Double parton scattering in QCD

Jonathan Gaunt (CERN)

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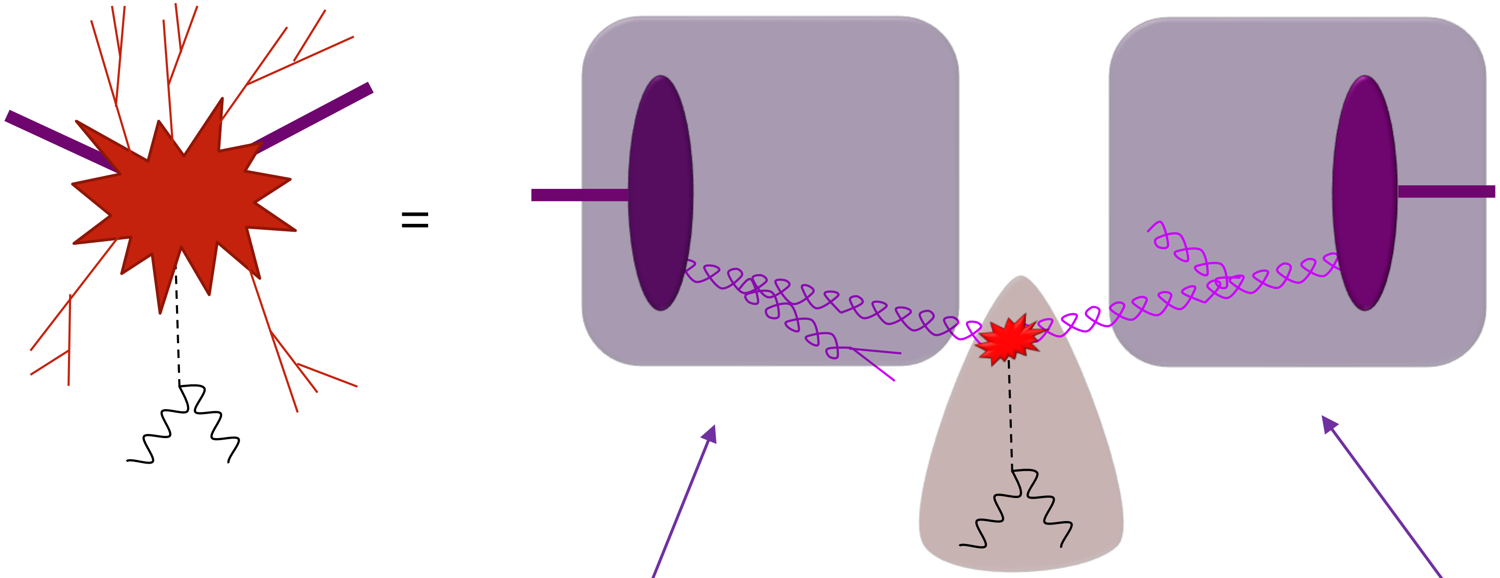
DIS2019, Torino, Italy, 08/04/19

Altarelli Award

FACTORISATION AT THE LHC

'Master formula' for computing cross sections at hadron colliders:



+ corrections suppressed by Λ/Q

$$\sigma_{pp\to H(ZZ)+X} = D_i(x_1, Q^2) \otimes \hat{\sigma}_{ik\to H(ZZ)}(\hat{s} = x_1 x_2 s) \otimes D_k(x_2, Q^2)$$

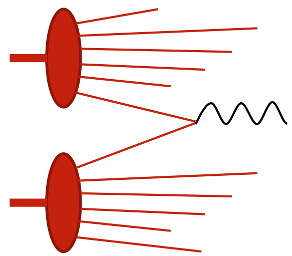
PDF — Hard cross section — PDF

Sum over all possibilities for particles accompanying *H*

HISTORY OF THE FACTORISATION FORMULA

Parton Model [Feynman, 1969, Drell, Yan, 1970]

- At large Q, partons seen as free particles in proton
- No scale variations in PDFs

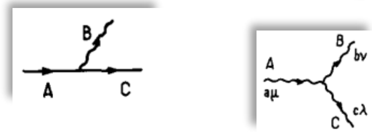


QCD-Improved Parton Model [Altarelli, Parisi, 1977, Dokshitzer, 1977, Gribov, Lipatov, 1972,…]

- QCD radiation possible at all scales
- PDFs vary logarithmically with scale according to DGLAP equations

$$\frac{dq^i(x,t)}{dt} = \frac{\alpha(t)}{2\pi}\int_x^1 \frac{dy}{y}\left[q^i(y,t)\,P_{qq}\left(\frac{x}{y}\right) + G(y,t)\,P_{qG}\left(\frac{x}{y}\right)\right],$$

$$\frac{dG(x,t)}{dt} = \frac{\alpha(t)}{2\pi}\int_x^1 \frac{dy}{y}\left[\sum_{i=1}^{2f} q^i(y,t)\,P_{Gq}\left(\frac{x}{y}\right) + G(y,t)\,P_{GG}\left(\frac{x}{y}\right)\right].$$

