# Parton distribution functions from lattice QCD

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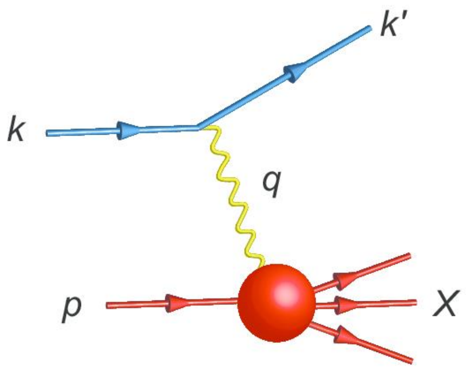
# Introduction

- **Deep-inelastic lepton-nucleon scattering**
  - DIS X-sec is an incoherent sum of virtual photon-quark partonic X-sec, weighted by the probability of finding each quark in the proton with a given longitudinal mom. fraction




$$\frac{d^2\sigma}{dx dQ^2} \sim \sum_{q,\bar{q}} \int_x^1 \frac{dz}{z} f_q(z) \hat{\sigma}_{q\gamma^* \to X}(x/z)$$

Parton distribution function (PDF), non-perturbative, universal

Perturbatively calculable, asymptotic freedom of QCD

# Introduction

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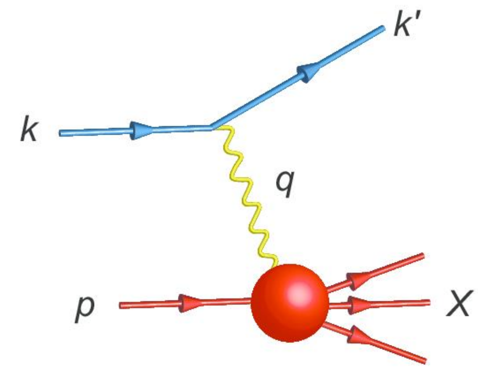


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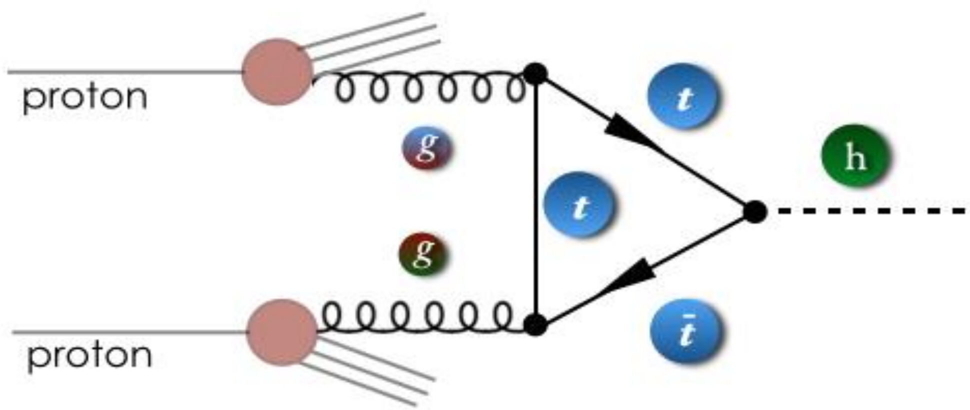
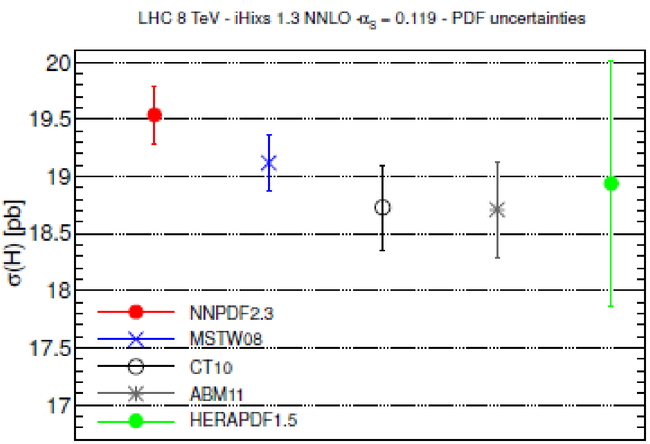
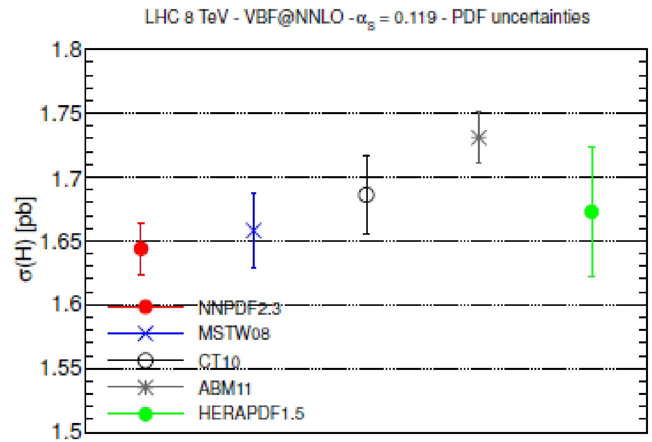
- ***PP* Collision (LHC)**



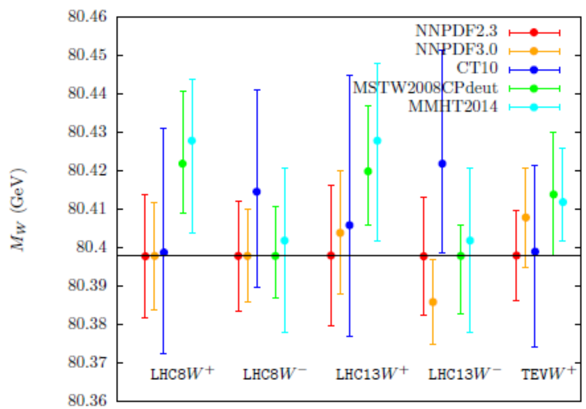
# Introduction

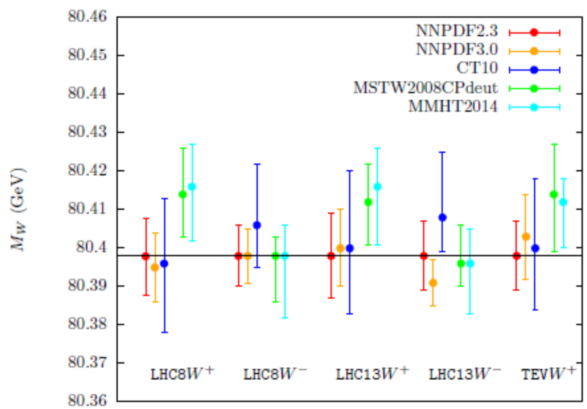
- Precise knowledge of PDFs helps
  - Acquire a better understanding of the SM
    - Higgs production and self coupling




