Non-eikonal corrections to multi-particle production in the CGC

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Outline

- eikonal scattering in pA collisions
- relaxing the eikonal approximation: finite width target
- from pA to pp
- single and double inclusive gluon production in pp

Dilute-Dense Scattering within CGC

High energy pA scattering within the CGC :

- Semi-classical approximation :
  - dense target $\equiv$ classical background field $\mathcal{A}_a^\mu(x) = O\left(\frac{1}{g}\right)$ at weak coupling g
  - dilute projectile $\equiv$ color charge $J_a^\mu(x) = O(g)$
- Eikonal approximation:
  - take the high energy limit $s \to \infty$.
  - drop power-suppressed contributions.

Coupling between the projectile and the target $\to \int d^4x \, J_\mu^a(x) \, \mathcal{A}_a^\mu(x)$

In the semi-classical approximation, the eikonal limit can be obtained by either boosting the projectile or the target or both...

Dilute-Dense Scattering

Boosting the target:

$$A_a^\mu(x) \mapsto \begin{cases} \gamma_t\, A_a^- \left(\gamma_t\, x^+,\ \frac{x^-}{\gamma_t},\ \mathbf{x}\right) \\ \frac{1}{\gamma_t} A_a^+ \left(\gamma_t\, x^+,\ \frac{x^-}{\gamma_t},\ \mathbf{x}\right) \\ A_a^i \left(\gamma_t\, x^+,\ \frac{x^-}{\gamma_t},\ \mathbf{x}\right) \end{cases}$$

- $A_a^- \gg A_a^i \gg A_a^+$ in a generic gauge
- in the light-cone gauge:

  $A_a^\mu(x) = \delta^{\mu-}\delta(x^+)A_a^-(\mathbf{x})$

  target is localized at $x^+ = 0$

  independent of x^-

Boosting the projectile :

$$J_a^\mu(x) \mapsto \begin{cases} \frac{1}{\gamma_p}\, J_a^- \left(\frac{x^+}{\gamma_p},\ \gamma_p x^-,\ \mathbf{x}\right) \\ \gamma_p\, J_a^+ \left(\frac{x^+}{\gamma_p},\ \gamma_p x^-,\ \mathbf{x}\right) \\ J_a^i \left(\frac{x^+}{\gamma_p},\ \gamma_p x^-,\ \mathbf{x}\right) \end{cases}$$

- $J_a^+ \gg J_a^i \gg J_a^-$
- slow x^+ dependence due to Lorentz time dilation

  $J_a^\mu(x) \propto \delta^{\mu+}\delta(x^-)\rho^a(\mathbf{x})$

  projectile is localized at $x^- = 0$

Corrections beyond eikonal accuracy

At the level of the background field, the eikonal approximation amounts to

1. $\mathcal{A}_a^\mu(x) \simeq \delta^{\mu-}\mathcal{A}_a^-(x)$
2. $\mathcal{A}_a^\mu(x) \simeq \mathcal{A}_a^\mu(x^+, \mathbf{x})$
3. $\mathcal{A}_a^\mu(x) \propto \delta(x^+)$

Relaxing any of these approximations will give correction to the strict eikonal limit! Three sources of corrections to eikonal approximation:

1. other components of the target background field $\mathcal{A}_a^\mu(x)$
2. dynamics of the target : x^- dependence of $\mathcal{A}_a^\mu(x)$
3. Finite width L^+ of the target along x^+

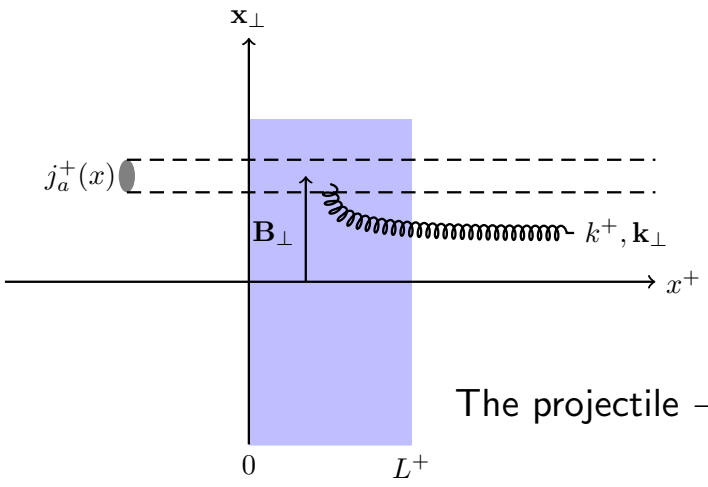
When the target is a large nucleus, the dominant contribution beyond the eikonal accuracy is obtained by relaxing the 3rd approximation because of the $A^{1/3}$ nuclear enhancement of the finite width target!

