Testing collinear factorization in a spectator model with mass corrections

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HAMPTON UNIVERSITY
THE STANDARD OF EXCELLENCE

Jefferson Lab
Thomas Jefferson National Accelerator Facility

Accessing PDFs at small Q²

Hard scattering reactions: Picture of the nucleons (partons)

Examples:

- Deep Inelastic Scattering (DIS): $e^- + p \to e^- + X$
- Semi-inclusive Deep Inelastic Scattering: $e^- + p \to e^- + h + X$

perturbative PDF

Relies on factorization:

$$\sigma^{\rm DIS} \propto \hat{\sigma} \otimes f + \mathcal{O}(\mu^2/Q^2)$$

$$\sigma^{\rm SIDIS} \propto \hat{\sigma} \otimes f \otimes D^h + \mathcal{O}(\mu^2/Q^2)$$

Fragmentation Function (FF)

- Kinematical effects $\mu = m$
- Dynamical effects $\mu = \Lambda_{\rm QCD}$

- Use data at JLab (but also HERMES & COMPASS)
- Need theory control of corrections

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Hadron Mass Effects

Let's consider an example for Pion Mass effects at JLab.

Accardi et al JHEP 0911, 084 (2009)

Jefferson Lab experiments:

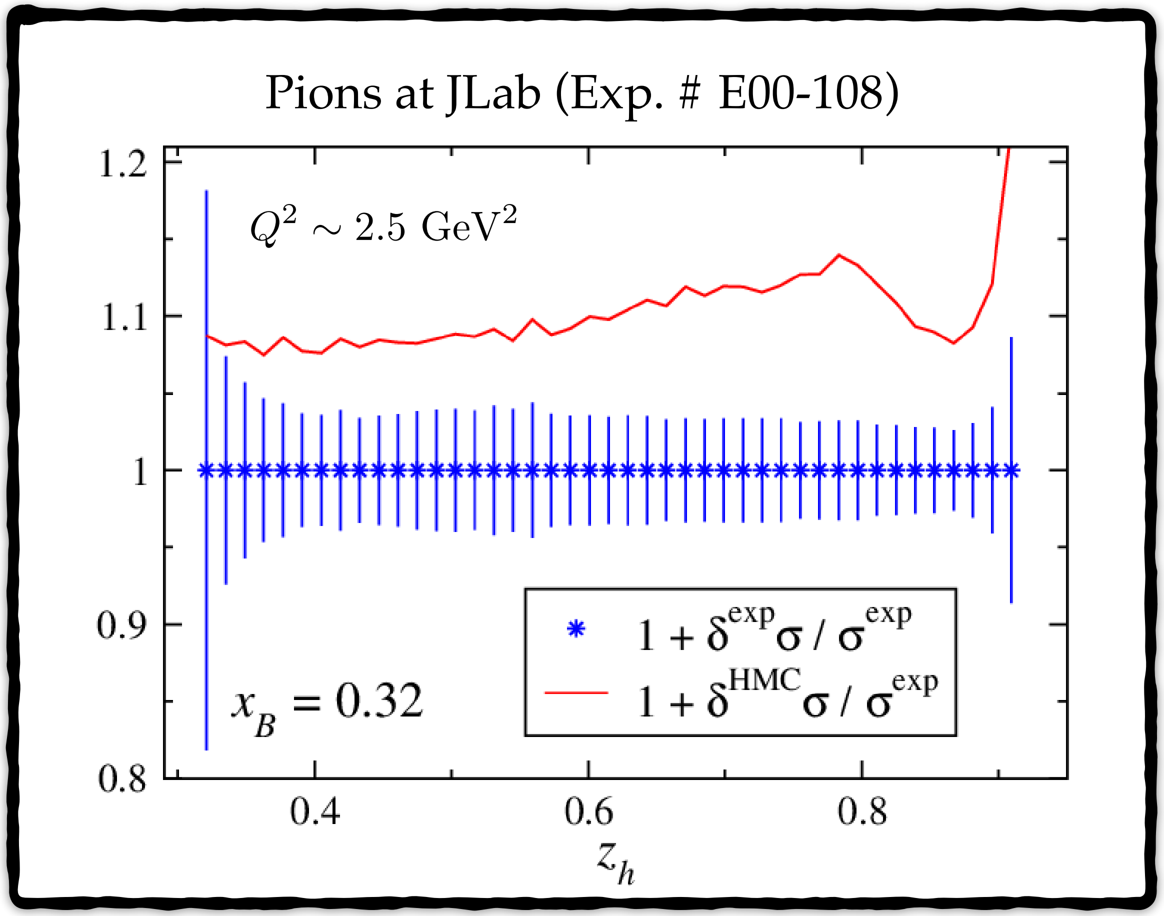
- Usually low Q^2.
- $1/Q^2$ corrections have to be controlled.

$O(m^2/Q^2)$ = Hadron Mass Corrections (HMCs)

$$m = M_P, m_\pi$$

$$x_B \longrightarrow \xi_h \equiv \xi\left(1+\frac{m_h^2}{\zeta_h Q^2}\right)$$

$$z_h \longrightarrow \zeta_h = \frac{z_h}{2}\frac{\xi}{x_B}\left(1+\sqrt{1-\frac{4x_B^2 M^2 m_h^2}{zh z_h^2\ Q^4}}\right)$$



$$m_\pi \sim 0.14 \text{ GeV}$$

$$\xi = \frac{2x_B}{1+\sqrt{1+4x_B^2 M^2/Q^2}}$$

Guerrero et al JHEP 1509, 169 (2015)

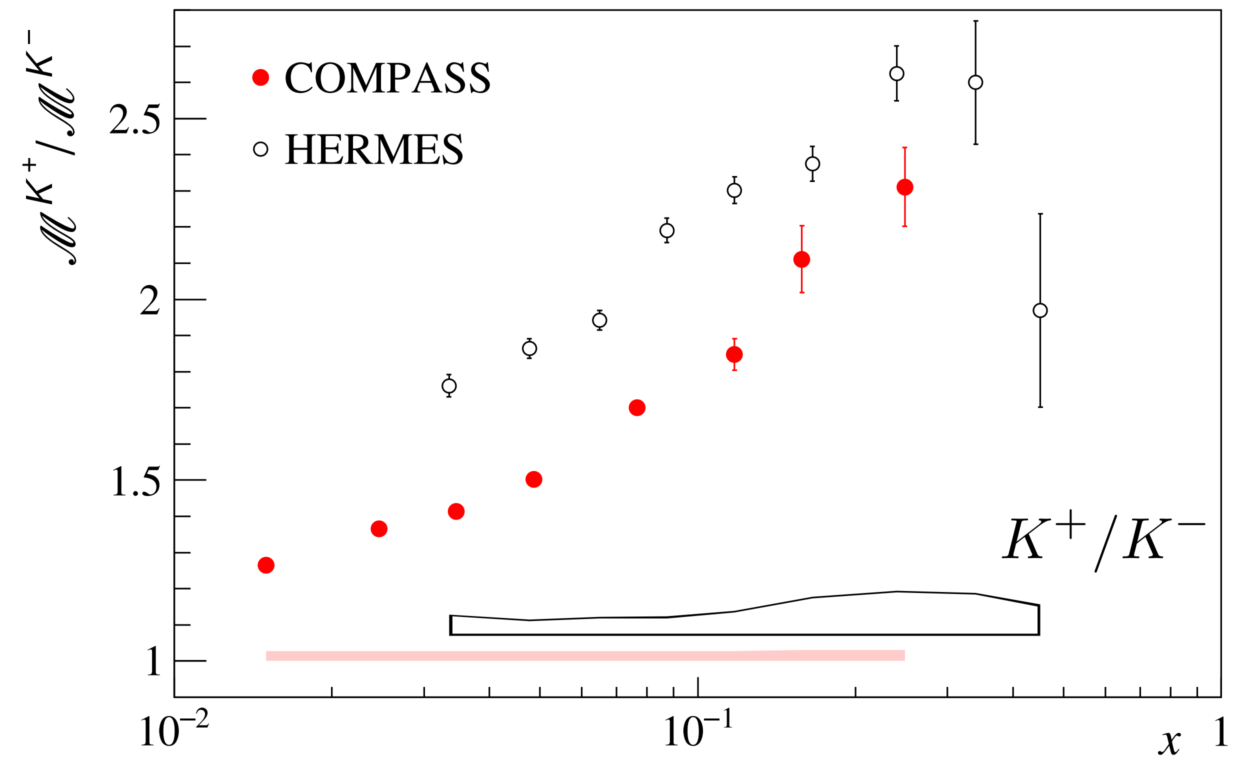
Integrated Kaon Multiplicities: SIDIS on Deuteron

Experimentally
HERMES, COMPASS: $M^K_{exp} = \dfrac{\int_{exp} dQ^2 \int_{0.2}^{0.8} dz_h \frac{dN^K}{dx_B dQ^2 dz_h}}{\int_{exp} dQ^2 \frac{dN^e}{dx_B dQ^2}}$

proxy to access s-pdf

COMPASS collaboration, PLB 767 (2017) 133

- Size discrepancy between these two measurements
- Even worst for the case of sum: $K^+ + K^-$
  - discrepancy in size and shape



Where does this discrepancy come from?
Is it real or apparent?

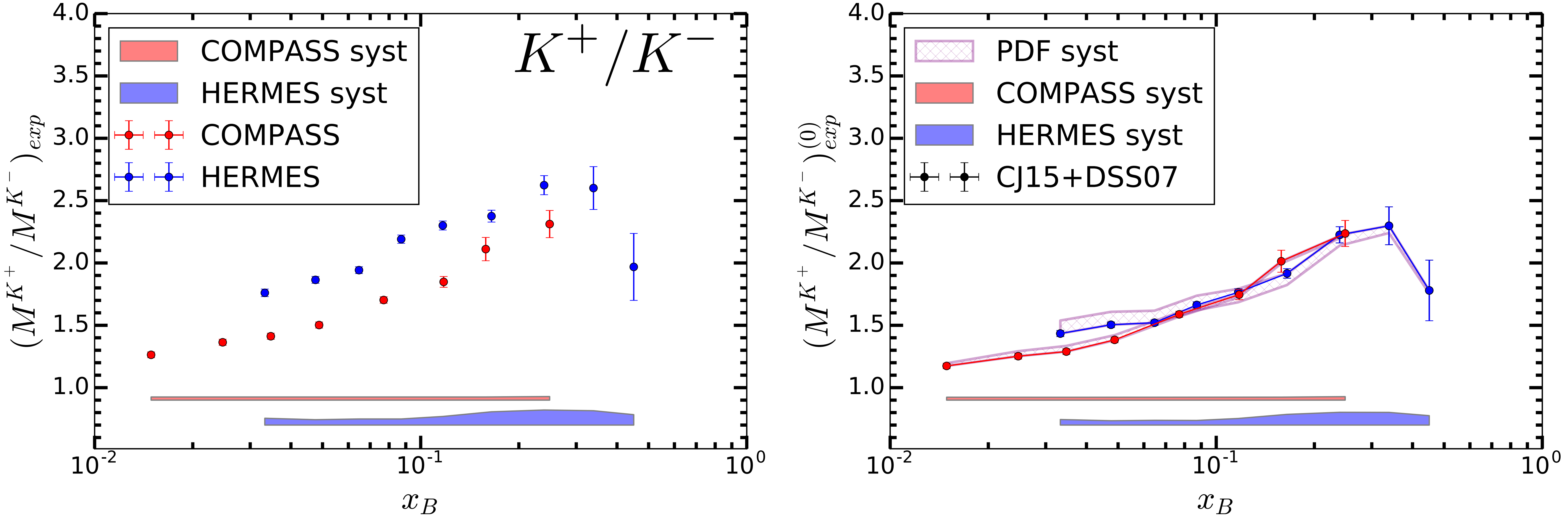
“Removing” Mass Corrections

Guerrero, Accardi PRD 97 (2018) 114012

$$r_{\exp} = \left(\frac{M^{K^+}}{M^{K^-}}\right)_{\exp}$$

- COMPASS: $r^{(0)}_{\exp} \equiv r_{\exp} \times R_{HMC}$
- HERMES: $r^{(0)}_{\exp} \equiv r_{\exp} \times R_{HMC} \times R^{H\to C}_{evo}$

Original data | “Massless” evolved data



- HERMES & COMPASS fully compatible after removing HMCs.

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Collinear factorization with masses in a spectator model

Factorization:

- *"controllable"* approximation

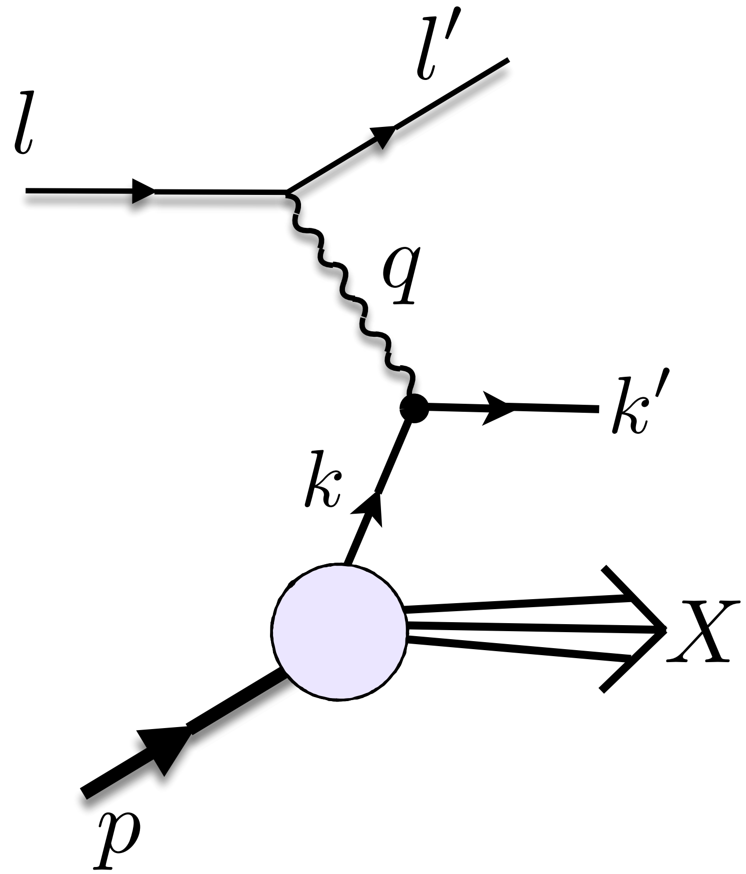
Our goal:

- Extend the range of validity of factorization

- **Test factorization in a simple model**:
  - Analytic Calculations
  - Full vs factorized cross-section
  - PDFs: calculated vs. fitted
- **Start simple: first DIS (2-body phase space)**
  - Then SIA: 3 body phase space
  - Then SIDIS: coupled final state and initial state kinematics

