Double parton scattering: theory progress

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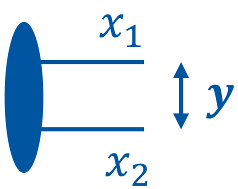
DGS framework: DPDs

Diehl, Gaunt, Schönwald, JHEP 1706 (2017) 083

Use DPDs in y space, operator definition for bare (quark-quark) DPD:



$$F_{bare}(x_i, \boldsymbol{y}) = 2p^+ \int dy^- \int \frac{dz_1^-}{2\pi}\frac{dz_2^-}{2\pi} e^{i(x_1 z_1^- + x_2 z_2^-)p^+} \langle p|\mathcal{O}(0,z_2)\mathcal{O}(y,z_1)|p\rangle$$

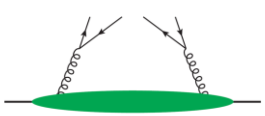
$$\mathcal{O}\,(y,z_i) = \bar{q}\big(y - \tfrac{1}{2}z_i\big)\,W^\dagger\big(y - \tfrac{1}{2}z_i\big)\frac{\gamma^+}{2}W\,\big(y + \tfrac{1}{2}z_i\big)\,q\big(y + \tfrac{1}{2}z_i\big)\big|_{z_i^+ = y^+ = 0}$$

(c.f. single PDF: $D_{bare}(x) = \int \frac{dz^-}{2\pi} e^{ixz^- p^+}\langle p|\mathcal{O}(0,z)|p\rangle$)

This includes both perturbative and nonperturbative mechanisms for generation of the parton pair (no field theoretic way to distinguish them!)

At small **y**, perturbative mechanism is dominant:

Diehl, Ostermeier and Schäfer (JHEP 1203 (2012))



$$F_{a_1 a_2}(x_1, x_2, y, \mu) = \frac{1}{\pi y^2}\sum_{a_0}\int_{x_1+x_2}^{1}\frac{dz}{z^2}\,\underbrace{V_{a_1 a_2, a_0}\left(\frac{x_1}{z}, \frac{x_2}{z}, a_s(\mu), \log\frac{\mu^2 y^2}{b_0^2}\right)}_{\text{Perturbative splitting kernel}}\,\overbrace{f_{a_0}(z,\mu)}^{\text{PDF}}$$

V known up to two loops Diehl, Gaunt, Plößl, Schäfer, arXiv:1902.08019

Renormalise two bilinears $\mathcal{O}$ separately:

Usual AP kernels

$$\mu_i \frac{dF(x_i, \boldsymbol{y}; \mu_i)}{d\mu_i} = P \otimes_{x_i} F$$

DGS framework: total cross section

Combine DPDs to form DPS cross section, insert cut-off to regularize cross section:

$$\sigma_{\text{DPS}} = \int d^2y \boxed{\Phi^2(\nu y)} F(x_1, x_2; y) F(\bar{x}_1, \bar{x}_2; y)$$

Must satisfy $\Phi(u) \to 1$ for $u \gg 1$, and $\Phi(u) \to 0$ as $u \to 0$. E.g.: $\Phi(u-1)$.

Use subtraction term in sum of SPS and DPS to avoid double counting:

$$\sigma_{\text{tot}} = \sigma_{\text{DPS}} + \sigma_{\text{SPS}} - \sigma_{\text{sub}}$$

$\sigma_{sub} =$ DPS cross section with both DPDs replaced by fixed order splitting expression

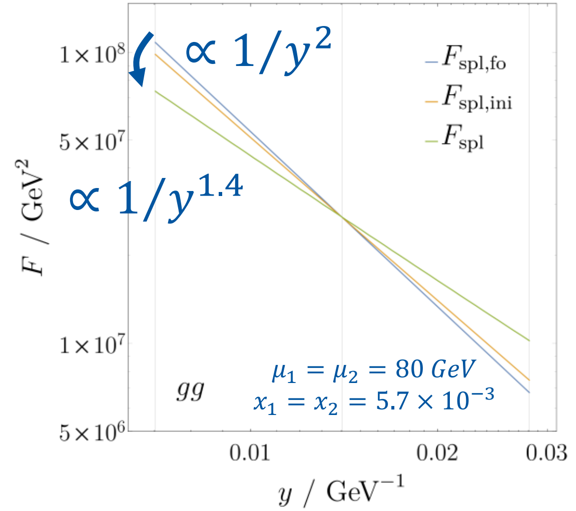
$$F_{a_1 a_2}(x_1, x_2, y, \mu) \Rightarrow \frac{1}{\pi y^2} \sum_{a_0} \int_{x_1+x_2}^{1} \frac{dz}{z^2} V_{a_1 a_2, a_0}\left(\frac{x_1}{z}, \frac{x_2}{z}, a_s(\mu), \log\frac{\mu^2 y^2}{b_0^2}\right) f_{a_0}(z, \mu)$$

ν dependence cancelled order by order between σ_{DPS} and σ_{sub}

Important: σ_{DPS} is not really 'meaningful' on its own. Can only measure $\sigma_{tot} = \sigma_{DPS} + \sigma_{SPS} - \sigma_{sub}$

On power counting grounds: $\sigma_{DPS} \propto \nu^2$. Very strong dependence on ν! Indicative of the fact that σ_{DPS} is not meaningful quantity alone. Need to combine with σ_{sub} & σ_{SPS} to get meaningful quantity.

Diehl, Gaunt, Schönwald, JHEP 1706 (2017) 083



HOWEVER: some scenarios where evolution effects skew y shapes causes DPDs to fall much shallower than $1/y^2$. Then:

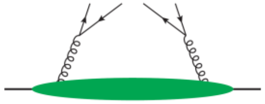
- 'bulk' of DPD at large $y \gg 1/Q$.
- dependence of σ_{DPS} on ν is much weaker.
- σ_{DPS} is more of a meaningful quantity on it's own – don't need σ_{sub}, or σ_{SPS} up to order containing first '1v1' type loop!

These scenarios are the best ones to look for DPS!

Model of DPDs

Construct model of DPDs, with 'intrinsic' and 'splitting' components:

$$F^{ij}(x_1, x_2, y, \mu) = F^{ij}_{\text{spl}}(x_1, x_2, y, \mu) + F^{ij}_{\text{int}}(x_1, x_2, y, \mu)$$



[Perturbative splitting expression] × [large y suppression]

[Product of PDFs] × [smooth transverse profile]

Study DPS luminosity (analogue of usual PDF luminosity for SPS)

$$\mathcal{L}_{a_1 a_2 b_1 b_2}(x_i, \bar{x}_i, \mu_i, \nu) = \int d^2\boldsymbol{y}\, \Phi^2(y\nu)\, F_{a_1 a_2}(x_i, \boldsymbol{y}; \mu_i)\, F_{b_1 b_2}(\bar{x}_i, \boldsymbol{y}; \mu_i)$$

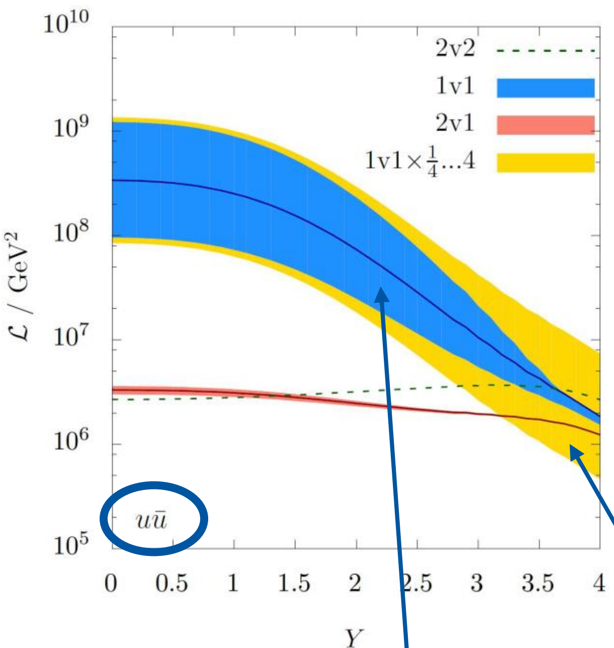
For cut-off function we use $\Phi(u) = \theta(u - b_0)$ $b_0 = 2e^{-\gamma_E} = 1.1229...$