$$\boxed{\mathcal{A}^\mu = \delta^{\mu-}\delta(x^+)\mathcal{A}^-(\mathbf{x}) \to \mathcal{A}^\mu = \delta^{\mu-}\mathcal{A}^-(x^+, \mathbf{x})}$$

Finite width target: relaxing the eikonal approximation

[T.A., N. Armesto, G. Beuf, M. Martinez, C.A. Salgado - 2014]
[T.A., N. Armesto, G. Beuf, A. Moscoso - 2015]

Consider a finite width target :



The target $\rightarrow \mathcal{A}^\mu(x) \equiv \delta^{\mu-}\mathcal{A}_a^-(x^+, \mathbf{x})$

The projectile $\rightarrow$ $j_a^\mu(x) \propto \delta^{\mu+}\delta(x^-)\,\rho^b(\mathbf{x}-\mathbf{B})$

The single inclusive gluon cross section for pA:

$$(2\pi)^3\,(2k^+)\,\frac{d\sigma}{dk^+\,d^2\mathbf{k}} = \int d^2\mathbf{B} \sum_{\lambda\,\text{phys.}} \left\langle \left\langle |\mathcal{M}_\lambda^a(\underline{k}, \mathbf{B})|^2 \right\rangle_p \right\rangle_A$$

gluon production amplitude

Finite width target: relaxing the eikonal approximation (II)

In the light cone gauge

$$\boxed{\mathcal{M}^a_\lambda(\underline{k},\mathbf{B}) = \varepsilon^{i*}_\lambda\,(2k^+)\lim_{x^+\to+\infty}\int d^2\mathbf{x}\int dx^-\;e^{ik\cdot x}\;\int d^4y\;G^{i-}_R(x,y)_{ab}\;j^+_b(y)}$$

$G^{\mu\nu}_R(x,y)_{ab}$ is the background retarded gluon propagator

Conveniently, the $(i-)$ component of the the background retarded propagator can be written in terms of the scalar background propagator:

$$G^{i-}_R(x,y)_{ab} = \int\frac{\mathrm{d}k^+}{2\pi}\,e^{-ik^+(x^--y^-)}\,\frac{i}{2(k^++i\epsilon)^2}\,\mathcal{G}^{i-}_{k^+}(\underline{x};\underline{y})_{ab}$$

$$\boxed{\mathcal{G}^{i-}_{k^+}(\underline{x};\underline{y})^{ab} = \frac{i}{k^++i\epsilon}\,\partial_{\mathbf{y}^i}\,\mathcal{G}^{ab}_{k^+}(\underline{x};\underline{y})}$$

$\mathcal{G}^{ab}_{k^+}(\underline{x};\underline{y})$ satisfies the scalar Green's eq. whose solution can be written formally as a path integral

$$\mathcal{G}^{ab}_{k^+}(\underline{x};\underline{y}) = \theta(x^+-y^+)\int_{\mathbf{z}(y^+)=\mathbf{y}}^{\mathbf{z}(x^+)=\mathbf{x}}\mathcal{D}\mathbf{z}(z^+)\;\exp\left[\frac{ik^+}{2}\int_{y^+}^{x^+}dz^+\;\dot{\mathbf{z}}^2(z^+)\right]U^{ab}\Big(x^+,y^+,\big[\mathbf{z}(z^+)\big]\Big)$$

