \ln^2 \frac{k_\perp^2}{\tilde{\mu}_c^2} \right) \right. \\ &\quad + \frac{1}{8} \gamma(\xi) \ln \frac{\zeta_u^2}{\tilde{\mu}_c^2} \left( \ln \frac{\zeta_u^2}{k_\perp^2} - 3 \ln \frac{k_\perp^2}{\tilde{\mu}_c^2} \right) - \frac{1}{4} (\xi^2 - 2\gamma(\xi) \ln \xi \bar{\xi}^2 - 3\xi + 2) \ln \frac{\zeta_u^2}{k_\perp^2} - \frac{1}{4} (\xi^2 - 2\gamma(\xi) \ln \xi + 3\xi - 2) \ln \frac{\zeta_u^2}{\tilde{\mu}_c^2} \\ &\quad \left. - \frac{1}{4} (\xi^2 + 2\gamma(\xi) \ln \xi + \xi - 6) \ln \frac{k_\perp^2}{\tilde{\mu}_c^2} + \frac{\gamma(\xi)}{2} \ln^2 \xi \bar{\xi} - \gamma(\xi) \text{Li}_2(-\xi/\bar{\xi}) - \frac{\pi^2 \gamma(\xi)}{24} + (\xi - 2)(2 + \ln \xi) \right\}, \end{aligned}$$

$$\hat{F}_q \Big|_E = -\frac{e^2 \alpha_s C_F}{2\pi} \delta(z) \frac{1}{k_\perp^2} \left\{ \gamma(\xi) \left[ \left( -\frac{2}{\epsilon_c} + \ln \frac{\mu^2}{\tilde{\mu}_c^2} \right) - 2 \left( -\frac{2}{\epsilon_c} + \ln \frac{\mu^2}{\tilde{\mu}_c^2} \right) + 2 \left( 1 + \ln \frac{\xi \mu^2}{k_\perp^2} \right) \right] - \xi^2 \right\}.$$

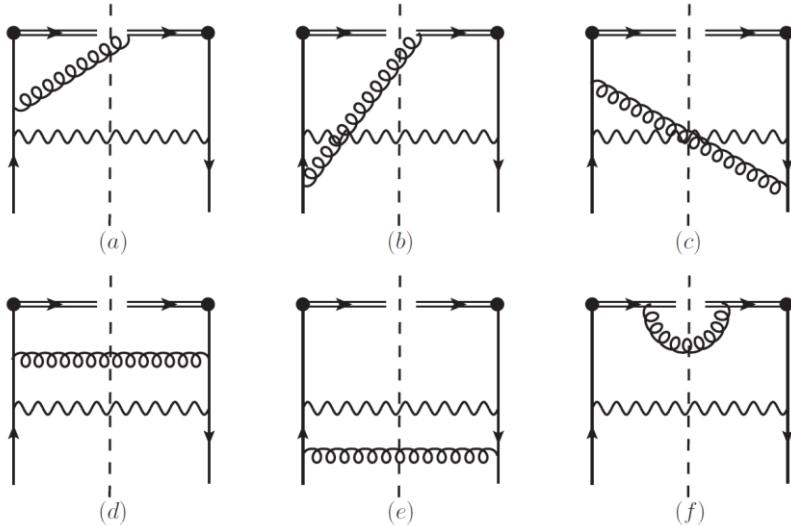
$$\tilde{\mu}_c^2 = 4\pi\mu_c^2 e^{-\gamma_E}, \quad \tilde{b}^2 = \frac{1}{4}b^2 e^{2\gamma_E}$$