Inclusive DIS in an spectator model

- **Nature**

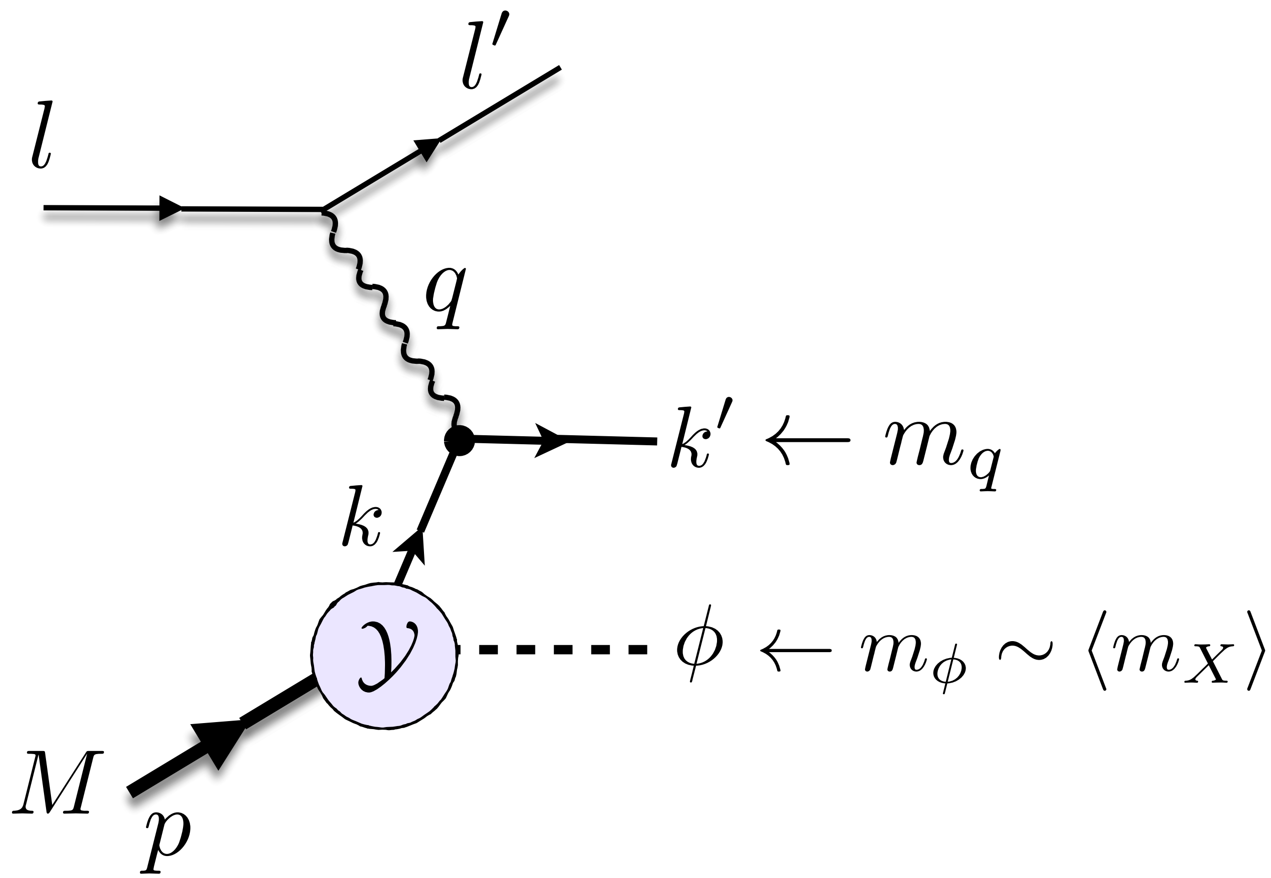


$$\mathcal{Y} = ig(k^2)\mathbb{1}$$

"dipole form factor"

Bacchetta et al, PRD 78 (2008) 074010

- **Yukawa Theory + Form Factors**



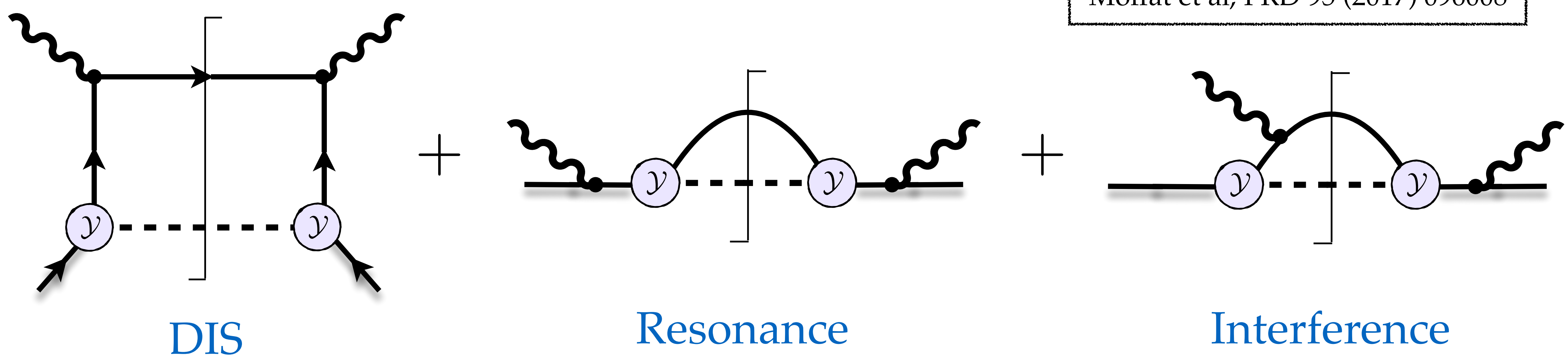
$$g(k^2) = g\frac{k^2 - m_q^2}{|k^2 - \Lambda^2|^2}$$

- Simulates confinement
- Smoothly suppresses the high-k_T region.

DIS in the spectator model

- **Gauge invariance:** needs also quasi-elastic photon-proton scattering.

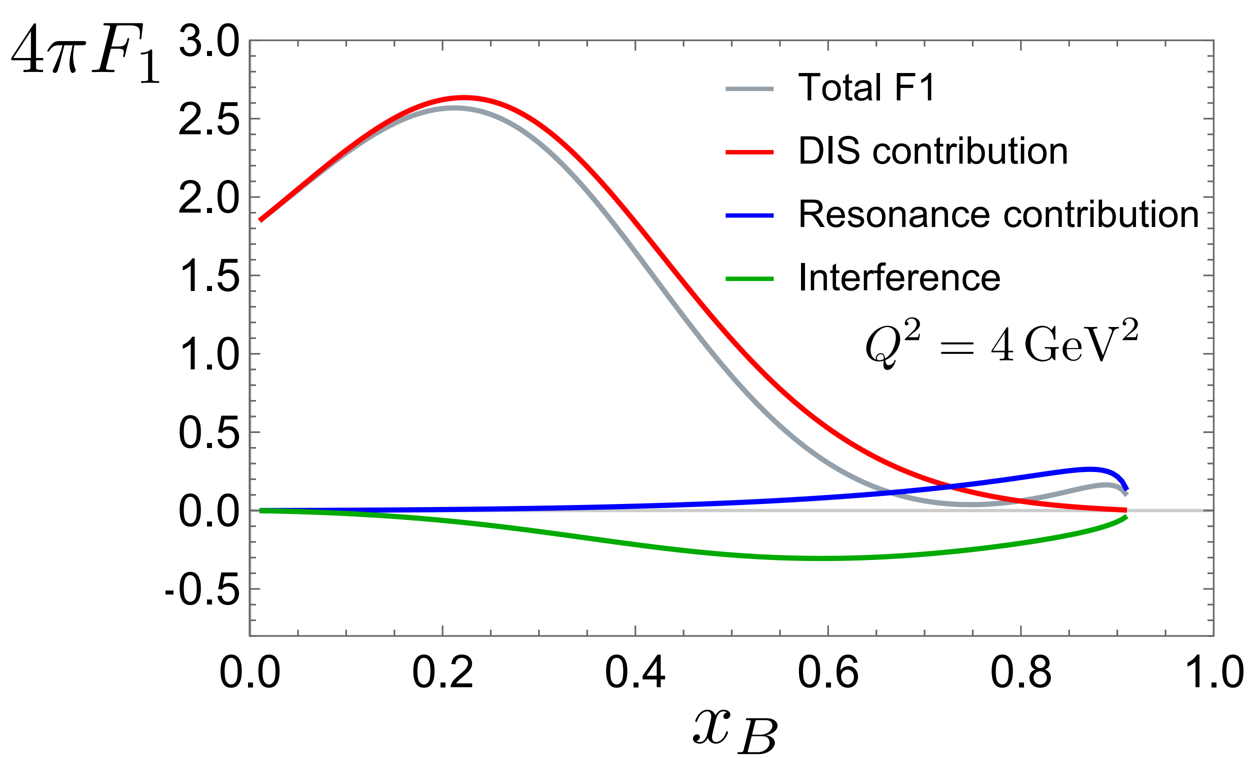
Moffat et al, PRD 95 (2017) 096008



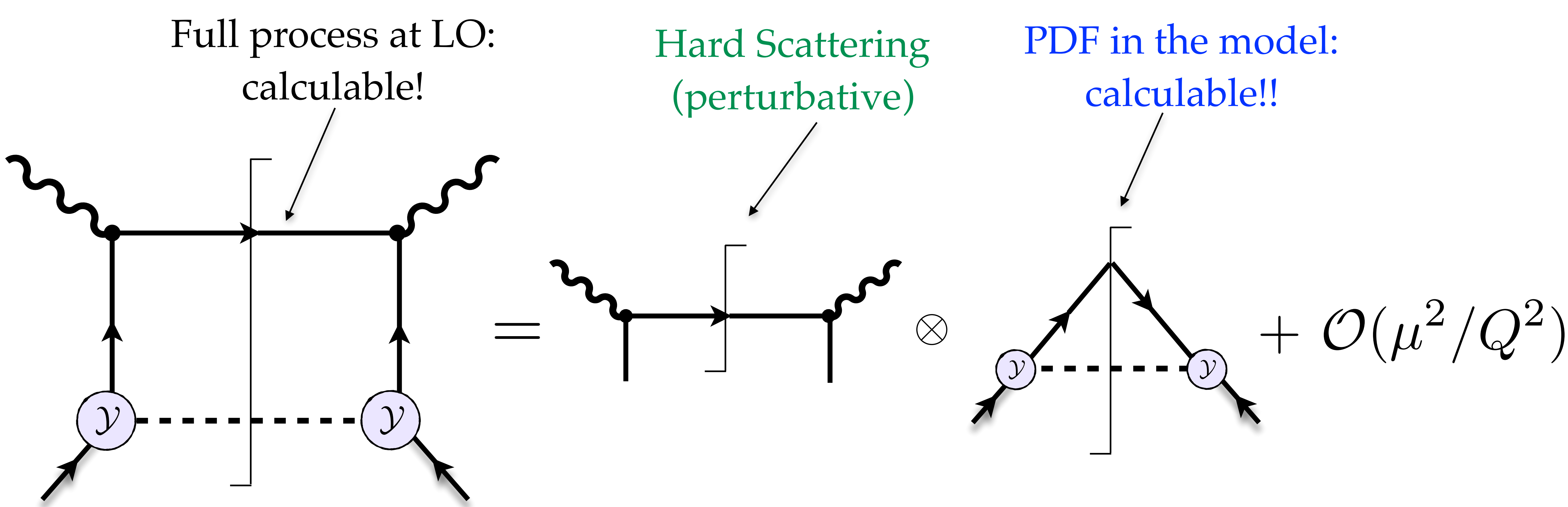
- **Gauge invariant individual contributions.**

JG, Alberto Accardi - in prep.

  - Each diagram not G.I. by itself, but contains different physics
  - Use longitudinal and transverse projectors to extract G.I. contributions of each process
  - Non-negligible interference contribution even at small x_B



Collinear Factorization for DIS diagram



$$F_1(x_B, Q^2) \approx (2\pi)\,\delta(x-\bar{x}) \otimes \varphi_q(x)$$

depends on the kinematical approximations

$$\Phi(x) = -\frac{(x-1)^3\left(-(x-1)\Lambda^2 + x\left(M^2(3x-1)+m_\phi^2\right)+4Mm_q x + 2m_q^2\right)}{6\left(x\left(M^2(x-1)+m_\phi^2\right)-(x-1)\Lambda^2\right)^3}$$

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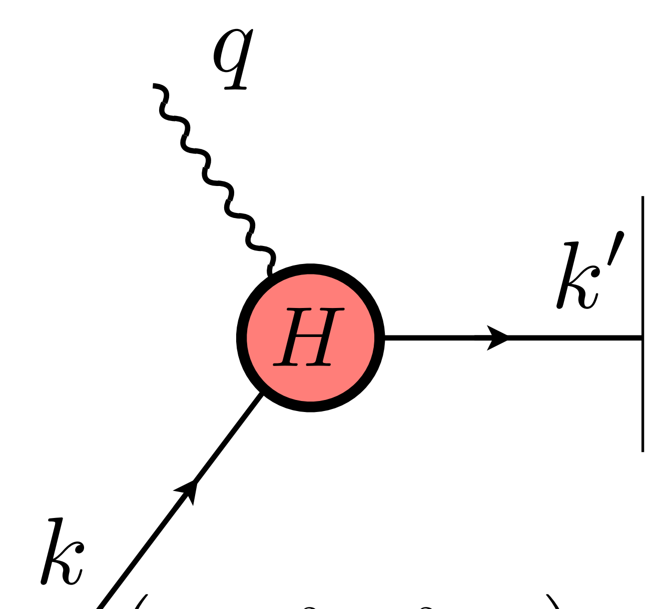
Hard scattering: 4-momentum conservation at LO

Exact:

$$x = \frac{\xi}{2}\left(1 + \frac{m_q^2 - k^2}{Q^2} + \sqrt{\left(1 + \frac{m_q^2 - k^2}{Q^2}\right)^2 + 4\frac{k^2 + k_T^2}{Q^2}}\right)$$

$$= \xi\left(1 + \frac{m_q^2 + k_T^2}{Q^2} - \frac{(m_q^2 + k_T^2)(k^2 + k_T^2)}{Q^4} + \mathcal{O}(\frac{\mu^6}{Q^6})\right)$$



$$k = \left(xp^+, \frac{k^2 + k_T^2}{2x}, \mathbf{k_T}\right)$$

- k_T^2 enters at O(1/Q^2)
- In SIDIS: can be absorbed in v'^{2}
- light cone virtuality k^2 enters at O(1/Q^4)

$$x = \bar{x}(\overbrace{x_B, Q^2, M^2, m_q^2}^{\text{External}}; \overbrace{k^2, k_T^2}^{\text{Internal}})$$

similar to AOT

Aivazis, Olness & Tung
PRD 50 (1994) 3085

Kinematic approximations:

$x =$

	M	m_q	k^2	k_T^2
x_B	0	0	0	0
ξ	✓	0	0	0
$\xi_q \equiv \xi(1 + \frac{m_q^2}{Q^2})$	✓	✓	0	0
$\xi_q^{\rm T} \equiv \xi(1 + \frac{m_q^2}{Q^2} + \frac{\langle k_T^2 \rangle}{Q^2})$	✓	✓	0	✓ $\langle k_T^2 \rangle$

← analogous to χ (ACOT-χ)

Nadolsky & Tung,
PRD 79 (2009) 113014

Averages definitions

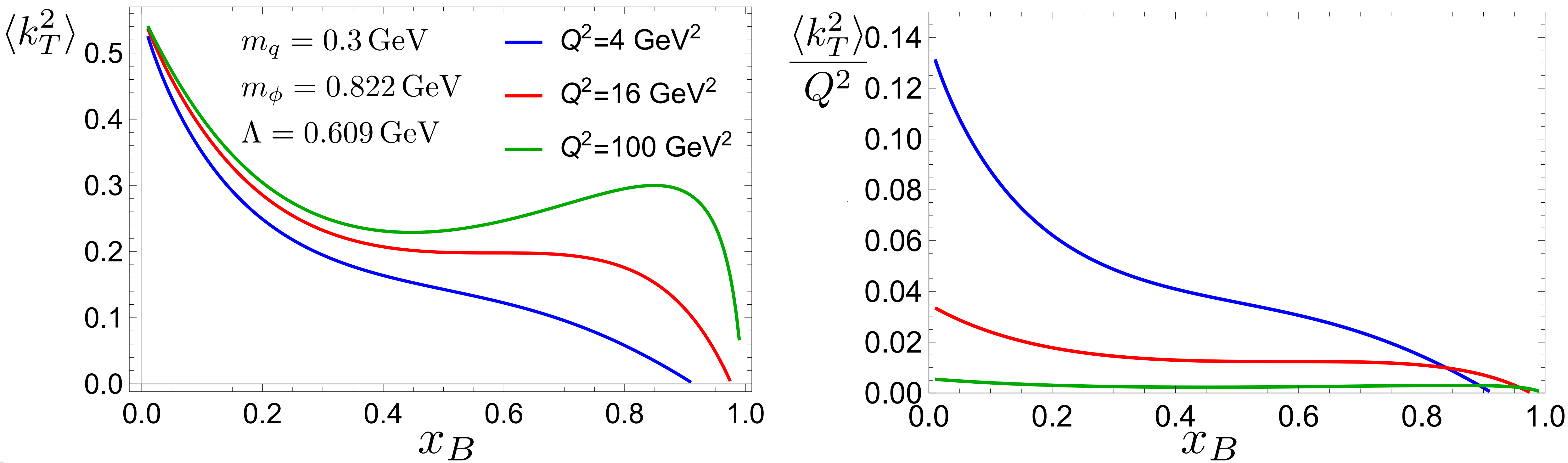
- **DIS:**

$$\langle \mathcal{O} \rangle (x_B, Q^2) = \frac{\int_0^{k_{T,\max}^2} dk_T^2\, \mathcal{O}(x_B, Q^2, k_T) \mathcal{F}_1^{\mathrm{DIS}}(x_B, Q^2, k_T)}{\int_0^{k_{T,\max}^2} dk_T^2\, \mathcal{F}_1^{\mathrm{DIS}}(x_B, Q^2, k_T)}$$

- **Factorized process PDF:**

$$\langle \mathcal{O} \rangle_{\mathrm{PDF}} (x, Q^2) = \frac{\int_0^{\infty} dk_T^2\, \mathcal{O}(x, Q^2, k_T) \Phi(x, Q^2, k_T)}{\int_0^{\infty} dk_T^2\, \Phi(x, Q^2, k_T)}$$

Example:



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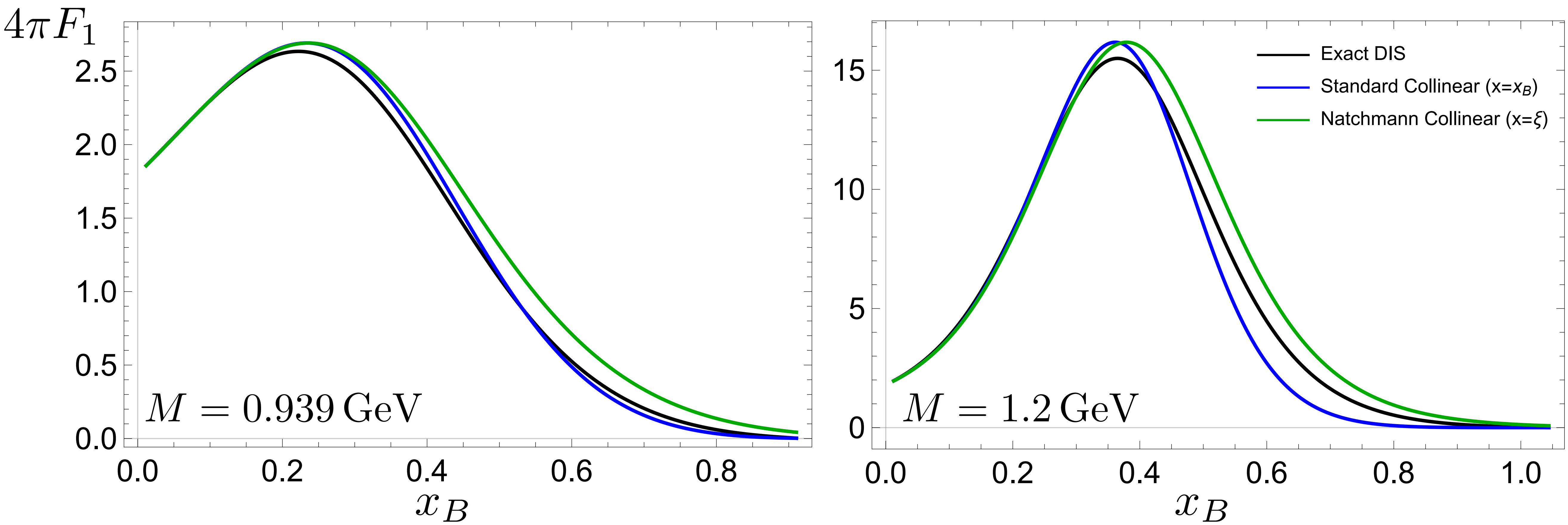
DIS Structure Function



$Q^2 = 4\,\mathrm{GeV}^2$

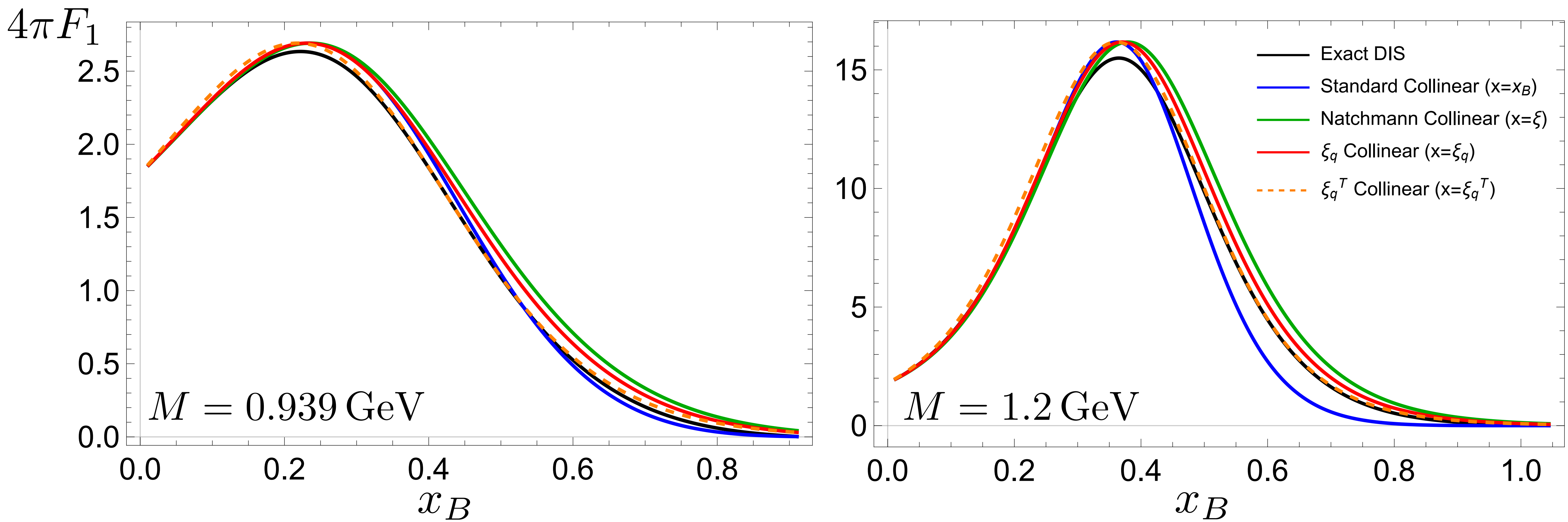
$m_q = 0.3\,\mathrm{GeV}$

$m_\phi = 0.822\,\mathrm{GeV}$

$\Lambda = 0.609\,\mathrm{GeV}$

- $x=x_B$ unstable with respect to the mass of the target
- $x=\xi$ stabilize (similar slope to exact F_1)

DIS Structure Function



$Q^2 = 4\,\mathrm{GeV}^2$

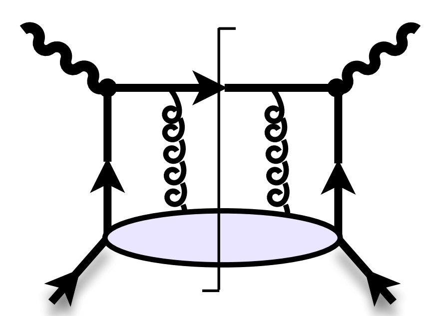
$m_q = 0.3\,\mathrm{GeV}$

$m_\phi = 0.822\,\mathrm{GeV}$

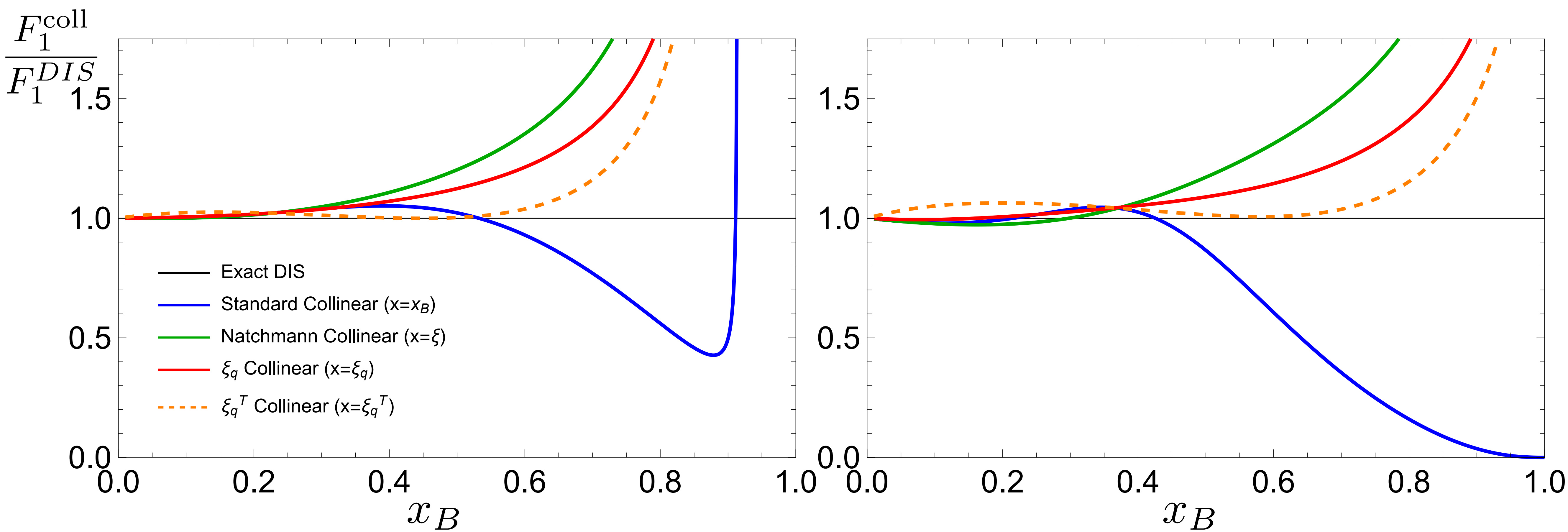
$\Lambda = 0.609\,\mathrm{GeV}$

- x=x_B unstable with respect to the mass of the target
- x=ξ stabilize (similar slope to exact F_1)
- x=ξ_q closest to exact F_1 stable solution considering only external observables
- Remaining gap: closed by <k_T^2> contributions
  - <k_T^2> inaccesible in DIS directly
  - but contributes to Twist-4 (makes the q-A-A-q diagram gauge covariant)