    - W mass determination




  - Disentangle new physics effects from SM

# Introduction

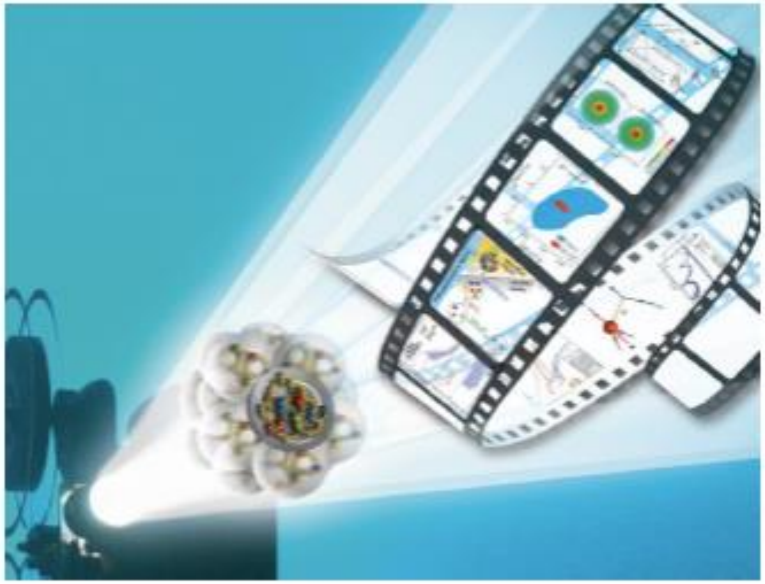
- Precise knowledge of PDFs helps
  - Three-dim. imaging of the nucleon

**Electron Ion Collider:**
**The Next QCD Frontier**

Imaging of the proton

*How are the sea quarks and gluons, and their spins, distributed in space and momentum inside the nucleon?*

EIC White Paper, 1212.1701



- *Ab initio* determination of PDFs from lattice QCD is desirable and provides complementary information to global analysis program

# Introduction

- Parton picture arises in high-energy collisions where hadrons/probe move nearly at the speed of light, or with infinite momentum

- Parton physics usually formulated in terms of light-cone quantization [Dirac, Wilson]
  - light-cone coordinates $\xi^{\pm} = (t \pm z)/\sqrt{2}$
  - Example: [Collins and Soper, NPB 82']

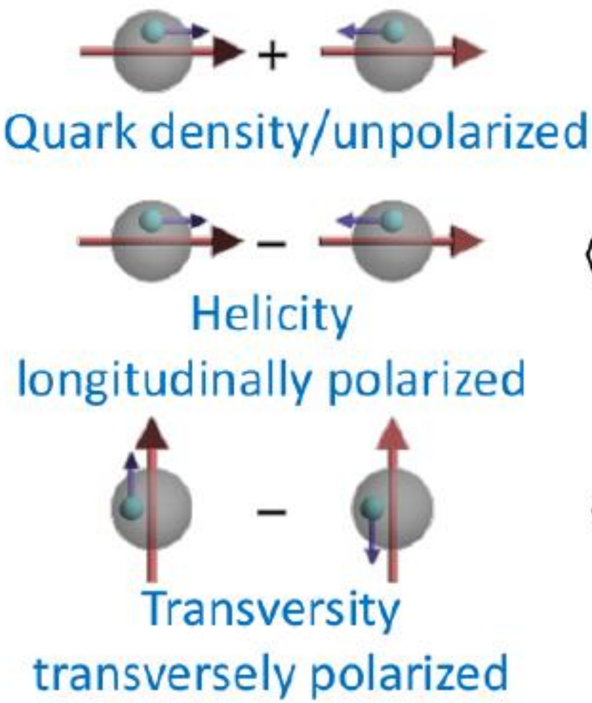
$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P|\overline{\psi}(\xi^-)\gamma^+ \exp\left(-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\psi(0)|P\rangle$$

  - Difficult to access from ***ab initio*** approaches like lattice QCD

$$t^2 - x^2 = 0 \rightarrow -\tau^2 - x^2 = 0$$

# Introduction

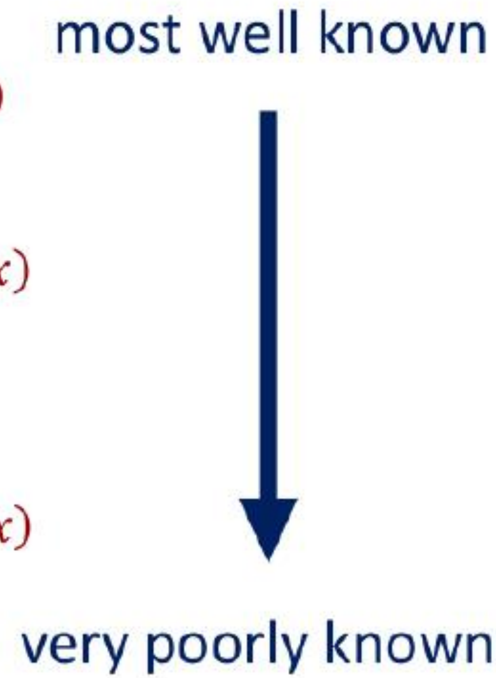
- Lattice effort on calculating Mellin moments of PDFs




$$\langle x^n \rangle_q = \int_{-1}^{1} dx\, x^n q(x)$$

$$\langle x^n \rangle_{\Delta q} = \int_{-1}^{1} dx\, x^n \Delta q(x)$$

$$\langle x^n \rangle_{\delta q} = \int_{-1}^{1} dx\, x^n \delta q(x)$$




- PDFs can be fully reconstructed if all their moments are known

# Introduction

- Lattice effort on calculating Mellin moments of PDFs

Full details of lattice-QCD calculations of higher moments of unpolarized and polarized PDFs.