$$d = 4 - \epsilon$$

# Factorization of TMD fracture function

## ■ Factorization in impact parameter space

- Real corrections of quark fracture function from initial quark



$$\begin{aligned}
\hat{F}_q|_a^R &= \frac{e^2 \alpha_s C_F}{\pi k_\perp^2} \left\{ -\frac{2\gamma(\xi)}{\epsilon_c^2} \delta(z) + \frac{\gamma(\xi)}{\epsilon_c} \delta(z) \left( \ln \frac{\tilde{\mu}_c^2}{\zeta_u^2 \bar{\xi}^2} - 2 \ln \frac{\tilde{\mu}_c^2 \xi}{k_\perp^2} + 1 - 2 \frac{\bar{\xi}}{\gamma(\xi)} \right) \right. \\
&\quad \left. - \frac{\delta(z)}{2} \left[ \xi^2 \left( \ln \frac{\tilde{\mu}_c^2}{\zeta_u^2 \bar{\xi}^2} - 2 \ln \frac{\tilde{\mu}_c^2 \xi}{k_\perp^2} \right) + \gamma(\xi) \left( -\frac{1}{2} \ln (\tilde{b}^2 \tilde{\mu}_c^2) \ln \frac{\tilde{\mu}_c^2}{\tilde{b}^2 \zeta_u^4 \bar{\xi}^4} + \ln \frac{\tilde{\mu}_c^2}{k_\perp^2} \ln \frac{\tilde{\mu}_c^2 \xi^2}{k_\perp^2} + \ln^2 \xi + \frac{\pi^2}{4} \right) \right] \right. \\
&\quad \left. + \gamma(\xi) \left[ - \left( \frac{1}{z} \right)_+ \left( \ln \tilde{b}^2 k_\perp^2 + \ln \frac{x}{\xi \bar{\xi}^2} \right) - \left( \frac{\ln z}{z} \right)_+ + \frac{1}{\bar{\xi}} \left( \ln \tilde{b}^2 k_\perp^2 + \ln \frac{xz}{\xi \bar{\xi}^2} \right) \right] \right\} + \mathcal{O}(b^2), \\
\hat{F}_q|_b^R &= \frac{e^2 \alpha_s C_F}{\pi k_\perp^2} \left\{ \frac{4\gamma(\xi)}{\epsilon_c^2} \delta(z) + \frac{2}{\epsilon_c} \left[ \delta(z) \left( \gamma(\xi) \ln \frac{\tilde{\mu}_c^2 \xi}{k_\perp^2} - \xi^2 \right) + \frac{1}{\bar{z}} \left( -\gamma(\xi) \left( \frac{1}{z} \right)_+ + x\bar{\xi} + x + 2 \right) \right] \right. \\
&\quad \left. + \frac{1}{\bar{z}} \left[ \left( -\gamma(\xi) \left( \frac{1}{z} \right)_+ + x\bar{\xi} + x + 2 \right) \left( \ln \frac{\tilde{\mu}_c^2}{k_\perp^2} + \ln \frac{\xi \bar{\xi}}{z \bar{z}} \right) + \left( \frac{1}{z} \right)_+ (\xi^2 - \gamma(\xi) \ln z) + \gamma(\xi) \left( \frac{\ln z}{z} \right)_+ - \xi^2 \right] \right. \\
&\quad \left. + \delta(z) \ln \frac{\tilde{\mu}_c^2 \xi}{k_\perp^2} \left( \frac{\gamma(\xi)}{2} \ln \frac{\tilde{\mu}_c^2 \xi}{k_\perp^2} - \xi^2 \right) + \frac{\pi^2}{12} \gamma(\xi) \delta(z) \right\} + \mathcal{O}(b^2), \\
\hat{F}_q|_c^R &= \frac{e^2 \alpha_s C_F}{\pi k_\perp^2} \left[ \frac{\xi x}{\bar{z}} \left( \frac{2}{\epsilon_c} + \ln \frac{\tilde{\mu}_c^2}{k_\perp^2} \right) + \left( \frac{\xi x}{\bar{z}} + \frac{\bar{x}}{2} \right) \ln \frac{\xi x}{z \bar{z}^2} + \frac{\bar{x}}{2} \ln \frac{xz}{\xi \bar{\xi}^2} + \frac{\gamma(\xi)x + \xi}{\xi \bar{z}} - \frac{\bar{z}}{\bar{\xi}} + \xi - x \right] + \mathcal{O}(b^2), \\
\hat{F}_q|_d^R &= \frac{e^2 \alpha_s C_F}{\pi k_\perp^2 \bar{\xi}^2} \left[ -\gamma(\xi) z \left( \ln \tilde{b}^2 k_\perp^2 + \ln \frac{xz}{\xi \bar{\xi}^2} + 1 \right) + \xi x \right] + \mathcal{O}(b^2), \\
\hat{F}_q|_e^R &= \frac{e^2 \alpha_s C_F}{\pi k_\perp^2} \left[ -z \left( 1 + \frac{x^2}{\bar{z}^2} \right) \left( \frac{2}{\epsilon_c} + \ln \frac{\tilde{\mu}_c^2}{k_\perp^2} + \ln \frac{\xi x}{z \bar{z}^2} \right) + \frac{\xi \bar{\xi}}{\bar{z}} \right] + \mathcal{O}(b^2), \\
\hat{F}_q|_f^R &= \frac{e^2 \alpha_s C_F}{\pi k_\perp^2} \delta(z) \left( \frac{2}{\epsilon_c} \gamma(\xi) + \gamma(\xi) \ln (\tilde{b}^2 \tilde{\mu}_c^2) - \xi^2 \right) + \mathcal{O}(b^2).
\end{aligned}$$