Qiu, PRD 42 (1990) 30

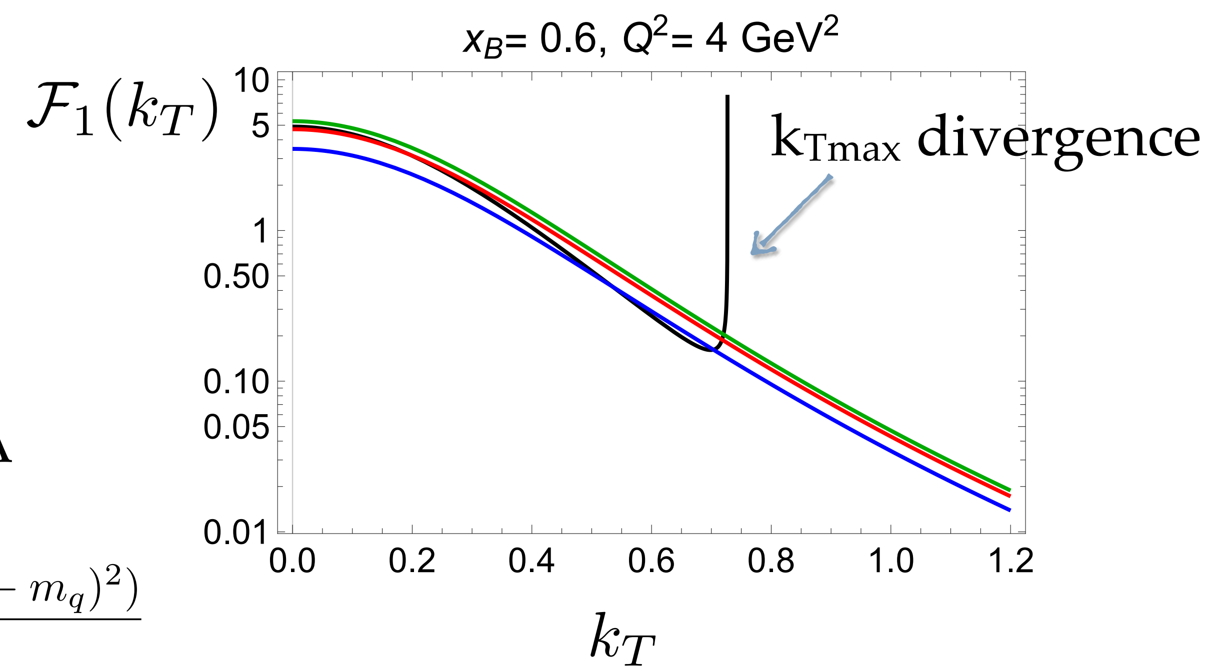


Structure Function: DIS vs. Collinear (ratio)

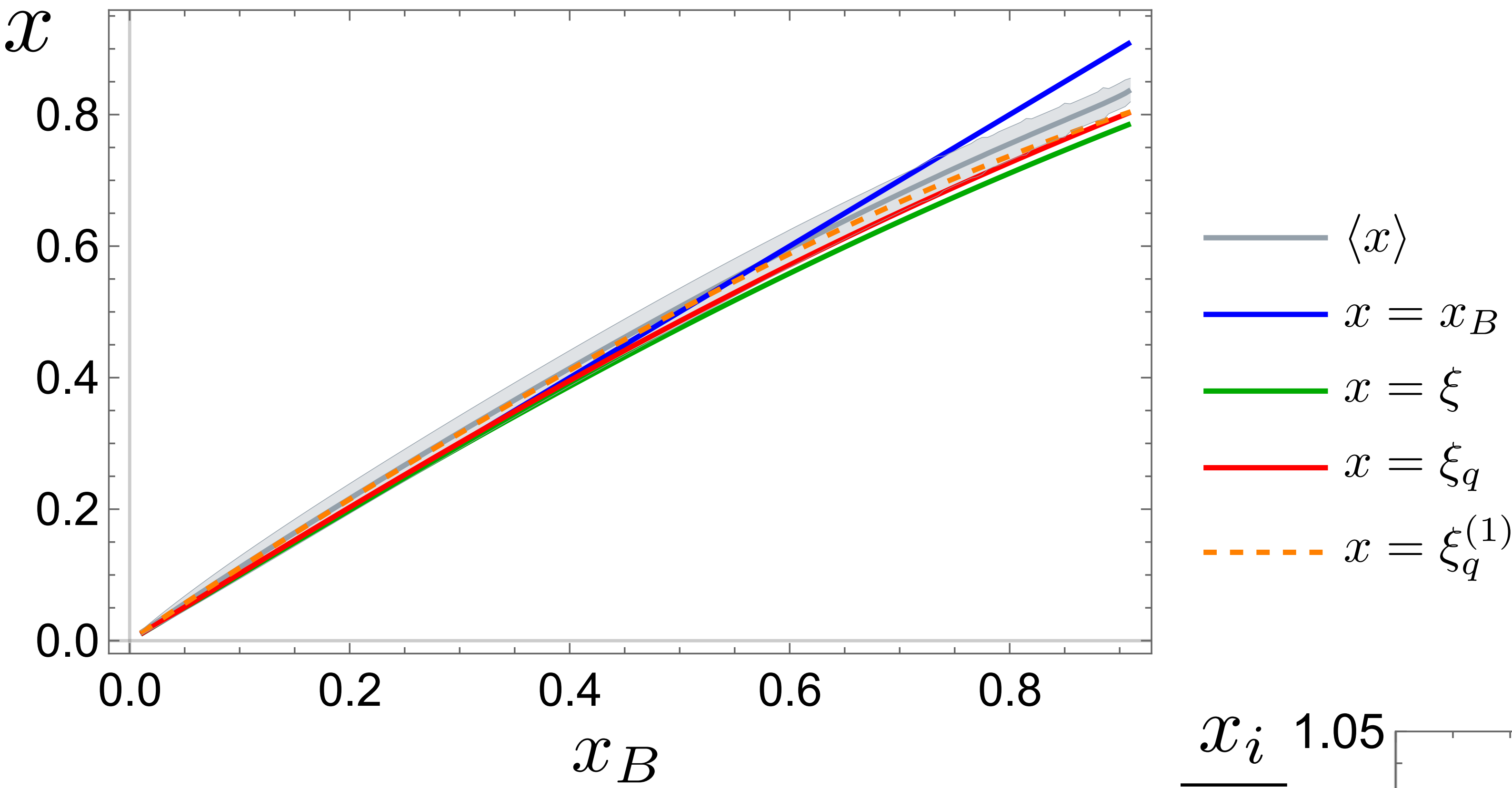


- Factorization approximation **breaks** $x_B > 0.6$:
  - b/c does not respect momentum conservation in transverse direction
  - Threshold independent of M
  - Need to check dependence on m_q, m_ϕ, Λ

$$k_{T,\max}^2 = \frac{(W^2 - (m_\phi + m_q)^2)(W^2 - (m_\phi - m_q)^2)}{4W^2}$$



Light cone fraction x: average and comparison



$Q^2 = 4\,\mathrm{GeV}^2$

$M = 0.939\,\mathrm{GeV}^2$

$m_q = 0.3\,\mathrm{GeV}$

$m_\phi = 0.822\,\mathrm{GeV}$

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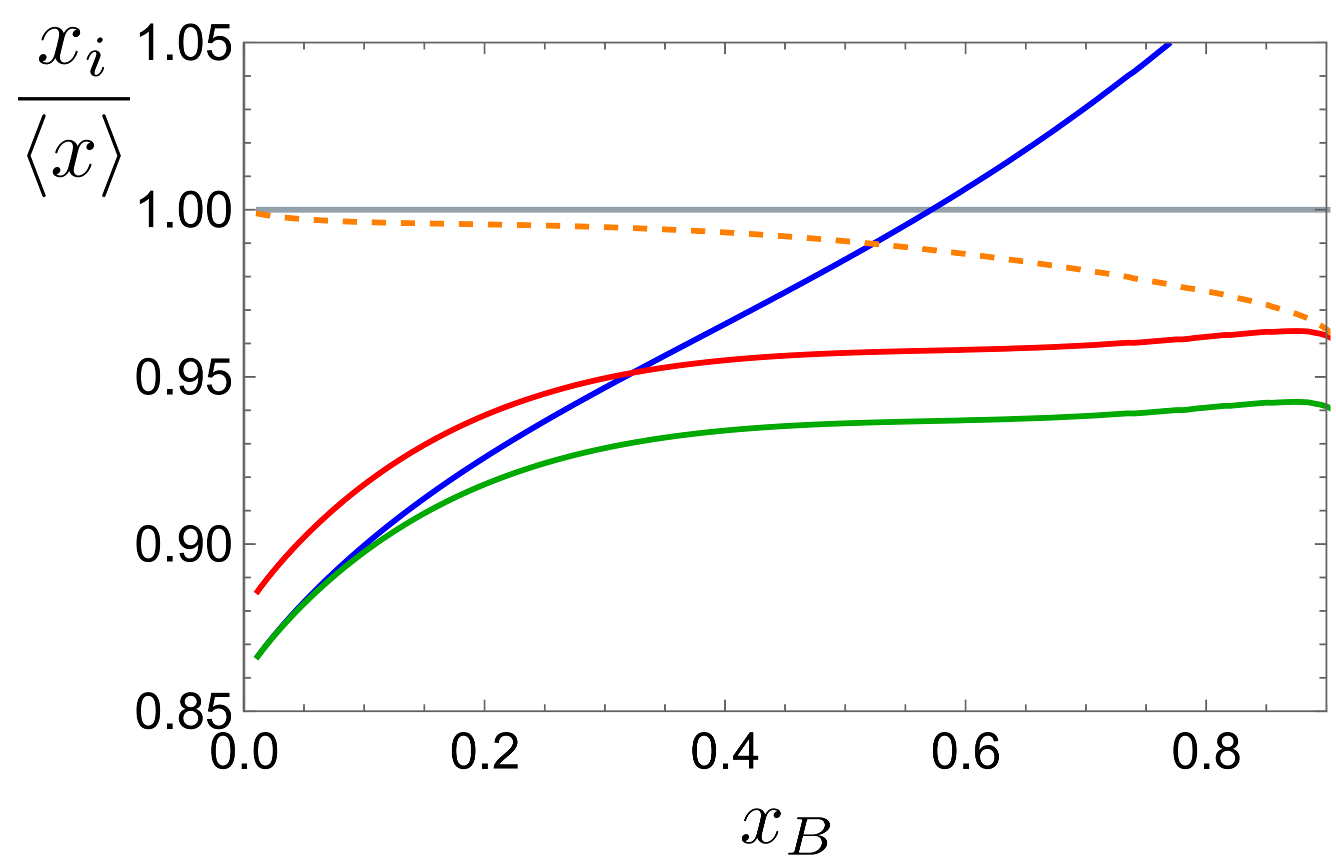
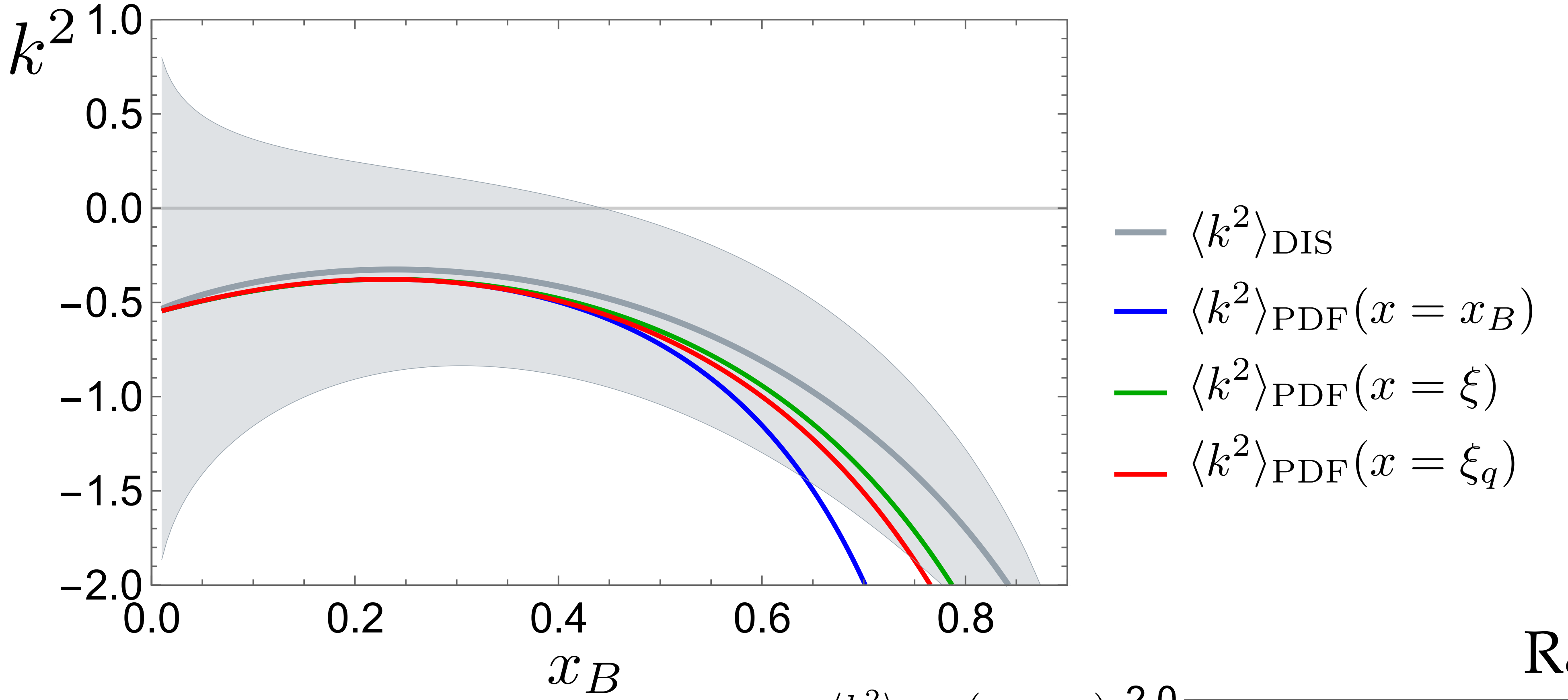
Ratio:



Highlights:

- $x=\xi_q$ captures the process kinematics (~95 % of the <x>)
- k_T^2/Q^2 largely fills the remaining gap

Average virtuality (DIS)