| Ref. | Observables | Sea quarks | Valence quarks | Renormalization |
|---|---|---|---|---|
| LHPC '10[b] [262] | $\langle x\rangle_{u^+-d^+}$, $\langle x^2\rangle_{u^--d^-}$, $g_A$, $\langle x\rangle_{\Delta u^- - \Delta d^-}$, $\langle x^2\rangle_{\Delta u^+ - \Delta d^+}$ | 2+1 staggered | domain wall | one-loop $Z_{\mathcal{O}}/Z_A$ |
| $\chi$QCD '09 [280] | $\langle x\rangle_{u^+,d^+,s^+}$ (superseded by $\chi$QCD '15), $\langle x^2\rangle_{u^-,d^-,s^-}$ | quenched | Wilson | one loop |
| LHPC '08 [295] | superseded by LHPC '10 | | | |
| QCDSF '05c [313] | $\langle x^2\rangle_{\Delta u^+ - \Delta d^+}$ | 2 clover | clover | Rome–Southampton |
| QCDSF '05b [93] | $\langle x\rangle_{u^+-d^+}$, $\langle x^2\rangle_{u^--d^-}$, $\langle x^3\rangle_{u^+-d^+}$ | quenched | clover | Rome–Southampton |
| QCDSF '05a[a] [314] | $\langle x\rangle_{u^+-d^+}$, $\langle x^2\rangle_{u^--d^-}$, $\langle x^3\rangle_{u^+-d^+}$ | 2 clover | clover | one loop |
| LHPC and SESAM '02 [279] | $\langle x\rangle_{u^+-d^+}$, $\langle x^2\rangle_{u^--d^-}$, $\langle x^3\rangle_{u^+-d^+}$, $g_A$, $\langle x\rangle_{\Delta u^- - \Delta d^-}$, $\langle x^2\rangle_{\Delta u^+ - \Delta d^+}$ | 2 Wilson and quenched | Wilson | one loop |
| QCDSF '01 [315] | $\langle x^2\rangle_{\Delta u^+ - \Delta d^+}$ | quenched | clover | Rome–Southampton |
| QCDSF '96 [100] | $\langle x\rangle_{u^+-d^+}$, $\langle x^2\rangle_{u^--d^-}$, $\langle x^3\rangle_{u^+-d^+}$, $g_A$, $\langle x^2\rangle_{\Delta u^+ - \Delta d^+}$ | quenched | Wilson | one loop |

- Difficulties in going beyond the first few moments
  - Power divergent mixing between high and low moments operators
  - Increasing stochastic noise for operators with many derivatives

# Introduction

- Obstacle to access PDFs on the lattice
  - They are defined on the light-cone

- However, they were originally introduced by Feynman as the infinite momentum limit of frame-dependent quantities

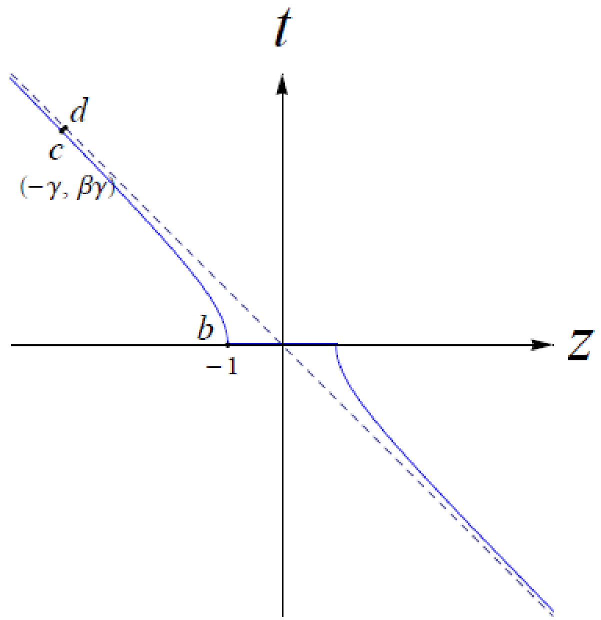
$$q(x) = \lim_{P_z \to \infty} \tilde{q}(x, P_z)$$

- Boost to infinite momentum leads to light-cone correlations

- If we can construct a $\tilde{q}(x, P_z)$ such that it is calculable on the lattice, and all $P_z$-dependence can be systematically removed

- Then we can calculate $q(x)$!

# Introduction



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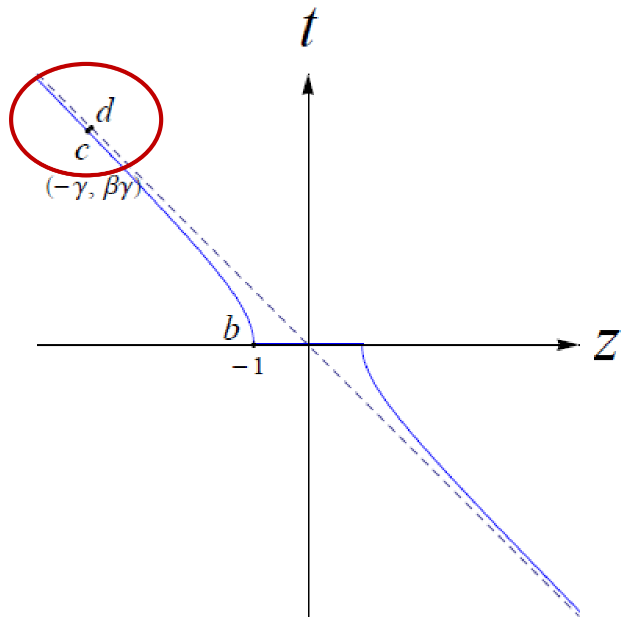

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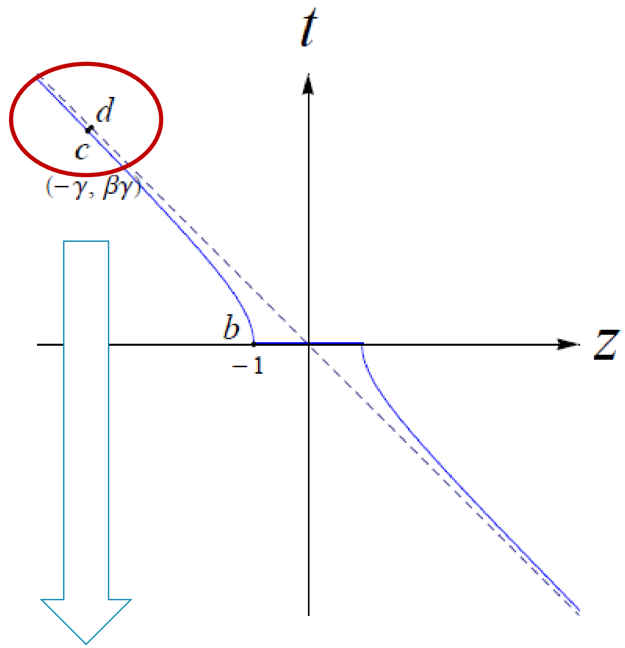
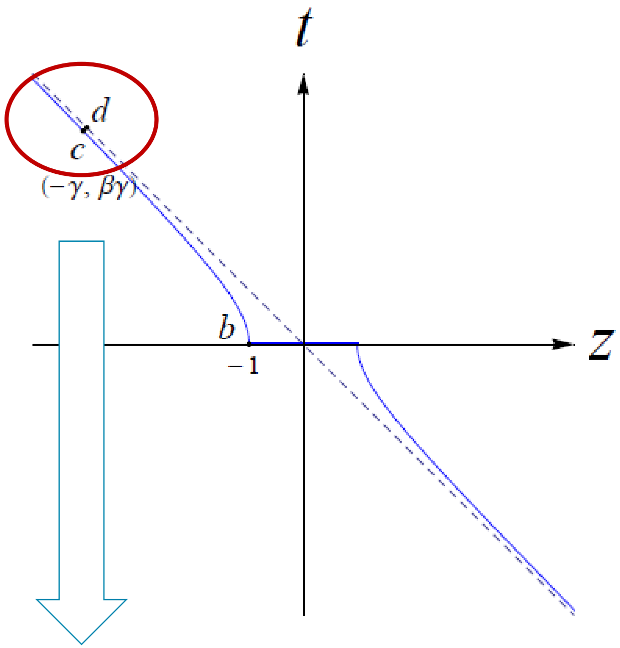