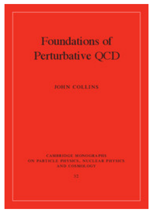


Altarelli, Parisi, Nucl.Phys. B126 (1977) 298-318

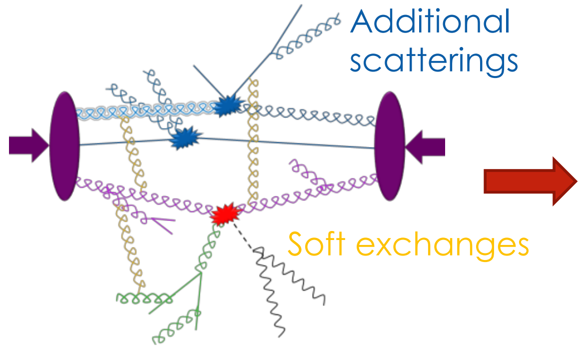
Full proof of collinear factorisation for hadronic collisions

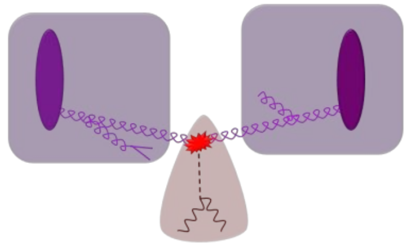
[Collins, Soper, Sterman, 1985, 1988, Bodwin, 1985, Collins, 2011]

- Demonstration that effect of additional scatterings and soft exchanges cancel, at least for inclusive cross section for colour singlet production.

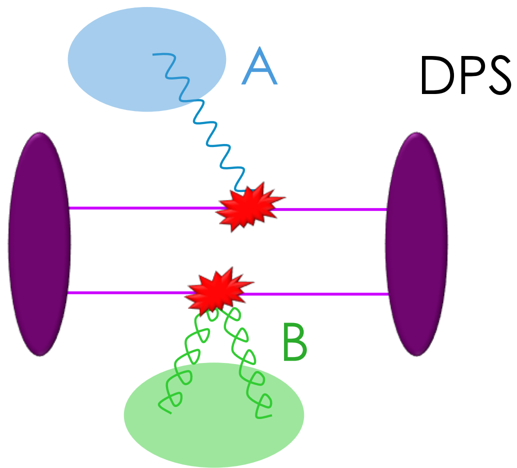


~600 pages





DOUBLE PARTON SCATTERING

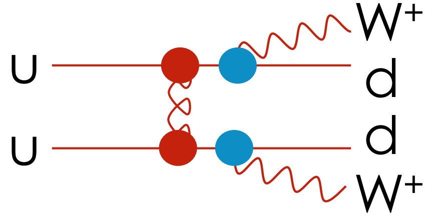


If final state of interest can be divided into two sets, each with associated hard scale, then a particularly important power correction term is double parton scattering (DPS).

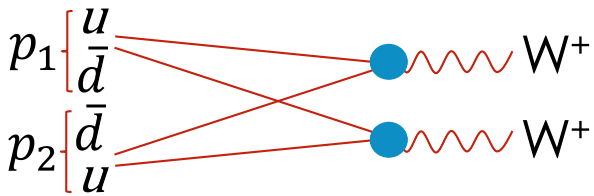
Why is DPS of particular interest/importance?

DPS can compete with single scattering (SPS) if SPS is suppressed by small/multiple couplings (e.g. same sign WW) and in certain phase space regions (small $q_{T,A}$, $q_{T,B}$).

SPS:



DPS:



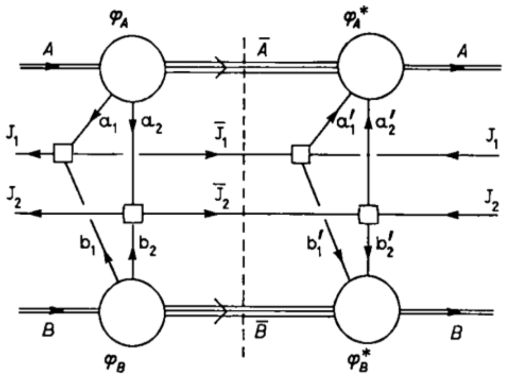
DPS becomes more important relative to SPS (and other power-suppressed mechanisms) as the collider energy grows – most important at LHC and future colliders (e.g. HE-LHC).

DPS tells us new information about proton structure - namely, correlations between partons!

FIRST STEPS IN DPS THEORY

What is the factorisation formula for DPS? Does factorisation even apply for DPS!?

