Non-eikonal corrections to multi-particle production in the CGC

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eikonal scattering in pA collisions
relaxing the eikonal approximation: finite width target
from pA to pp
single and double inclusive gluon production in pp
High energy pA scattering within the CGC:

- **Semi-classical approximation**:
  - dense target $\equiv$ classical background field $A_\mu^a(x) = O\left(\frac{1}{g}\right)$ at weak coupling $g$
  - dilute projectile $\equiv$ color charge $J_\mu^a(x) = O(g)$

- **Eikonal approximation**:
  - take the high energy limit $s \to \infty$.
  - drop power-suppressed contributions.

Coupling between the projectile and the target $\to \int d^4x \; J_\mu^a(x) \cdot A_\mu^a(x)$

In the semi-classical approximation, the eikonal limit can be obtained by either boosting the projectile or the target or both...
Dilute-Dense Scattering

Boosting the target:

\[ A_{a}^{\mu}(x) \mapsto \begin{cases} 
\gamma_{t} A_{a}^{-} \left( \gamma_{t} x^{+}, \frac{x^{-}}{\gamma_{t}}, x \right) \\
\frac{1}{\gamma_{t}} A_{a}^{+} \left( \gamma_{t} x^{+}, \frac{x^{-}}{\gamma_{t}}, x \right) \\
A_{a}^{i} \left( \gamma_{t} x^{+}, \frac{x^{-}}{\gamma_{t}}, x \right) 
\end{cases} \]

- \( A_{a}^{-} \gg A_{a}^{i} \gg A_{a}^{+} \) in a generic gauge
- \( A_{a}^{-} \gg A_{a}^{i} \gg A_{a}^{+} \) in the light-cone gauge:

\[ A_{a}^{\mu}(x) = \delta^{\mu -} \delta(x^{+}) A_{a}^{-}(x) \]

target is localized at \( x^{+} = 0 \)

independent of \( x^{-} \)

Boosting the projectile:

\[ J_{a}^{\mu}(x) \mapsto \begin{cases} 
\frac{1}{\gamma_{p}} J_{a}^{-} \left( \frac{x^{+}}{\gamma_{p}}, \gamma_{p} x^{-}, x \right) \\
\gamma_{p} J_{a}^{+} \left( \frac{x^{+}}{\gamma_{p}}, \gamma_{p} x^{-}, x \right) \\
J_{a}^{i} \left( \frac{x^{+}}{\gamma_{p}}, \gamma_{p} x^{-}, x \right) 
\end{cases} \]

- \( J_{a}^{+} \gg J_{a}^{i} \gg J_{a}^{-} \)
- slow \( x^{+} \) dependence due to Lorentz time dilation

\[ J_{a}^{\mu}(x) \propto \delta^{\mu +} \delta(x^{-}) \rho^{a}(x) \]

projectile is localized at \( x^{-} = 0 \)
Corrections beyond eikonal accuracy

At the level of the background field, the eikonal approximation amounts to

1. \( A_\mu^a(x) \simeq \delta^\mu - A_a^-(x) \)
2. \( A_\mu^a(x) \simeq A_\mu^a(x^+, x) \)
3. \( A_a^\mu(x) \propto \delta(x^+) \)

Relaxing any of these approximations will give correction to the strict eikonal limit! Three sources of corrections to eikonal approximation:

1. other components of the target background field \( A_\mu^a(x) \)
2. dynamics of the target: \( x^- \) dependence of \( A_\mu^a(x) \)
3. Finite width \( L^+ \) of the target along \( x^+ \)

When the target is a large nucleus, the dominant contribution beyond the eikonal accuracy is obtained by relaxing the 3rd approximation because of the \( A^{1/3} \) nuclear enhancement of the finite width target!

\[
A^\mu = \delta^\mu - \delta(x^+)A^- (x) \rightarrow A^\mu = \delta^\mu - A^- (x^+, x)
\]
Consider a finite width target:

\[ j^\mu_a(x) \propto \delta^\mu - \rho^b(x - B) \]

The target \( \rightarrow A^\mu(x) \equiv \delta^\mu - A_\perp(x^+, x) \).