- Systematic connection through large momentum effective theory (LaMET) [Ji, PRL 13', Sci. China Phys. Mech. Astron. 14']

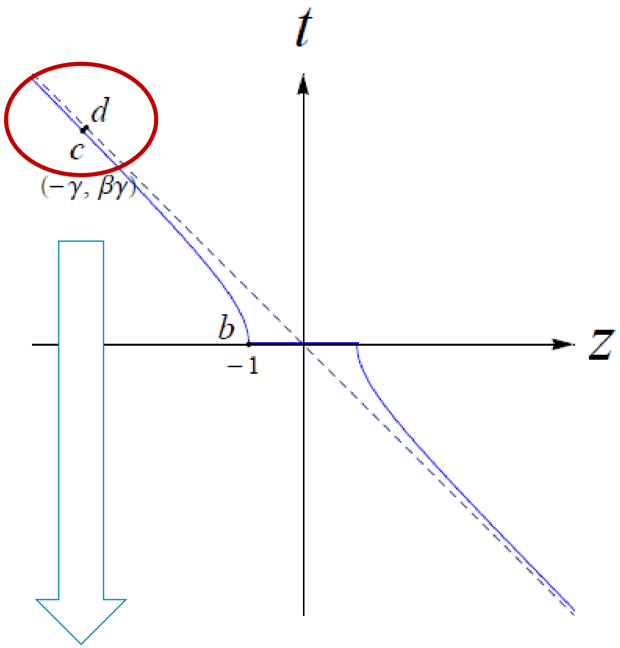
# Introduction




- Systematic connection through large momentum effective theory (LaMET) [Ji, PRL 13′, Sci. China Phys. Mech. Astron. 14′]
  - Appropriately chosen $\tilde{q}(x, P_z)$ can be calculated on the Euclidean lattice, e.g.

$$\tilde{q}(x,\Lambda,P^z)=\int_{-\infty}^{\infty}\frac{dz}{4\pi}e^{izk^z}\langle P|\overline{\psi}(0,0_\perp,z)\gamma^z\exp\left(-ig\int_0^z dz' A^z(0,0_\perp,z')\right)\psi(0)|P\rangle$$

# Introduction

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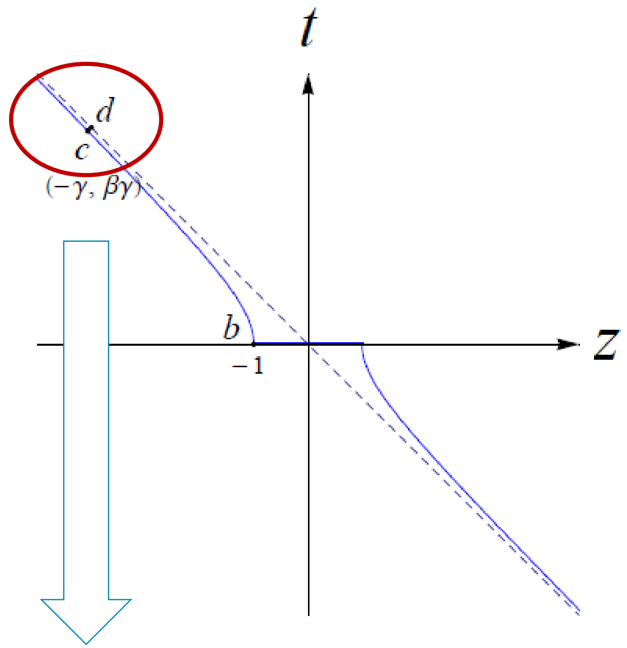
  - A finite but large $P_z$ already offers a good approximation, where (leading) frame-dependence can be removed through a factorization formula

$$\tilde{q}(x, P_z, p_z^R, \mu_R) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y}, r, \frac{yP_z}{\mu}, \frac{yP_z}{p_z^R}\right) q(y, \mu) + \mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{P_z^2}\right)$$

# Introduction




- Systematic connection through large momentum effective theory (LaMET) [Ji, PRL 13′, Sci. China Phys. Mech. Astron. 14′]