with the Wilson line

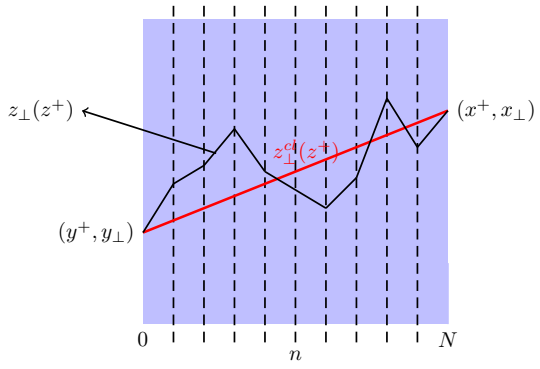
$$\mathcal{U}^{ab}\Big(x^+,y^+,\big[\mathbf{z}(z^+)\big]\Big) \;=\; \mathcal{P}_+\exp\left\{ig\int_{y^+}^{x^+}dz^+\;T\cdot\mathcal{A}^-\Big(z^+,\mathbf{z}(z^+)\Big)\right\}^{ab}$$

following the Brownian trajectory $\mathbf{z}(z^+)$.

Expanding the background propagator

(i) discretize the background propagator.

(ii) Perturbative expansion around free classical path:



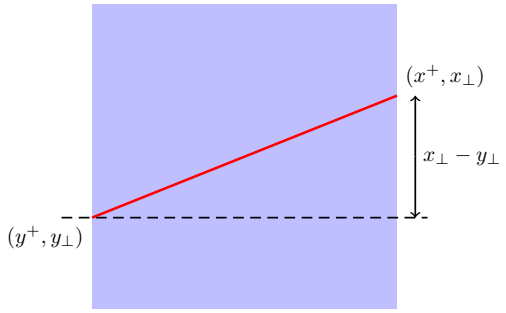
eikonal limit : $\frac{k^+}{(x^+-y^+)} \gg Q_\perp^2$ in the problem

$\mapsto$ large k^+ limit (classical free path!)

$\Rightarrow$ perturbative expansion around the free classical path:

$\mathbf{z}_n = \mathbf{z}_n^{\rm cl} + \mathbf{u}_n$ with $\mathbf{z}_n^{\rm cl} = \mathbf{y} + \frac{n}{N}(\mathbf{x} - \mathbf{y})$

(iii) Expansion around the initial transverse position:



The first expansion is performed for fixed initial and final positions.

In the large k^+ limit , the result has to be re-expanded since $\mathbf{z}^{\rm cl}(z^+) - \mathbf{y}$ is small at each step.

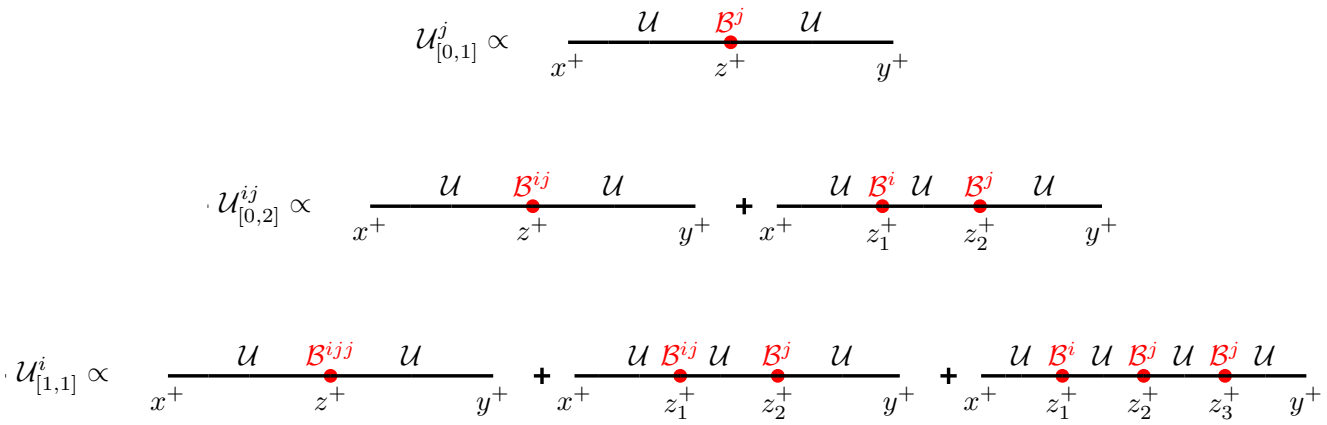
Background scalar propagator at NNE accuracy