The projectile \( \rightarrow j^\mu_a(x) \propto \delta^{\mu+} \delta(x^-) \rho^b(x - B) \).

The single inclusive gluon cross section for pA:

\[
(2\pi)^3 (2k^+) \frac{d\sigma}{dk^+ d^2k} = \int d^2B \sum_{\lambda \text{phys.}} \left\langle \left\langle |M^\lambda_\perp(k, B)|^2 \right\rangle_p \right\rangle_A
\]

gluon production amplitude
In the light cone gauge

\[ \mathcal{M}^2_\lambda(k, B) = \varepsilon^*_\lambda (2k^+) \lim_{x^+ \to +\infty} \int d^2 x^\perp \int dx^- e^{ik \cdot x} \int d^4 y \, G^{-}_R(x, y)_{ab} \, j^+_b(y) \]

\( G^{\mu\nu}_R(x, y)_{ab} \) is the background retarded gluon propagator

Conveniently, the \((i-)\) component of the background retarded propagator can be written in terms of the scalar background propagator:

\[ G^{-}_R(x, y)_{ab} = \frac{ik^+}{2(k^+ + i\epsilon)^2} G^{-}_k(x; y)_{ab} \]

\[ G^{ab}_k(x; y) = \frac{i}{k^+ + i\epsilon} \partial_y^i G^{ab}_k(x; y) \]

\( G^{ab}_k(x; y) \) satisfies the scalar Green's eq. whose solution can be written formally as a path integral

\[ G^{ab}_k(x; y) = \theta(x^+ - y^+) \int_{z(y^+) = y}^{z(x^+) = x} Dz(z^+) \exp \left[ \frac{ik^+}{2} \int_{y^+}^{x^+} dz^+ \, z^2(z^+) \right] U^{ab}(x^+, y^+, [z(z^+)]) \]

with the Wilson line

\[ U^{ab}(x^+, y^+, [z(z^+)]) = \mathcal{P}_+ \exp \left\{ ig \int_{y^+}^{x^+} dz^+ \, T \cdot A^- (z^+, z(z^+)) \right\}^{ab} \]

following the Brownian trajectory \( z(z^+) \).
Expanding the background propagator

(i) discretize the background propagator.

(ii) Perturbative expansion around free classical path:

\[
\begin{align*}
\text{eikonal limit : } & \frac{k^+}{(x^+ - y^+)} \gg Q^2_\perp \text{ in the problem} \\
\implies & \text{large } k^+ \text{ limit (classical free path!)} \\
\Rightarrow & \text{perturbative expansion around the free classical path: } \\
& z_n = z_n^{\text{cl}} + u_n \text{ with } z_n^{\text{cl}} = y + \frac{n}{N}(x - y)
\end{align*}
\]

(iii) Expansion around the initial transverse position:

The first expansion is performed for fixed initial and final positions.
In the large \( k^+ \) limit, the result has to be re-expanded since \( z^{\text{cl}}(z^+) - y \) is small at each step.
After all:

\[
\int d^2 \mathbf{x} \ e^{-i \mathbf{k} \cdot \mathbf{x}} \ G^{ab}_{k^+}(x_+; y_-) = \theta(x^+-y^+) \ e^{-i \mathbf{k} \cdot \mathbf{y}} \ e^{-i k^- (x^+-y^+)} \left\{ U(x^+, y^+, y) \right. \\
+ \frac{(x^+-y^+)}{k^+} \left[ k^i U^{i}_{[0,1]}(x^+, y^+, y) + \frac{i}{2} U^{i}_{[1,0]}(x^+, y^+, y) \right] \\
+ \frac{(x^+-y^+)^2}{(k^+)^2} \left[ k^i k'^j U^{ij}_{[0,2]}(x^+, y^+, y) + \frac{i}{2} k^i U^{i}_{[1,1]}(x^+, y^+, y) - \frac{1}{4} U^{i}_{[2,0]}(x^+, y^+, y) \right] \left\}^{ab} \right.
\]

- \( U(x^+, y^+, y) \equiv \) standard Wilson lines that appears only at the eikonal level as expected.