First step: parton model calculation. Pioneering calculations by Politzer, 1980, Paver, Treleani, 1982, Mekhfi, 1985:

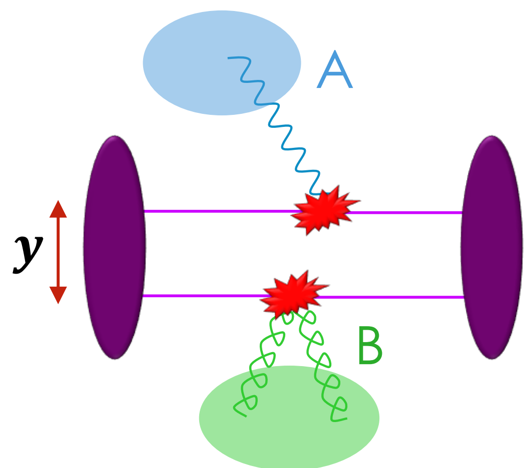


Double parton distributions (DPDs)

$$\sigma_{DPS}^{(A,B)} = \int F_{ik}(x_1, x_2, \mathbf{y}) F_{jl}(x'_1, x'_2, \mathbf{y}) \hat{\sigma}_{ij}^{A} \hat{\sigma}_{kl}^{B} dx_i dx'_i d^2\mathbf{y}$$

Paver, Treleani, Nuovo Cim. A70 (1982) 215
Mekhfi, Phys.Rev. D32 (1985) 2371
Diehl, Ostermeier, Schafer, JHEP 1203 (2012) 089

$\mathbf{y}$ is transverse separation between partons:



FIRST STEPS IN DPS THEORY

Ignoring correlations between partons, one obtains:

$$\sigma_{DPS}^{(A,B)} = \int F_{ik}(x_1, x_2, \boldsymbol{y})\, F_{jl}(x'_1, x'_2, \boldsymbol{y})\; \hat{\sigma}_{ij}^{A}\, \hat{\sigma}_{kl}^{B}\, dx_i dx'_i d^2\boldsymbol{y} \quad \xrightarrow{F_{ik}(x_1, x_2, \boldsymbol{y}) \to D_i(x_1) D_k(x_2) G(y)} \quad \sigma_{DPS}^{(A,B)} = \sigma_{SPS}^{(A)} \sigma_{SPS}^{(B)} / \sigma_{eff}$$

'DPS pocket formula'

This simplified approach is the basis for:

- many phenomenological investigations of DPS
- parton shower models of MPI (although many improvements over pocket formula approach in these models!)

See e.g. Sjöstrand, Skands, JHEP 0403 (2004) 053, Eur.Phys.J. C39 (2005) 129-154, Corke, Sjöstrand, JHEP 1105 (2011) 009

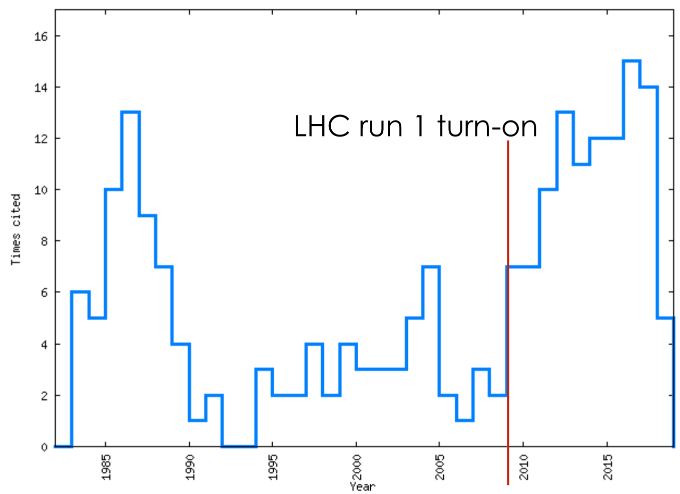
But what about developments towards full theoretical description of DPS in QCD?

RECENT THEORY PROGRESS

Not much until ~2010, then huge burst of theoretical activity! Possibly spurred on by turn-on of LHC.

Many groups making progress on the theory: JG, Stirling, Diehl, Schafer, Blok, Dokshitzer, Frankfurt, Strikman, Manohar, Waalewijn, Ryskin, Snigirev…

Citations for Paver, Treleani paper



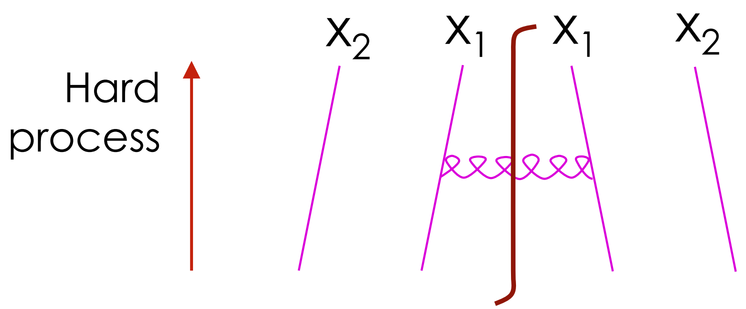
DPS was my PhD project! Worked with my supervisor James Stirling…

…and then later with Markus Diehl, Andreas Schäfer and others.

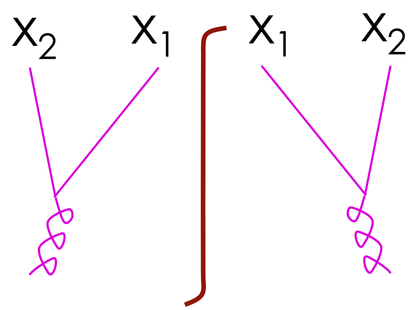
I will touch on a few highlights from my work.

ADDING QCD RADIATION

Imagine that we start at the hard scattering and ‘go backwards’ towards longer distance scales. What kind of QCD effects can occur?