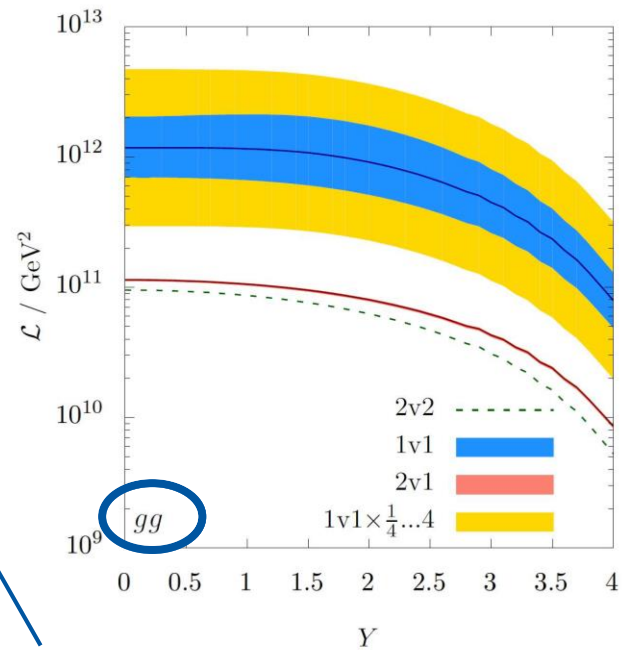
DPS luminosities

$Q_A = Q_B = 80\ \text{GeV}, \sqrt{s} = 14\ \text{TeV}$

Vary scale ν between $Q/2$ and 2Q

Here: plot luminosities against rapidity of one hard system (other kept central)





Need SPS contribution up to order containing double box, and subtraction.

Actual ν variation

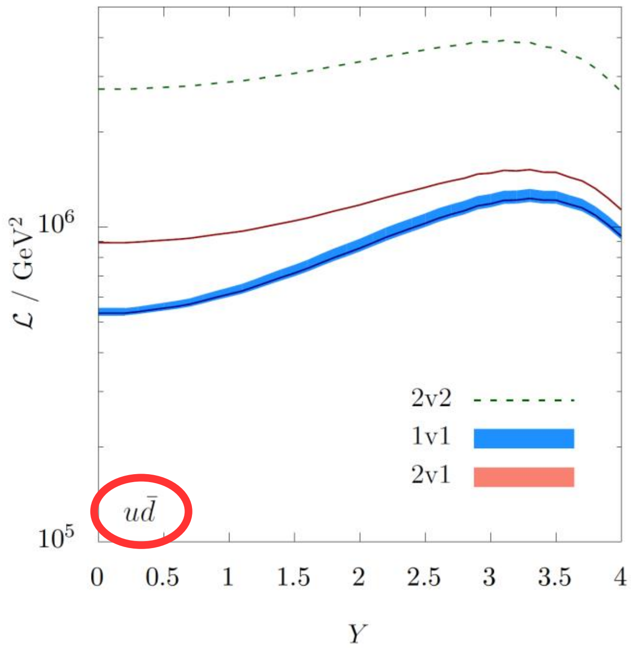
Naïve power counting expectation for ν variation $\propto \nu^2$

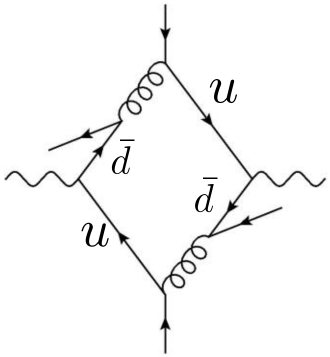
DPS luminosities

Some situations where ν variation is reduced, & don't need SPS and subtraction up to order containing double box.

Examples:

(1) When the parton pairs in the relevant DPDs cannot be produced in a single leading-order splitting (e.g. $u\bar{d}$)





Relevant for same sign WW production!

Here splitting effects are not so pronounced.

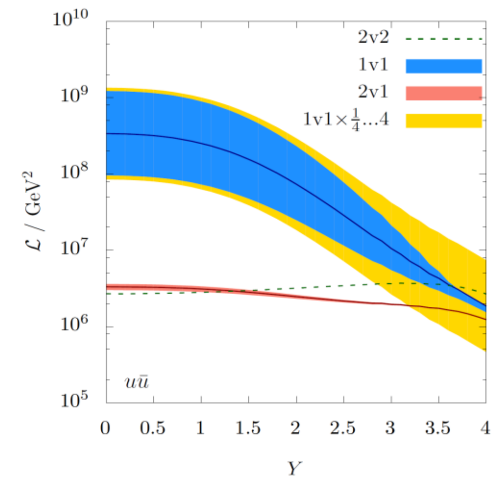
DPS luminosities

(2) When low x values in the DPDs are probed

E.g. Low mass Drell-Yan or heavy quark production at (HE-)LHC, with hard systems widely separated in rapidity

Here can have a significant contribution from pure splitting processes (where these can legitimately be thought of as DPS)

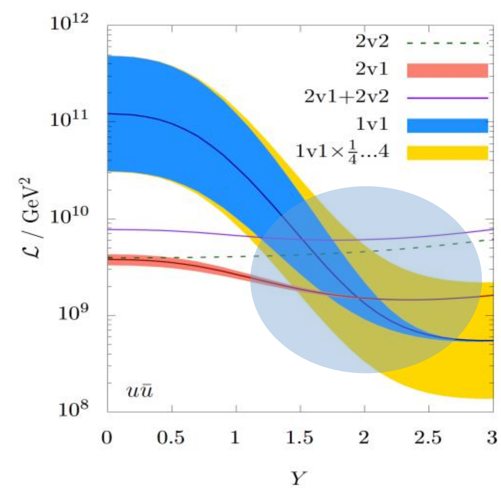
By combining studies of different types of processes, could probe size of splitting effects and compare with theory. More studies needed!



$\sqrt{s} = 14 \text{ TeV} \rightarrow \sqrt{s} = 27 \text{ TeV}$

$Q = 80\text{GeV} \rightarrow Q = 40\text{GeV}$

One particle at Y, another particle at -Y



Transverse momentum in DPS

Small $\boldsymbol{q}_i$ region particularly important for DPS – DPS & SPS same power

Parton model analysis: $$\frac{d\sigma^{(A,B)}}{d^2\boldsymbol{q}_1 d^2\boldsymbol{q}_2} \sim \int d^2\boldsymbol{y}\, d^2\boldsymbol{z}_i\, e^{-i\boldsymbol{z}_1\cdot\boldsymbol{q}_1 - i\boldsymbol{z}_2\cdot\boldsymbol{q}_2} \underbrace{F(\boldsymbol{z}_1,\boldsymbol{z}_2,\boldsymbol{y})}_{\text{DTMDs}}\, F(\boldsymbol{z}_1,\boldsymbol{z}_2,\boldsymbol{y})$$

Diehl, Ostermeier, Schafer, JHEP 1203 (2012) 089

QCD treatment of transverse momentum in DPS (including DGS-style double counting subtraction) developed in Buffing, Diehl, Kasemets JHEP 1801 (2018) 044. DPS cross section in QCD:

$$\frac{d\sigma_{\mathrm{DPS}}}{dx_1 dx_2 d\overline{x}_1 d\overline{x}_2 d^2\boldsymbol{q}_1 d^2\boldsymbol{q}_2} = \frac{1}{C} \cdot \sum_{a_1,a_2,b_1,b_2} \widehat{\sigma}_{a_1 b_1}(Q_1,\mu_1)\,\widehat{\sigma}_{a_2 b_2}(Q_2,\mu_2) \times \int \frac{d^2\boldsymbol{z}_1}{(2\pi)^2}\frac{d^2\boldsymbol{z}_2}{(2\pi)^2} d^2\boldsymbol{y} \cdot e^{-i\boldsymbol{q}_1\boldsymbol{z}_1 - i\boldsymbol{q}_2\boldsymbol{z}_2} W_{a_1a_2b_1b_2}(\overline{x}_i, x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \nu),$$

Cut-off functions

$$W_{a_1a_2b_1b_2}(\overline{x}_i, x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \nu) = \Phi(\nu \boldsymbol{y}_+)\,\Phi(\nu \boldsymbol{y}_-) \times \sum_R \eta_{a_1a_2}(R)\, {}^{R}F_{b_1b_2}(\overline{x}_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \overline{\zeta}) \cdot {}^{R}F_{a_1a_2}(x_i, \boldsymbol{z}_i, \boldsymbol{y}; \mu_i, \zeta).$$

Dependence on ren. scales μ_i AND a rapidity scale ζ.

