B Meson Light Cone Distribution Amplitudes and Application

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Outline

- Part I: Light-Cone Distribution Amplitudes (LCDAs)
  - Definitions and Classification
  - Constraints
  - One-loop evolution
  - Models
  - Application
- Part II: Study of rare decay $B \to \ell \nu_\ell \gamma$
  - Motivation
  - Theoretical Framework
  - Predictions and Outlook

Motivations

Many high energy processes can be written as a convolution of perturbative and nonperturbative parts.
Schematically,

$$\mathcal{A} = [C] \otimes [\phi] \tag{1}$$

where $[C]$ corresponds to a high energy process which can be computed perturbatively, $[\phi]$ encodes the nonperturbative information of the hadronic state and is called distribution amplitude.
At present, $[\phi]$ is not directly accessible theoretically.
We can still study its evolution and asymptotic behavior, build model to fit experimental/lattice data.

Part I: definitions of leading two-particle B-meson LCDAs

The leading two-particle B-meson LCDAs (more generally pseudoscalar in HQET) are constructed from the matrix element of the renormalized nonlocal operator involving a heavy quark field $h_v(0)$ and a light (anti)quark with a light-like separation: [A. G. Grozin and M. Neubert, (1997)]

$$\langle 0|\, \bar{q}(nz)\Gamma[nz,0]h_v(0)\,|\bar{B}(v)\rangle = \\ = -\frac{i}{2}F_B(\mu)\,\mathrm{Tr}\left\{\gamma_5\Gamma P_+\left[\Phi_+(z,\mu) - \frac{1}{2}\not{n}\left(\Phi_+(z,\mu) - \Phi_-(z,\mu)\right)\right]\right\} \quad (2)$$

where $n^\mu = (1,0,0,-1)$ is the light-cone vector, $v^\mu = (1,0,0,0)$ is the heavy quark velocity, $P_+ = \frac{1}{2}(1+\not{v})$, $F_B(\mu)$ is the B-meson decay constant in HQET which can be matched to the physical one. $\Phi_+(z,\mu)$ and $\Phi_-(z,\mu)$ are called leading and sub-leading-twist two-particle B-meson DAs. [M. Beneke and T. Feldmann, (2001)]

Part I: three-particle B-meson LCDAs

The general three-particle B-meson LCDAs are constructed by including the gluon field strength tensor and projecting out different Lorentz structures. There are 8 independent ones in total.

[H. Kawamura, J. Kodaira, C. F. Qiao and K. Tanaka (2001); B. Geyer and O. Witzel, (2005)]

In practice, the field strength tensor is often contracted with n^μ which reduces the number of independent Lorentz structures to 6.

- ☺ The original basis of the three-particle DAs is convenient because of the simple Lorentz structures.
- ☹ It is not suitable for QCD factorization as DAs in this basis have no definite collinear twist $t = d - s$. Hence terms with different power counting get mixed.

We rewrite the three-particle LCDAs with definite collinear twist and conformal spin $j = \frac{1}{2}(d + s)$. [V. M. Braun, YJ and A. N. Manashov (2017)]

The collinear twist t determines the power counting in QCD factorization while the conformal spin j is related to the evolution of LCDAs and their small momentum behavior.

Part I: B-meson LCDAs of definite twists

Two-particle LCDAs:

- 1 collinear twist-2 DA: Φ_+
- 1 collinear twist-3 DA: Φ_-
- 1 collinear twist-4 DA: G_+
- 1 collinear twist-5 DA: G_-

$\Big\}$ not independent

Three-particle LCDAs:

- 1 collinear twist-3 DA: Φ_3
- 3 collinear twist-4 DAs: Φ_4, Ψ_4, $\widetilde{\Psi}_4$
- 3 collinear twist-5 DAs: Φ_5, Ψ_5, $\widetilde{\Psi}_5$
- 1 collinear twist-6 DA: Φ_6

An example: [V. M. Braun, YJ and A. N. Manashov (2017)]

$$2F_B(\mu)\Phi_3(z_1, z_2; \mu) = \langle 0| \bar{q}(z_1) g G_{\mu\nu}(z_2) n^\nu \not{n} \gamma_\perp^\mu \gamma_5 h_v(0) |\bar{B}(v)\rangle \ , \ldots \quad (3)$$

which is related to the DAs in the original definition

$$\Phi_3 = \Psi_A - \Psi_V \ , \ldots \quad (4)$$

LCDAs in momentum space

DAs in momentum space are usually preferred in phenomenology. The momentum and coordinate space DAs are related by the Fourier transform:

$$F(z,\mu) = \int_0^\infty d\omega\, \mathrm{e}^{-i\omega z} f(\omega,\mu)\,,$$
$$f(\omega,\mu) = \frac{1}{2\pi}\int_{-\infty}^\infty dz\, \mathrm{e}^{i\omega z}\, F(z-i\epsilon,\mu)\,. \tag{5}$$

Asymptotic behavior of LCDAs at small momentum are (roughly) determined by the conformal spin of each constituent (quark/gluon). In particular, [V. M. Braun and I. E. Filyanov, (1990)]

$$\phi_3(\omega_1,\omega_2) \sim \omega_1\omega_2^2\,,\ \phi_4(\omega_1,\omega_2) \sim \omega_2^2\,,\ \psi_4(\omega_1,\omega_2) \sim \widetilde{\psi}_4(\omega_1,\omega_2) \sim \omega_1\omega_2\,,$$

from pion studies.

Part I: Constraints of LCDAs

Not all higher-twist DAs are independent. In particular,

- Φ_- is related to $\{\Phi_+, \Phi_3\}$ and $\{\Phi_+, \Phi_4, \Psi_4, \bar{\Lambda}\}$
- G_+ is related to $\{\Phi_+, \Phi_-, \Psi_4, \bar{\Lambda}\}$ $\quad \bar{\Lambda} = m_B - m_b$
- G_- is related to $\{\Phi_+, \Phi_-, \Psi_5, \bar{\Lambda}\}$

[H. Kawamura, J. Kodaira, C. F. Qiao and K. Tanaka, (2001); V. M. Braun, YJ and A. N. Manashov, (2017)]

In addition, we also have the first two moments of leading-twist DA:

[A. G. Grozin and M. Neubert, (1997)]

$$\int_0^\infty d\omega\, \omega\, \phi_+(\omega) = \frac{4}{3}\bar{\Lambda}\,, \quad \int_0^\infty d\omega\, \omega^2 \phi_+(\omega) = 2\bar{\Lambda}^2 + \frac{2}{3}\lambda_E^2 + \frac{1}{3}\lambda_H^2\,, \tag{6}$$

and normalization of higher-twist DAs:

$$\Phi_3(\underline{z}=0) = \frac{1}{3}(\lambda_E^2 - \lambda_H^2)\,, \qquad \Phi_4(\underline{z}=0) = \frac{1}{3}(\lambda_E^2 + \lambda_H^2)\,,$$
$$\Psi_4(\underline{z}=0) = \frac{1}{3}\lambda_E^2\,, \qquad \widetilde{\Psi}_4(\underline{z}=0) = \frac{1}{3}\lambda_H^2\,, \tag{7}$$

with $\lambda_E^2/\lambda_H^2 \sim 0.5$ by QCDSR. [A. Grozin and M. Neubert, (1997); T. Nishikawa and K. Tanaka, (2014)]

Part I: Evolution of LCDAs

RGE
$$\left(\mu\frac{\partial}{\partial\mu}+\beta(\alpha_s)\frac{\partial}{\partial\alpha_s}+\frac{\alpha_s}{2\pi}\mathcal{H}\right)f(\underline{z},\mu)=0\,, \tag{8}$$

where $\mathcal{H}$ is an integral operator (evolution kernel) for LCDAs.
$\mathcal{H}$ for leading twist DA $\phi_+(z,\mu)$ is known.

[A. G. Grozin and M. Neubert, (1997); M. Braun, D. Y. Ivanov and G. P. Korchemsky, (2004)]

The one-loop evolution of $\Phi_+(z,\mu)$ can be solved analytically.

[G. Bell, T. Feldmann, Y.-M. Wang and M. W. Y. Yip, (2013)]

$$\Phi_+(z,\mu)=-\frac{1}{z^2}\int_0^\infty ds\, s\, \mathrm{e}^{is/z}\,\eta_+(s,\mu)\,,$$

$$\eta_+(s,\mu)=R(s,\mu,\mu_0)\eta_+(s,\mu_0)\,,\qquad R(s,\mu,\mu_0)\propto s^{\frac{2C_F}{\beta_0}\ln\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}} \tag{9}$$

The evolution of twist-3 and twist-4 LCDAs can also be solved analytically with $N_c^{-2}\sim 10^{-1}$ corrections.

[V. M. Braun, A. N. Manashov and N. Offen, (2015); V. M. Braun, YJ and A. N. Manashov, (2017)]

Coming soon: two-loop evolution of the leading twist DA $\Phi_+(z,\mu)$ (last missing piece for NLL accuracy; nice properties!)

Part I: Models Building for LCDAs

The LCDAs must satisfy all known constraints (EOMs, normalization). Models are constructed at $\mu_0 = 1\ \mathrm{GeV}$ and evolved to different scales.

Exponential Model: [A. Grozin and M. Neubert, (1997); A. Khodjamirian, T. Mannel and N. Offen (2007)]

$$\phi_+(\omega, \mu_0) = \omega_0^{-2} \omega\, \mathrm{e}^{-\omega/\omega_0}\,, \ldots \tag{10}$$

simple and easy to implement, have expected small momentum behaviors, evolution of leading-twist has analytical form. Only one free parameter ω_0.

Duality Model: [A. Khodjamirian, T. Mannel and N. Offen (2007); V. Braun, YJ and A. Manashov, (2017)]

$$\phi_+(\omega, \mu_0) \propto \omega(2\omega_0 - \omega)^p \theta(2\omega_0 - \omega)\,, \ldots \tag{11}$$

larger parameter space, evolution can only be done numerically ...

Generalized Exponential Model: [M. Beneke, V. M. Braun, YJ and Y. B. Wei, (2018)]

$$\phi_+(\omega, \alpha, \beta, \mu_0) = \frac{\Gamma(\beta)}{\Gamma(\alpha)} \frac{\omega}{\omega_0^2} \mathrm{e}^{-\omega/\omega_0} U(\beta - \alpha, 3 - \alpha, \omega/\omega_0)\,,\ \alpha, \beta > 1 \tag{12}$$

easy to implement, evolution of leading-twist has analytical form, larger parameter space.

Applications

B meson LCDAs can be applied to B-meson decay processes that can be factorized into perturbative and nonperturbative parts.
Some recent developments using higher-twist DAs:

- $B \to \pi, K$ [C-D. Lü, Y-L. Shen, Y- M. Wang, Y-B. Wei, (2018)]
- $B \to \pi, K, \rho, K^*, D^*$ [N. Gubernari, A. Kokulu, D. Van Dyk, (2018)]
- $B \to \gamma \ell \nu_\ell$ [Y-M Wang, Y-L Shen, (2018); M. Beneke, V. Braun, YJ, Y-B. Wei, (2018)]

Goal

We investigate the subleading power contributions of $B \to \gamma \ell \nu_\ell$ and provide more accurate predictions for its branching ratio.

Motivation

- The simplest process for probing B meson LCDAs
- Leading power estimate may be too crude for analyzing experimental data
- Sub-leading contributions are interesting in their own right (end-point divergences etc)

Part II: B meson Radiative Leptonic Decay

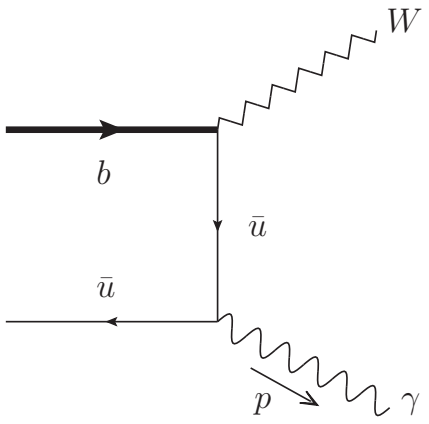


Figure : Leading contribution to $B \to \gamma \ell \nu_\ell$ decay.

The hadronic tensor $T_{\mu\nu}$ for radiative leptonic B-meson decay $B^- \to \gamma \ell \bar{\nu}_\ell$ can be written in terms of two form factors F_V and F_A:

$$T_{\mu\nu}(p,q) = -i \int d^4x\, e^{ipx} \langle 0| T\{ j_\mu^{em}(x)\, \bar{u}(0)\gamma_\nu(1-\gamma_5) b(0)\} |B^-(p+q)\rangle$$

$$= \quad \epsilon_{\mu\nu\tau\rho} p^\tau v^\rho F_V + i\big[-g_{\mu\nu}(pv) + v_\mu p_\nu \big] F_A - i \frac{v_\mu v_\nu}{(pv)} f_B m_B + \cancel{p_\mu\text{-terms}}\,.$$

Here $(p+q) = m_B v$ is the B-meson momentum in the B meson rest frame.