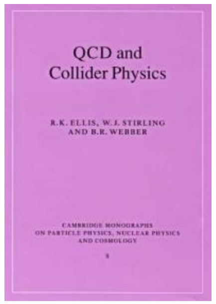
Simple emission from one leg: very similar to single scattering case, can be treated in same way

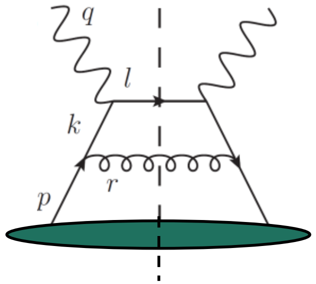


‘1→2 splitting’: new effect. How should we treat these?

ADDING QCD RADIATION: 1V1 DIAGRAMS

What did we do for single scattering?



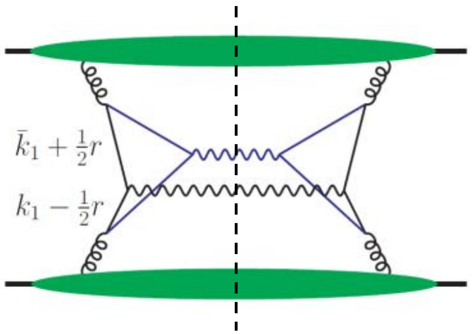


$P_{qq}(z)$

$$\frac{d^2\sigma}{dxdQ^2}|_{qe\to egq,a} = \int_0^1 dz \int^{Q^2} \frac{dk_\perp^2}{k_\perp^2}\frac{\alpha_s}{2\pi}C_F\left[\frac{1+z^2}{1-z}\right]\frac{d^2\sigma}{dxdQ^2}|_{qe\to eq}(z)$$ + non divergent

Logarithmic divergence.

'Absorb' into PDF, up to scale μ_F



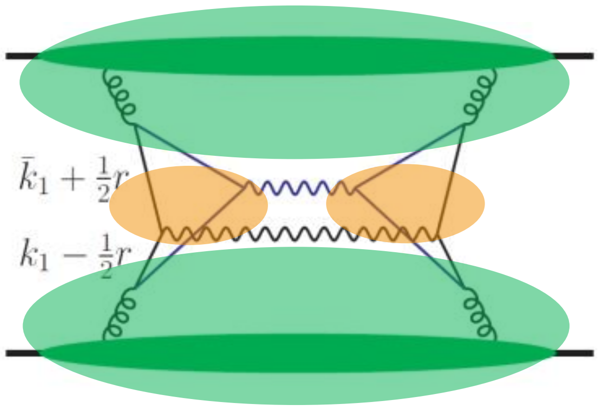
Let's try to apply this approach to this diagram with two $1 \to 2$ splittings ('1v1' diagram). Looking for some piece proportional to $(\Lambda^2/Q^4)\log^2(Q^2/\Lambda^2)$

DPS is power suppressed   Two splittings

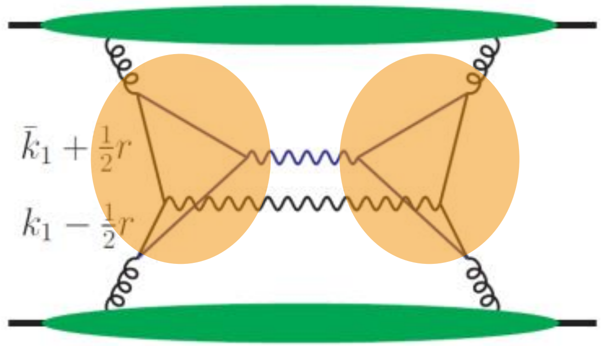
ADDING QCD RADIATION: 1V1 DIAGRAMS

First observation: this can also be viewed as a loop correction to SPS!

DPS view:



SPS view:



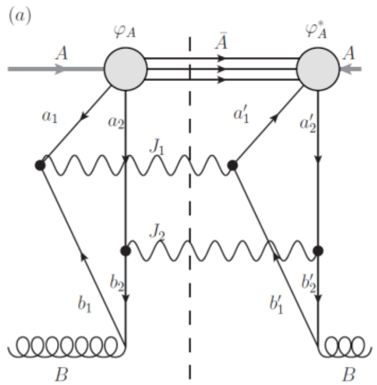
Second observation: there is no 'natural' piece of this graph (or any 1v1 graph) that is proportional to $(\Lambda^2/Q^4)\log^2(Q^2/\Lambda^2)$. No power corrections in massless QCD!

JG and Stirling, JHEP 1106 048 (2011)

Just remove 1v1 graphs from DPS and regard as SPS?