# Factorization of TMD fracture function

## ■ Factorization in impact parameter space

All the double poles are cancelled among the sum of virtual and real diagrams. The remaining divergent part contains only a collinear single pole:

$$\hat{F}_q \Big|_{div.} = \frac{e^2 \alpha_s C_F}{\pi} \frac{1}{k_\perp^2} \left( -\frac{2}{\epsilon_c} \right) \gamma(\xi/y) \left( \frac{1+y^2}{(1-y)_+} + \frac{3}{2} \delta(1-y) \right), \quad y = x + \xi, \quad \int_\xi^1 dy \frac{T(y)}{(1-y)_+} = \int_\xi^1 dy \frac{T(y) - T(1)}{1-y} + T(1) \ln(1-\xi)$$

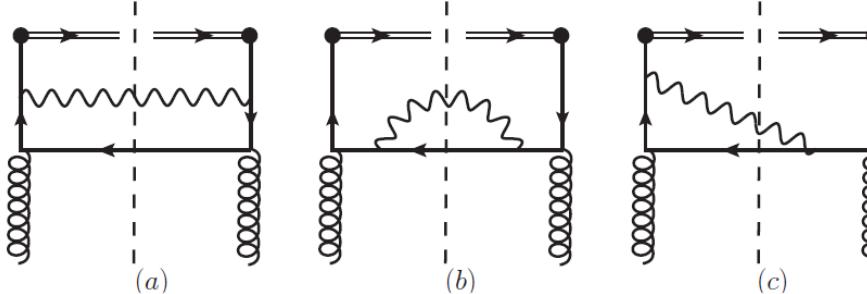
The quark-in-quark PDF:  $f_{q/q}(x) = \delta(1-x) + \frac{\alpha_s}{2\pi} \left( -\frac{2}{\epsilon_c} + \ln \frac{e^{\gamma_E} \mu^2}{4\pi \mu_c^2} \right) C_F \left( \frac{1+x^2}{(1-x)_+} + \frac{3}{2} \delta(1-x) \right) + \mathcal{O}(\alpha_s^2)$

$$\begin{aligned} \rightarrow \hat{\mathcal{H}}_q^{(1)}(x, \xi, b, k_\perp) = & \frac{e^2 \alpha_s C_F}{\pi k_\perp^2} \left\{ -\gamma(\xi/y) \frac{1+y^2}{(\bar{y})_+} \ln \frac{\mu^2}{k_\perp^2} - 2\gamma(\xi) \left( \frac{1}{\bar{y}} \right)_+ \left( \ln \tilde{b}^2 k_\perp^2 + \ln \frac{y-\xi}{\xi \bar{\xi}^2} + \frac{1}{y} \ln \frac{\xi \bar{\xi}}{y} \right) \right. \\ & + \delta(\bar{y}) \left[ \gamma(\xi) \left( \ln \bar{\xi} \left( 2 \ln \tilde{b}^2 k_\perp^2 - \ln \bar{\xi} \xi^2 \right) - \frac{1}{2} \ln^2 \left( \bar{\xi}^2 \zeta_u^2 \tilde{b}^2 e^{-1} \right) - 2 \text{Li}_2 \left( -\frac{\xi}{\bar{\xi}} \right) - \frac{3\pi^2}{2} - \frac{7}{2} \right) + 3\bar{\xi} + 1 \right] \\ & + \left( \bar{\xi} + \frac{2}{\bar{\xi}} - \frac{\bar{y}}{\bar{\xi}^2} + 1 \right) \ln \frac{\bar{y}(y-\xi)}{\xi \bar{\xi}^2} + \frac{2(\bar{\xi}+1)(y-\xi)+4}{y} \ln \frac{\xi \bar{\xi}}{y \bar{y}} + \left( 2y - \xi - (y+1) \frac{(y-\xi)^2}{y^2} \right) \ln \frac{\xi(y-\xi)}{y^2 \bar{y}} \\ & \left. + \frac{\gamma(\xi)}{\bar{\xi}^2} (\bar{\xi} - \xi + y) \ln \tilde{b}^2 k_\perp^2 + \frac{2\gamma(\xi)}{y} \ln \bar{y} + \frac{3\bar{y}+1}{\bar{\xi}} - \frac{2\bar{y}}{\bar{\xi}^2} + 2\xi - \frac{\xi^2}{y^2} - \frac{\xi}{y} - y \right\} + \mathcal{O}(\alpha_s^2). \end{aligned}$$