- Parton model is an effective theory for the nucleon moving at large momentum

# Related proposals

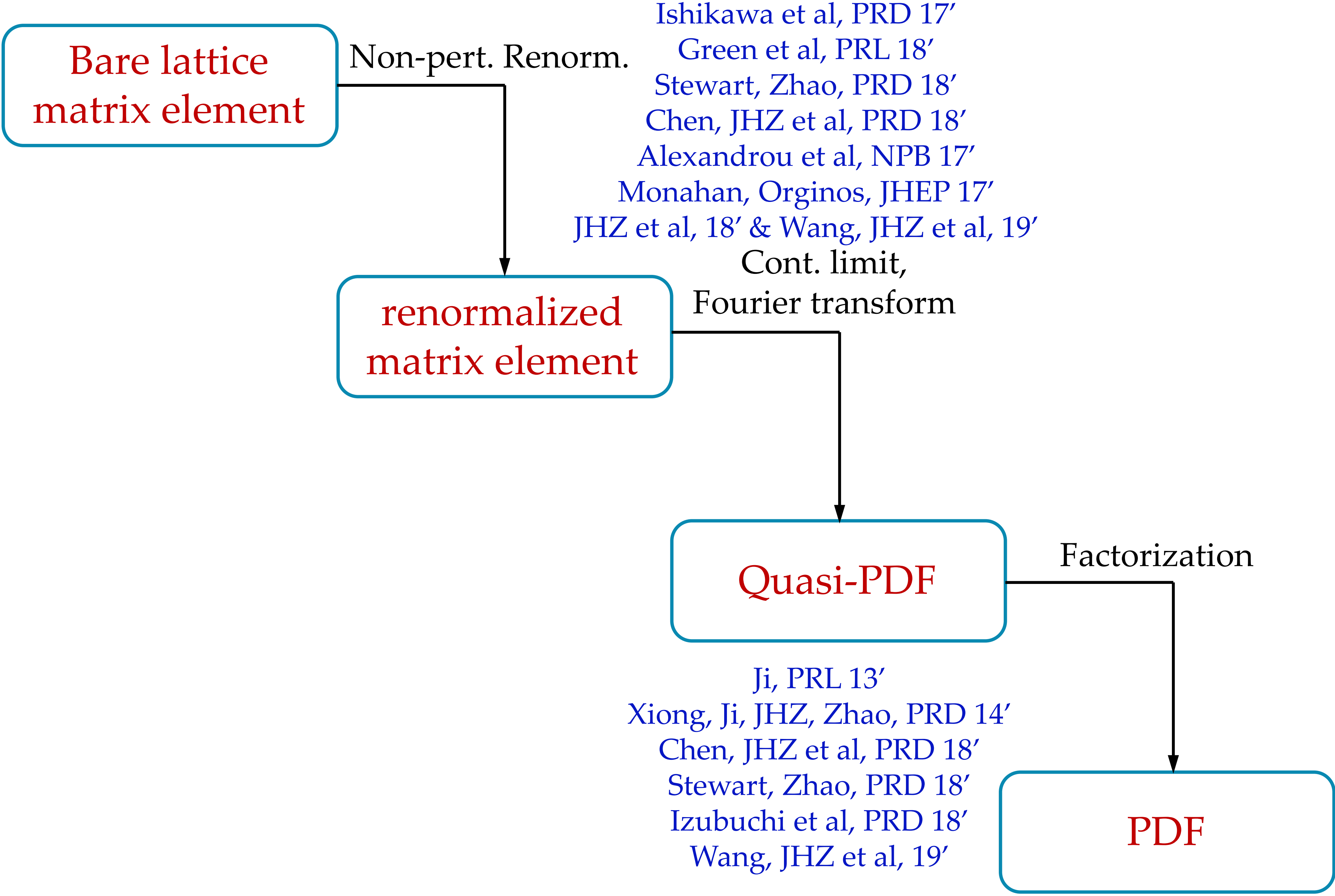
- **They all share the same property of computing correlations at spacelike separations**

- Current-current correlation functions
- [Liu and Dong, PRL 94′]
- [Detmold and Lin, PRD 06′]
- [Braun and Müller, EPJC 08′]
- [Davoudi and Savage, PRD 12′]
- [Chambers et al., PRL 17′]

- Lattice cross sections
- [Ma and Qiu, 14′ & PRL 17′]

- Ioffe-time /pseudo-distribution
- [Radyushkin, PRD 17′]

# PDFs from LaMET



# PDFs from LaMET

Ji, JHZ, Zhao, PRL 18'
Ishikawa et al, PRD 17'
Green et al, PRL 18'
Stewart, Zhao, PRD 18'
Chen, JHZ et al, PRD 18'
Alexandrou et al, NPB 17'
Monahan, Orginos, JHEP 17'
JHZ et al, 18' & Wang, JHZ et al, 19'

Bare lattice matrix element → Non-pert. Renorm. → renormalized matrix element

$$\tilde{h}(z, P_z, a^{-1}) = \frac{1}{2P^0}\langle P|O_{\gamma^t}(z)|P\rangle$$

$$O_\Gamma(z) = \bar{\psi}(z)\Gamma U(z,0)\psi(0)$$

$$U(z,0) = P\exp\left(-ig\int_0^z dz' A_z(z')\right)$$

$$\tilde{h}_R(z, P_z, p_z^R, \mu_R) = Z^{-1}(z, p_z^R, a^{-1}, \mu_R)\tilde{h}(z, P_z, a^{-1})\Big|_{a\to 0}$$

$$Z(z, p_z^R, a^{-1}, \mu_R) = \frac{\sum_s \langle p,s|O_{\gamma^t}(z)|p,s\rangle}{\sum_s \langle p,s|O_{\gamma^t}(z)|p,s\rangle_{\text{tree}}}\Bigg|_{\substack{p^2=-\mu_R^2 \\ p_z = p_z^R}}$$