After all:

$$\int d^2\mathbf{x}\; e^{-i\mathbf{k}\cdot\mathbf{x}}\; \mathcal{G}^{ab}_{k^+}(\underline{x};\underline{y}) = \theta(x^+-y^+)\; e^{-i\mathbf{k}\cdot\mathbf{y}}\; e^{-ik^-(x^+-y^+)} \Bigg\{ \mathcal{U}(x^+,y^+,\mathbf{y})$$
$$+ \frac{(x^+-y^+)}{k^+}\left[\mathbf{k}^i\, \mathcal{U}^i_{[0,1]}(x^+,y^+,\mathbf{y}) + \frac{i}{2}\,\mathcal{U}_{[1,0]}(x^+,y^+,\mathbf{y})\right]$$
$$+ \frac{(x^+-y^+)^2}{(k^+)^2}\left[\mathbf{k}^i\mathbf{k}^j\mathcal{U}^{ij}_{[0,2]}(x^+,y^+;\mathbf{y}) + \frac{i}{2}\mathbf{k}^i\mathcal{U}^i_{[1,1]}(x^+,y^+;\mathbf{y}) - \frac{1}{4}\mathcal{U}_{[2,0]}(x^+,y^+;\mathbf{y})\right]\Bigg\}^{ab}$$

- $\mathcal{U}(x^+,y^+,\mathbf{y}) \equiv$ standard Wilson lines that appears only at the eikonal level as expected.
- $\mathcal{U}_{[\alpha,\beta]}(x^+,y^+,\mathbf{y}) \equiv$ decorated Wilson lines that only appears beyond eikonal accuracy.
- The subscripts
  - α stands for the order of the expansion around the classical path.
  - β stands for the order of the expansion around the initial transverse position.

Structure of the decorated Wilson lines



with

$$
\begin{aligned}
\mathcal{B}^i(z^+, \mathbf{y}) &\equiv igT \cdot \partial_{\mathbf{y}^i} \mathcal{A}^-(z^+, \mathbf{y}), \\
\mathcal{B}^{ij}(z^+, \mathbf{y}) &\equiv igT \cdot \partial_{\mathbf{y}^i} \partial_{\mathbf{y}^j} \mathcal{A}^-(z^+, \mathbf{y}), \\
\mathcal{B}^{ijl}(z^+, \mathbf{y}) &\equiv igT \cdot \partial_{\mathbf{y}^i} \partial_{\mathbf{y}^j} \partial_{\mathbf{y}^l} \mathcal{A}^-(z^+, \mathbf{y}),
\end{aligned}
$$

Dilute target limit and the modified Lipatov vertex

[T.A., A. Dumitru - 2015]

go from pA → pp:

• *dilute limit of the target:*
expand the standard & decorated Wilson lines to first order in the background field.

• expressions simplify: for example the first decorated dipole becomes

$$U^{i,ab}_{[0,1]}(x^+, y^+, y) = \int_{y^+}^{x^+} dz^+ \frac{z^+ - y^+}{x^+ - y^+} \left[igT^e_{ab} \partial_{y_i} A^{-,e}(z^+, y) \right]$$

• summing up all the NEik and NNEik terms in the dilute target limit, one gets

$$\boxed{\mathcal{M} \propto \left[\frac{(k-q)^i}{(k-q)^2} - \frac{k^i}{k^2} \right] \left\{ 1 + i \frac{k^2}{2k^+} x^+ - \frac{1}{2} \left(\frac{k^2}{2k^+} x^+ \right)^2 \right\}}$$

• $O(1)$ term → eikonal Lipatov vertex.

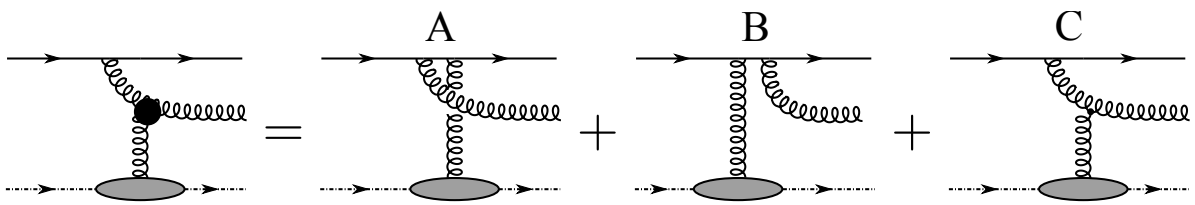
• we get *the Lipatov vertex at NNEik accuracy.*

• the form suggests exponentiation.