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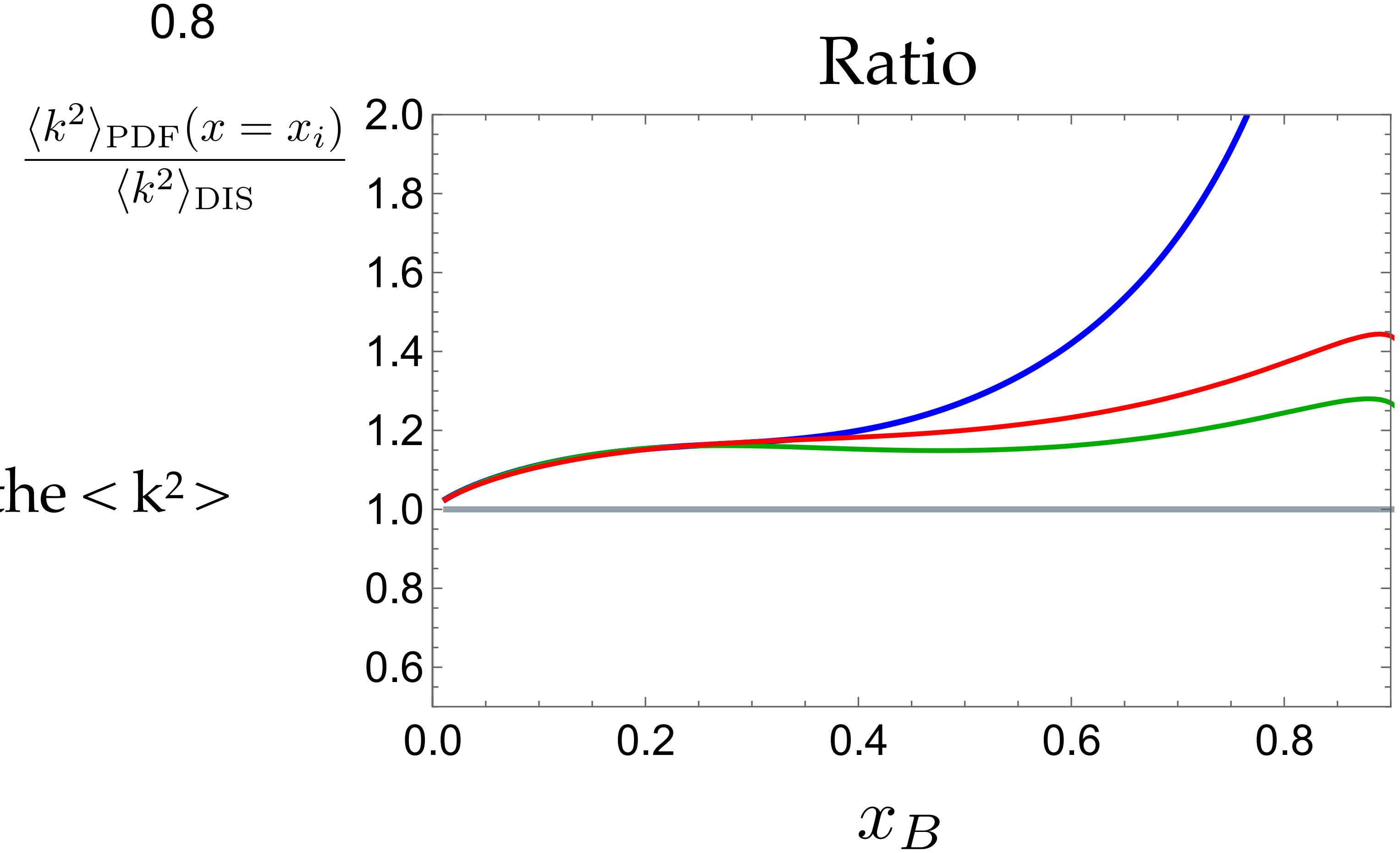
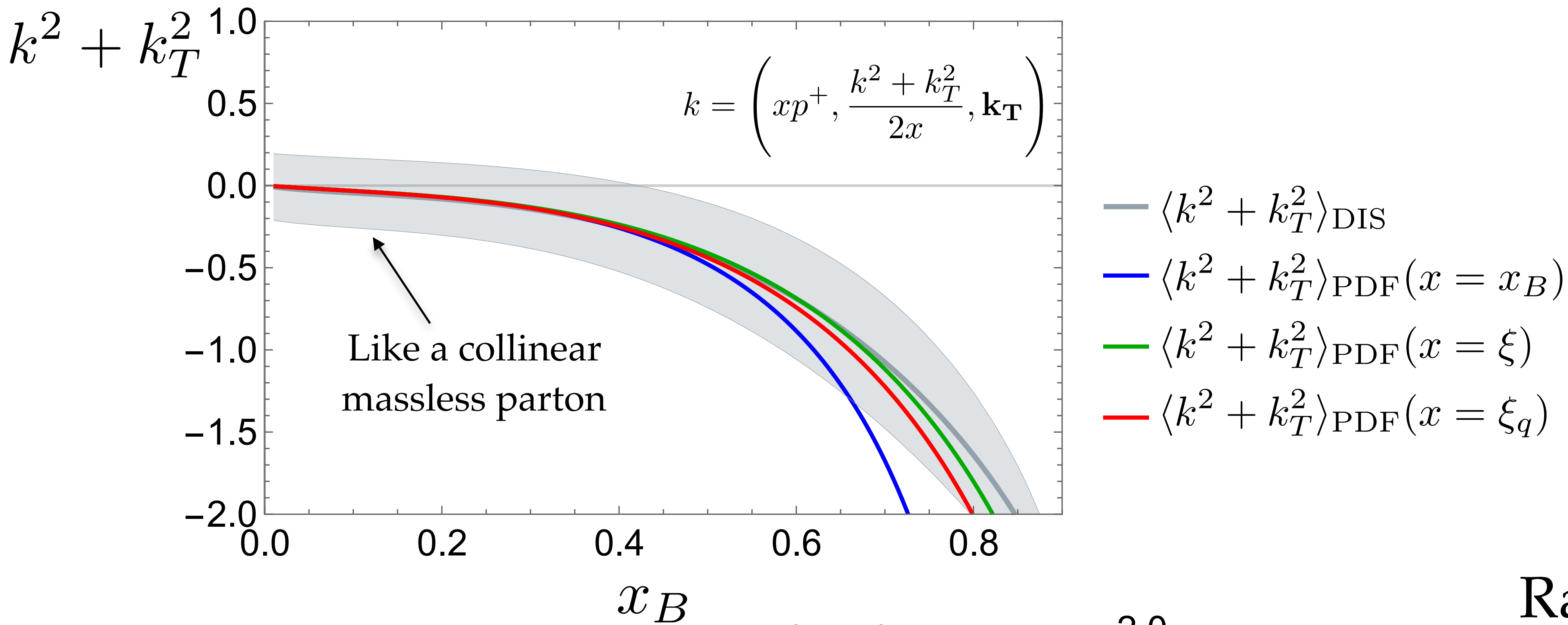


Highlights:

- k^2 negative (bound state)
- $x=\xi$ and $x=\xi_q$ capture most of the $< k^2 >$

Average "light-cone virtuality" (DIS)



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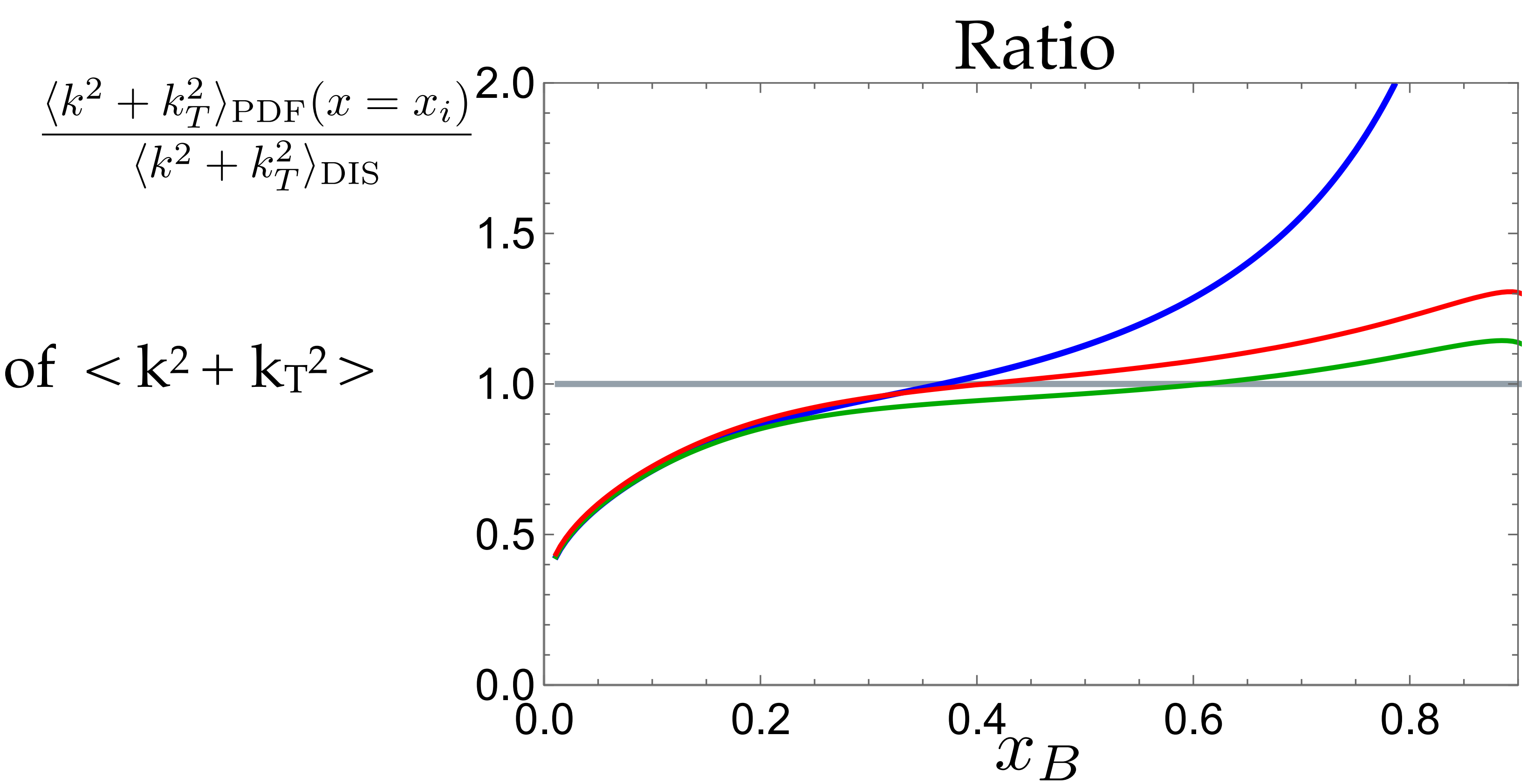

Highlights:

- $x=\xi$ and $x=\xi_q$ capture most of $< k^2 + k_T^2 >$

Concluding Remarks

- Hadron Mass Corrections non-negligible:
  - Kaons at HERMES vs. COMPASS largely reconciled
  - Proposed scheme phenomenologically successful.

- Use spectator model to study the proposed sub-asymptotic Q^2 kinematic approximations (in DIS, for now).
  - $x = x_B$ (neglect all masses) — NOT GOOD
  - $x=\xi$ (keep target mass only) — better
  - $x=\xi_q = \xi(1+m_q^2/Q^2)$ (also final state quark mass) — best
  - would also need to include k_T^2/Q^2
    - possible in CF at twist 4, or with TMD formalism

- Collinear Factorization does not respect transverse momentum conservation
  - valid only up to a (Q^2-dependent) maximum x_B value.

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Thank you!

Backup slides

Gauge invariant decomposition

- **Polarization vectors:** scalar and transverse polarization vectors to q and p form orthonormal basis in Minkowski space

$$\varepsilon_0^\mu(p,q) = \frac{\hat{p}^\mu}{\sqrt{\hat{p}^2}}$$

$$\varepsilon_\pm^\mu(p,q) = \frac{1}{\sqrt{2}}(0,\mp 1,-i,0)$$

$$\varepsilon_q^\mu(p,q) = \frac{q^\mu}{\sqrt{-q^2}}\ ,$$

$$\hat{p}^\mu = p^\mu - \frac{p\cdot q}{q^2}q^\mu$$

- **Parity-invariant orthogonal projectors:**

$$P_L^{\mu\nu}(p,q) = \varepsilon_0^\mu(p,q)\varepsilon_0^{\nu *}(p,q)$$

$$P_T^{\mu\nu}(p,q) = \varepsilon_+^\mu(p,q)\varepsilon_+^{\nu *}(p,q) + \varepsilon_-^\mu(p,q)\varepsilon_-^{\nu *}(p,q)$$

$$P_S^{\mu\nu}(p,q) = \varepsilon_q^\mu(p,q)\varepsilon_q^{\nu *}(p,q)$$

$$P_{\{0q\}}^{\mu\nu}(p,q) = \varepsilon_0^\mu(p,q)\varepsilon_q^{\nu *}(p,q) + \varepsilon_q^\mu(p,q)\varepsilon_0^{\nu *}(p,q)$$

AOT, PRD 50 (1994) 3085

Gauge invariant decomposition

- **Polarization vectors:** scalar and transverse polarization vectors to q and p form orthonormal basis in Minkowski space

$$\varepsilon_0^\mu(p,q) = \frac{\hat{p}^\mu}{\sqrt{\hat{p}^2}}$$

$$\varepsilon_\pm^\mu(p,q) = \frac{1}{\sqrt{2}}(0,\mp 1,-i,0)$$

$$\varepsilon_q^\mu(p,q) = \frac{q^\mu}{\sqrt{-q^2}} \ ,$$

$$\hat{p}^\mu = p^\mu - \frac{p\cdot q}{q^2} q^\mu$$

$$\hat{g}^{\mu\nu} = g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}$$

- **Parity-invariant orthogonal projectors:**

$$P_L^{\mu\nu}(p,q) = \frac{\hat{p}^\mu \hat{p}^\nu}{\hat{p}^2}$$

$$P_T^{\mu\nu}(p,q) = -\hat{g}^{\mu\nu} + \frac{\hat{p}^\mu \hat{p}^\nu}{\hat{p}^2}$$

$$P_S^{\mu\nu}(p,q) = -\frac{q^\mu q^\nu}{q^2}$$

$$P_{\{0q\}}^{\mu\nu}(p,q) = \frac{\hat{p}^\mu q^\nu + \hat{p}^\nu q^\mu}{\sqrt{-q^2 \hat{p}^2}}$$

- Orthogonal basis for parity-invariant rank-2 tensors:

Gauge invariant!!!

$$A^{\mu\nu}(p,q) = \frac{1}{2} P_T^{\mu\nu} a_T(x_B,Q^2) + P_L^{\mu\nu} a_L(x_B,Q^2) + P_S^{\mu\nu} a_S(x_B,Q^2) - \frac{1}{2} P_{\{0q\}}^{\mu\nu} a_{\{0q\}}$$