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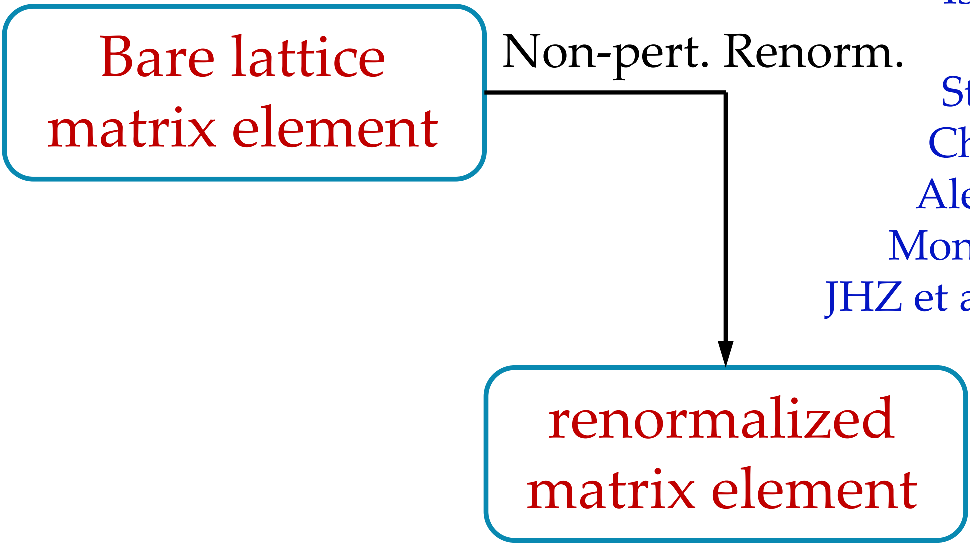
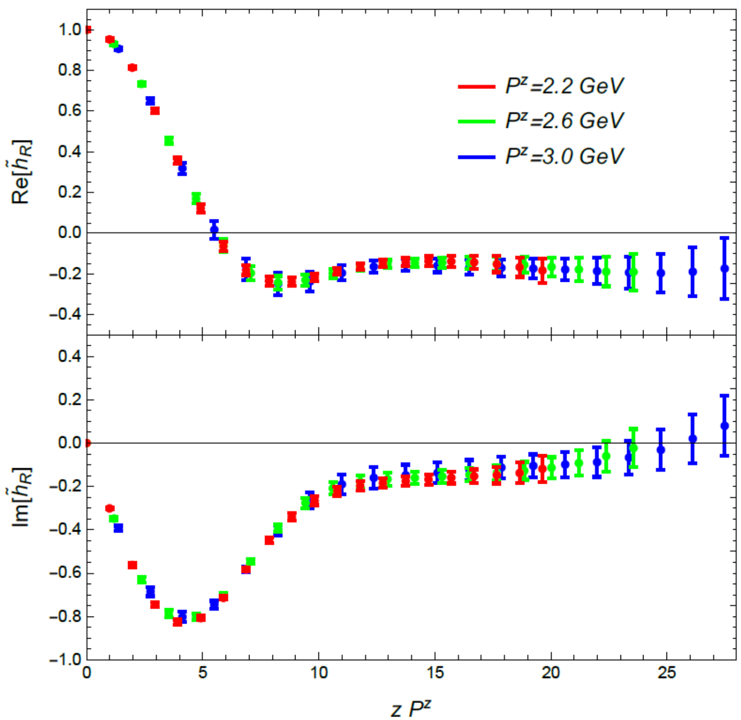
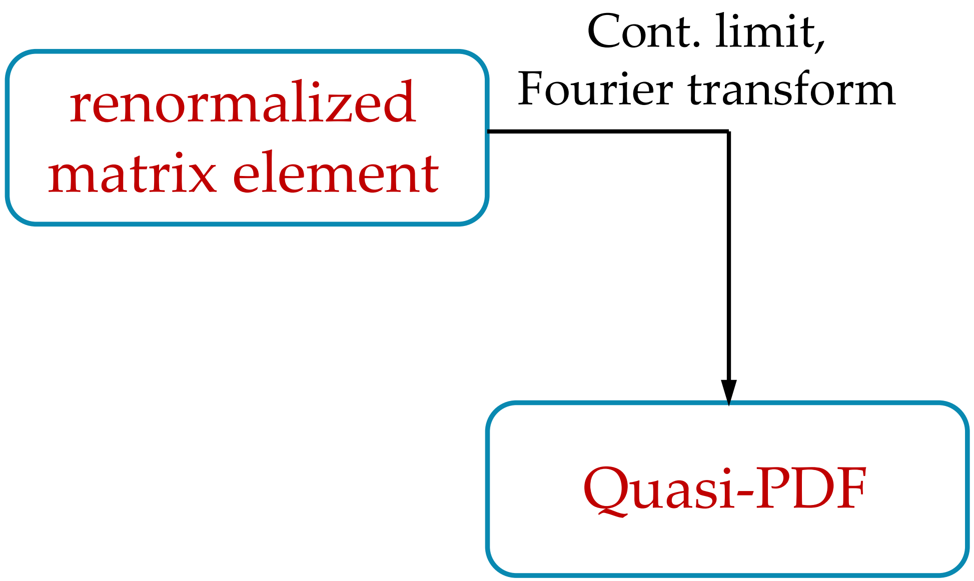


# PDFs from LaMET




$$\tilde{q}_R(x, P_z, p_z^R, \mu_R) = P_z \int \frac{dz}{2\pi} \, e^{ixP_z z} \tilde{h}_R(z, P_z, p_z^R, \mu_R)$$

# PDFs from LaMET

$$\tilde{q}_R(x, P_z, p_z^R, \mu_R) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y}, r, \frac{yP_z}{\mu}, \frac{yP_z}{p_z^R}\right) q(y, \mu) + \mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{P_z^2}\right)$$

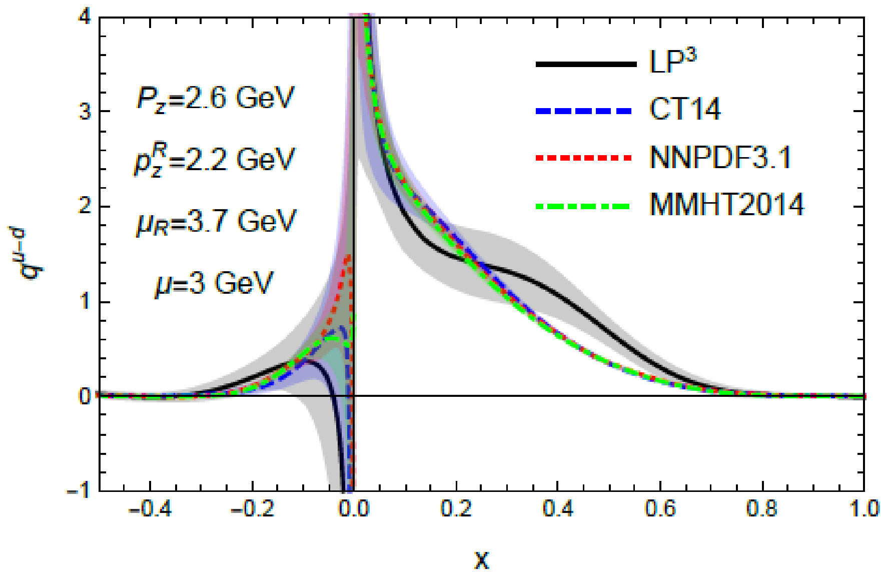
Quasi-PDF

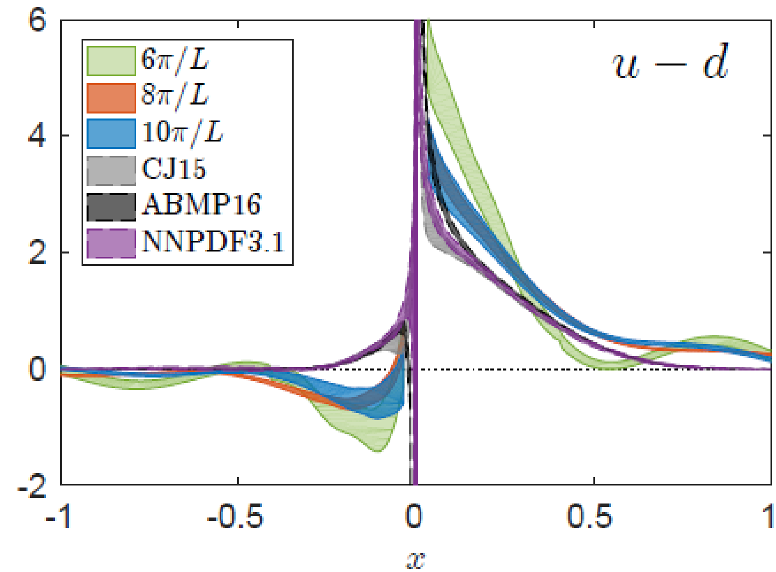
Factorization

PDF

Ji, PRL 13'
Xiong, Ji, JHZ, Zhao, PRD 14'
Chen, JHZ et al, PRD 18'
Stewart, Zhao, PRD 18'
Izubuchi et al, PRD 18'
Wang, JHZ et al, 19'