JG and Stirling, JHEP 1106 048 (2011)
Blok et al. Eur.Phys.J. C72 (2012) 1963
Manohar, Waalewijn Phys.Lett. B713 (2012) 196

ADDING QCD RADIATION: 2V1 DIAGRAMS



JG, JHEP 1301 (2013) 042
Blok et al., Eur.Phys.J. C72 (2012) 1963
Ryskin, Snigirev, Phys.Rev.D83:114047,2011

At the same time, graphs with one $1 \to 2$ splitting ('2v1' diagrams) do have a $(\Lambda^2/Q^4)\log(Q^2/\Lambda^2)$ piece:

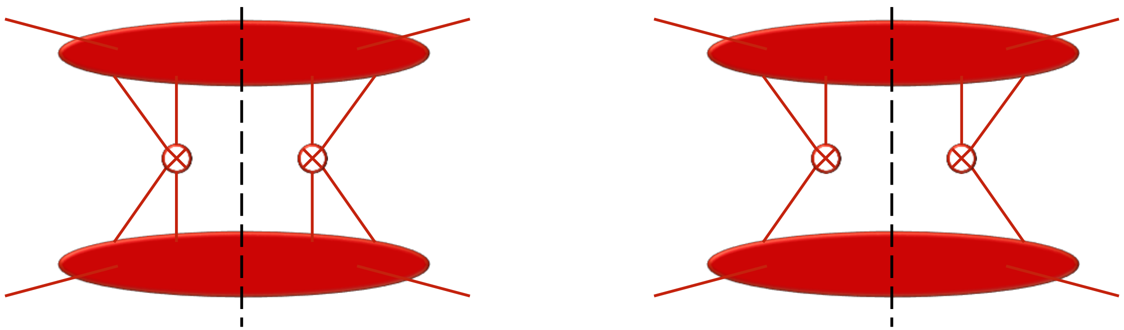
$$\sigma_{1v2}(s) = \sum_{s_i s_i' t_i t_i'} \int dx_1 dx_2 dy_1 dy_2 \hat{\sigma}_{\bar{q}q\to\gamma*}^{s_1,t_1;s_1',t_1';\mu_1}(\hat{s} = x_1 y_1 s)\hat{\sigma}_{q\bar{q}\to\gamma*}^{s_2,t_2;s_2',t_2';\mu_2}(\hat{s} = x_2 y_2 s)$$
$$\times \quad \Gamma_A^{s_1 s_2, s_1' s_2'}(x_1, x_2; \boldsymbol{b} = \boldsymbol{0}) \quad \left[\frac{\alpha_s}{2\pi} P_{g\to q\bar{q}}^{\lambda\to t_2 t_1, t_2' t_1'}(y_2)\,\delta(1 - y_1 - y_2) \int_{\Lambda^2}^{Q^2} \frac{d\boldsymbol{J_1}^2}{\boldsymbol{J_1}^2}\right]$$

$\propto \Lambda^2$

'$1 \to 2$' Altarelli-Parisi splitting function

Divergent logarithm

So…then we include the 2v1 graphs in DPS, but not the 1v1 graphs? Drawback of this approach: no 'factorisation' in the usual sense, with individual DPDs for each hadron. Appropriate hadronic objects involve both protons at once:



Blok et al. Eur.Phys.J. C72 (2012) 1963
Manohar, Waalewijn Phys.Lett. B713 (2012) 196

A NEW SCHEME

Alternative idea: *can* regard some of the 1v1 loop diagrams as DPS if we want: only the sum of SPS+DPS to produce a particular final state is meaningful!

Split these diagrams up into DPS/SPS according to value of partonic separation y, and be careful to avoid double counting.

This is the basis for the the DGS framework for computing DPS:

Diehl, Gaunt, Schönwald, JHEP 1706 (2017) 083

Step1: Insert cutoff into DPS cross section:

$$\sigma_{\rm DPS} = \int d^2y\, \Phi^2(\nu y)\, F(x_1, x_2; y) F(\bar{x}_1, \bar{x}_2; y)$$

DPDs including both parton pairs generated by perturbative $1 \to 2$ splittings, and "intrinsic" pairs

If one just combined this with SPS naively, would have double counting up to scale ν!

A NEW SCHEME

Step 2: For total cross section for production of *AB*, include a subtraction term to remove double counting.

$$\sigma_{tot} = \sigma_{DPS} + \sigma_{SPS} - \sigma_{sub}$$

Subtraction term constructed by adapting techniques used by Collins in the SPS factorisation treatment, and satisfies by construction:

$$\sigma_{tot}(y \sim 1/Q) \approx \sigma_{SPS} \qquad \sigma_{tot}(y \sim 1/\Lambda) \approx \sigma_{DPS}$$