Part II: B meson Radiative Leptonic Decay

The differential decay width is given by the form factors:

$$\frac{d\Gamma}{dE_\gamma} = \frac{\alpha_{\rm em} G_F^2 |V_{ub}|^2}{6\pi^2} m_B E_\gamma^3 \left(1 - \frac{2E_\gamma}{m_B}\right) \left(\left|F_V\right|^2 + \left|F_A + \frac{e_\ell f_B}{E_\gamma}\right|^2\right)$$

For large photon energies [M. Beneke and J. Rohrwild, (2011)]

$$\begin{aligned} F_V(E_\gamma) &= \underbrace{\frac{e_u f_B m_B}{2E_\gamma \lambda_B(\mu)} R(E_\gamma, \mu)}_{\text{leading power contribution } F_{V/A}^{\rm tw-2}} + \underbrace{\xi(E_\gamma) + \Delta\xi(E_\gamma)}_{\substack{\text{power suppressed} \\ \text{\& } \gamma \text{ from b-quark vertex}}}, \\ F_A(E_\gamma) &= \frac{e_u f_B m_B}{2E_\gamma \lambda_B(\mu)} R(E_\gamma, \mu) + \xi(E_\gamma) - \Delta\xi(E_\gamma), \end{aligned} \tag{13}$$

where $1/\lambda_B$ is the inverse moment of the twist-2 DA.
$R(E_\gamma)$ is the radiative correction which equals 1 at tree-level, one-loop corrections are known [M. Beneke and J. Rohrwild, (2011)]. We study $\xi(E_\gamma)$ and $\Delta\xi(E_\gamma)$.

Hard-collinear vs. Soft contributions

Power suppressed corrections are divided into several parts:

$$\xi(E_\gamma) = \xi^{\rm ht} + \xi^{\rm soft}_{\rm (NLO)} + \xi^{\rm soft}_{\rm (tw-3,4)} + \xi^{\rm soft}_{\rm tw-5,6}\,,$$
$$\Delta\xi(E_\gamma) = \Delta\xi^{\rm ht} + \Delta\xi^{\rm soft}_{\rm (tw-3,4)} + \Delta\xi^{\rm soft}_{\rm (tw-5,6)} \tag{14}$$

$\xi^{\rm ht}$ appears when the virtuality of the quark propagator that joins the weak and EM vertex is hard-collinear, $m_b\Lambda$. $\xi^{\rm soft}_{(\ldots)}$ arises when the virtuality $E_\gamma\omega$ enters the soft region, Λ^2.
The leading-power contribution $F^{\rm tw-2}_{V/A}$ depends on the twist-2 DA through the so-called jet function $J(E_\gamma,\mu)$ which arises when the quark propagator is hard-collinear. Its modification for the soft region gives rise to $\xi^{\rm soft}_{\rm (NLO)}$ [Y.-M. Wang, (2016)].

Higher twist correction $\xi^{\rm ht}$

The $\xi^{\rm ht}$ correction is further split into two parts $\xi^{\rm ht} = \xi^{\rm ht}_{1/E_\gamma} + \xi^{\rm ht}_{1/m_b}$. Using the light-cone expansion of the light quark propagator and definitions of B-meson LCDAs we find the higher-twist $1/E_\gamma^2$ contributions to be

$$\xi^{\rm ht}_{1/E_\gamma} = \frac{e_u f_B m_B}{4E_\gamma^2}\left\{-1 + 2\int_0^\infty d\omega\, \ln\omega\, \phi_-^{\rm t3}(\omega) - 2\int_0^\infty \frac{d\omega_2}{\omega_2}\,\phi_4(0,\omega_2)\right\},$$

$$\Delta\xi^{\rm ht}_{1/E_\gamma} = \frac{e_u f_B m_B}{4E_\gamma^2}\,, \qquad (15)$$

$\phi_-^{\rm t3}(\omega)$ is genuine twist-3 components of $\phi_-(\omega)$

ξ_{1/m_b} is obtained by expanding the nonlocal operator $\bar{q}(x)\Gamma\vec{D}_\xi h_v(0)$ to $\frac{1}{m_b}$

$$\xi^{\rm ht}_{1/m_b} = \frac{e_u f_B m_B}{4m_b E_\gamma}\left\{\frac{\bar{\Lambda}}{\lambda_B} - 2 + 2\int_0^\infty \frac{d\omega_1}{\omega_1}\int_0^\infty \frac{d\omega_2}{\omega_1+\omega_2}\,\phi_3(\omega_1,\omega_2)\right\},$$

$$\Delta\xi^{\rm ht}_{1/m_b} = \frac{e_b f_B m_B}{2E_\gamma m_b}\,, \qquad (16)$$

b-quark photon emission [M. Beneke and J. Rohrwild, (2011)]

$\xi^{\rm ht}_{1/E_\gamma}$ and $\xi^{\rm ht}_{1/m_b}$ can be written into different forms by EOMs.

Key message: No endpoint divergence!

Light-Cone Sum Rules and soft corrections

Soft contributions arise when the virtuality of the quark propagator becomes small. In this case, we apply the LCSR to take it into account. LCSR can be understood as matching of two different representations for the correlation function $T_{\mu\nu}$ — from the QCD side in terms of quarks and gluons DOF vs. physical intermediate hadronic state.

[V. M. Braun and A. Khodjamirian, (2013); I. I. Balitsky, V. M. Braun and A. V. Kolesnichenko, (1989)]

For the simplest sum rule, matching the QCD factorization and the hadronic approach at large photon virtuality $-p^2$ allows us to get the following formula for the soft-contribution

$$\xi^{\rm soft}_{B\to\gamma}(E_\gamma) = \frac{1}{\pi}\int_0^{s_0}\frac{ds}{s}\left[\frac{s}{m_\rho^2}e^{-(s-m_\rho^2)/M^2}-1\right]\mathrm{Im}\,F^{\rm QCD}_{B\to\gamma^*}(E_\gamma,s)\,, \qquad (17)$$

where s_0 is an effective continuum threshold. $M^2 \in (1,2)\ \mathrm{GeV}^2$ is the Borel parameter suppressing higher states. The sensitivity of the soft contribution to M^2 indicates the accuracy of the method.