# Isovector quark unpolarized PDF



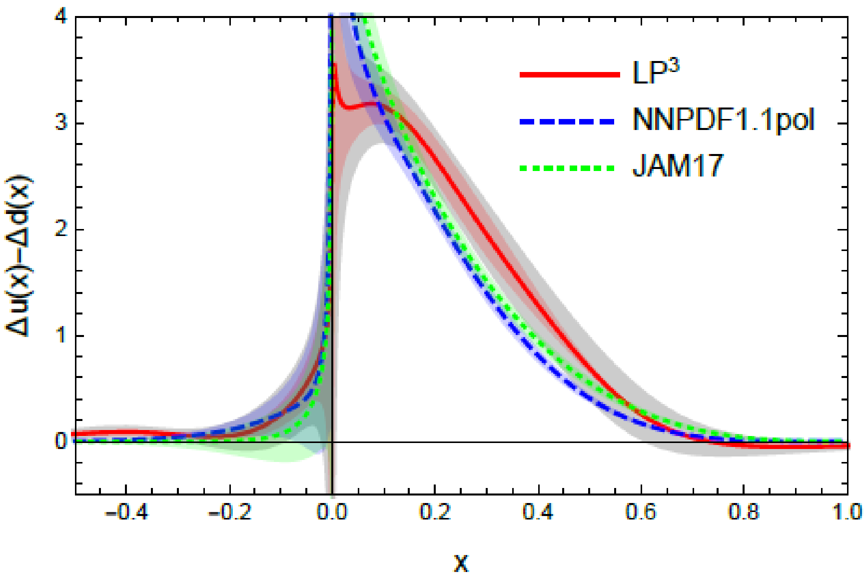




Liu, JHZ et al, 1803.04393 & 1807.06566, $m_\pi \approx 135\ MeV, a = 0.09\ fm, L \approx 5.8 fm$

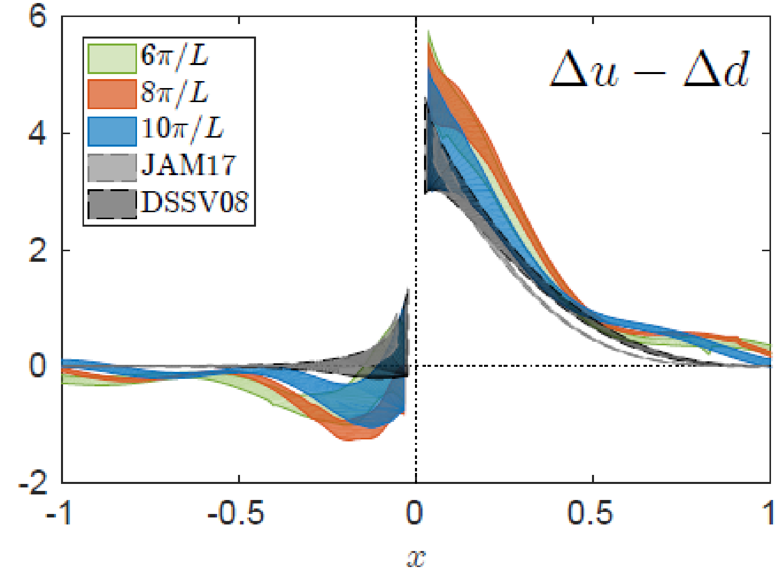
Alexandrou et al, 1803.02685, $m_\pi \approx 130\ MeV,\ a = 0.094\ fm, L \approx 4.5 fm$

- Systematics
  - Higher-order +higher-twist corrections
  - Continuum extrapolation
  - Discretization errors
  - Finite volume effects

# Isovector quark helicity PDF




Lin, JHZ et al, 1807.07431, $m_\pi \approx 135\ MeV, a = 0.09\ fm, L \approx 5.8 fm$

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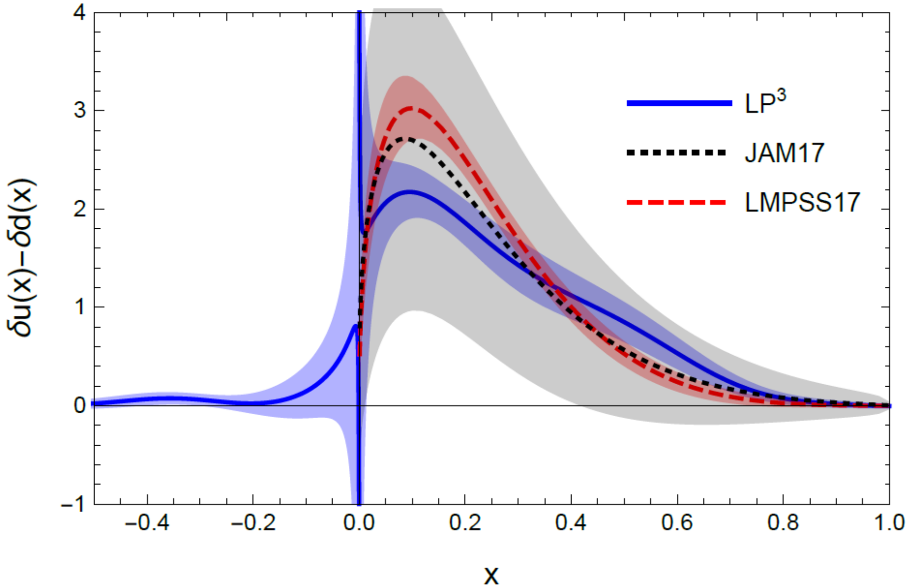
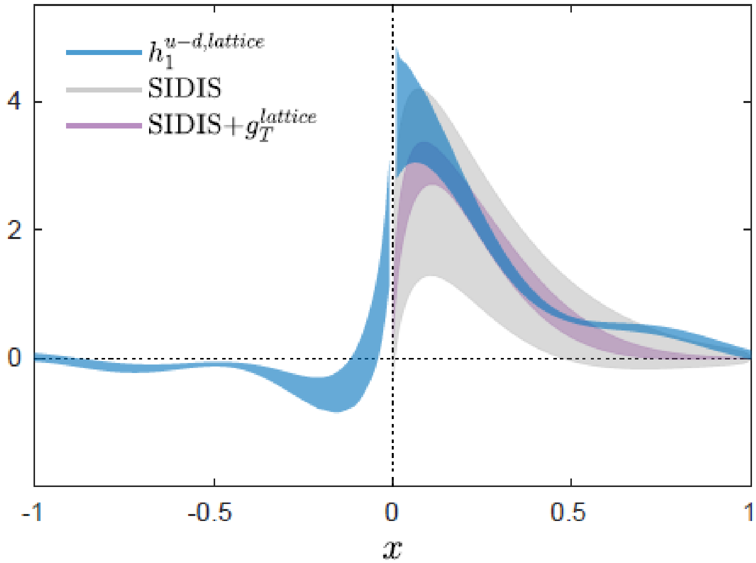
# Isovector quark transversity PDF