- \( U_{[\alpha, \beta]}(x^+, y^+, y) \equiv \) decorated Wilson lines that only appears beyond eikonal accuracy.

- The subscripts
  - \( \alpha \) stands for the order of the expansion around the classical path.
  - \( \beta \) stands for the order of the expansion around the initial transverse position.
Structure of the decorated Wilson lines

\[ \mathcal{U}_{[0,1]}^i \propto \begin{array}{c} \mathcal{U} \\ x^+ \end{array} \begin{array}{c} B^j \\ z^+ \end{array} \begin{array}{c} \mathcal{U} \\ y^+ \end{array} \]

\[ \mathcal{U}_{[0,2]}^{ij} \propto \begin{array}{c} \mathcal{U} \\ x^+ \end{array} \begin{array}{c} B^{ij} \\ z^+ \end{array} \begin{array}{c} \mathcal{U} \\ y^+ \end{array} + \begin{array}{c} \mathcal{U} \\ x^+ \end{array} \begin{array}{c} B^i \\ z_1^+ \end{array} \begin{array}{c} \mathcal{U} \\ z_2^+ \end{array} \begin{array}{c} B^j \\ y^+ \end{array} \]

\[ \mathcal{U}_{[1,1]}^i \propto \begin{array}{c} \mathcal{U} \\ x^+ \end{array} \begin{array}{c} B^{ij} \\ z^+ \end{array} \begin{array}{c} \mathcal{U} \\ z_1^+ \end{array} \begin{array}{c} \mathcal{U} \\ z_2^+ \end{array} \begin{array}{c} \mathcal{U} \\ y^+ \end{array} + \begin{array}{c} \mathcal{U} \\ x^+ \end{array} \begin{array}{c} B^i \\ z_1^+ \end{array} \begin{array}{c} B^j \\ z_2^+ \end{array} \begin{array}{c} \mathcal{U} \\ z_3^+ \end{array} \begin{array}{c} \mathcal{U} \\ y^+ \end{array} \]

with

\[ B^i(z^+, y) \equiv igT \cdot \partial_y A^-(z^+, y), \]
\[ B^{ij}(z^+, y) \equiv igT \cdot \partial_y \partial_y A^-(z^+, y), \]
\[ B^{ijl}(z^+, y) \equiv igT \cdot \partial_y \partial_y \partial_y A^-(z^+, y), \]
Dilute target limit and the modified Lipatov vertex

[ T.A., A. Dumitru - 2015 ]

go from $pA \rightarrow pp$:

- **dilute limit of the target:**
  expand the standard & decorated Wilson lines to first order in the background field.

- expressions simplify: for example the first decorated dipole becomes

  $$U_{[0,1]}^{i,ab}(x^+, y^+, y) = \int_{y^+}^{x^+} dz^+ \frac{z^+-y^+}{x^+-y^+} [igT_{ab}^{e} \partial_y^i A^{-e}(z^+, y)]$$

- summing up all the NEik and NNEik terms in the dilute target limit, one gets

  $$\mathcal{M} \propto \left[ \frac{(k-q)^i}{(k-q)^2} - \frac{k^i}{k^2} \right] \left\{ 1 + i \frac{k^2}{2k^+} x^+ - \frac{1}{2} \left( \frac{k^2}{2k^+} x^+ \right)^2 \right\}$$

- $O(1)$ term $\rightarrow$ **eikonal Lipatov vertex**.

- we get **the Lipatov vertex at NNEik accuracy**.

- the form suggests exponentiation.
The total amplitude reads

\[ i(M_A + M_B + M_C) \propto \int \frac{d^2 q}{(2\pi)^2} L^i(k, q) e^{ik \cdot x^+} A^-_a(k^-, q) e^{-iq \cdot x_1} \]

with \( L^i(k, q) \) is the standard Lipatov vertex

\[
L^i(k, q) = \frac{(k - q)^i}{(k - q)^2} - \frac{k^i}{k^2}
\]

and the non-eikonal Lipatov vertex being

\[
L_{NE}^i(k, q) = \left[ \frac{(k - q)^i}{(k - q)^2} - \frac{k^i}{k^2} \right] e^{ik \cdot x^+}
\]
Single inclusive gluon production in pA collisions (eikonal accuracy):

\[
\frac{d\sigma}{d^2kd\eta} \propto \int_{zx\bar{z}y} e^{ik(z-\bar{z})} A^i(x - z) A^\dagger(\bar{z} - y) \langle \rho^a(x) \rho^b(y) \rangle_p \left\langle [U_x - U_z]^{ac}[U^\dagger_{\bar{z}} - U^\dagger_y]^{cb} \right\rangle_T
\]