Soft contribution of higher twist DAs

The twist-2 soft corrections (NLO) reads,

$$\xi^{\mathrm{t2-soft}}_{(\mathrm{NLO})} = \frac{e_u f_B m_B}{2E_\gamma}\left[1+\alpha_s g(E_\gamma,\mu)+\mathcal{O}(\alpha_s^2)\right]U(E_\gamma,\mu_h,\mu)$$
$$\times \int_0^{\frac{s_0}{2E_\gamma}} d\omega' \left[\frac{2E_\gamma}{m_\rho^2}\,\mathrm{e}^{-(2E_\gamma\omega'-m_\rho^2)/M^2}-\frac{1}{\omega'}\right]\phi_+^{\mathrm{eff}}(\omega',\mu)$$

the α_s correction to the jet function is included in ϕ_+^{eff} [Y.-M. Wang, (2016)] .
The twist-3,4 soft contributions read, [M. Beneke, V. Braun, YJ, Y-B. Wei, (2018)]

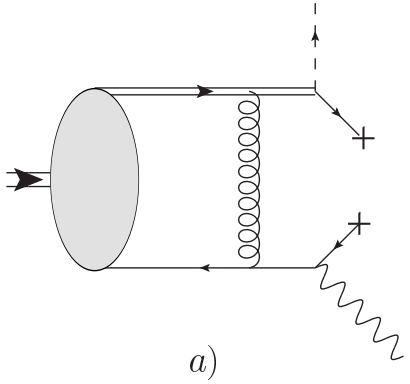
$$\xi^{\mathrm{soft}}_{(\mathrm{tw-3,4})}(E_\gamma) = \frac{e_u m_B f_B}{4E_\gamma}\int_0^{\frac{s_0}{2E_\gamma}} d\omega \left[\frac{2E_\gamma}{m_\rho^2}\,\mathrm{e}^{\frac{m_\rho^2-2E_\gamma\omega}{M^2}}-\frac{1}{\omega}\right]\left(\frac{\Xi_1(\omega)}{E_\gamma}+\frac{\Xi_2(\omega)}{m_b}\right)$$

$$\Delta\xi^{\mathrm{soft}}_{(tw-3,4)}(E_\gamma) = \frac{e_u m_B f_B}{4E_\gamma^2}\int_0^{\frac{s_0}{2E_\gamma}} d\omega \left[\frac{2E_\gamma}{m_\rho^2}\,\mathrm{e}^{\frac{m_\rho^2-2E_\gamma\omega}{M^2}}-\frac{1}{\omega}\right]\omega\,\phi_+(\omega)\,,$$

where Ξ_1, Ξ_2 depend on higher twist DAs.

Factorizable twist-5,6 four-particle contribution

We also include factorizable twist-5,6 4-particle contributions using factorization approximation. They arise as products of lower twist DAs and quark condensates.



$$\Rightarrow\ T^{(\mathrm{a})}_{\mu\nu}(p,q) = \frac{ie_u f_B m_B g^2 C_F \langle\bar{u}u\rangle}{48 p^2 E_\gamma}\,\mathrm{Tr}\left[\not{p}\gamma_\mu\gamma_\nu P_L \not{v}\right]\int_0^\infty d\omega\,\frac{\phi_-(\omega)}{p^2-2E_\gamma\omega}$$

The real photon limit $p^2 \to 0$ cannot be taken directly. 6 diagrams in total.

Dispersion relation improvement is necessary and we find

$$\xi^{\mathrm{soft}}_{(\mathrm{tw}-5,6)} = \frac{e_u g^2 C_F \langle\bar{u}u\rangle f_B m_B}{48 E_\gamma^2\, m_\rho^2}\left\{ \mathrm{e}^{\frac{m_\rho^2}{M^2}} \int_0^{\frac{s_0}{2E_\gamma}} \frac{d\omega}{\omega}\left(\mathrm{e}^{-\frac{2E_\gamma\omega}{M^2}} - 1\right)\phi_-^{\mathrm{WW}}(\omega) \right.$$
$$\left. + \int_{\frac{s_0}{2E_\gamma}}^{\infty} \frac{d\omega}{\omega}\left(\frac{m_\rho^2}{2E_\gamma\omega} - \mathrm{e}^{\frac{m_\rho^2}{M^2}}\right)\phi_-^{\mathrm{WW}}(\omega) - \frac{5}{\lambda_B}\mathrm{e}^{\frac{m_\rho^2}{M^2}} \right\},$$

$$\Delta\xi^{\mathrm{soft}}_{(\mathrm{tw}-5,6)} = -\frac{e_u g_s^2 C_F \langle\bar{u}u\rangle f_B m_B}{48 E_\gamma^2\, m_\rho^2 \lambda_B}\mathrm{e}^{m_\rho^2/M^2}$$

with terms of $\mathcal{O}(1/E_\gamma^3)$ excluded. ϕ_-^{WW} is the twist-2 component in $\phi_-(\omega)$.

Ansatz for higher-twist DAs

We propose the general ansatz for the LCDAs as follows

$$\phi_+(\omega) = \omega\, f(\omega)\,,$$
$$\phi_3(\omega_1,\omega_2) = -\frac{1}{2}\varkappa(\lambda_E^2-\lambda_H^2)\,\omega_1\omega_2^2\, f'(\omega_1+\omega_2)\,,$$
$$\phi_4(\omega_1,\omega_2) = \frac{1}{2}\varkappa(\lambda_E^2+\lambda_H^2)\,\omega_2^2\, f(\omega_1+\omega_2)\,,$$

and other relavent higher twist DAs are obtained from EOMs. This gives

$$\xi^{\rm ht}(E_\gamma) = -\frac{e_u f_B m_B}{2E_\gamma^2}\left\{\frac{2(\lambda_E^2+2\lambda_H^2)}{6\bar\Lambda^2+2\lambda_E^2+\lambda_H^2}+\frac{1}{2}\right\}$$
$$+\frac{e_u f_B m_B}{4m_b E_\gamma}\left\{\frac{\bar\Lambda}{\lambda_B}-2+\frac{4(\lambda_E^2-\lambda_H^2)}{6\bar\Lambda^2+2\lambda_E^2+\lambda_H^2}\right\},$$

without knowing the explicit expression for function $f(\omega)$.
Note that the soft contributions of the higher-twist DAs still require the explicit form of $f(\omega)$.