# Factorization of TMD fracture function

## ■ Factorization in impact parameter space

- Real corrections of quark fracture function from initial gluon



$$\hat{F}_q|_a^{R,g} = \frac{e^2 \alpha_s}{2\pi k_\perp^2} \left[ -(\bar{z}^2 + z^2) \left( \frac{x^2}{\bar{z}^2} + 1 \right) \left( \frac{2}{\epsilon_c} + \ln \frac{\tilde{\mu}_c^2}{k_\perp^2} + \ln \frac{\xi x}{z \bar{z}^2} \right) + \frac{\xi \bar{x}}{\bar{z}} + 4xz \right] + \mathcal{O}(b^2),$$

Factorized into the quark-in-gluon PDF

$$f_{q/g}(x) = \frac{\alpha_s}{2\pi} \left( -\frac{2}{\epsilon_c} + \ln \frac{\mu^2}{\tilde{\mu}_c^2} \right) \frac{x^2 + (1-x)^2}{2}$$

$$\hat{F}_q|_b^{R,g} = \frac{e^2 \alpha_s}{2\pi k_\perp^2} \left[ -(\bar{x}^2 + x^2) \left( \frac{z^2}{\bar{x}^2} + 1 \right) \left( \frac{2}{\epsilon_c} + \ln \frac{\tilde{\mu}_c^2}{k_\perp^2} + \ln \frac{\xi^3 x}{z \bar{x}^2} \right) + \frac{\xi}{\bar{x}} (\xi + x^2) + 4x\bar{x} - 3\xi x \right] + \mathcal{O}(b^2),$$

Factorized into the quark-to-photon FF

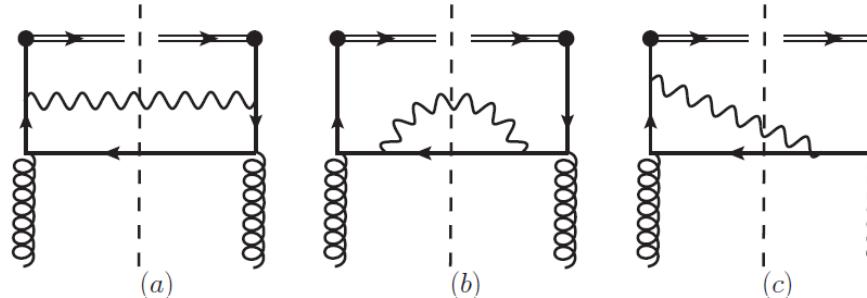
$$D_q(z) = \frac{e^2}{8\pi^2} \left( -\frac{2}{\epsilon_c} + \ln \frac{\mu^2}{\tilde{\mu}_c^2} \right) \frac{1 + (1-z)^2}{z}$$

$$\hat{F}_q|_c^{R,g} = \frac{e^2 \alpha_s}{2\pi k_\perp^2} \left[ (x^2 + z^2) \ln \frac{xz}{\bar{x}\bar{z}} - \frac{(-\bar{\xi} + x^2 + z^2)(\bar{\xi} - 2xz)}{\bar{x}\bar{z}} \right] + \mathcal{O}(b^2).$$