Dilute target limit and the modified Lipatov vertex

[P. Agostini, T.A., N. Armesto - 2019]

- calculate the diagrams by keeping the phase $e^{ik^- x^+}$ which is taken to be 1 in the eikonal limit.



The total amplitude reads

$$i(\mathcal{M}_A + \mathcal{M}_B + \mathcal{M}_C) \propto \int \frac{d^2q}{(2\pi)^2} L^i(k,q) e^{ik^- x_1^+} A_a^-(k^-,q) e^{-iq\cdot x_1}$$

with $L^i(k,q)$ is the standard Lipatov vertex

$$\boxed{L^i(k,q) = \frac{(k-q)^i}{(k-q)^2} - \frac{k^i}{k^2}}$$

and the non-eikonal Lipatov vertex being

$$\boxed{L^i_{\mathrm{NE}}(k,q) = \left[\frac{(k-q)^i}{(k-q)^2} - \frac{k^i}{k^2}\right] e^{ik^- x^+}}$$

Noneikonal single inclusive gluon production

Single inclusive gluon production in pA collisions (eikonal accuracy):

$$\frac{d\sigma}{d^2k d\eta} \propto \int_{zx\bar{z}y} e^{ik(z-\bar{z})}\, A^i(x-z)\, A^j(\bar{z}-y) \left\langle \rho^a(x)\rho^b(y)\right\rangle_P \left\langle [U_z - U_x]^{ac}[U^\dagger_{\bar{z}} - U^\dagger_y]^{cb}\right\rangle_T$$

- *projectile averaging:* in x-space $\rightarrow \langle \rho^a(x)\rho^b(y)\rangle_P = \delta^{ab}\mu^2(x,y)$
  in p-space $\rightarrow \langle \rho^a(k)\rho^b(p)\rangle_P = \delta^{ab}\mu^2(k,p) = \delta^{ab}\, T\left(\frac{k-p}{2}\right) F\left[(k+p)R\right]$

T $\rightarrow$ transverse momentum dependent distribution of the color charge densities
F $\rightarrow$ soft form factor which is peaked when its argument vanihes

- *dilute target limit* $\rightarrow U_{ab}(x) \approx 1 + igT^c_{ab}\int_{x^+q} e^{iqx}\, A^-_c(x^+,q)$

$$\left.\frac{d\sigma}{d^2q d\eta}\right|_{\text{dilute}} \propto \int_{x_1^+x_2^+q_1q_2} L^i(k,q_1)\, L^i(k,q_2)\, \mu^2\left[k-q_1, k-q_2\right]\left\langle A^-_c(x_1^+,q_1)A^-_{\bar{c}}(x_2^+,q_2)\right\rangle_T$$

- *go from eikonal to non-eikonal:* $L^i(k,q) \rightarrow L^i_{\text{NE}}(\underline{k},q;x^+)$

- *target averaging:*

$\left\langle A^-_c(x_1^+,q_1)A^-_{\bar{c}}(x_2^+,q_2)\right\rangle_T = \delta^{c\bar{c}} n(x_1^+)\frac{1}{2\lambda^+}\Theta\left(\lambda^+ - |x_1^+ - x_2^+|\right)(2\pi)^2\delta^{(2)}(q_1-q_2)|a(q_1)|^2$

$\lambda^+ \equiv$ color correlation length in the target ($\lambda^+ \ll L^+$)

$n(x^+) \equiv$ 1-d target density along longitudinal direction ($n(x^+) = n_0$ for $0 \leq x^+ \leq L^+$ and 0 elsewhere)

$a(q) \equiv$ functional form of the potential in p-space (Yukawa type $\rightarrow |a(q)|^2 = \frac{m^2}{(q^2+m^2)^2}$)

Noneikonal single inclusive gluon production

After all said and done:

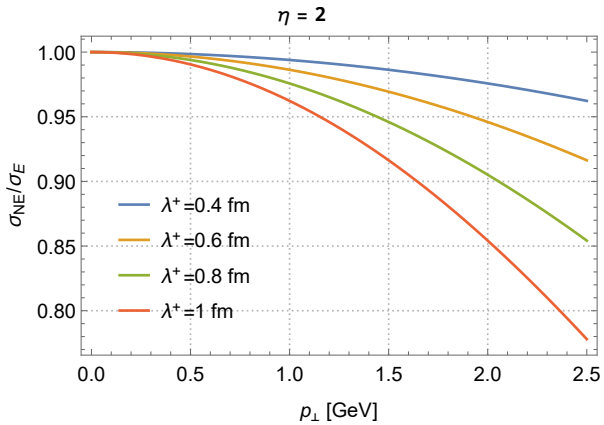
$$\left.\frac{d\sigma}{d^2k d\eta}\right|_{\text{dilute}}^{\text{NE}} \propto \mathcal{G}_1^{\text{NE}}(k^-;\lambda^+)\int_q \mu^2\left[k-q,q-k\right]L^i(k,q)L^i(k,q)\left|a(q)\right|^2$$

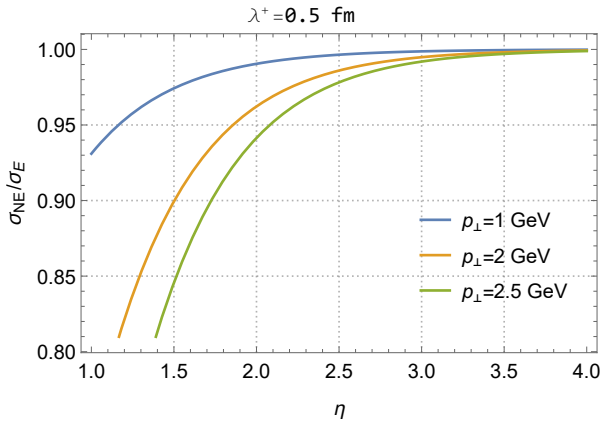
the function that encodes the non-eikonal effects

$$\boxed{\mathcal{G}_1^{\text{NE}}(k^-;\lambda^+) = \frac{1}{k^-\lambda^+}\sin(k^-\lambda^+)}$$

in the eikonal limit:

$$\lim_{(k^-\lambda^+)\to 0}\mathcal{G}_1^{\text{NE}}(k^-;\lambda^+) = 1$$





($N_c = 3$, $m = 0.2$ GeV, $\mu^2(k,q) = \delta^{(2)}(k+q)$ with a projectile size $S_\perp = 4GeV^{-2}$, and regulate the denominators that give rise to infrared divergencies by substituting the corresponding squared transverse momenta $l^2 \to l^2 + m_g^2$ where we have used the numerical value $m_g^2 = 0.2$GeV.)

Double inclusive gluon production and glasma graphs

Same procedure can be adopted to calculate the double inclusive gluon production.

The double inclusive gluon production X-section for dilute-dense scattering:

$$\frac{d\sigma}{d^2k_1 d\eta_1 d^2k_2\eta_2} \propto \int_{z_1 z_2 x_1 x_2 \bar{z}_1 \bar{z}_2 y_1 y_2} e^{ik_1(z_1-\bar{z}_1)+ik_2(z_2-\bar{z}_2)} A^i(x_1-z_1)A^i(\bar{z}_1-y_1)A^j(x_2-z_2)A^j(\bar{z}_2-y_2)$$
$$\times \left\langle \rho^{a_1}_{x_1}\rho^{a_2}_{x_2}\rho^{b_1}_{y_1}\rho^{b_2}_{y_2}\right\rangle_P \left\langle [U_{z_1}-U_{x_1}]^{a_1 c}[U^\dagger_{\bar{z}_1}-U^\dagger_{y_1}]^{cb_1}[U_{z_2}-U_{x_2}]^{a_2 d}[U^\dagger_{\bar{z}_2}-U^\dagger_{y_2}]^{db_2}\right\rangle_T$$

- *projectile averaging:* pair wise Wick contraction & use the same two color charge correlator.
- *dilute target limit*
- *from eikonal to non-eikonal:* $L(k,q) \to L_{\rm NE}(k,q)$
- *target averaging:* pair wise Wick contraction & use the same two field correlator.