- **projectile averaging:**
  - in x-space \( \langle \rho^a(x) \rho^b(y) \rangle_p = \delta^{ab} \mu^2(x, y) \)
  - in p-space \( \langle \rho^a(k) \rho^b(p) \rangle_p = \delta^{ab} \mu^2(k, p) = \delta^{ab} T\left(\frac{k-p}{2}\right) F[(k+p)R] \)

\( T \rightarrow \) transverse momentum dependent distribution of the color charge densities

\( F \rightarrow \) soft form factor which is peaked when its argument vanishes

- **dilute target limit** \( \rightarrow U_{ab}(x) \approx 1 + igT_{ab}^c \int_{x+q} e^{iqx} A_c^-(x^+, q) \)

\[
\left. \frac{d\sigma}{d^2qd\eta} \right|_{\text{dilute}} \propto \int_{x_1^+ x_2^+ q_1 q_2} L^i(k, q_1) L^i(k, q_2) \mu^2 [k - q_1, k - q_2] \left\langle A_c^-(x_1^+, q_1) A_c^-(x_2^+, q_2) \right\rangle_T
\]

- **go from eikonal to non-eikonal:** \( L^i(k, q) \rightarrow L^i_{\text{NE}}(k, q; x^+) \)

- **target averaging:**

\[
\langle A_c^-(x_1^+, q_1) A_c^-(x_2^+, q_2) \rangle_T = \delta^{cc} n(x_1^+) \frac{1}{2\lambda^+} \Theta(\lambda^+ - |x_1^+ - x_2^+|) (2\pi)^2 \delta^{(2)}(q_1 - q_2) |a(q_1)|^2
\]

\( \lambda^+ \equiv \) color correlation length in the target \( (\lambda^+ \ll L^+) \)

\( n(x^+) \equiv 1\text{-d target density along longitudinal direction (} n(x^+) = n_0 \text{ for } 0 \leq x^+ \leq L^+ \text{ and } 0 \text{ elsewhere}) \)

\( a(q) \equiv \) functional form of the potential in p-space (Yukawa type \( \rightarrow |a(q)|^2 = \frac{m^2}{(q^2 + m^2)^2} \))
Noneikonal single inclusive gluon production

After all said and done:

\[
\frac{d\sigma}{d^2kd\eta}\bigg|_{\text{dilute}}^{\text{NE}} \propto G_1^{\text{NE}}(k^-; \lambda^+) \int_q \mu^2|k - q, q - k|L^i(k, q)L_i(k, q)|a(q)|^2
\]

the function that encodes the non-eikonal effects

\[
G_1^{\text{NE}}(k^-; \lambda^+) = \frac{1}{k^-\lambda^+} \sin(k^-\lambda^+)
\]

in the eikonal limit:

\[
\lim_{(k^-\lambda^+) \to 0} G_1^{\text{NE}}(k^-; \lambda^+) = 1
\]

\[\begin{array}{c}
\begin{array}{c}
\eta = 2 \\
0 \quad 0.5 \quad 1.0 \quad 1.5 \quad 2.0 \quad 2.5
\end{array}
\end{array}\]

\[\begin{array}{c}
\begin{array}{c}
\lambda^* = 0.4 \text{ fm} \\
\lambda^* = 0.6 \text{ fm} \\
\lambda^* = 0.8 \text{ fm} \\
\lambda^* = 1 \text{ fm}
\end{array}
\end{array}\]

\[\begin{array}{c}
\begin{array}{c}
p_\perp \text{[GeV]} \\
0 \quad 0.5 \quad 1.0 \quad 1.5 \quad 2.0 \quad 2.5 \quad 3.0 \quad 3.5 \quad 4.0
\end{array}
\end{array}\]

\[\begin{array}{c}
\begin{array}{c}
p_\perp \text{[GeV]} \\
1 \quad 1.5 \quad 2.0 \quad 2.5 \quad 3.0 \quad 3.5 \quad 4.0
\end{array}
\end{array}\]