Ansatz for twist-2 DA

Explicit expression for the function $f(\omega)$ is necessary for studying the soft contributions of the LCDAs and the evolution of twist-2 DA. We propose three models (all normalized to 1):

$$\phi_+^{(\mathrm{I})}(\omega,\mu_0) = \left[(1-b) + \frac{b\,\omega}{2\omega_0}\right]\frac{\omega}{\omega_0^2}\,\mathrm{e}^{-\omega/\omega_0}\,, \quad 0 \le b \le 1$$

$$\phi_+^{(\mathrm{II})}(\omega,\mu_0) = \frac{1}{\Gamma(2+a)}\frac{\omega^{1+a}}{\omega_0^{2+a}}\mathrm{e}^{-\omega/\omega_0}\,, \quad -0.5 < a < 1$$

$$\phi_+^{(\mathrm{III})}(\omega,\mu_0) = \frac{\sqrt{\pi}}{2\Gamma(3/2+a)}\frac{\omega}{\omega_0^2}\,e^{-\omega/\omega_0}\,U(-a,3/2-a,\omega/\omega_0)\,, \quad 0 < a < 0.5$$

all of which are special cases for the general ansatz,

$$\phi_+(\omega) = \frac{\Gamma(\beta)}{\Gamma(\alpha)}\frac{\omega}{\omega_0^2}\,e^{-\omega/\omega_0}U(\beta-\alpha,3-\alpha,\omega/\omega_0)\,,$$

$$\Longrightarrow\ \lambda_B = \frac{\alpha-1}{\beta-1}\omega_0\,, \qquad \widehat{\sigma}_1 = \psi(\beta-1) - \psi(\alpha-1) + \ln\frac{\alpha-1}{\beta-1}\,,\ldots$$

Final predictions

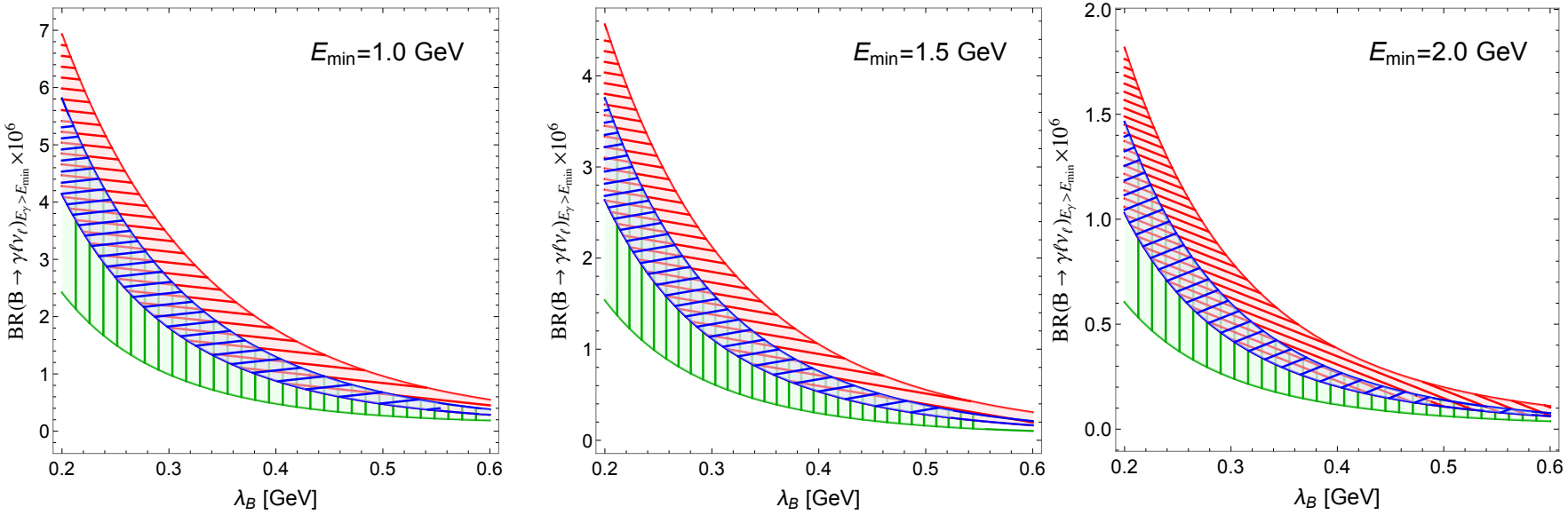


Figure : Predictions of $B \to \gamma \ell \nu_\ell$ branching fraction with different lower cut for the photon energy E_γ. Uncertainties from input parameters (scale dependences, m_b, etc) are negligible compared to the model uncertainties.

Note that the leading twist contribution dominates over the total soft contributions as well as the hard-collinear higher twist contributions, proving the validity of the twist/power expansion.

Prediction meets experimental data

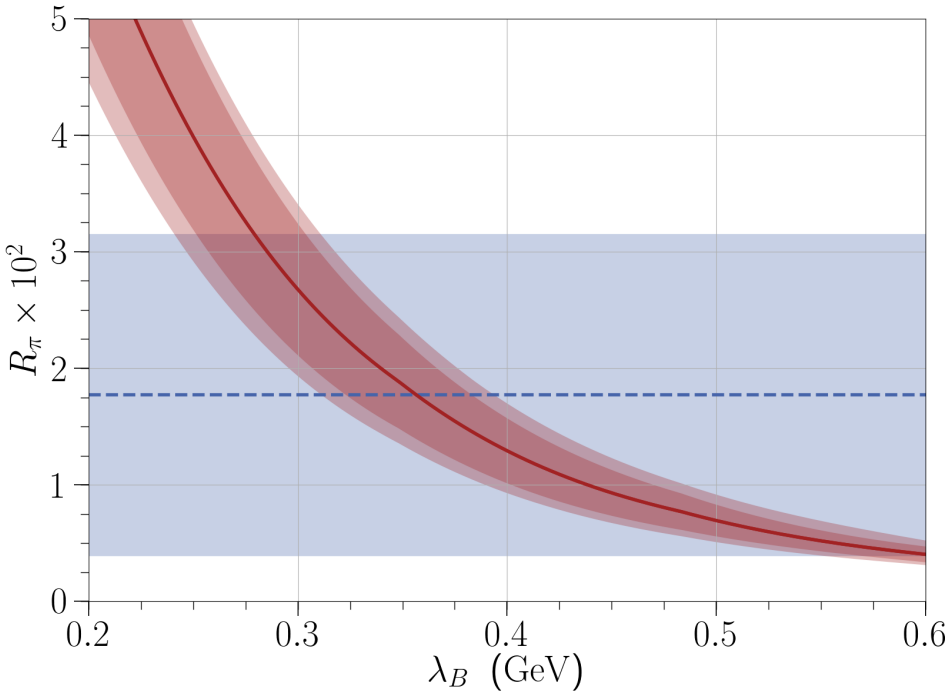


Figure : Theory prediction and 1σ uncertainty (red line and dark red band) for R_π vs. measured value and 1σ uncertainty (blue dashed line and band). The light red band contains full model dependence. Note that extrapolation is used for $E_\gamma > 1$ GeV.