The dilute limit with non-eikonal corrections:

$$\left.\frac{d\sigma}{d^2k_1 d\eta_1 d^2k_2\eta_2}\right|^{\rm NE}_{\rm dilute} \propto \int_{q_1 q_2} |a(q_1)|^2|a(q_2)|^2 \mathcal{G}^{\rm NE}_1(k_1^-;\lambda^+)\,\mathcal{G}^{\rm NE}_1(k_2^-;\lambda^+)\left\{ I^{(0)}_{2\rm tr} + \frac{1}{N_c^2-1}\left[I^{(1)}_{2\rm tr}+I^{(1)}_{1\rm tr}\right]\right\}$$

Glasma graphs / two particle correlations

In our set up:

- $k_1 - q_1$ and $k_2 - q_2$: momenta of the two gluons in the projectile.
- k_1 and k_2: momenta of the two gluons in the final state.
- q_1 and q_2: momenta transferred from the target to the projectile during the interaction.

In such a set up:

- (forward/backward) Bose enhancement of the gluons in the projectile $\Rightarrow F\left[|(k_1 - q_1) \mp (k_2 - q_2)|R\right]$
- (forward/backward) HBT correlations of the final state gluons $\Rightarrow F\left[|k_1 \mp k_2|R\right]$
- (forward/backward) Bose enhancement of the gluons in the target $\Rightarrow F\left[|q_1 \mp q_2|R\right]$

Glasma graphs / two particle correlations

identification of the terms:

$$I^{(0)}_{2\,\mathrm{tr}} = \Big(\mu^2\big[k_1-q_1,q_1-k_1\big]\,L^i(k_1,q_1)L^i(k_1,q_1)\Big)\,\Big(\mu^2\big[k_2-q_2,q_2-k_2\big]\,L^j(k_2,q_2)L^j(k_2,q_2)\Big)$$

- Square of the single inclusive production / uncorrelated production.

$$I^{(1)}_{2\,\mathrm{tr}} = \Big\{\mathcal{G}_2^{\mathrm{NE}}(k_1^-,k_2^-;L^+)\mu^2\big[k_1-q_1,q_2-k_1\big]\,\mu^2\big[k_2-q_2,q_1-k_2\big] \times L^i(k_1,q_1)L^i(k_1,q_2)\,L^j(k_2,q_2)L^j(k_2,q_1)\Big\} + (\underline{k}_2 \to -\underline{k}_2)$$

- $\underline{k} \equiv (k^+,k)$
- $\mu^2[k_1-q_1,q_2-k_1] \propto F\big[|q_1-q_2|R\big] \Rightarrow$ Bose enhancement of the target gluons.
- A new function appears that accounts for non-eikonal effects:

$$\boxed{\mathcal{G}_2^{\mathrm{NE}}(k_1^-,k_2^-;L^+) = \left\{\frac{2}{(k_1^- - k_2^-)L^+}\sin\left[\frac{(k_1^- - k_2^-)}{2}L^+\right]\right\}^2}$$

- in the eikonal limit:

$$\lim_{L^+\to 0}\mathcal{G}_2^{\mathrm{NE}}(k_1^-,k_2^-;L^+) = 1$$

Glasma graphs / two particle correlations

identification of the terms:

$$I^{(1)}_{1\,\mathrm{tr}} = \Big\{ \mu^2\big[k_1 - q_1, q_2 - k_2\big]\, \mu^2\big[k_2 - q_2, q_1 - k_1\big]\, L^i(k_1, q_1) L^i(k_1, q_1)\, L^j(k_2, q_2) L^j(k_2, q_2)$$
$$+ \mathcal{G}_2^{\mathrm{NE}}(k_1^-, k_2^-; L^+) \Big(\mu^2\big[k_1 - q_1, q_1 - k_2\big]\, \mu^2\big[k_2 - q_2, q_2 - k_1\big] + \frac{1}{2}\mu^2\big[k_1 - q_1, k_2 - q_2\big]\, \mu^2\big[q_2 - k_1, q_1 - k_2\big] \Big)$$
$$\times L^i(k_1, q_1) L^i(k_1, q_2)\, L^j(k_2, q_1) L^j(k_2, q_2) \Big\} + (\underline{k}_2 \to -\underline{k}_2)$$