(N_c = 3, m = 0.2 GeV, \(\mu^2(k, q) = \delta^{(2)}(k + q)\) with a projectile size \(S_\perp = 4 \text{GeV}^{-2}\), and regulate the denominators that give rise to infrared divergencies by substituting the corresponding squared transverse momenta \(l^2 \to l^2 + m_g^2\) where we have used the numerical value \(m_g^2 = 0.2 \text{GeV}^2\).)

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Non-eikonal corrections to multi-particle production in the CGC
Same procedure can be adopted to calculate the double inclusive gluon production.

The double inclusive gluon production $\sigma$-section for dilute-dense scattering:

$$\frac{d\sigma}{d^2k_1d\eta_1d^2k_2d\eta_2} \propto \int_{z_1x_1\eta_1\bar{z}_1y_1x_2\bar{z}_2y_2} e^{ik_1(z_1-\bar{z}_1)+ik_2(z_2-\bar{z}_2)} A^i(x_1-z_1) A^i(\bar{z}_1-y_1) A^i(x_2-z_2) A^i(\bar{z}_2-y_2)$$

$$\times \left\langle \rho_{x_1}^{a_1} \rho_{x_2}^{a_2} \rho_{y_1}^{b_1} \rho_{y_2}^{b_2} \right\rangle \left\langle \left[ U_{z_1} - U_{x_1} \right]^{a_1c} \left[ U_{\bar{z}_1}^{\dagger} - U_{y_1}^{\dagger} \right]^{cb_1} \left[ U_{z_2} - U_{x_2} \right]^{a_2d} \left[ U_{\bar{z}_2}^{\dagger} - U_{y_2}^{\dagger} \right]^{db_2} \right\rangle_T$$

- **projectile averaging**: pair wise Wick contraction & use the same two color charge correlator.
- **dilute target limit**
- **from eikonal to non-eikonal**: $L(k, q) \rightarrow L_{NE}(k, q)$
- **target averaging**: pair wise Wick contraction & use the same two field correlator.

The dilute limit with non-eikonal corrections:

$$\left. \frac{d\sigma}{d^2k_1d\eta_1d^2k_2d\eta_2} \right|_{\text{dilute}} \propto \int_{q_1q_2} a(q_1)^2 a(q_2)^2 G_1^{NE}(k_1^-; \lambda^+) G_1^{NE}(k_2^-; \lambda^+) \left\{ I_{2tr}^{(0)} + \frac{1}{N_c^2-1} \left[ I_{2tr}^{(1)} + I_{1tr}^{(1)} \right] \right\}$$
In our set up:

- $k_1 - q_1$ and $k_2 - q_2$: momenta of the two gluons in the projectile.
- $k_1$ and $k_2$: momenta of the two gluons in the final state.
- $q_1$ and $q_2$: momenta transferred from the target to the projectile during the interaction.

In such a set up:

- (forward/backward) Bose enhancement of the gluons in the projectile $\Rightarrow F[|(k_1 - q_1) \mp (k_2 - q_2)|R]$ 
- (forward/backward) HBT correlations of the final state gluons $\Rightarrow F[|k_1 \mp k_2|R]$ 
- (forward/backward) Bose enhancement of the gluons in the target $\Rightarrow F[|q_1 \mp q_2|R]$
Glasma graphs / two particle correlations

identification of the terms:

\[ I_{2tr}^{(0)} = \left( \mu^2[k_1 - q_1, q_1 - k_1] L^i(k_1, q_1)L^i(k_1, q_1) \right) \left( \mu^2[k_2 - q_2, q_2 - k_2] L^j(k_2, q_2)L^j(k_2, q_2) \right) \]

• Square of the single inclusive production / uncorrelated production.

\[ I_{2tr}^{(1)} = \left\{ G_{2NE}^2(k_1^-, k_2^-; L^+) \mu^2[k_1 - q_1, q_2 - k_1] \mu^2[k_2 - q_2, q_1 - k_2] \right\} \times L^i(k_1, q_1)L^i(k_1, q_2)L^j(k_2, q_2)L^j(k_2, q_1) + (k_2 \rightarrow -k_2) \]

• \( k \equiv (k^+, k) \)

• \( \mu^2[k_1 - q_1, q_2 - k_1] \propto F[|q_1 - q_2|R] \Rightarrow \) Bose enhancement of the target gluons.