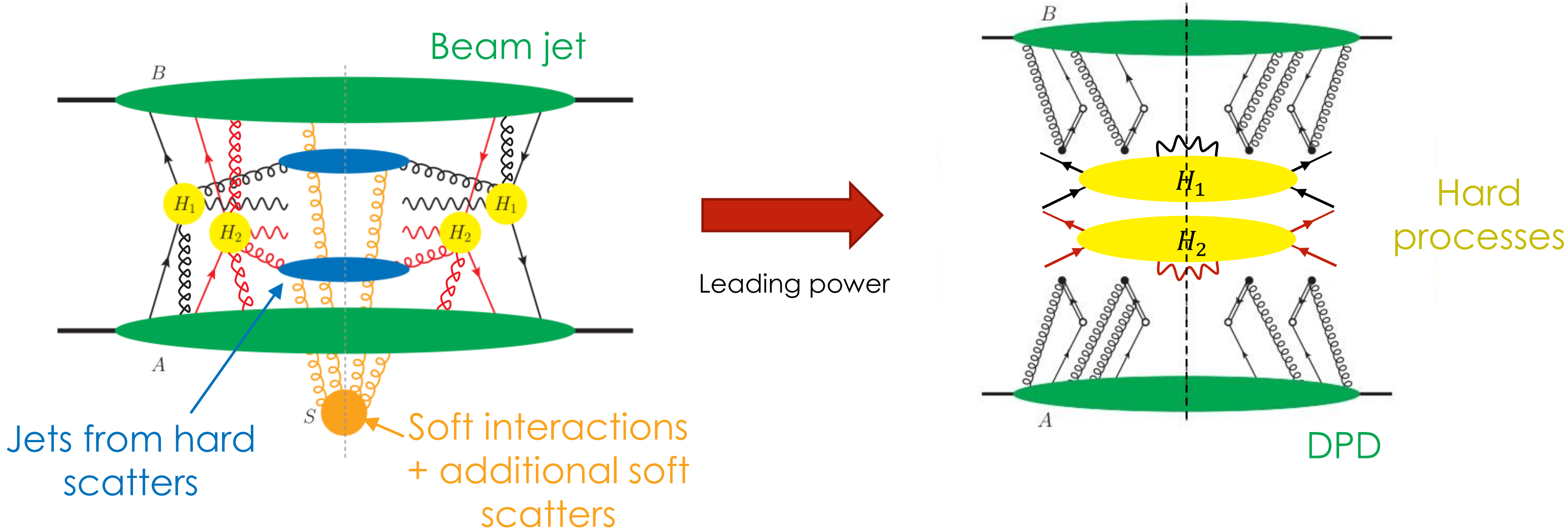
Some advantages:

- Retain concept of the DPD for an individual hadron with rigorous operator definition. Such a definition needed for lattice computations of DPDs (see e.g. Bali et al., JHEP 1812 (2018) 061)

- Resum logs in all diagrams where appropriate (2v2, 2v1 and 1v1).

FACTORISATION IN DPS

To prove factorisation for DPS inclusive cross section, need to show:

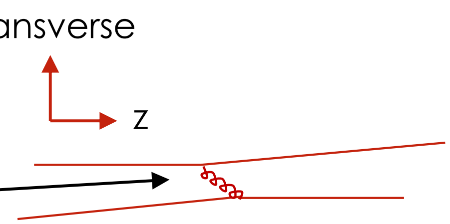


Key step: need to separate off all soft connections entangling beam and final state jets.

For ‘normal’ soft exchanges, this can be achieved via Ward identities:

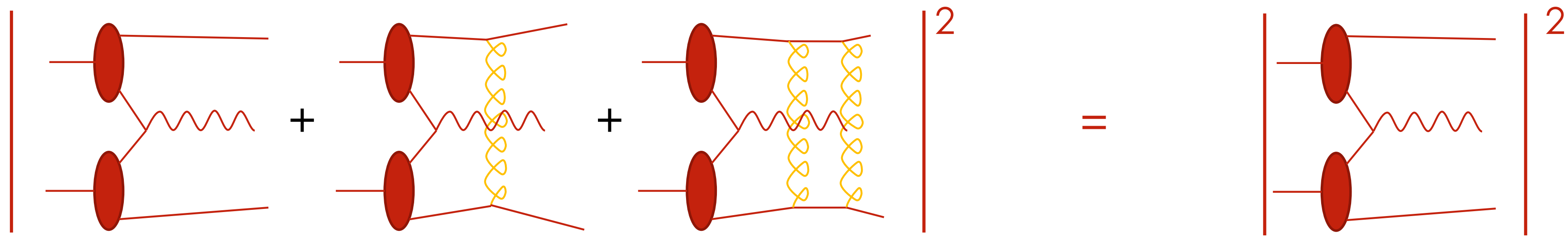
FACTORISATION: SOFT EXCHANGES

However, there is a particular type of soft exchange for which this doesn't work: Glauber exchanges. Soft particles mediating forward scattering.



Treatment of Glauber exchanges is the trickiest part of a factorisation proof!

Single scattering production of colour singlet V: Collins, Soper, Sterman showed that effect of Glauber exchanges cancels if we measure only properties of V, and sum over everything else!



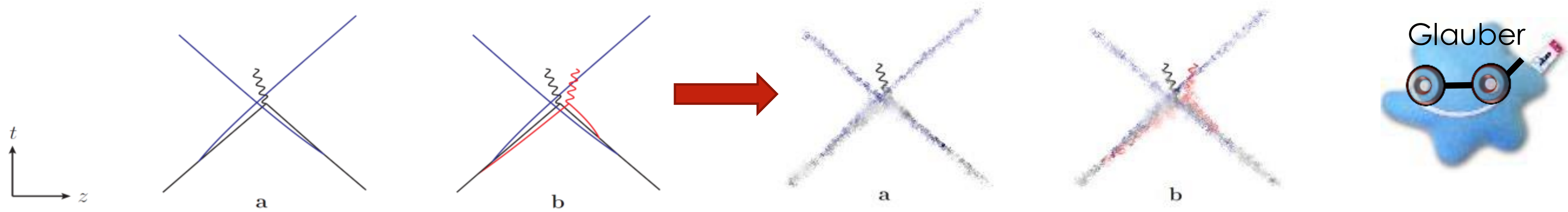
If one starts measuring properties of radiation accompanying V (e.g. global event shape variables), this argument breaks down!