Recent Belle experiment finds $\lambda_B = 0.36^{+0.25}_{-0.09}$ GeV with $\lambda_B > 0.24$ GeV at 90% CL incorporating an overall model dependence.

[M. Gelb *et al.* [Belle Collaboration] (2018).

Conclusion and Outlook

Conclusion

- We have given a comprehensive study of higher twist B meson LCDAs and their $B \to \gamma \ell \nu_\ell$ decay.
- We find the higher power corrections in $B \to \gamma \ell \nu_\ell$ are small but not negligible and without end-point divergences, evidence for the validity of the higher power/twist expansion.
- λ_B has been extracted based on our theory predictions.

Outlook for future work

- Two loop renormalization for the twist-2 DA is desirable to match the three-loop cusp anomalous dimension in the literature (*coming soon*).
- Re-derivation of EOMs in a cut-off scheme consistent with the treatment of the large momentum behavior of the twist-2 DA.
- An updated estimate of higher twist matrix elements.
- A detailed study of recent $B \to \gamma \ell \nu_\ell$ data including the two-loop evolution of twist-2 DA.

Backup Slides

Part I: definitions of three-particle B-meson LCDAs

The general three-particle B-meson LCDAs are constructed by adding the gluon field strength tensor and projecting out specific gamma structures:

$$
\begin{aligned}
&\langle 0|\, \bar{q}(nz_1) g G_{\mu\nu}(nz_2) \Gamma h_v(0)\, |\bar{B}(v)\rangle = \\
&= \frac{1}{2} F_B(\mu)\, \mathrm{Tr}\Big\{ \gamma_5 \Gamma P_+ \Big[(v_\mu \gamma_\nu - v_\nu \gamma_\mu)\big[\Psi_A - \Psi_V\big] - i\sigma_{\mu\nu}\Psi_V \\
&\quad - (n_\mu v_\nu - n_\nu v_\mu) X_A + (n_\mu \gamma_\nu - n_\nu \gamma_\mu)[W + Y_A] \\
&\quad - i\epsilon_{\mu\nu\alpha\beta} n^\alpha v^\beta \gamma_5 \widetilde{X}_A + i\epsilon_{\mu\nu\alpha\beta} n^\alpha \gamma^\beta \gamma^5 \widetilde{Y}_A \\
&\quad - (n_\mu v_\nu - n_\nu n_\mu) \not{n} W + (n_\mu \gamma_\nu - n_\nu \gamma_\mu) \not{n} Z \Big] \Big\}(z_1, z_2; \mu)\,. \qquad (18)
\end{aligned}
$$

In total there are 8 independent ones.

[H. Kawamura, J. Kodaira, C. F. Qiao and K. Tanaka (2001); B. Geyer and O. Witzel, (2005)]

In practice, the field strength tensor is often contracted with n^μ which reduces the number of independent LCDAs to 6.

Twist Expansion

Subleading power corrections at $\mathcal{O}(\alpha_s^0)$ (e.g. $1/E_\gamma^2$, $1/m_b E_\gamma$) are generated by higher twist operators

- twist-2 one 2-particle LCDA: $\phi_+(\omega,\mu)$
- twist-3 one 2-particle LCDA: $\phi_-(\omega,\mu)$ and one 3-particle LCDA $\phi_3(\omega_1,\omega_2,\mu)$
- twist-4 one 2-particle LCDA: $\underset{\sim}{g}_+(\omega,\mu)$ and three 3-particle LCDA $\phi_4(\omega_1,\omega_2,\mu)\,,\psi_4(\omega_1,\omega_2,\mu)\,,\tilde{\psi}_4(\omega_1,\omega_2,\mu)$

At one-loop order, mixing happens inclusively within the LCDAs listed above under renormalization V. M. Braun, YJ, and A. N. Manashov JHEP 1705 (2017) 022

Evolution of higher twist DAs

$$\Phi_3(\underline{z},\mu) = \int_0^\infty ds \Big[\eta_3^{(0)}(s,\mu)\, Y_3^{(0)}(s\,|\,\underline{z}) + \frac{1}{2}\int_{-\infty}^{\infty} dx\, \eta_3(s,x,\mu)\, Y_3(s,x\,|\,\underline{z})\Big],$$

where

$$Y_3(s,x\,|\,\underline{z}) = \frac{is^2}{z_1^2 z_2^3}\int_0^1 du\, u\bar{u}\, e^{is(u/z_1+\bar{u}/z_2)}\, {}_2F_1\left(\begin{matrix}-\frac{1}{2}-ix, -\frac{1}{2}+ix\\ 2\end{matrix}\Big|-\frac{u}{\bar{u}}\right),$$

$$Y_3^{(0)}(s\,|\,\underline{z}) = Y_3(s,x=i/2\,|\,\underline{z}) = \frac{is^2}{z_1^2 z_2^3}\int_0^1 du\, u\bar{u}\, e^{is(u/z_1+\bar{u}/z_2)}\,.$$

Doublet forms for three-particle twist-4 DAs.

[V. M. Braun, A. N. Manashov and N. Offen, (2015); V. M. Braun, YJ and A. N. Manashov, (2017)]

Higher twist LCDAs

Twist-2,3,4 LCDAs are *not* completely independent. They are related by tree-level EOMs V. M. Braun, YJ, and A. N. Manashov JHEP 1705 (2017) 022.
Moreover, neglecting the 4-particle DAs, which are beyond our current accuracy, the following identity holds V. M. Braun, YJ, and A. N. Manashov JHEP 1705 (2017) 022:

$$[\psi_4 + \widetilde{\psi}_4](\omega_1, \omega_2) - \omega_2 \frac{\partial}{\partial \omega_2}[\psi_4 + \widetilde{\psi}_4](\omega_1, \omega_2) = -2\omega_1 \frac{\partial}{\partial \omega_1}\phi_4(\omega_1, \omega_2)\,,$$

and therefore there are only two independent 3-particle twist-4 DAs at tree level.