- $\mu^2\big[k_1 - q_1, q_2 - k_2\big] \propto F\big[|(k_1 - q_1) - (k_2 - q_2)|R\big] \Rightarrow$ Bose enhancement of the projectile gluons (forward peak).
- $\mu^2\big[k_1 - q_1, q_1 - k_2\big] \propto F\big[|k_1 - k_2|R\big] \Rightarrow$ HBT correlations of the produced gluons.
- $\mu^2\big[k_1 - q_1, k_2 - q_2\big] \propto F\big[|(k - 1 - q_1) + (k_2 - q_2)|R\big] \Rightarrow$ Bose enhancement of the projectile gluons (backward peak).

The nature of $\mathcal{G}_2^{\rm NE}(k_1^-, k_2^-; L^+)$

In the double inclusive production X-section:

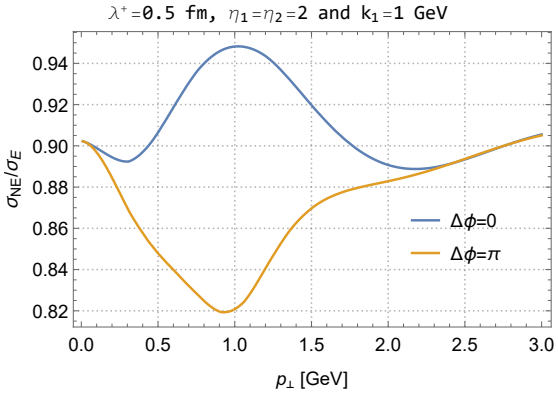
- certain terms are accompanied by $\mathcal{G}_2^{\rm NE}(k_1^-, k_2^-; L^+)$
- and their mirror images given by $(\underline{k}_2 \to -\underline{k}_2)$ are accompanied by $\mathcal{G}_2^{\rm NE}(k_1^-, -k_2^-; L^+)$.

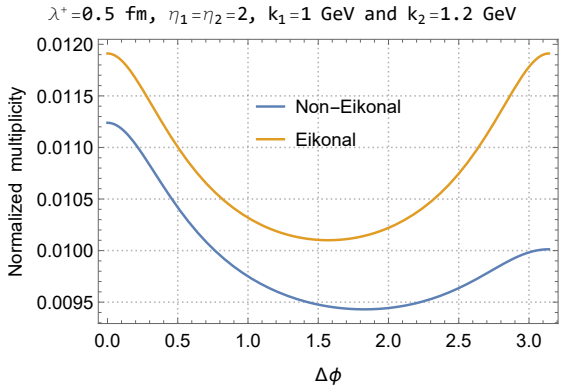
In certain kinematics the behavior of $\mathcal{G}_2^{\rm NE}(k_1^-, k_2^-; L^+)$ differs completely from $\mathcal{G}_2^{\rm NE}(k_1^-, -k_2^-; L^+)$:

- in the region where $k_1^- \sim k_2^-$ we get

$$\boxed{\mathcal{G}_2^{\rm NE}(k_1^-, k_2^-; L^+) \gg \mathcal{G}_2^{\rm NE}(k_1^-, -k_2^-; L^+)}$$

- *This asymmetry created by the non-eikonal effects immediately reminds the asymmetry between the forward and backward peaks of the ridge structure observed in two particle production.*





$L^+ = 6\ fm$

Remarks / discussion

- "accidental symmetry in CGC:" double inclusive X-section is symmetric under $k_2 \to -k_2$

$$\Downarrow$$

$$\boxed{\text{vanishing odd harmonics}}$$

- breaking the accidental symmetry with the density corrections to the projectile:
[A. Kovner, M. Lublinsky, V. Skokov - 2017 / Y. Kovchegov, V. Skokov 2018]

- non-eikonal corrections seem to be breaking the accidental symmetry!!

- *Can we generate non-zero v_3 from the non-eikonal corrections?*
P. Agostini, T.A. , N. Armesto, in preparation

- extension to pA
P. Agostini, T.A. , N. Armesto, in preparation