Evolution of DTMDS in all of these scales known at one loop.

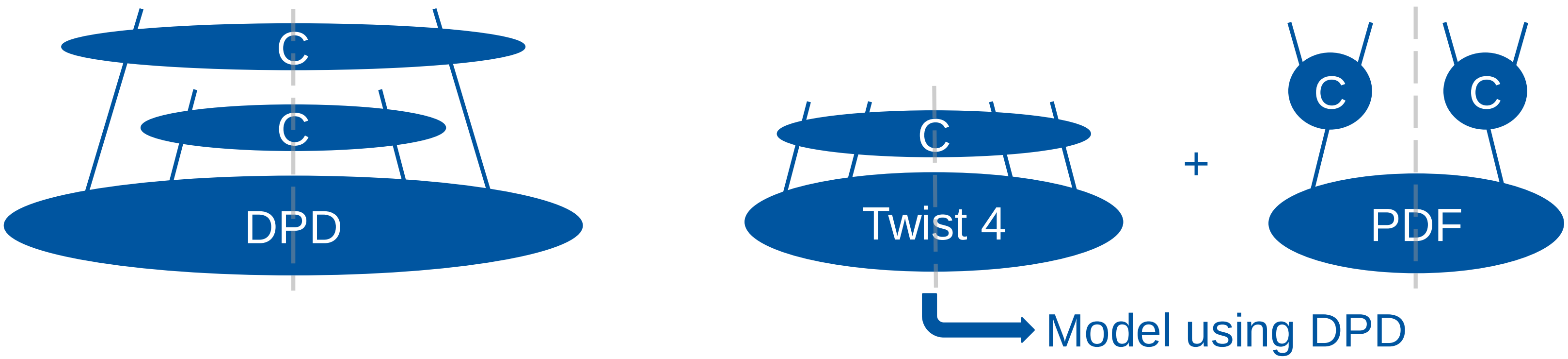
Transverse momentum in DPS

Still need some 'initial' expressions for the DTMDs. Function of many arguments $(x_i, \boldsymbol{y}, \boldsymbol{z}_i)$. Hopeless?

For perturbative $|\boldsymbol{q}_i| \gg \Lambda$ can expand DTMDs in terms of collinear quantities:

Large $\boldsymbol{y} \sim 1/\Lambda$:

Small $\boldsymbol{y} \sim 1/q_T \sim |\boldsymbol{z}_i|$:

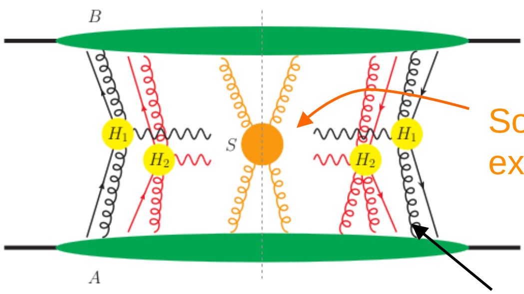


So then, need only DPDs and PDFs: very good prospects for phenomenology at perturbative $|\boldsymbol{q}_i|$!

Brief overview of transverse momentum in DPS given in JG, Kasemets, Advances in High Energy Physics, 2019, 3797394

Formal factorisation status of DPS

Formal factorization status of DPS producing two colour singlets is very good! [For coloured particle production at measured p_T, problems identified even in SPS case.]

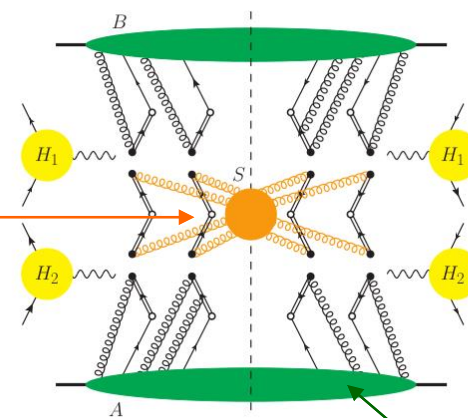


Soft and Glauber exchanges

Initial picture

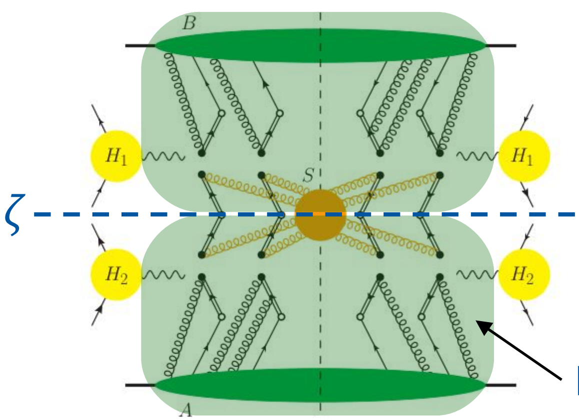
Extra (unphysically polarised) gluon connections to hard

Diehl, JG, Ostermeier, Plößl, Schafer, JHEP 1601 (2016) 076, Diehl, Ostermeier, Schafer, JHEP 1203 (2012) 089, Diehl, Nagar, arXiv:1812.09509.



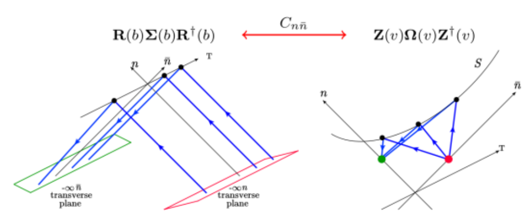
Soft factor

Collinear factor



DPDs/DTMDS

(Vladimirov, JHEP 1804 (2018) 045)



$$\sigma \sim F \otimes F \otimes \hat{\sigma} \otimes \hat{\sigma}$$

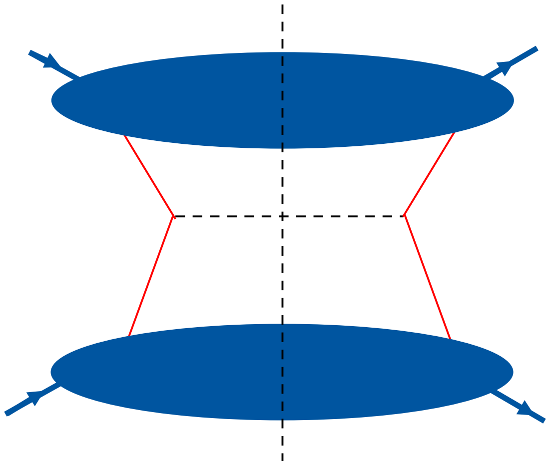
Interference contributions to proton-proton DPS

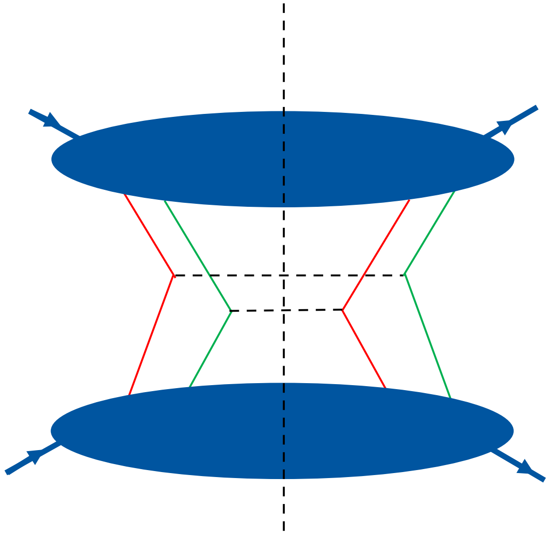
Mekhfi, Phys. Rev. D32 (1985) 2380
Diehl, Ostermeier and Schafer (JHEP 1203 (2012))
Manohar, Waalewijn, Phys.Rev. D85 (2012) 114009

SPS: One parton per proton ‘leaves’, interacts and ‘returns’.

To reform proton, parton must return with same quantum numbers.

No interference contributions to SPS cross section.





DPS: Here we have two partons per proton interacting.

Interference contributions to total cross section in which quantum numbers are swapped between parton legs. Complementary swap is required in other proton.

Can get interference contributions in colour, spin, flavour, and quark number. Also distributions associated with parton correlations!