Twist-2 leading power contribution (hard-collinear)

The leading power contribution [M. Beneke and J. Rohrwild, (2011)]

$$F_{V/A}^{\mathrm{tw}-2} = \frac{e_u f_B m_B}{2E_\gamma \lambda_B(\mu)} \underbrace{[1 + \alpha_s(g(E_\gamma,\mu) + J(E_\gamma,\mu)) + \mathcal{O}(\alpha_s^2)] U(E_\gamma,\mu_h,\mu)}_{R(E_\gamma,\mu)},$$

where $g(E_\gamma,\mu)$ comes from matching coefficients between SCET and QCD, $J(E_\gamma,\mu)$ is the hard-collinear jet function that depends on the logarithmic moments $(\widehat{\sigma}_1, \widehat{\sigma}_2)$ of the twist-2 DA:

$$\widehat{\sigma}_n = \int_0^\infty d\omega \frac{\lambda_B}{\omega} \ln^n \frac{\lambda_B e^{-\gamma_E}}{\omega} \phi_+(\omega),$$

U is the evolution factor that sums up large logarithms between different scales.

Higher twist corrections $1/(m_b E_\gamma)$

Using the operator identity

$$\bar{q}(x)\Gamma \overrightarrow{D}_\xi \, h_v(0) = \partial_\xi \bar{q}(x)\Gamma h_v(0) + i\int_0^1 du\, \bar{u}\, \bar{q}(x) x^\rho g G_{\rho\xi}(ux)\Gamma h_v(0) - \frac{\partial}{\partial x^\xi}\bar{q}(x)\Gamma h_v(0)$$

the $1/(m_b E_\gamma)$ corrections is found to be

$$\xi^{\rm ht}_{1/m_b} = \frac{e_u f_B m_B}{4 m_b E_\gamma}\left\{\frac{\bar{\Lambda}}{\lambda_B} - 2 + \int_0^\infty d\omega\, \ln\omega\, \phi_-^{\rm t3}(w) + 2\int_0^\infty \frac{d\omega_1}{\omega_1}\int_0^\infty \frac{d\omega_2}{\omega_2}\,\phi_3(\omega_1,\omega_2)\left\{1 - \frac{\omega_1}{\omega_2}\ln\frac{\omega_1+\omega_2}{\omega_1}\right\}\right\}$$

$$= \frac{e_u f_B m_B}{4 m_b E_\gamma}\left\{\frac{\bar{\Lambda}}{\lambda_B} - 2 + 2\int_0^\infty \frac{d\omega_1}{\omega_1}\int_0^\infty \frac{d\omega_2}{\omega_1+\omega_2}\,\phi_3(\omega_1,\omega_2)\right\},$$

$$\Delta\xi^{\rm ht}_{1/m_b} = 0\,,$$

Soft contribution of higher twist DAs

$$\Xi_1(\omega) = -\int_0^\omega d\omega_1 \int_{\omega-\omega_1}^\infty \frac{d\omega_2}{\omega_2}\frac{\partial}{\partial\omega_1}\left[\psi_4 + \widetilde{\psi}_4\right](\omega_1,\omega_2)$$
$$-\int_0^\omega d\omega_2 \int_{\omega-\omega_2}^\infty \frac{d\omega_1}{\omega_1}\frac{\partial}{\partial\omega_2}\left[\psi_4 + \widetilde{\psi}_4\right](\omega_1,\omega_2)$$
$$+\,2\int_\omega^\infty d\rho\,\phi_-^{\mathrm{t3}}(\rho) - 2\omega\phi_-^{\mathrm{WW}}(\omega) + 2\omega\phi_+(\omega) + \omega^2\frac{d}{d\omega}\phi_+(\omega)\,, \quad (19)$$

$$\Xi_2(\omega) = 2\int_0^\infty \frac{d\omega_2}{\omega_2}\,\phi_3(\omega,\omega_2) - 2\int_0^\omega d\omega_1 \int_{\omega-\omega_1}^\infty \frac{d\omega_2}{\omega_2^2}\,\phi_3(\omega_1,\omega_2)$$
$$+\int_\omega^\infty d\rho\,\phi_-^{\mathrm{t3}}(\rho) + (\bar{\Lambda}-\omega)\,\phi_+(\omega) - \omega\phi_-^{\mathrm{WW}}(\omega)\,. \quad (20)$$

Input parameters

μ_0	1 GeV		
$\Lambda_{\rm QCD}^{(4)}$	0.291552 GeV	$\alpha_s(\mu_0)$	0.348929
μ	(1.5 ± 0.5) GeV	μ_h	$m_b/2 \div 2m_b$
m_b	(4.8 ± 0.1) GeV	$\bar{\Lambda}$	$m_B - m_b$
λ_E^2/λ_H^2	0.5 ± 0.1	$2\lambda_E^2 + \lambda_H^2$	(0.25 ± 0.15) GeV^2
s_0	(1.5 ± 0.1) GeV^2	M^2	(1.25 ± 0.25) GeV^2
$\langle \bar{u}u \rangle(\mu_0)$	$-(240 \pm 15$ MeV$)^3$		
m_B	5.27929 GeV	m_ρ	0.77526 GeV
G_F	1.166378×10^{-5} GeV^{-2}	τ_B	$1.638 \times 10^{-12}s$
f_B	(192.0 ± 4.3) MeV *	$\lvert V_{ub}\rvert^{\rm excl}$	$(3.70 \pm 0.16) \times 10^{-3}$**

Table : Central values and ranges of all parameters used in this study. The four-flavor $\Lambda_{\rm QCD}$ parameter corresponds to $\alpha_s(m_Z) = 0.1185$ with decoupling of the bottom quark at the scale m_b.

* (S. Aoki et al., EPJC 77 (2017)) , ** C. Patrignani et al., Chin. Phys. C 40 (2016) no.10, 100001.