# Factorization of TMD fracture function

## ■ Factorization in impact parameter space

- Real corrections of quark fracture function from initial gluon



$$\hat{\mathcal{H}}_{gq}(x, \xi, z, b, k_\perp) = \frac{4\pi\alpha_s}{k_\perp^2} (x^2 + \bar{x}^2) \bar{x}z^2 \delta(\bar{x}z - \xi) + \mathcal{O}(\alpha_s^2).$$

$$\begin{aligned} \hat{\mathcal{H}}_g(x, \xi, b, k_\perp) = & \frac{e^2 \alpha_s}{2\pi k_\perp^2} \left\{ -(y^2 + \bar{y}^2) \left[ \left( \ln \frac{\mu^2}{k_\perp^2} + \ln \frac{\xi x}{\bar{y}y^2} \right) \left( \frac{x^2}{y^2} + 1 \right) + \frac{\xi^2}{y^2} \right] \right. \\ & - (\bar{x}^2 + x^2) \left( \frac{\bar{y}^2}{\bar{x}^2} + 1 \right) \left( \ln \frac{\mu^2}{k_\perp^2} + \ln \frac{\xi^3 x}{\bar{y}\bar{x}^2} \right) + 2(x^2 + \bar{y}^2) \ln \frac{x\bar{y}}{\bar{x}y} \\ & \left. - 2(x^2 + \bar{y}^2 - \xi) \frac{\bar{\xi} - 2x\bar{y}}{\bar{x}y} + \xi \left( \frac{y}{\bar{x}} + \frac{\bar{x}}{y} \right) + 8x\bar{y} \right\} + \mathcal{O}(\alpha_s^2). \end{aligned}$$

# Factorization of TMD fracture function

## ■ Evolution equations

- The renormalization scale evolution:

$$\frac{\partial \hat{F}_q(x, b, \xi, k_\perp, \zeta_u, \mu)}{\partial \ln \mu} = 2\gamma_F \hat{F}_q(x, b, \xi, k_\perp, \zeta_u, \mu), \quad \gamma_F = \frac{3\alpha_s C_F}{4\pi} + \mathcal{O}(\alpha_s^2)$$

- Collins-Soper equation:

$$\frac{\partial \hat{F}_q(x, b, \xi, k_\perp, \zeta_u, \mu)}{\partial \ln \zeta_u} = -\frac{\alpha_s C_F}{\pi} \left( \ln \frac{x^2 \zeta_u^2 b^2 e^{2\gamma_E - 1}}{4} \right) \hat{F}_q(x, b, \xi, k_\perp, \zeta_u, \mu) + \mathcal{O}(\alpha_s^2)$$

The evolution equations are the same with those for TMD PDFs!  
Useful for resummation of large logs in perturbative calculations.



# Contents

➤ Fracture functions and  $pp \rightarrow \gamma^*\gamma X$  at forward Rapidity

➤ Factorization of TMD fracture function

➤ Factorization of integrated fracture function

➤ Summary



# Factorization of integrated fracture function

When the transverse momentum of the struck parton  $k_{A\perp}$  is not observed or can be neglected, e.g., for semi-inclusive DIS in target fragmentation region, we need the integrated fracture functions:

$$F_q(x, \xi, k_\perp) = \sum_X \int \frac{dz}{4\pi} e^{ixP_A^\dagger z} \langle P_A | \bar{q}(0) \mathcal{L}_n(0) \gamma^+ | X, \gamma(k) \rangle \langle \gamma(k), X | \mathcal{L}_n^\dagger(zn) q(zn) | P_A \rangle$$

When  $k_\perp \gg \Lambda_{QCD}$ , the expected factorization form:

$$F_q(x, \xi, k_\perp) = \sum_{a=q, \bar{q}, g} \int \frac{dy}{y} f_{a/h}(y) \left[ \mathcal{H}_a(x/y, \xi/y, k_\perp) + \sum_b \int \frac{dz}{z} D_b(z) \mathcal{H}_{ab}(x/y, \xi/y, z, k_\perp) \right]$$