Correlated parton contributions to DPS

There are also contributions to the unpolarised p-p DPS cross section associated with correlations between partons:

e.g.

$$\Delta q_1 \Delta q_2 = \underbrace{q_1{\uparrow}\, q_2{\uparrow} + q_1{\downarrow}\, q_2{\downarrow}}_{\text{Same spin}} - \underbrace{q_1{\uparrow}\, q_2{\downarrow} - q_1{\downarrow}\, q_2{\uparrow}}_{\text{Opposing spin}}$$

Can construct limits on size of colour/spin correlated distributions (at LO) based on probabilistic picture of parton densities ('positivity bounds'):

Simple case: $|f_{\Delta q \Delta q}| \leq f_{qq}$

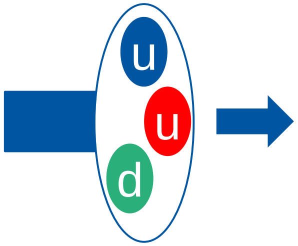
Diehl, Kasemets, JHEP 1305 (2013) 150
Kasemets, Mulders, Phys.Rev. D91 (2015) 014015

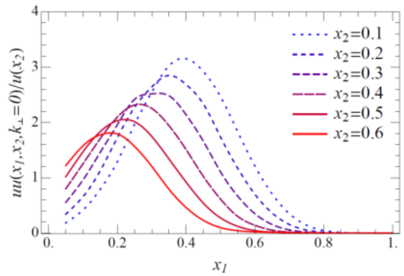
More general:

$$\rho = \frac{1}{4}\begin{pmatrix} f_{qq} + f_{\Delta q \Delta q} & -ie^{i\varphi_y} yM f_{\delta q q} & -ie^{i\varphi_y} yM f_{q \delta q} & 2e^{2i\varphi_y} y^2 M^2 f^t_{\delta q \delta q} \\ ie^{-i\varphi_y} yM f_{\delta q q} & f_{qq} - f_{\Delta q \Delta q} & 2 f_{\delta q \delta q} & -ie^{i\varphi_y} yM f_{q \delta q} \\ ie^{-i\varphi_y} yM f_{q \delta q} & 2 f_{\delta q \delta q} & f_{qq} - f_{\Delta q \Delta q} & -ie^{i\varphi_y} yM f_{\delta q q} \\ 2e^{-2i\varphi_y} y^2 M^2 f^t_{\delta q \delta q} & ie^{-i\varphi_y} yM f_{q \delta q} & ie^{-i\varphi_y} yM f_{\delta q q} & f_{qq} + f_{\Delta q \Delta q} \end{pmatrix} \qquad \sum_{\lambda_1' \lambda_2' \lambda_1 \lambda_2} v^*_{\lambda_1' \lambda_2'}\, \rho_{(\lambda_1' \lambda_2')(\lambda_1 \lambda_2)}\, v_{\lambda_1 \lambda_2} \geq 0$$

Knowledge of nonperturbative DPDs

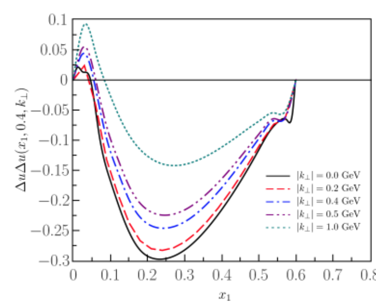
Model calculations:





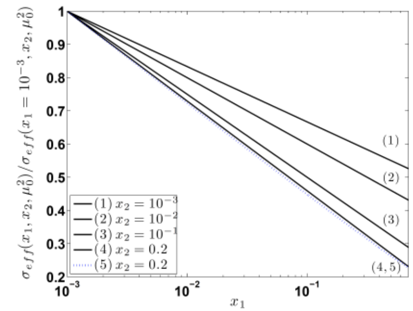
Bag model

[Phys. Rev. D 87, 034009 (2013), Manohar, Waalewijn, Chang]



Light-front CQM

[Rinaldi, Scopetta, Traini, Vento, JHEP 12 (2014) 028]



AdS/QCD

[Traini, Rinaldi, Scopetta, Vento, Phys. Lett. B 768 (2017) 270-273]

General message: factorisation of DPD into separate x_1, x_2, $\boldsymbol{y}$ pieces fails strongly at high x_i, low μ_i where these models are relevant.

Momentum and number sum rules:

[JG, Stirling, JHEP 1003 (2010) 005

Diehl, Plößl, Schafer, Eur.Phys.J. C79 (2019) no.3, 253]

Construction of DPDs to satisfy rules in e.g. JG, Stirling, JHEP 1003 (2010) 005, Golec-Biernat et al. Phys.Lett. B750 (2015) 559-564, Diehl, JG, Lang, Plößl, Schafer, to appear

$$\sum_{j_2} \int_0^{1-x_1} \mathrm{d}x_2\, x_2\, F^{j_1 j_2}(x_1, x_2; \mu) = (1-x_1) f^{j_1}(x_1; \mu)$$

$$\int_0^{1-x_1} \mathrm{d}x_2\, F^{j_1 j_{2,v}}(x_1, x_2; \mu) = \left(N_{j_{2,v}} + \delta_{j_1, \overline{j_2}} - \delta_{j_1, j_2}\right) f^{j_1}(x_1; \mu)$$

$$F(x_1, x_2; \mu) = \int d^2\boldsymbol{y}\Phi(\mu y) F(x_1, x_2, \boldsymbol{y}; \mu) + \mathcal{O}(\alpha_s)$$

Knowledge of nonperturbative DPDs

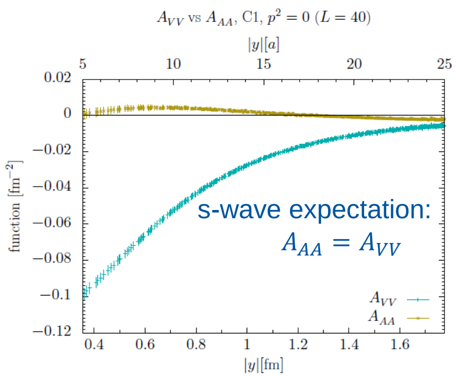
Of course, best theory input would be from lattice calculations!

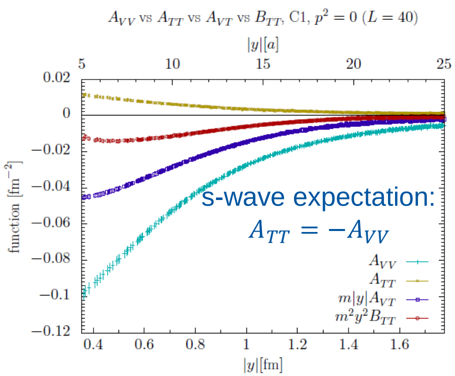
Ongoing programme to compute DPD Mellin moments. Results so far only for the pion, but calculation with proton is WIP.

Bali, Bruns, Castagnini, Diehl, JG, Gläßle, Schäfer, Sternbeck

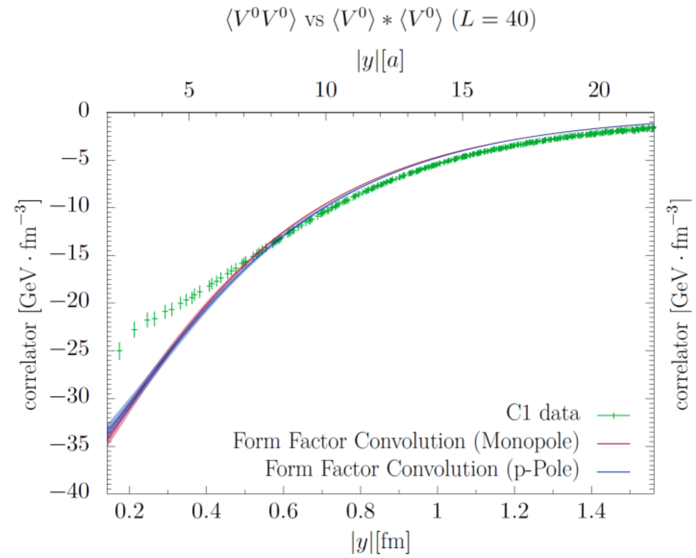
Test of classical s-wave picture of the pion:

$$-A_{VV} \sim u^+d^+ + u^-d^- + u^+d^- + u^-d^+$$
$$+A_{AA} \sim u^+d^+ + u^-d^- - u^+d^- - u^-d^+$$
$$-A_{TT} \sim u^{\vec{s}}d^{\vec{s}} + u^{-\vec{s}}d^{-\vec{s}} - u^{\vec{s}}d^{-\vec{s}} - u^{-\vec{s}}d^{\vec{s}}$$