Relative importance of different contributions

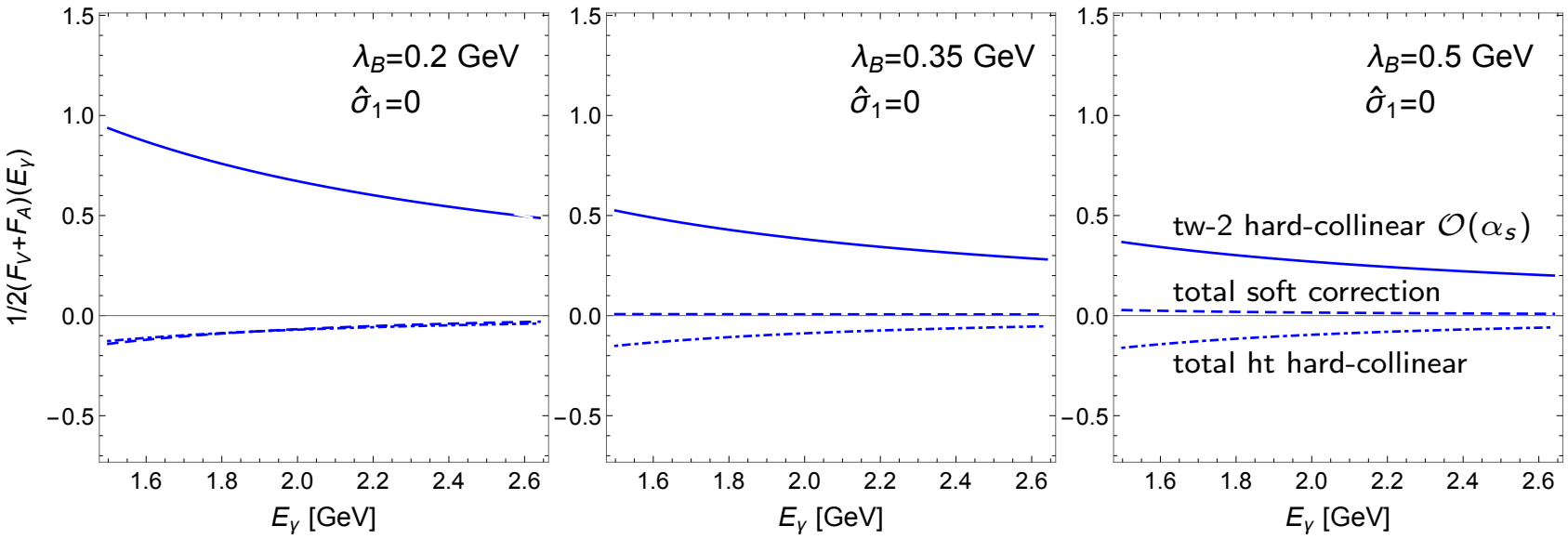


Figure : Relative importance of various contributions.

Final predictions

We present our final predictions for F_V after summing up all the contributions

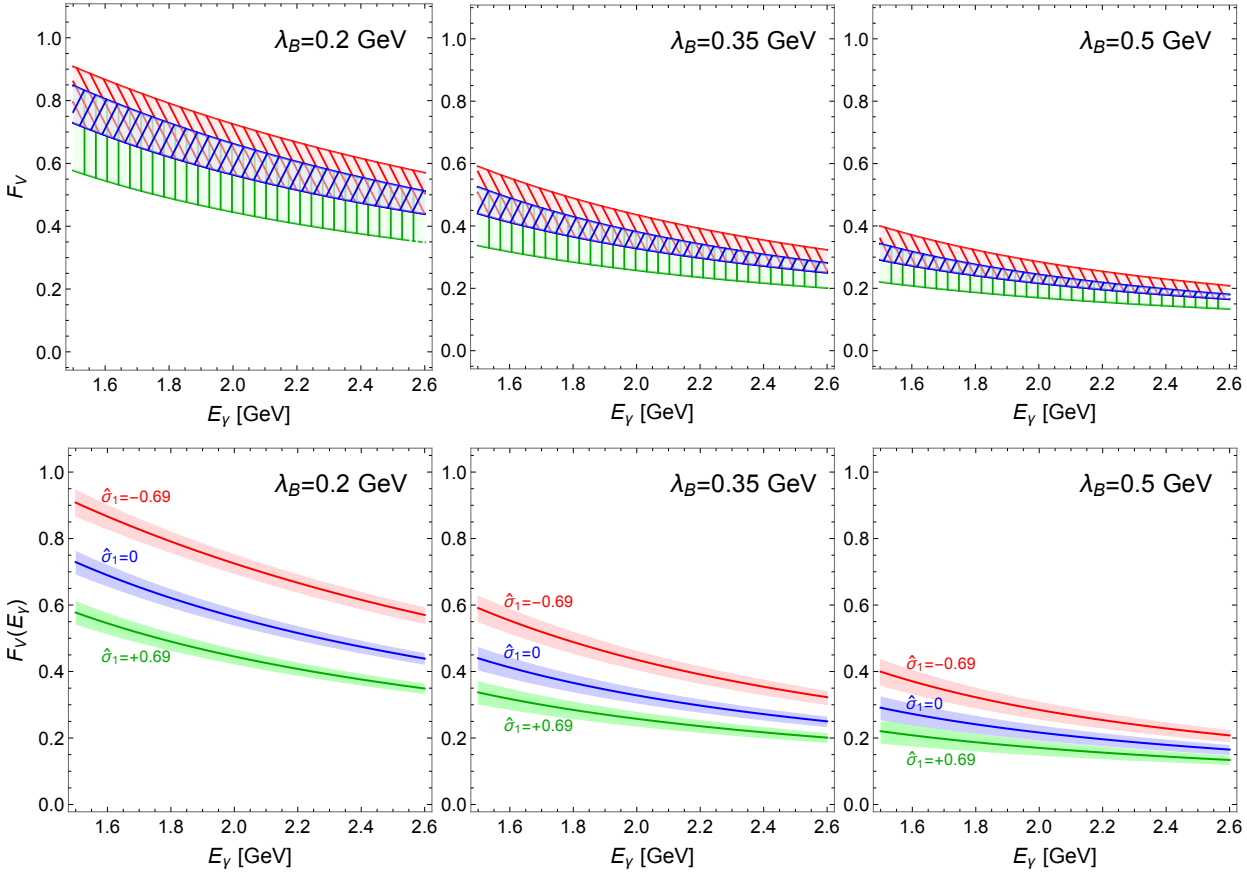


Figure : Vector form factor $F_V(E_\gamma)$. The shaded regions on the upper panels show the variation for a given model with the range of model parameters. The uncertainty due to the physical parameters is shown on the lower panels for $\widehat{\sigma}_1 = 0, \pm 0.69$ corresponding to the boundary of the model ranges and the simple exponential ansatz.