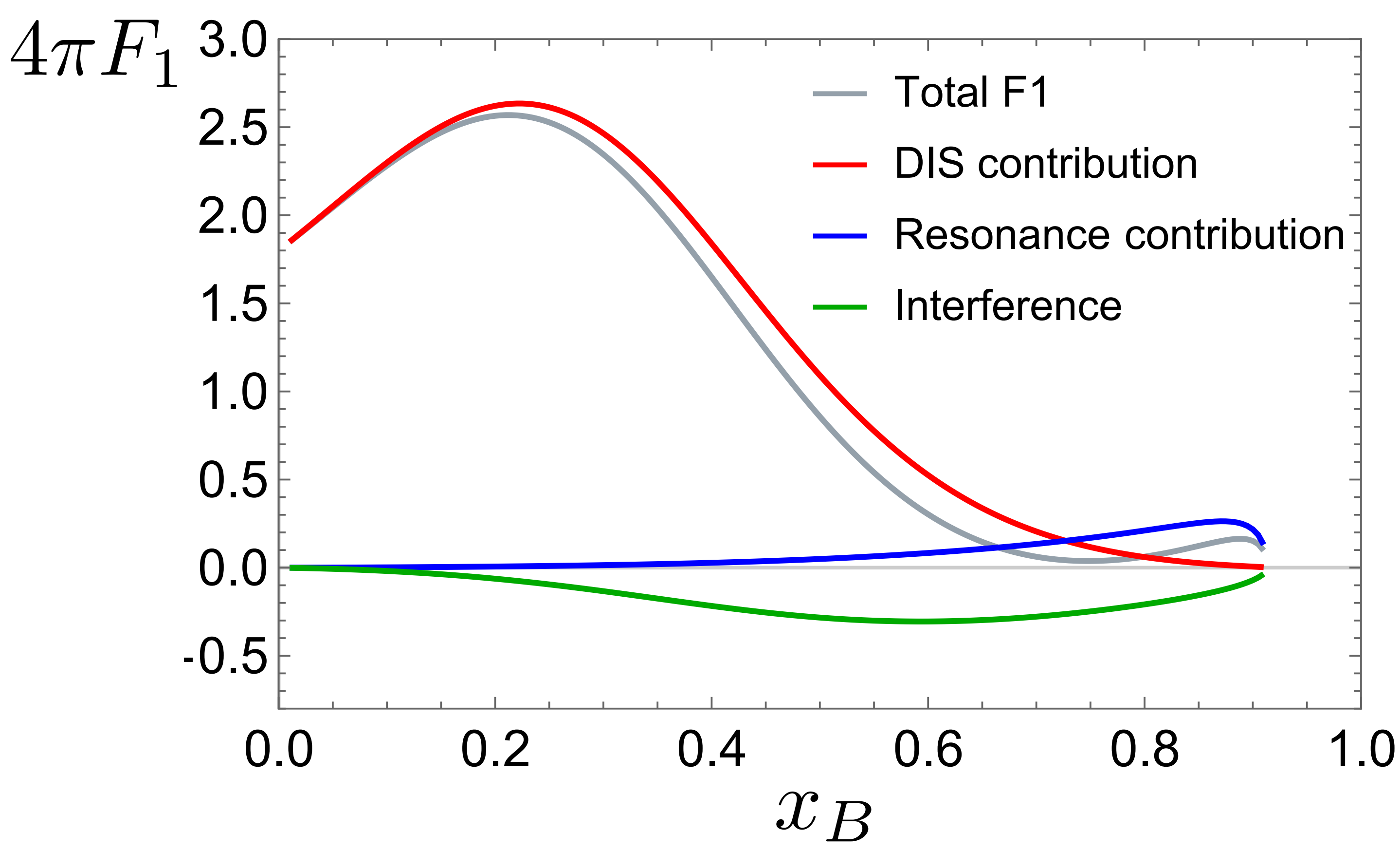
Gauge breaking

Gauge invariant decomposition

- **Hadronic tensor:** propose that decomposition for each process

$$W^{(j),\mu\nu}_{\text{inv.}}(p,q) = \frac{1}{2}P^{\mu\nu}_T F^{(j)}_T(x_B,Q^2) + P^{\mu\nu}_L F^{(j)}_L(x_B,Q^2) \qquad \text{j} = \text{DIS, INT, RES}$$

$$W^{(j),\mu\nu}_{\text{g.b}}(p,q) = P^{\mu\nu}_S F^{(j)}_S(x_B,Q^2) - \frac{1}{2}P^{\mu\nu}_{\{0q\}} F^{(j)}_{\{0q\}}$$



Highlights:

- F_1^{DIS} and F_1^{RES} are positive
- Non-negligible interference contribution even at small x_B

$$F_L = -F_1 + \frac{\rho_B^2}{2x_B}F_2 \qquad \rho_B^2 = 1 + 4x_B^2\frac{M^2}{Q^2}$$

$$F_T = 2F_1$$

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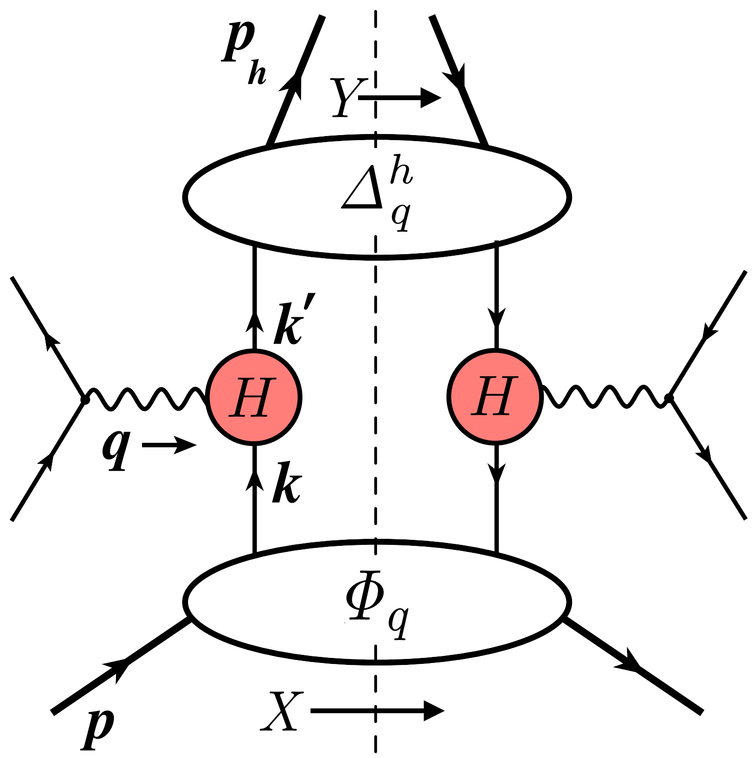
Collinear factorization with masses



1) Expand the correlators:

$$\Phi_q = k^+ \left[\phi_2(k) \bar{\not{n}} + \mathcal{O}(1/k^+) \right]$$

$$\Delta_q = k'^- \left[\delta_2(k') \not{n} + \mathcal{O}(1/k'^-) \right]$$

Leading contribution (LT)

contribute to Higher-Twist (HT) terms

2) Expand the hadronic tensor:

$$2MW^{\mu\nu} = \int d^4k \; d^4k' \; \mathrm{Tr}\left[\Phi_q(p,k) \, \gamma^\mu \, \Delta_q^h(k', p_h) \, \gamma^\nu \right] \; \delta^{(4)}(k + q - k')$$

$$= \int d^4k \; d^4k' \; \phi_2(k) \delta_2(k') \mathrm{Tr}\left[k^+ \bar{\not{n}} \gamma^\mu \, k'^- \not{n} \, \gamma^\nu \right] \; \delta^{(4)}(k + q - k') + \mathrm{HT}$$

Note: $q_\mu W^{\mu\nu} = 0$

3) Approximate overall 4-momentum:

$$k \approx \widetilde{k} \qquad k' \approx \widetilde{k}'$$

Collinear momenta

- (p,q) frame: p and q are collinear and have zero transverse momentum

$$\left.\begin{aligned}\widetilde{k}^2 &= v^2 \\ \widetilde{k}' &= v'^2\end{aligned}\right\}$$

Physically: "Average virtualities"

Fragmenting parton collinear to hadron

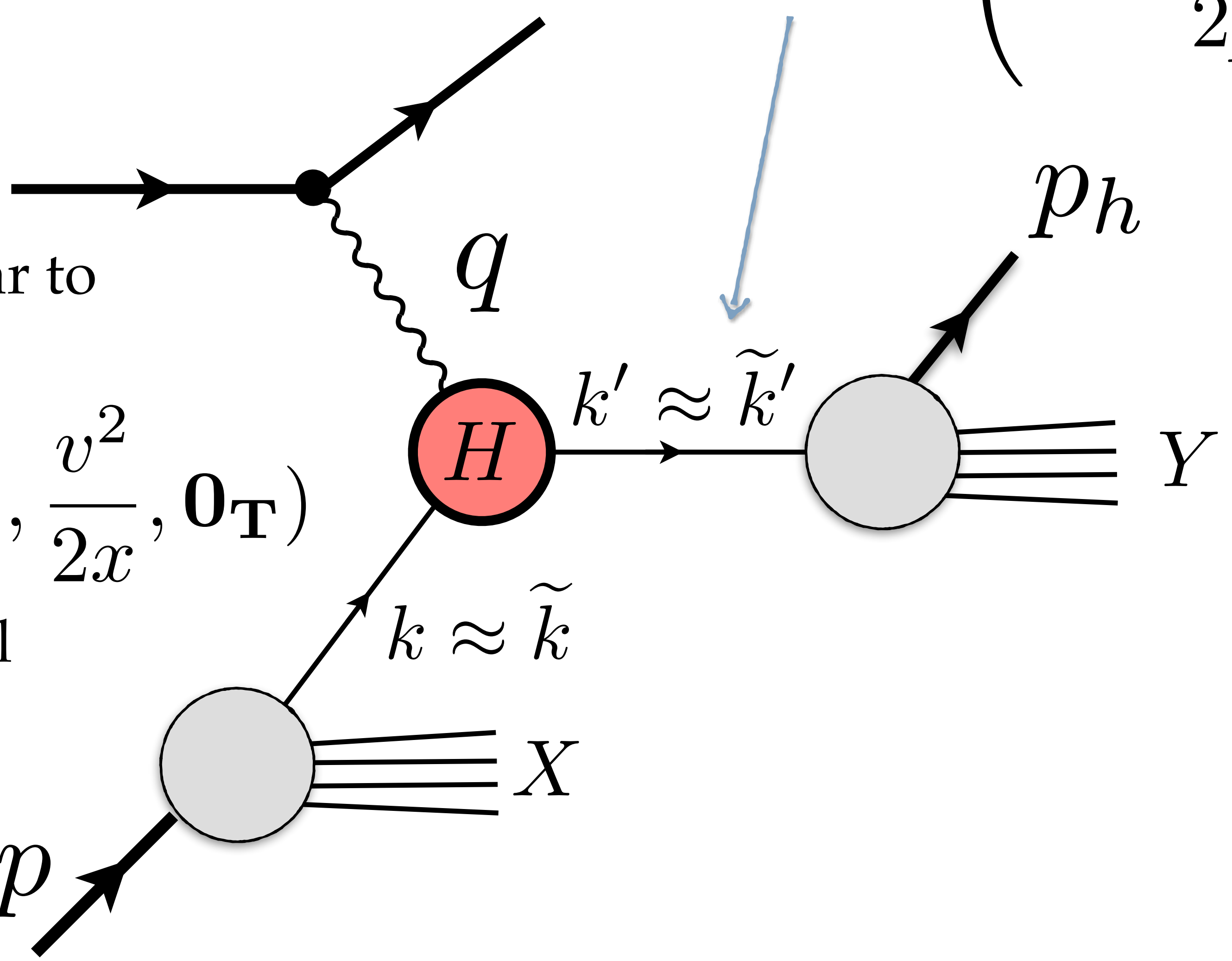
$$\widetilde{k}' = \left(\frac{v'^2 + (\boldsymbol{p}_{h\perp}/z)^2}{2p_h^-/z}, \frac{p_h^-}{z}, \frac{\boldsymbol{p}_{h\perp}}{z} \right)$$

Approx.:
Parton collinear to proton…

$$\widetilde{k} = \left(xp^+, \frac{v^2}{2x}, \mathbf{0_T}\right)$$

… and on-shell

$$\boxed{v^2 = 0}$$



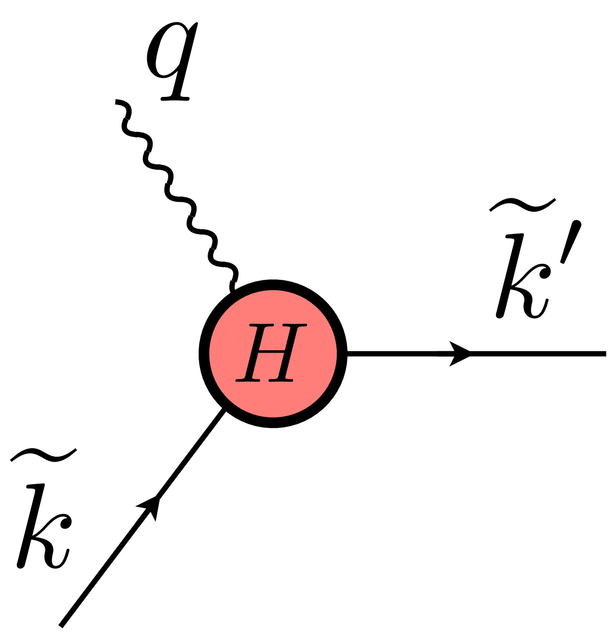
Fragmentation into a **massive** hadron

$$\boxed{v'^2 = ?}$$

need to match partonic & hadronic kinematics

Matching Hadronic and Partonic Kinematics at LO

Hard scattering: 4-momentum conservation at LO



$$x = \xi\left(1 + \frac{v'^2}{Q^2}\right)$$
$$z = \zeta_h$$

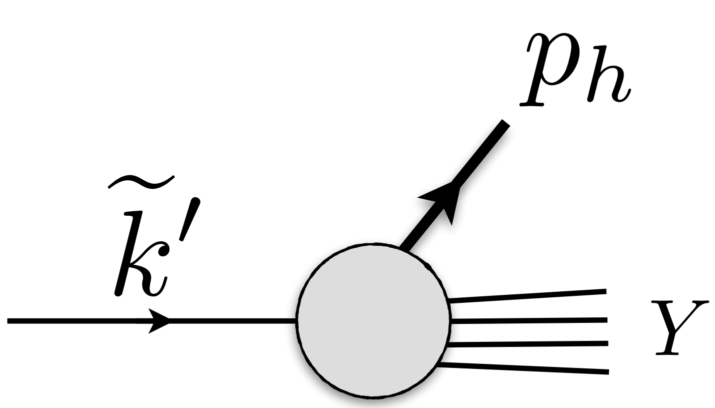
Bjorken limit:

$$x = x_B$$

$$z = z_h$$

Fragmenting blob: momentum conservation in + direction

$$\widetilde{k}'^+ = p_h^+ + Y^+ \geq p_h^+$$



$$v'^2 \geq \frac{m_h^2}{z} \underset{\text{LO}}{=} \frac{m_h^2}{\zeta_h}$$

Standard choice:

$$v'^2 = 0$$

Albino et al. Nucl. Phys. B803 (2008) 42-104

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Collinear factorization with masses: LO case

4) Let 3 integrations out of 4 act in correlator:

$$2MW^{\mu\nu} = \sum_q e_q^2 \int \frac{dx}{x}\frac{dz}{z}\, q(x)\, \mathcal{H}^{\mu\nu}(x,z)\, D_q(z) + \mathrm{HT}$$

$$q(x) = \int dk^- d^2k_T \phi_2(k)$$ ← **PDF**

$$D_q(z) = (z/2)\int dk'^+ d^2k'_T \delta_2(k)$$ ← **FF**

$$\mathcal{H}^{\mu\nu}(x,z) = \frac{1}{2z}\mathrm{Tr}\left[\not{k}_0\gamma^\mu \not{k}'_0\gamma^\nu\right]$$ ← **Hard scattering coefficient**

$$\times\ \delta\left(k_0^+ + q^+ - \frac{v'^2}{2k_0'^-}\right)\delta\left(\frac{v^2}{2k_0^+} + q^- - k_0'^-\right)\delta^{(2)}(\boldsymbol{k'_{0T}})$$

$$x = \xi_h \equiv \xi\left(1 + \frac{m_h^2}{\zeta_h Q^2}\right)$$

$$z = \zeta_h$$

$$k_0 \equiv \widetilde{k}|_{v=0}$$

$$k'_0 \equiv \widetilde{k}'|_{v=0}$$

Leading Order (LO) Cross-Sections at finite Q².