JG, JHEP 1407 (2014) 110
Zeng, JHEP 1510 (2015) 189
Schwartz, Yan, Zhu, Phys.Rev. D97 (2018) no.9, 096017

GLAUBER CANCELLATION IN DPS

In JHEP 1601 (2016) 076 (Diehl, JG, Schäfer, Ostermeier, Plößl) we adapted the methodology of Collins, Soper, Sterman to show that Glauber exchanges also cancel for DPS production of two colourless systems.

Full proof is very technical, but can get some insight as to why it works by looking at spacetime pictures of single and double scattering:



Other important steps towards factorisation proof made in Diehl, Ostermeier, Schafer, JHEP 1203 (2012) 089 Vladimirov, JHEP 1804 (2018) 045, Diehl, Nagar, arXiv:1812.09509.

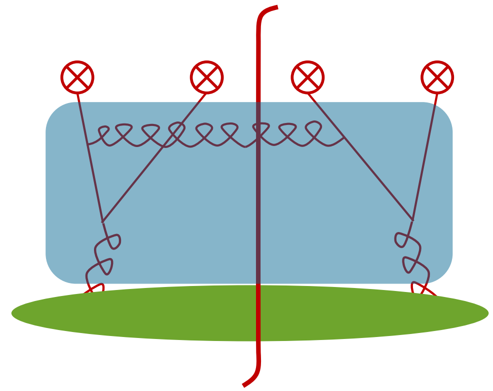
GOING TO NLO

Another advantage of DGS framework for computing DPS is that it can be formulated at all orders, with corrections that can be practicably computed!

What perturbative ingredients do we need for NLO DPS cross sections?

- NLO corrections to partonic cross sections: already known for many processes from SPS calculations ✓
- NLO 'usual' Altarelli-Parisi splitting functions - needed for evolution of $F(\boldsymbol{y})$: already known since the 80s ✓
- NLO corrections to 1 → 2 splitting

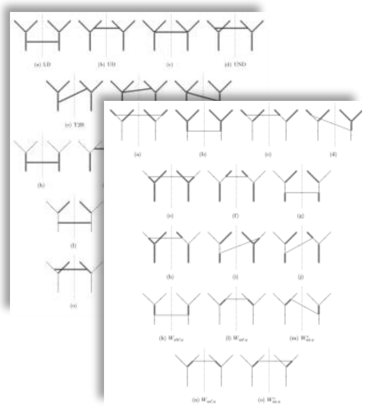
$$F_{a_1 a_2}(x_1, x_2, y, \mu) = \frac{1}{\pi y^2} \sum_{a_0} \int_{x_1+x_2}^{1} \frac{dz}{z^2} V_{a_1 a_2, a_0}\left(\frac{x_1}{z}, \frac{x_2}{z}, a_s(\mu), \log\frac{\mu^2 y^2}{b_0^2}\right) f_{a_0}(z, \mu)$$



GOING TO NLO

Recently we computed the NLO corrections to V for all flavour channels (Diehl, Gaunt, Plößl, Schäfer, arXiv:1902.08019).

Turned out to require modern methods for computing multiloop Feynman integrals:



Graphs

Integration-by-parts identities

$$I_1(a_1,a_2,a_3,a_4) = \int \frac{d^{d-2}\boldsymbol{k}_1 d^{d-2}\boldsymbol{k}_2}{\prod_{i=1..4} D_i^{a_i}}$$

'Master integrals'

Differential equations, boundary conditions from method of regions

Results for bare graphs!

Opens the way for the first full NLO computations of DPS!

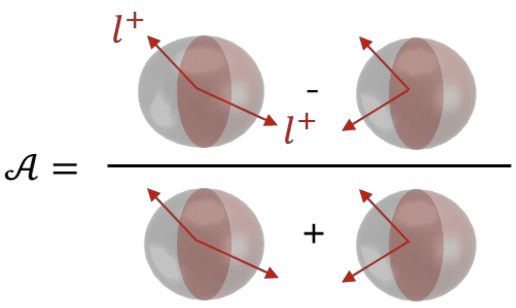
In fact, computing ingredients needed for NLO DPS computations was one of my planned PhD tasks. Finally done just over 10 years later!

NEXT STEPS

Now that QCD DPS framework is established, put it into practice!