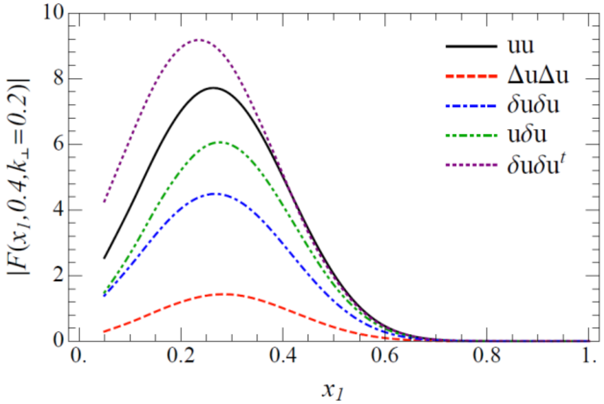


Factorisation test:



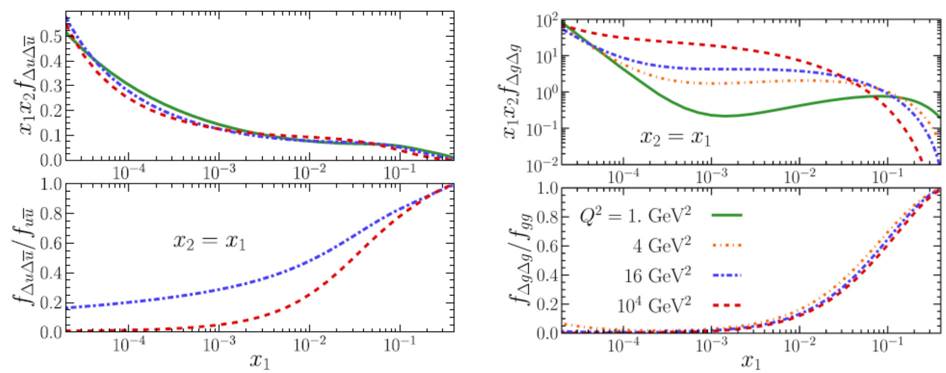
Preliminary, from talk by Christian Zimmermann at MPI@LHC 2018

Numerics of spin/colour correlation distributions



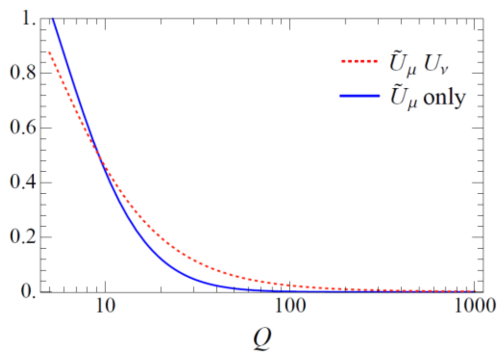
Model calculations indicate spin correlations are large at large x

Chang, Manohar, Waalewijn, Phys.Rev. D87 (2013) no.3, 034009



Evolution decreases relative importance of polarised distributions, especially at small x

Diehl, Kasemets, Keane, JHEP 1405 (2014) 118



Colour correlations are suppressed by a Sudakov factor, but can be non-negligible at lower scales $\lesssim 10\ \mathrm{GeV}$

Mekhfi and Artru, Phys.Rev. D37 (1988) 2618–2622
Manohar, Waalewijn, Phys.Rev. D85 (2012) 114009
Diehl, Ostermeier, Schafer, JHEP 1203 (2012) 089

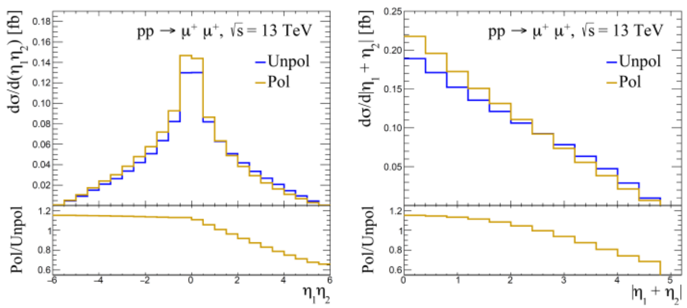
Spin correlation effects in cross sections

Evolution quickly ‘washes out’ spin correlations → difficult to find observables to measure them.

Significant effect in double open charm production [Echevarria, Kasemets, Mulders Pisano, JHEP 04 (2015) 034], but shape of polarised contribution very similar to unpolarised.

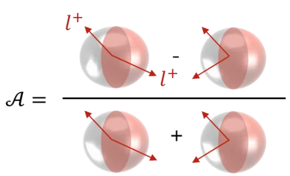
Recently identified that there may be hope of measuring spin polarisation effects in same-sign WW [Cotogno, Kasemets, Myska,1809.09024]

Good process in terms of spin polarisation:
- involves quarks.
- Ws couple only to left-handed quarks



Input at 1 GeV for polarised DPD chosen to saturate bounds & yield maximum effect

$$f_{\Delta a \Delta b}(x_1, x_2, \boldsymbol{y}; Q_0) = (-1)^n f_{ab}(x_1, x_2, \boldsymbol{y}; Q_0)$$



$\lvert\eta_i\rvert$	> 0	> 0.6	> 1.2
A	0.07	0.11	0.16
σ [fb]	0.51	0.29	0.13

Few percent effect on lepton pseudorapidity asymmetry

A Monte Carlo implementation of the DGS framework

Ideally would like to have a parton shower implementation of DGS framework - allows generation of exclusive DPS processes, can apply arbitrary cuts.

DGS framework has been implemented in the Monte Carlo code dShower (Baptiste, JG, Ostrolenk):

- Backward evolution from hard process using homogeneous double DGLAP equations:

$$\mathrm{d}\mathcal{P}_{ij}^{\mathrm{ISR}} = \mathrm{d}\mathcal{P}_{ij} \exp\left(-\int_{Q^2}^{Q_h^2} \mathrm{d}\mathcal{P}_{ij}\right)$$

$$\mathrm{d}\mathcal{P}_{ij} = \frac{\mathrm{d}Q^2}{Q^2}\left(\sum_{i'}\int_{x_1}^{1-x_2}\frac{\mathrm{d}x_1'}{x_1'}\frac{\alpha_s(p_\perp^2)}{2\pi}P_{i'\to i}\left(\frac{x_1}{x_1'}\right)\frac{F_{i'j}(x_1',x_2,\boldsymbol{y},Q^2)}{F_{ij}(x_1,x_2,\boldsymbol{y},Q^2)}\right.$$
$$\left.+\sum_{j'}\int_{x_2}^{1-x_1}\frac{\mathrm{d}x_2'}{x_2'}\frac{\alpha_s(p_\perp^2)}{2\pi}P_{j'\to j}\left(\frac{x_2}{x_2'}\right)\frac{F_{ij'}(x_1,x_2',\boldsymbol{y},Q^2)}{F_{ij}(x_1,x_2,\boldsymbol{y},Q^2)}\right)$$

Guided by some DPD set – can include momentum and number sum rule constraints and other correlations

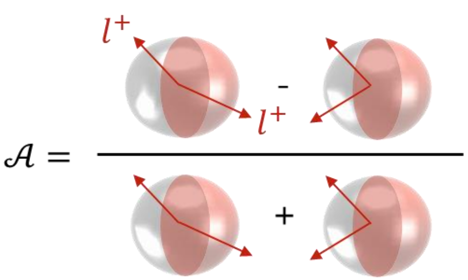
- $2 \to 1$ 'mergings' in backward evolution forced at scale μ_y, with probability given by splitting DPD/total DPD.

A Monte Carlo implementation of the DGS framework

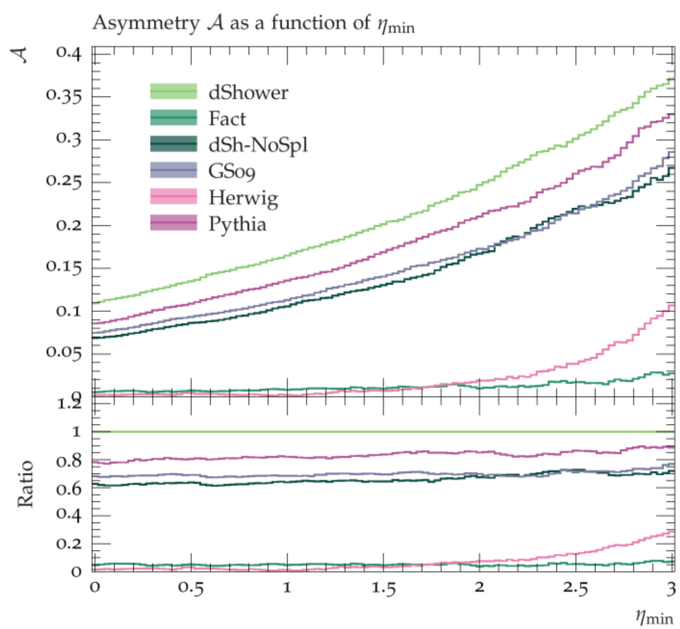
So far, no heavy quark thresholds implemented.