- **With Hadron Masses (unpolarized and polarized):**

Scale dependent Jacobian

Finite Q² scaling variables

$$\sigma_h = \frac{2\pi\alpha^2}{Q^4}\frac{y^2}{1-\varepsilon} J_h(\xi,\zeta_h,Q^2)\sum_q e_q^2\, q(\xi_h,Q^2)\, D_q^h(\zeta_h,Q^2)$$

$$\Delta\sigma_h = \frac{4\pi\alpha^2}{Q^4}\frac{y^2\sqrt{1-\varepsilon^2}}{1-\varepsilon} J_h(\xi,\zeta_h,Q^2)\sum_q e_q^2\, \Delta q(\xi_h,Q^2)\, D_q^h(\zeta_h,Q^2)$$

- Bjorken limit: $\left(\frac{M^2}{Q^2},\frac{m_h^2}{Q^2}\right)\to 0$

$$\xi_h \equiv \xi\left(1+\frac{m_h^2}{\zeta_h Q^2}\right)$$

$$\sigma_h^{(0)} = \frac{2\pi\alpha^2}{Q^4}\frac{y^2}{1-\varepsilon}\sum_q e_q^2\, q(x_B,Q^2)\, D_q^h(z_h,Q^2)$$

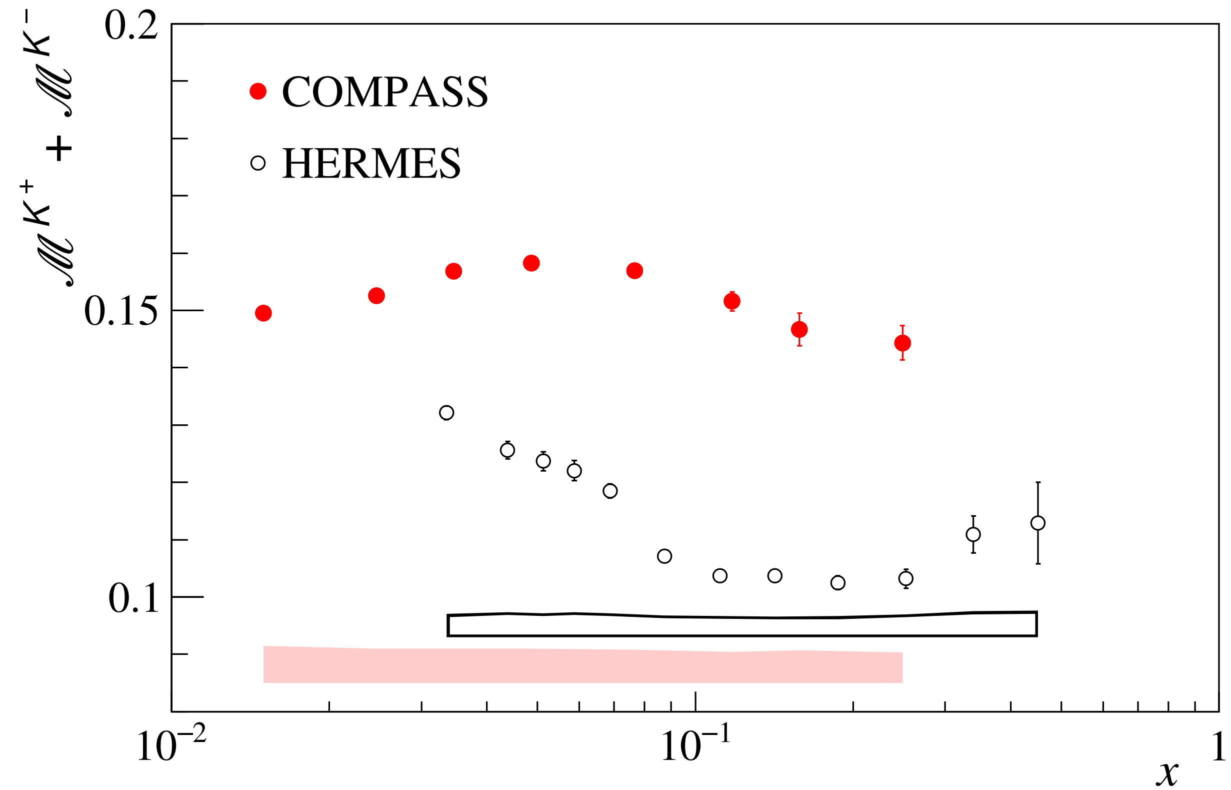
Parton model definition

Integrated Kaon Multiplicities: SIDIS on Deuteron

Experimentally
HERMES, COMPASS: $M^K_{exp} = \frac{\int_{exp} dQ^2 \int_{0.2}^{0.8} dz_h \frac{dN^K}{dx_B dQ^2 dz_h}}{\int_{exp} dQ^2 \frac{dN^e}{dx_B dQ^2}}$

proxy to access s-pdf

- **HERMES:**
  - Claim very different s-quark shape compared to CTEQ6L.
  - Strange PDF may not be what we think!
- **But COMPASS:**
  - Different x_B dependence
  - COMPASS overall value higher.

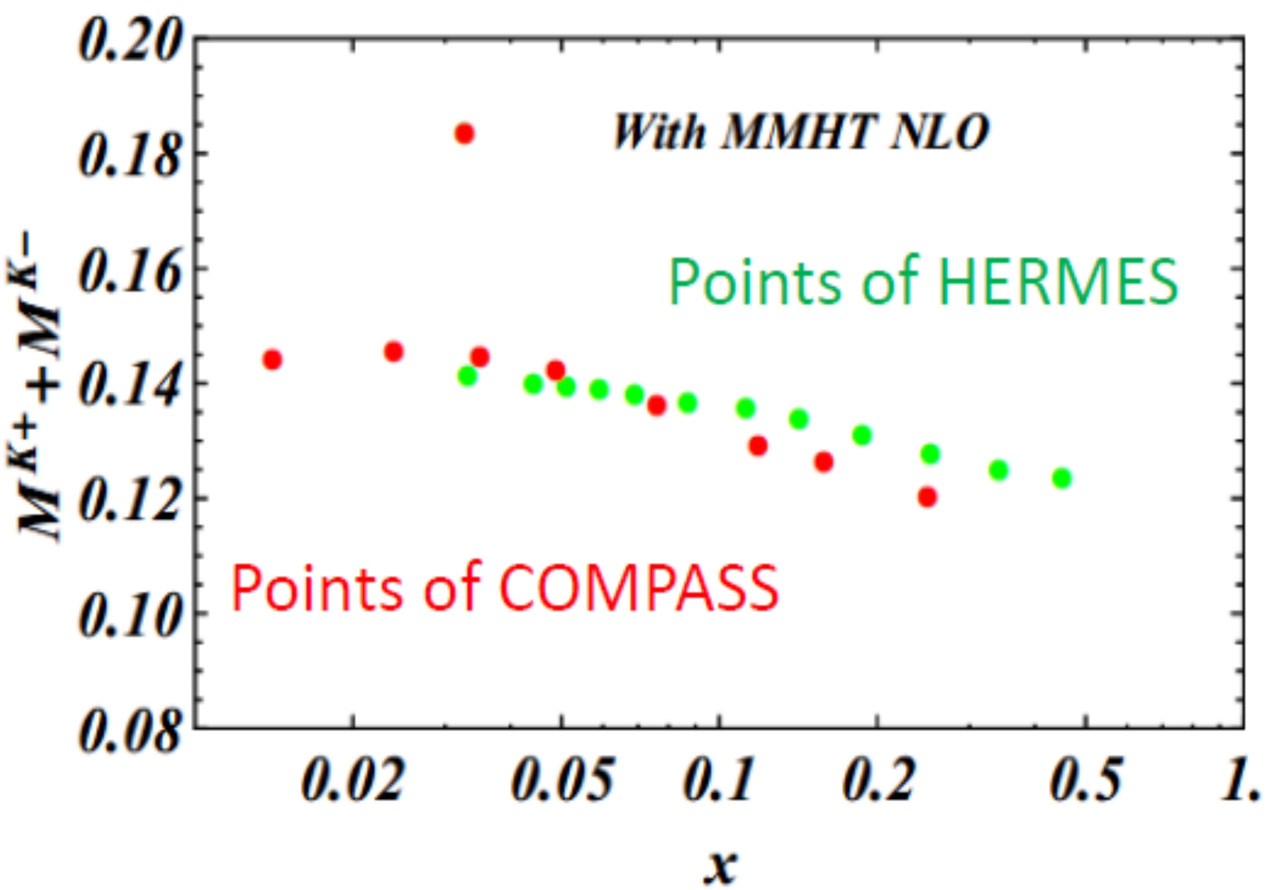


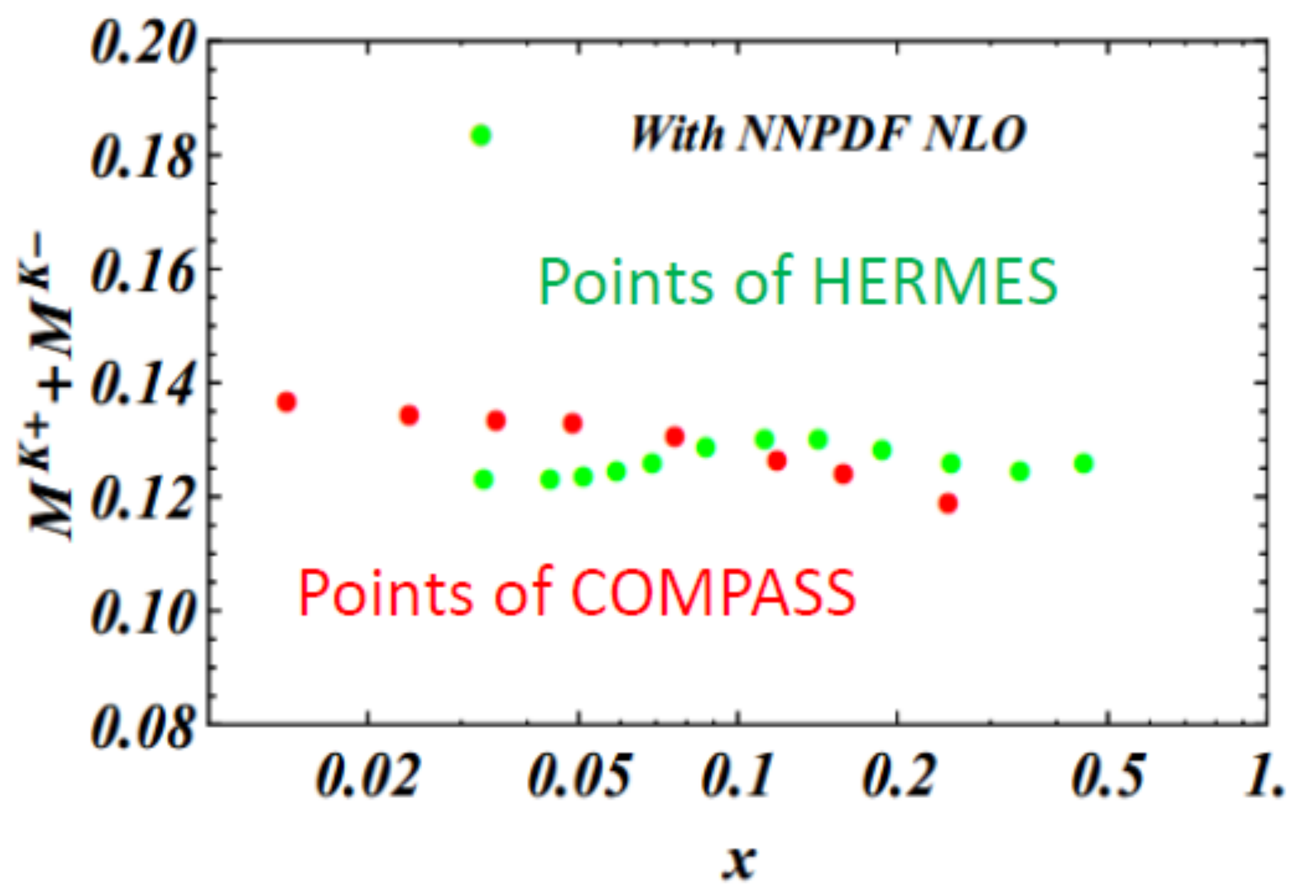
Where does this discrepancy come from?
Is it real or apparent?