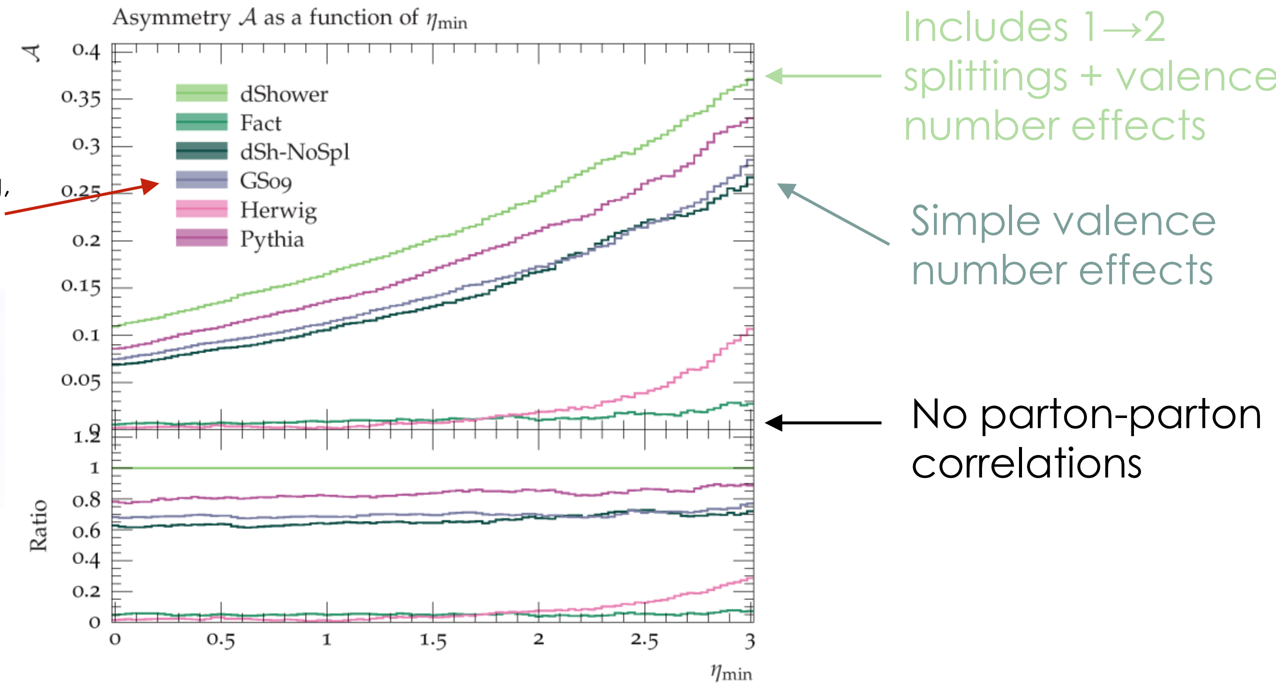
- Find processes and regions where effects of 1→2 splittings and other QCD correlations are significant!
- Provide tools for others to compute DPS cross sections under DGS framework.

$$pp \to W^+W^+ \to e^+\nu_e\mu^+\nu_\mu$$



JG, Kom, Kulesza, Stirling, Eur.Phys.J. C69 (2010) 53-65

JG, Stirling, JHEP 1003 (2010) 005



Cabouat, JG, Ostrolenk

Can find out more about DPS/MPI on Thursday morning!

MANY THANKS TO…

- The selection committee for this award, as well as the sponsors (Centro Fermi, the European Physical Journal, World Scientific)
- Guido Altarelli for his pioneering work in QCD!
- Markus Diehl
- My collaborators, both junior and senior (Gunnar Bali, Daniel Boer, Baptiste Cabouat, Shireen Gangal, Tomas Kasemets, Anna Kulesza, Steve Kom, Rafal Maciula, Piet Mulders, Peter Plößl, Kay Schönwald, Max Stahlhofen, Andreas Schäfer, Antoni Szczurek, Frank Tackmann, Tom van Daal, Jon Walsh, Bryan Webber, Christian Zimmermann…)
- MPI@LHC community
- My family!

Finally, a huge thank you to James Stirling, who guided and encouraged me during the early stages of my physics career.

Colossal contribution to DIS physics and QCD collider physics in general.

THE DIAGRAMMATIC STRUCTURE OF DEEP INELASTIC SCATTERING IN ASYMPTOTICALLY FREE THEORIES

W.J. STIRLING
Department of Applied Mathematics and Theoretical Physics, University of Cambridge, England

Received 21 March 1978

We investigate the struct… ple scalar gluon model form… gluon theory. It is found tha… dard QCD result for the non… strahlung model is presented…

IMPLICATIONS OF NEW DEEP INELASTIC SCATTERING DATA FOR PARTON DISTRIBUTIONS

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Received 23 December 1987

PHENOMENOLOGY OF THE ANOMALOUS GLUON CONTRIBUTION TO POLARIZED LEPTOPRODUCTION

G. Altarelli and W.J. Stirling

Abstract

The gluon contribution, in… and its phenomenological imp… and its x distribution. We stu… helicity. We compare the requ… parton densi… constraints an… by the combi… experimental…

…an attempt to resolve the …ple QCD fits are obtained …istributions derived from

Spin-dependent parton distributions from polarized structure function data

T. Gehrmann[1], W.J. Stirling[2]

…LE, UK

MRST2001: partons and α_S from precise deep inelastic scattering and Tevatron jet data

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EVIDENCE FOR MULTIPLE PARTON INTERACTIONS FROM THE OBSERVATION OF MULTI-MUON EVENTS IN DRELL–YAN EXPERIMENTS

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Received 15 January 1987

Like-Sign W Boson Production at the LHC as a Probe of Double Parton Scattering

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← Long-standing interest in DPS!