DPD set is $n_f = 3$ set from JHEP 1706 (2017) 083 + modifications to very approximately account for number & momentum sum rule constraints (adapted from JG, Stirling, JHEP 1003 (2010) 005).

$$pp \rightarrow W^+W^+ \rightarrow e^+\nu_e\mu^+\nu_\mu$$



JG, Kom, Kulesza, Stirling, Eur.Phys.J. C69 (2010) 53-65



Cabouat, JG, Ostrolenk

Includes 1→2 splittings + valence number effects

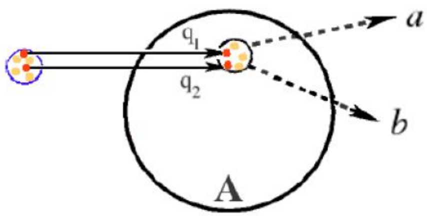
Simple valence number effects

No parton-parton correlations

DPS in pA collisions

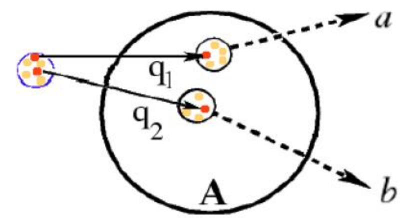
Two contributions to DPS in pA collisions:

See e.g. Strikman, Treleani, Phys. Rev. Lett. 88, 031801 (2002).
d’Enterria, Snigirev, Adv.Ser.Direct.High Energy Phys. 29 (2018) 159-187



$A\ \sigma_{pN}^{DPS}$

$\int d^2y\, F(y)F(y)$: 2v2, 2v1,1v1

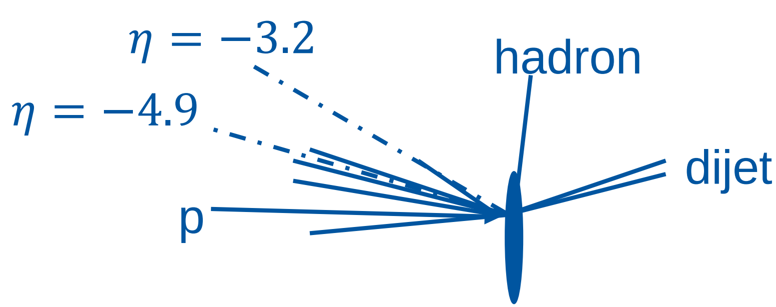


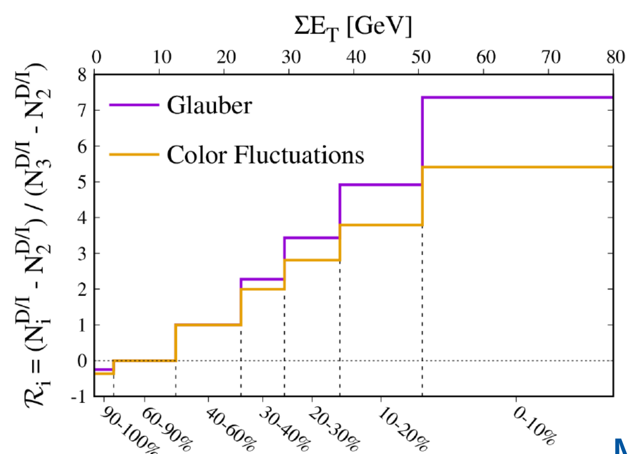
$\sim A^{4/3}\ \sigma_{pN}^{DPS} \dfrac{\sigma_{eff}}{14[mb]\pi}$

$\int d^2y\, F(y)$: 2v2, 2v1

Idea to separate the two mechanisms: use different centrality dependence of two terms: $\dfrac{d\sigma_{pA}^{DPS}}{d^2b} = \sigma_{pN}^{DPS} T(b) + \sigma_a \sigma_b T^2(b)$

Alvioli, Azarkin, Blok, Strikman, arXiv:1901.11266





More 2-nucleon contribution

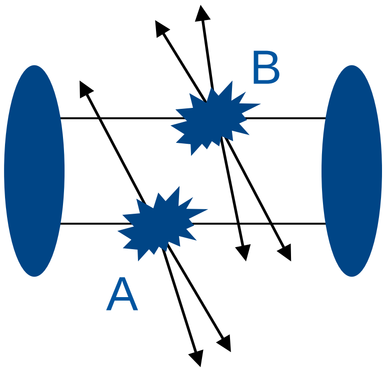
More central

Summary

- DGS framework: cut-off in DPS cross section + subtraction to avoid double counting. Various advantages (retains individual DPDs per proton…)
- DPS luminosities:
  - generically very large 1v1 with large uncertainty – have to compute SPS and subtraction up to two-loop.
  - some scenarios where DPS more 'prominent' – processes at small x, processes with systems separated in rapidity, same sign WW.
- Theory framework has also been extended to the case of measured transverse momentum. Significant simplifications for perturbative $\boldsymbol{q}_i$.
- Interference & correlated parton contributions in spin, colour, flavour, parton type. Typically washed away quickly by evolution. Possible chance to detect spin correlations in WW?
- Some guide on NP shape of DPDs from models. First info from lattice computations emerging.
- Shower implementation of DGS framework in development.
- pA DPS has two contributions, probe DPDs in different way. Potential prospects to separate the two pieces using centrality dependence.

CERN

DPS: Basic features



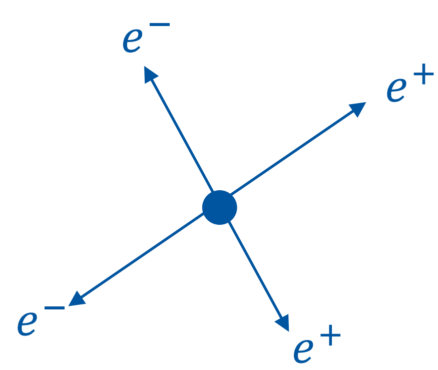
Double Parton Scattering (DPS) = two separate hard interactions in a single proton-proton collision

In terms of total cross section for production of AB, DPS is power suppressed with respect to single parton scattering (SPS) mechanism: $\frac{\sigma_{DPS}}{\sigma_{SPS}} \sim \frac{\Lambda^2}{Q^2}$

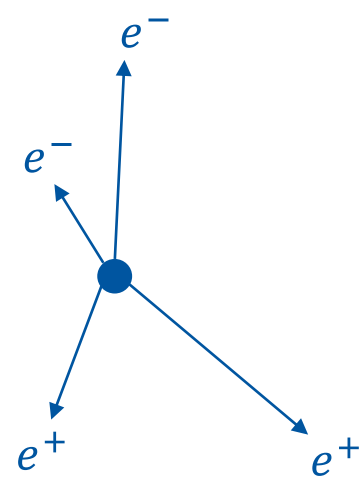
DPS populates the final state phase space in a different way from SPS. In particular, it tends to populate the region of small $\mathbf{q}_A$, $\mathbf{q}_B$ – competitive with SPS in this region.

e.g. pp → $\underbrace{e+\ e-}_{A}\ \underbrace{e+\ e-}_{B}$

DPS:



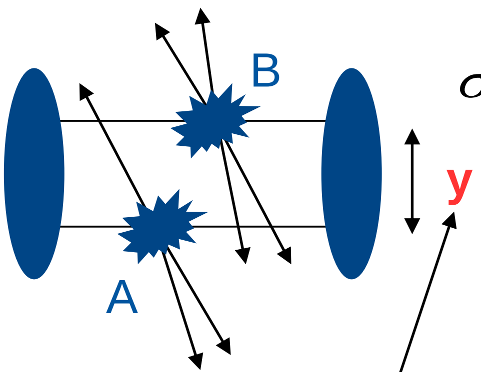
SPS:



DPS becomes more important relative to SPS as the collider energy grows, and we probe smaller *x* values where there is a larger density of partons.