# Factorization of integrated fracture function

$$\begin{aligned}
 & \text{Diagram 1} + \text{Diagram 2} = \frac{e^2 \alpha_s C_F}{\pi k_\perp^2} \left\{ \gamma(\xi) \left( \delta(z) + \frac{x}{\bar{\xi}(z)_+} \right) \ln \frac{\mu^2}{k_\perp^2} + \delta(z) [(\ln \xi + 2)\gamma(\xi) - \xi^2] - \left( \frac{1}{z} \right)_+ [\xi^2 - \gamma(\xi) \ln \xi \bar{\xi}^2] \frac{x}{\bar{\xi}} - \left( \frac{\ln x z}{z} \right)_+ \frac{x \gamma(\xi)}{\bar{\xi}} \right\},
 \end{aligned}$$
  

$$\begin{aligned}
 & \text{Diagram 3} + \text{Diagram 4} = \frac{e^2 \alpha_s C_F}{\pi k_\perp^2} \left\{ \left( -\frac{2}{\epsilon_c} + \ln \frac{k_\perp^2}{\tilde{\mu}_c^2} \right) \left[ \delta(z) (\gamma(\xi) \ln \bar{\xi} + 2 - \xi) + \xi + \frac{\gamma(\xi)}{(z)_+} - 2 \right] + \delta(z) \left[ \frac{\gamma(\xi)}{4} \ln^2 \xi + \left( \xi - 2 - \frac{3}{2} \gamma(\xi) \ln \bar{\xi} \right) \ln \xi + \xi^2 \ln \bar{\xi} + 2\xi - 4 \right. \right. \\
 & \quad \left. \left. + \frac{\pi^2 \gamma(\xi)}{6} + \frac{\gamma(\xi)}{2} \text{Li}_2(\xi) + \frac{\gamma(\xi)}{2} \text{Li}_2(-\bar{\xi}/\xi) \right] + \left( \frac{1}{z} \right)_+ \left[ \xi^2 + \frac{\gamma(\xi)}{2} \ln \frac{\bar{z}^2}{x \xi^2 \bar{\xi}^2} \right] + \frac{\gamma(\xi)}{2} \left( \frac{\ln x}{z} \right)_+ + \gamma(\xi) \left( \frac{\ln z}{z} \right)_+ + (2 - \xi) \ln \frac{\xi \bar{\xi}}{z \bar{z}} \right\}
 \end{aligned}$$
  

$$\begin{aligned}
 & \text{Diagram 5} = \frac{e^2 \alpha_s C_F}{\pi} \delta(z) \frac{1}{k_\perp^2} \left[ \frac{1}{2} \gamma(\xi) \ln \frac{\xi \mu^2}{k_\perp^2} + \xi \bar{\xi} \left( \frac{2}{\epsilon_c} + \ln \frac{\xi \tilde{\mu}_c^2}{k_\perp^2} \right) + \frac{3}{2} \xi \bar{\xi} + 1 \right]
 \end{aligned}$$

Light-cone singularities canceled between real and virtual diagrams.

$$\begin{aligned}
 & \text{Diagram 6} = \frac{e^2 \alpha_s C_F}{\pi k_\perp^2} \left[ \frac{\xi x}{\bar{z}} \left( \frac{2}{\epsilon_c} + \ln \frac{\tilde{\mu}_c^2}{k_\perp^2} \right) + \left( \frac{\xi x}{\bar{z}} + \frac{\bar{x}}{2} \right) \ln \frac{\xi x}{z \bar{z}^2} + \frac{\bar{x}}{2} \ln \frac{x z}{\xi \bar{\xi}^2} + \frac{\gamma(\xi)x + \xi}{\bar{\xi} \bar{z}} - \frac{\bar{z}}{\bar{\xi}} + \xi - x \right]
 \end{aligned}$$
  

$$\begin{aligned}
 & \text{Diagram 7} = \frac{e^2 \alpha_s C_F}{\pi k_\perp^2 \bar{\xi}^2} \left[ \gamma(\xi) z \left( \ln \frac{\mu^2}{k_\perp^2} + \ln \frac{\xi \bar{\xi}^2}{x z} - 1 \right) + \xi(x - \xi z) \right]
 \end{aligned}$$
  

$$\begin{aligned}
 & \text{Diagram 8} = \frac{e^2 \alpha_s C_F}{\pi k_\perp^2} \left[ -z \left( 1 + \frac{x^2}{\bar{z}^2} \right) \left( \frac{2}{\epsilon_c} + \ln \frac{\tilde{\mu}_c^2}{k_\perp^2} + \ln \frac{\xi x}{z \bar{z}^2} \right) + \frac{\xi \bar{\xi}}{\bar{z}} \right]
 \end{aligned}$$