Integrated Kaon Multiplicities: SIDIS on Deuteron

- Theoretical prediction at NLO:

Plots from Chung-Wen Kao:
Talk at DIS 2018 conference





- MMHT+DSS17
- NNPDF +DSS17

- Theoretical prediction: both sets should be close
- Q^2 evolution does not explain the discrepancy
- **Other effects?**

HERMES & COMPASS data: direct comparison

"Theoretical correction ratios" → Produce approximate "massless" parton model multiplicities

Make data directly comparable

Largely insensitive to D_K normalization

- COMPASS:

$$M^{h(0)}_{exp} \equiv M^h_{exp} \times R^h_{HMC}$$

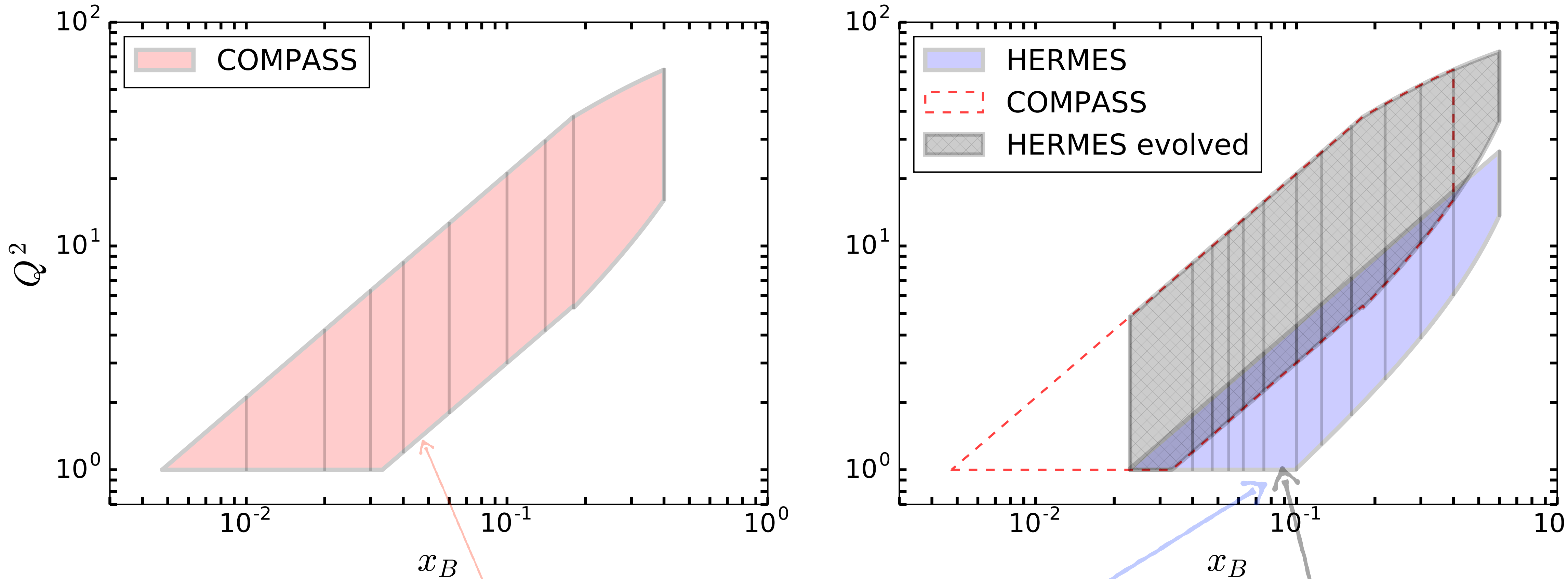
- HERMES:

$$M^{h(0)}_{exp} \equiv M^h_{exp} \times R^h_{HMC} \times R^{H\to C}_{evo}$$

HMC correction ratio

HERMES to COMPASS evolution

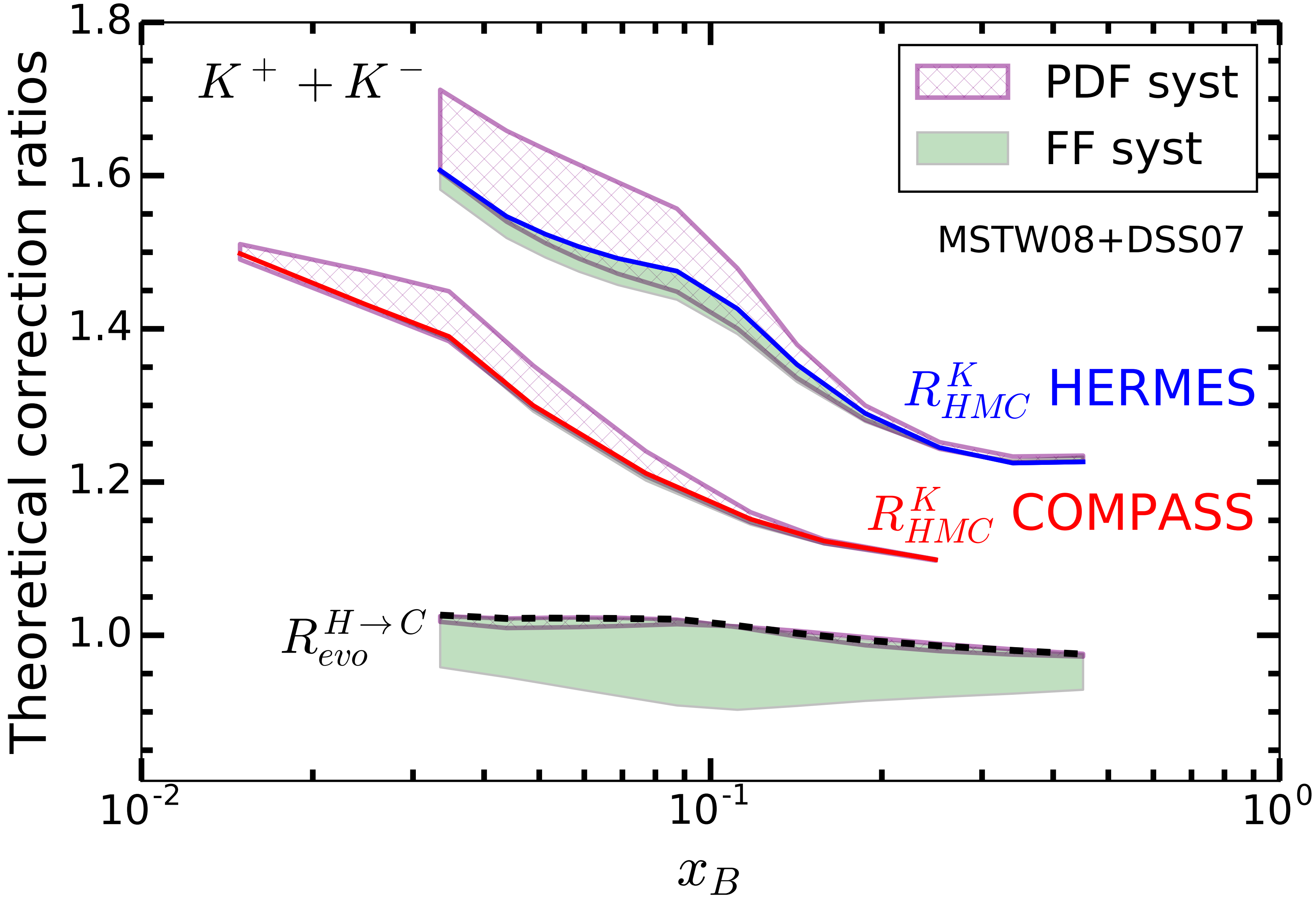
HERMES & COMPASS data: direct comparison



- HMC ratio $$R^h_{HMC} = \frac{M^{h(0)}}{M^h}$$

- Evolution ratio (HERMES to COMPASS) $$R^{H\to C}_{evo} = \frac{M^{h(0)}(x_B^{HERMES})\Big|_{\text{COMPASS cuts}}}{M^{h(0)}(x_B^{HERMES})\Big|_{\text{HERMES cuts}}}$$

Correction ratios



- Hadron mass effects dominant over evolution effects
- At COMPASS smaller HMCs than at HERMES.

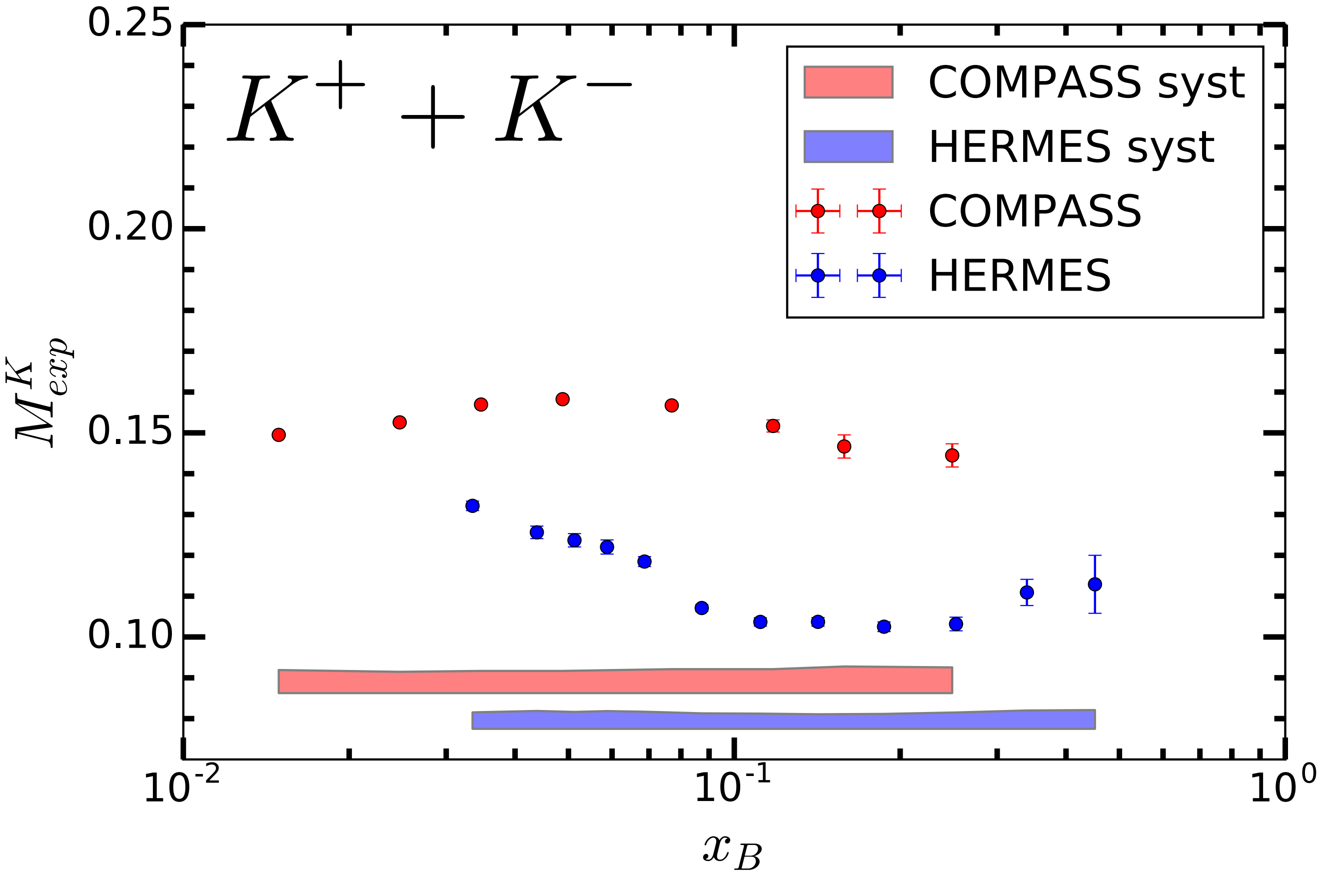
Direct Data Comparison: $K^+ + K^-$

Guerrero, Accardi PRD 97 (2018) 114012

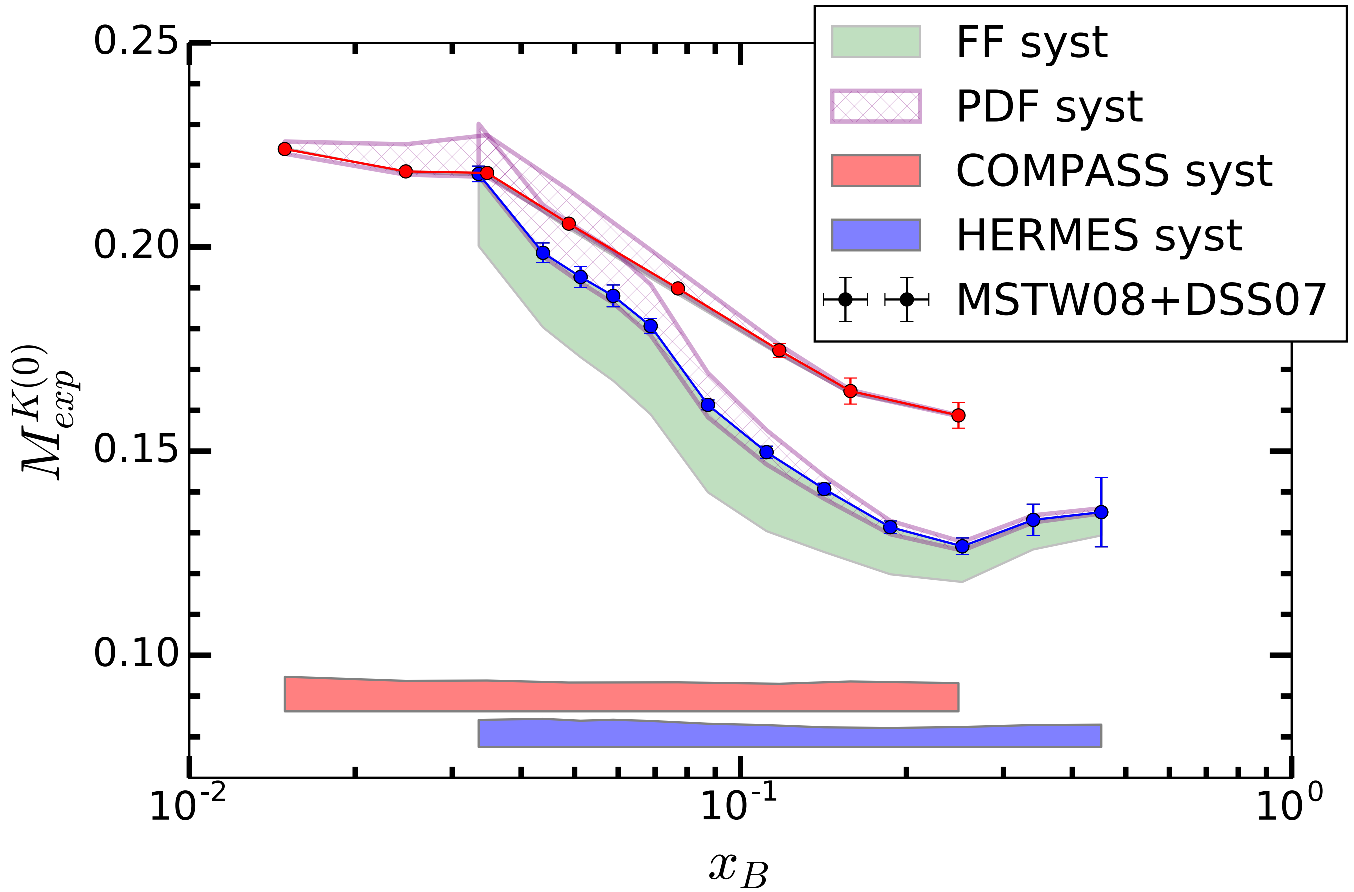
$$M^K_{\rm exp} = (M^{K^+} + M^{K^-})_{\rm exp}$$

- COMPASS: $M^{K(0)}_{\rm exp} \equiv M^K_{\rm exp} \times R_{HMC}$
- HERMES: $M{\rm exp}^{K(0)} \equiv M^K_{\rm exp} \times R_{HMC} \times R^{H\to C}_{evo}$

Original data



"Massless" evolved data



- HMCs reduce the discrepancy in size