Inclusive cross section for DPS

Postulated form for integrated double parton scattering cross section based on analysis of lowest order Feynman diagrams / parton model considerations:



Symmetry factor

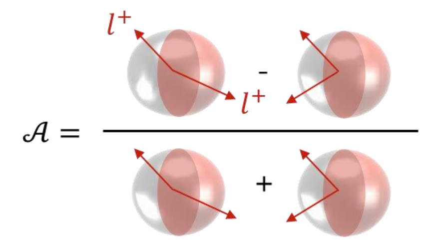
$$\sigma_D^{(A,B)} = \frac{m}{2} \sum_{i,j,k,l} \int \overbrace{F_h^{ik}\left(x_1, x_2, \mathbf{y}; Q_A, Q_B\right) F_h^{jl}\left(x_1', x_2', \mathbf{y}; Q_A, Q_B\right)}^{\text{Collinear double parton distribution (DPD)}}$$
$$\times \underbrace{\hat{\sigma}_{ij}^{A}\left(x_1, x_1'\right) \hat{\sigma}_{kl}^{B}\left(x_2, x_2'\right)}_{\text{Parton level cross sections}} \mathrm{d}x_1\, \mathrm{d}x_1' \mathrm{d}x_2\, \mathrm{d}x_2' \mathrm{d}^2 \mathbf{y}$$

y = separation in transverse space between the two partons

Paver, Treleani, Nuovo Cim. A70 (1982) 215.
Mekhfi, Phys. Rev. D32 (1985) 2371.
Blok, Dokshitzer, Frankfurt, Strikman, Phys.Rev. D83 (2011) 071501
Diehl, Ostermeier and Schäfer (JHEP 1203 (2012))

Future prospects at HL-LHC

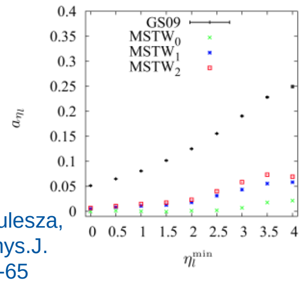
In same-sign WW, important observable



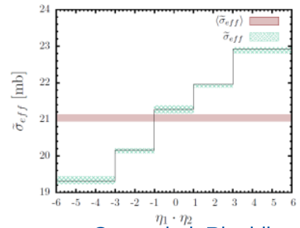
$a_{\eta l} \neq 0$ only if there are correlations in DPS

Various effects can cause a few per cent asymmetry:

x-space correlations, valence number effects



Gaunt, Kom Kulesza, Stirling, Eur.Phys.J. C69 (2010) 53-65



Ceccopieri, Rinaldi, Scopetta, Phys. Rev. D 95, 114030 (2017)

Polarised contributions

$\lvert\eta_i\rvert$	> 0	> 0.6	> 1.2
$a_{\eta l}$	0.07	0.11	0.16
σ [fb]	0.51	0.29	0.13

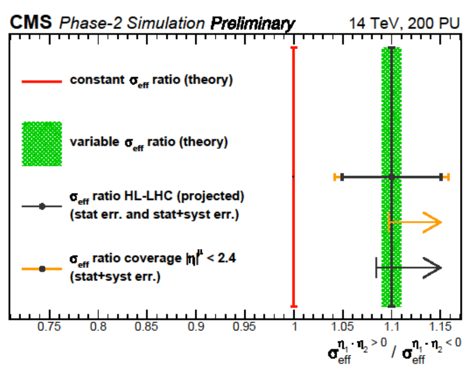
Cotogno, Kasemets, Myska,1809.09024

$1 \to 2$ splitting effects

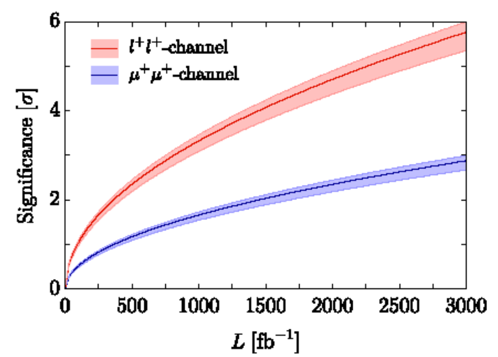
$$a_{\eta l} \approx 0.03$$

JG et al., CERN HL/HE-LHC YR

Asymmetry of a few per cent is observable at the HL-LHC – would be definitive sign of correlations!



CMS-TDR-017-003



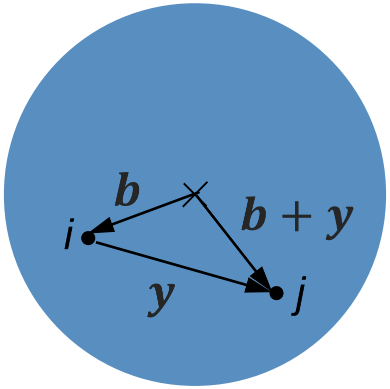
Cotogno, Kasemets, Myska,1809.09024

Simplifying assumptions for DPS cross section

If one ignores correlations between partons in the proton:

$$F^{ij}(x_1, x_2, \boldsymbol{y}) = \int d^2\boldsymbol{b}\, D^i(x_1, \boldsymbol{b})\; D^j(x_1, \boldsymbol{b} + \boldsymbol{y})$$

Impact parameter dependent PDFs (FT of GPD)



Common 'lore': approximately valid at low *x*, due to the large population of partons at such x values.

Further approximation that is often made: $D^i(x_1, \boldsymbol{b}) = D^i(x_1) G(\boldsymbol{b})$

$$\Longrightarrow \quad F^{ij}(x_1, x_2, \boldsymbol{y}) = D^i(x_1)\; D^j(x_2) \int d^2\boldsymbol{b} G(\boldsymbol{b})(\boldsymbol{b} + \boldsymbol{y})$$

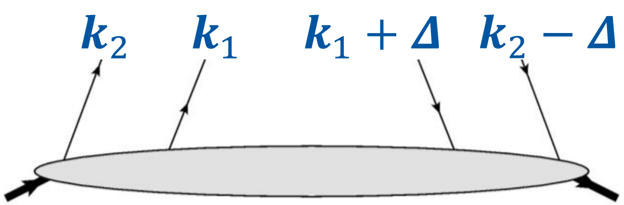
$$\Longrightarrow \quad \sigma_D^{(A,B)} = \frac{\sigma_S^{(A)} \sigma_S^{(B)}}{\sigma_{eff}}$$

"Pocket formula"

Almost all phenomenological estimates of DPS use this equation

DPDs in Δ-space

One can also consider Δ-space DPDs, where all divergences regularised using dimreg + $\overline{MS}$, and compute matching onto PDFs:



$$F_{a_1 a_2}(x_1, x_2, \Delta, \mu) = \sum_{a_0} \int_{x_1+x_2}^{1} \frac{dz}{z^2} \boxed{W_{a_1 a_2, a_0}\left(\frac{x_1}{z}, \frac{x_2}{z}, a_s(\mu), \log\frac{\mu^2}{\Delta^2}\right)} f_{a_0}(z, \mu)$$