• A new function appears that accounts for non-eikonal effects:

\[ G_{2NE}^2(k_1^-, k_2^-; L^+) = \left\{ \frac{2}{(k_1^- - k_2^-) L^+} \sin \left[ \frac{(k_1^- - k_2^-) L^+}{2} \right] \right\}^2 \]

• in the eikonal limit:

\[ \lim_{L^+ \rightarrow 0} G_{2NE}^2(k_1^-, k_2^-; L^+) = 1 \]
Glasma graphs / two particle correlations

identification of the terms:

\[ l^{(i)}_1 = \left\{ \begin{array}{l} \mu^2 [k_1 - q_1, q_2 - k_2] \mu^2 [k_2 - q_2, q_1 - k_1] L^i(k_1, q_1) L^i(k_1, q_1) L^i(k_2, q_2) L^i(k_2, q_2) \\ + G_2^{NE}(k_1^{-}, k_2^{-}; L^+) \left( \frac{1}{2} \mu^2 [k_1 - q_1, k_2 - q_2] + \frac{1}{2} \mu^2 [q_2 - k_1, q_1 - k_2] \right) \right\} + (k_2 \rightarrow -k_2) \]

- \[ \mu^2 [k_1 - q_1, q_2 - k_2] \propto F[|(k_1 - q_1) - (k_2 - q_2)|R] \Rightarrow \text{Bose enhancement of the projectile gluons (forward peak).} \]

- \[ \mu^2 [k_1 - q_1, q_1 - k_2] \propto F[|k_1 - k_2|R] \Rightarrow \text{HBT correlations of the produced gluons.} \]

- \[ \mu^2 [k_1 - q_1, k_2 - q_2] \propto F[|(k - 1 - q_1) + (k_2 - q_2)|R] \Rightarrow \text{Bose enhancement of the projectile gluons (backward peak).} \]
The nature of $G_{2}^{\text{NE}}(k_{1}^{-}, k_{2}^{-}; L^{+})$

In the double inclusive production $X$-section:

- certain terms are accompanied by $G_{2}^{\text{NE}}(k_{1}^{-}, k_{2}^{-}; L^{+})$
- and their mirror images given by $(k_{2} \rightarrow -k_{2})$ are accompanied by $G_{2}^{\text{NE}}(k_{1}^{-}, -k_{2}^{-}; L^{+})$.

\textit{In certain kinematics the behavior of $G_{2}^{\text{NE}}(k_{1}^{-}, k_{2}^{-}; L^{+})$ differs completely from $G_{2}^{\text{NE}}(k_{1}^{-}, -k_{2}^{-}; L^{+})$:}

- in the region where $k_{1}^{-} \sim k_{2}^{-}$ we get

$$G_{2}^{\text{NE}}(k_{1}^{-}, k_{2}^{-}; L^{+}) \gg G_{2}^{\text{NE}}(k_{1}^{-}, -k_{2}^{-}; L^{+})$$

- \textit{This asymmetry created by the non-eikonal effects immediately reminds the asymmetry between the forward and backward peaks of the ridge structure observed in two particle production.}

$L^{+} = 6 \text{ fm}$
• "accidental symmetry in CGC:" double inclusive X-section is symmetric under $k_2 \rightarrow -k_2$

\[ \downarrow \]

vanishing odd harmonics

• breaking the accidental symmetry with the density corrections to the projectile:

• non-eikonal corrections seem to be breaking the accidental symmetry!!

• *Can we generate non-zero $v_3$ from the non-eikonal corrections?*

• extension to pA