LP3, $m_\pi \approx 135\ MeV, a = 0.09\ fm, L \approx 5.8 fm$

Alexandrou et al, 1807.00232, $m_\pi \approx 130\ MeV,\ a = 0.094\ fm, L \approx 4.5 fm$

- Systematics
  - Higher-order +higher-twist corrections
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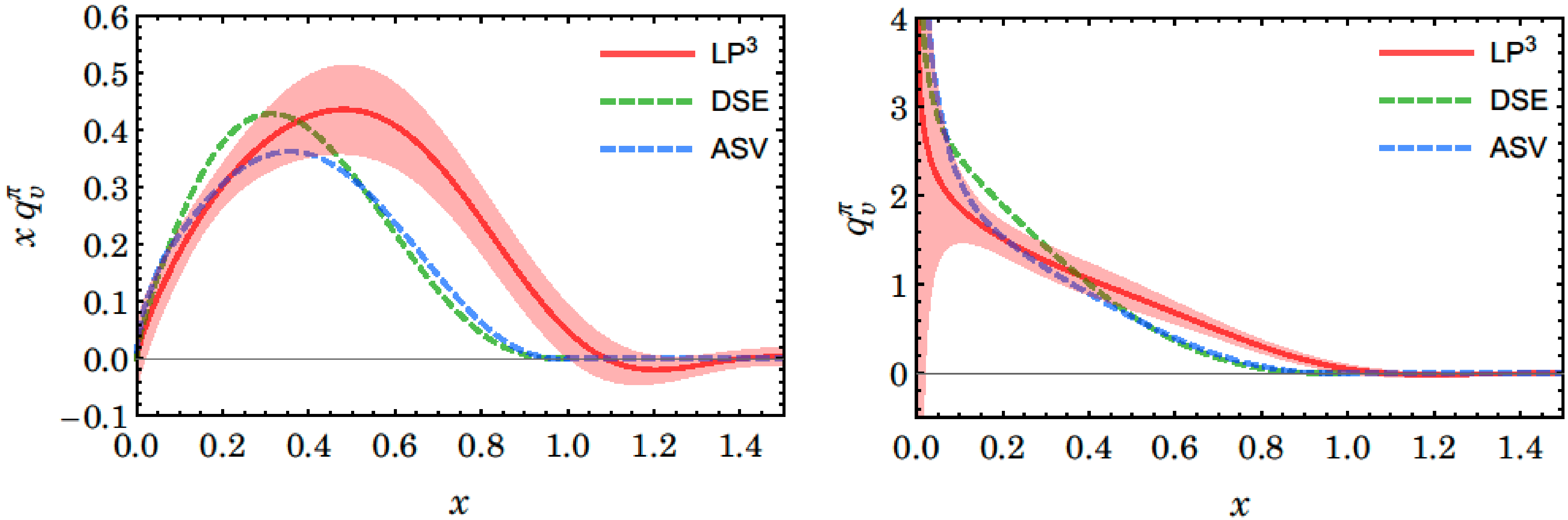
# Summary and outlook

- Recent years have witnessed rapid progress on direct computations of x-dependence of hadron structure from lattice QCD
- **Large momentum effective theory** & related proposals
- Applications to nucleon PDFs (isovector quark), as well as meson PDFs & DAs have yielded encouraging results
- Flavor-singlet quark PDF and Gluon PDF
  - Renormalization and factorization
    - [JHZ et al, 1808.10824] (see also [Li, Ma, Qiu, 1809.01836])
    - [Wang, JHZ et al, 1904.00978]
- GPDs, TMDs …

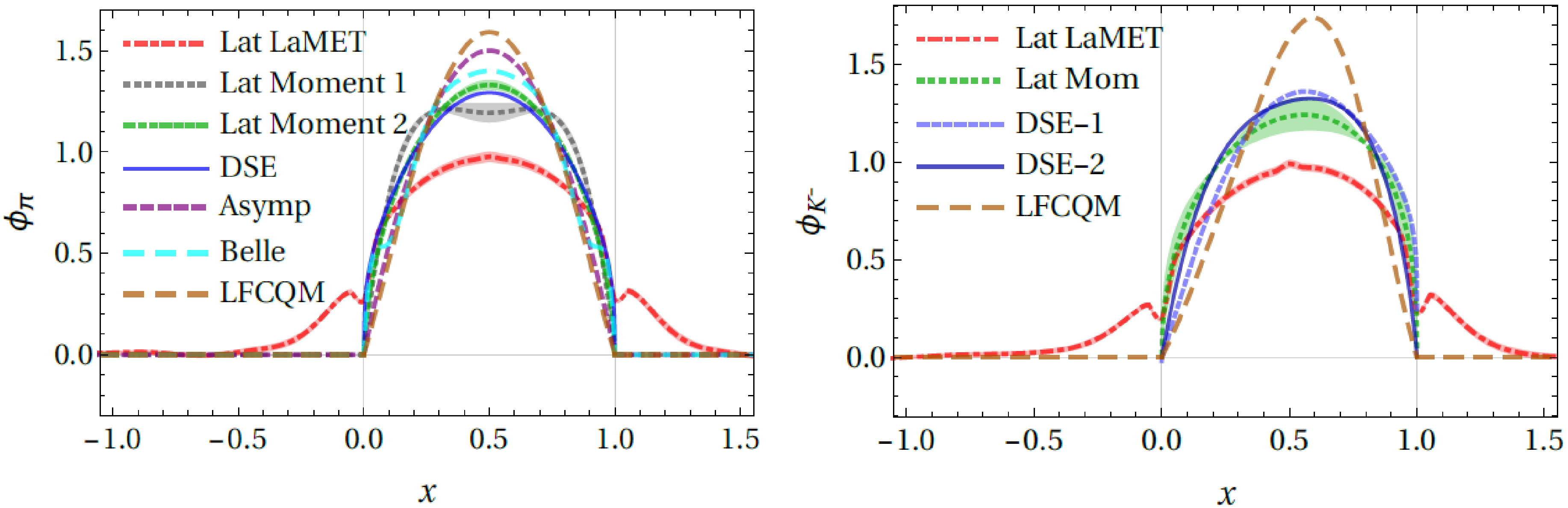
# Backup Slides

# Pion valence quark PDF




JHZ et al, 1804.01483, $m_\pi \approx$ 310 $MeV, a = 0.12\ fm, L \approx 3fm$

# Meson distribution amplitudes




Chen, JHZ et al, 1712.10025, $m_\pi \approx 310\ MeV, a = 0.12\ fm, L \approx 3 fm$