# Factorization of integrated fracture function

The divergent term is correctly factorized into the quark PDF:

$$F_q(x, \xi, k_\perp) \Big|_{div.} = \frac{\alpha_s e^2}{\pi k_\perp^2} \left( -\frac{2}{\epsilon_c} \right) \gamma(\xi/y) P_{qq}(y), \quad P_{qq}(y) = C_F \left( \frac{1+y^2}{(1-y)_+} + \frac{3}{2} \delta(1-y) \right), \quad y = x + \xi$$

→ 
$$\begin{aligned} \mathcal{H}_q^{(1)} = & \frac{e^2 \alpha_s C_F}{\pi k_\perp^2} \left\{ \ln \frac{\mu^2}{k_\perp^2} \left[ \gamma(\xi) \gamma(\bar{y}/\bar{\xi}) \left( \frac{1}{(\bar{y})_+} - \delta(\bar{y}) \ln \bar{\xi} \right) - \gamma(\xi/y) \frac{1+y^2}{(\bar{y})_+} \right] \right. \\ & + \delta(\bar{y}) \left[ \gamma(\xi) \left( -3 \ln \xi \ln \bar{\xi} + 2 \ln \bar{\xi} + \frac{1}{2} \ln^2 \xi + \text{Li}_2 \left( -\frac{\bar{\xi}}{\xi} \right) + \text{Li}_2(\xi) + \frac{\pi^2}{6} - 4 \right) - 4\bar{\xi} (\ln \bar{\xi} - 2) + \xi \right] \\ & + \gamma(\xi) \left( \frac{1}{\bar{y}} \right)_+ \left( 2 \frac{\bar{y}}{\bar{\xi}} \ln \frac{y-\xi}{\xi \bar{\xi}^2} - \ln \frac{(y-\xi)^2}{y^2 \bar{\xi}^2} - \frac{2\xi^2}{y^2 \gamma(\xi)} \right) + \left( 2y - \xi - (y+1) \frac{(y-\xi)^2}{y^2} \right) \ln \frac{\xi(y-\xi)}{y^2 \bar{y}} \\ & \left. + \left( \bar{y} + \xi - \gamma(\xi) \frac{\bar{y}}{\bar{\xi}^2} \right) \ln \frac{\bar{y}(y-\xi)}{\xi \bar{\xi}^2} - 2\bar{\xi} \ln \frac{y}{\xi \bar{\xi}} + \frac{2}{\bar{\xi}} \ln \bar{y} - 2 \ln \frac{y \bar{y}}{\xi \bar{\xi}} + \frac{\xi^2 - \xi \bar{\xi}}{y} + \frac{5\bar{y} + 3}{\bar{\xi}} - \frac{3\bar{y}}{\bar{\xi}^2} + \frac{\xi^2}{y^2} - 3 \right\} + \mathcal{O}(\alpha_s^2) \end{aligned}$$

The hard coefficients for the gluon initial state are the same with those in  $b$ -space, because there is no U.V. divergence, i.e.,  $\mathcal{H}_g = \hat{\mathcal{H}}_g + \mathcal{O}(\alpha_s^2)$ ,  $\mathcal{H}_{gq} = \hat{\mathcal{H}}_{gq} + \mathcal{O}(\alpha_s^2)$

The  $\mu$ -evolution equation:

$$\frac{\partial F_q(x, \xi, k_\perp, \mu)}{\partial \ln \mu} = \frac{\alpha_s}{\pi} \int_x^{1-\xi} \frac{dy}{y} P_{qq}(x/y) F_q(y, \xi, k_\perp, \mu) + \dots$$

Same as that for the standard quark collinear PDF!



# Contents

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# Summary

- ◆ The hadronic tensor for the diffractive lepton pair and photon production in hadron-hadron collisions can be factorized in terms of TMD or integrated fracture functions for different kinematic regions.
- ◆ Explicit one loop calculations show that both the TMD and integrated fracture functions can be further factorized into parton distribution and fragmentation functions when the transverse momentum is large.
- ◆ The factorizations are done in momentum and impact parameter space respectively. Perturbative coefficients are extracted.
- ◆ The evolution equations for TMD and integrated fracture functions are derived. They take exactly the same form as those for quark TMD and collinear PDFs respectively.

Thank you!