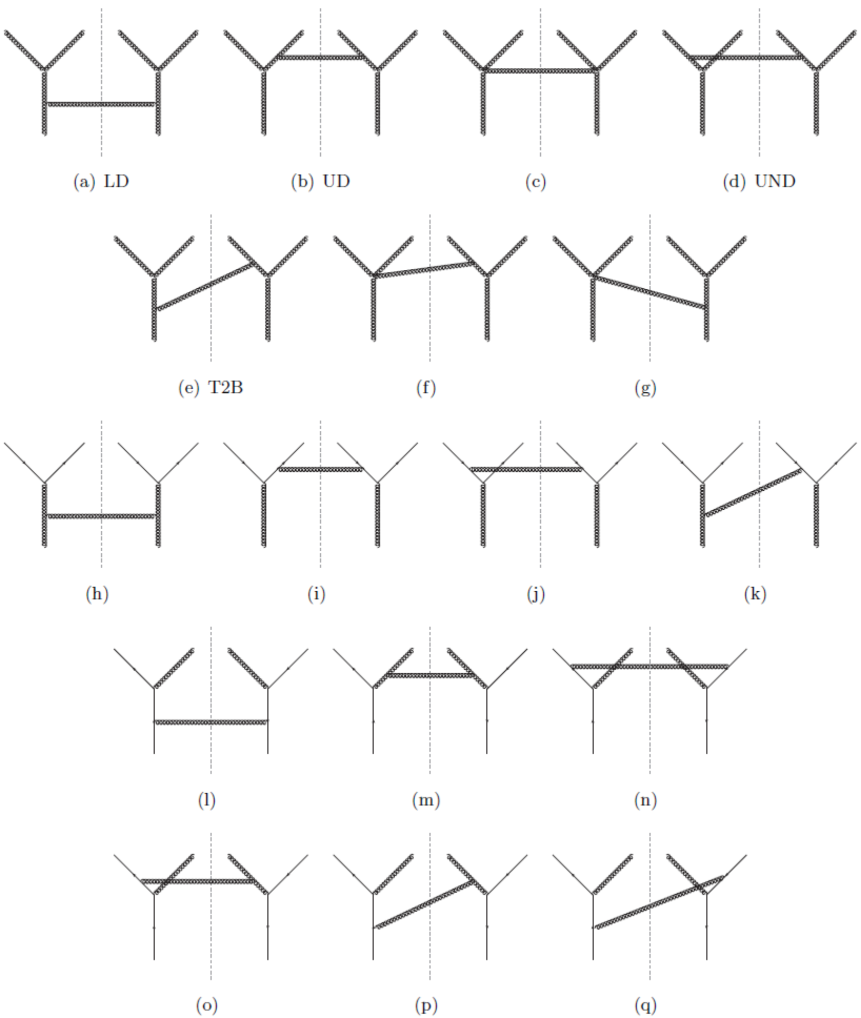
Evolution of Δ-space DPDs involves an inhomogeneous 1 → 2 splitting term:

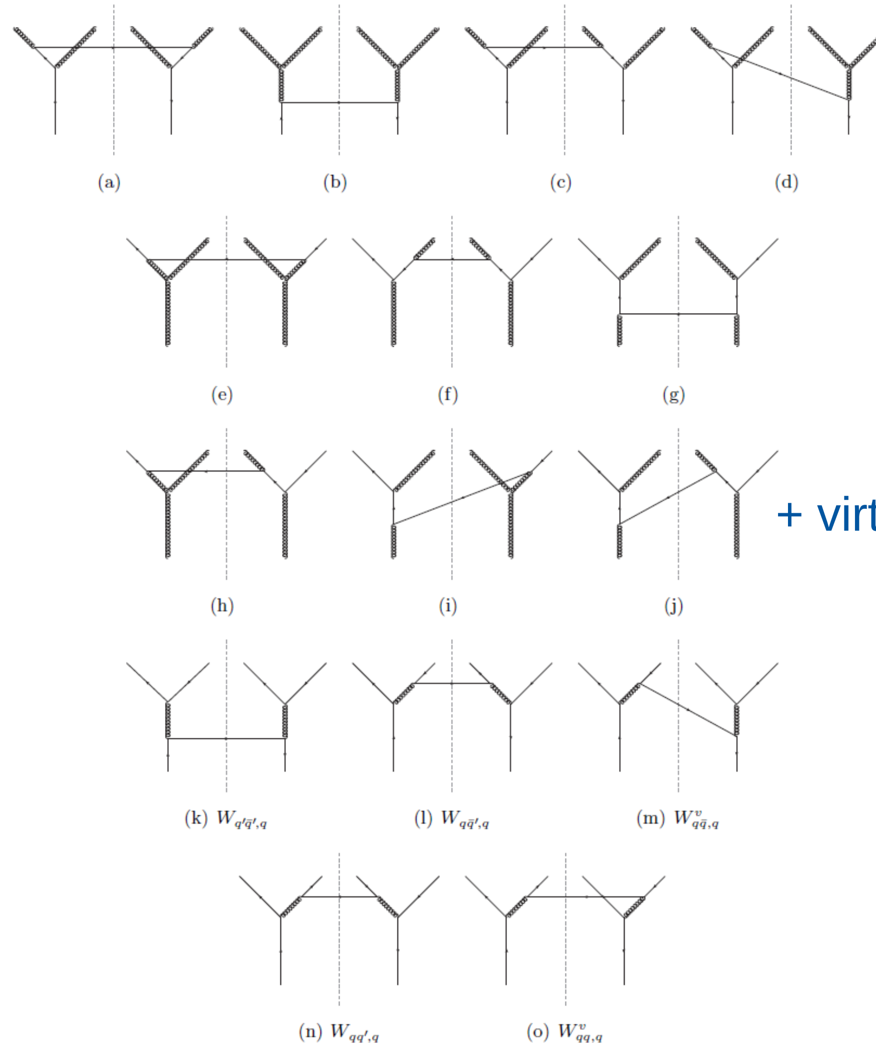
$$\frac{d}{d\ln\mu^2} F_{a_1 a_2}(x_1, x_2, \Delta, \mu) = \sum_{a_0} \int_{x_1+x_2}^{1} \frac{dz}{z^2} \boxed{P_{a_1 a_2, a_0}\left(\frac{x_1}{z}, \frac{x_2}{z}, a_s(\mu)\right)} f_{a_0}(z, \mu)$$
$$+ \{\text{homogeneous terms}\}\,,$$

We compute <u>also</u> W and P at NLO.

Graphs

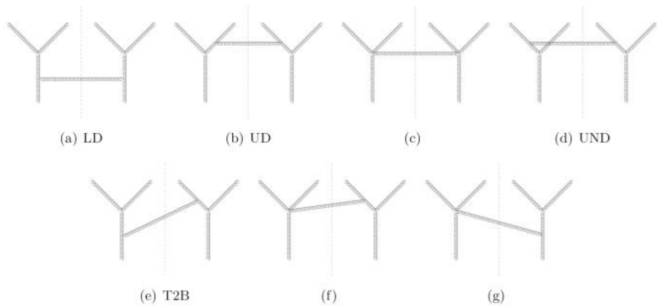
In light-cone gauge, graphs to compute:





+ virtuals

Method



Compute graph expressions (`FORM`, `FeynCalc`). Integrate over minus components using contours.

[Kuipers, Ueda, Vermaseren, Vollinga, Comput. Phys. Commun. 184 (2013) 1453-1467]
[Shtabovenko, Mertig, Orellana, Comput. Phys. Commun. 207 (2016) 432-444]

$$D_1 = \frac{(\boldsymbol{k}_1+\boldsymbol{\Delta})^2}{x_1} + \frac{(\boldsymbol{k}_2-\boldsymbol{\Delta})^2}{x_2} + \frac{(\boldsymbol{k}_1+\boldsymbol{k}_2)^2}{x_3} \qquad D_2 = \frac{\boldsymbol{k}_1^2}{x_1} + \frac{\boldsymbol{k}_2^2}{x_2} + \frac{(\boldsymbol{k}_1+\boldsymbol{k}_2)^2}{x_3}$$

$$D_3 = (\boldsymbol{k}_1+\boldsymbol{\Delta})^2 \qquad D_4 = \boldsymbol{k}_2^2 \qquad \tilde{D}_4 = \boldsymbol{k}_1^2 \qquad \tilde{D}_5 = (\boldsymbol{k}_1+\boldsymbol{k}_2)^2$$

$$I_1(a_1,a_2,a_3,a_4) = \int \frac{d^{d-2}\boldsymbol{k}_1 d^{d-2}\boldsymbol{k}_2}{\prod_{i=1..4} D_i^{a_i}} \qquad I_2(a_1,a_2,a_3,a_4,a_5) = \int \frac{d^{d-2}\boldsymbol{k}_1 d^{d-2}\boldsymbol{k}_2}{\prod_{i=1...3} D_i^{a_i} \prod_{i=4...5} \tilde{D}_i^{a_i}}$$

$I_1(1,1,0,0), I_1(0,1,1,0), I_1(1,1,1,0),$
$I_1(1,0,1,1), I_1(1,1,1,1), I_1(2,1,1,1)$

$I_2(0,1,1,0,1), I_2(1,1,1,1,0)$

Integration-by-parts reduction to master integrals (`LiteRed`)

[Lee, J. Phys. Conf. Ser. 523 (2014)]

$$\begin{bmatrix} \frac{\partial I_1(1,1,0,0)}{\partial x_1} \\ \frac{\partial I_1(0,1,1,0)}{\partial x_1} \\ \frac{\partial I_1(1,1,1,0)}{\partial x_1} \\ \frac{\partial I_1(1,0,1,1)}{\partial x_1} \\ \frac{\partial I_1(1,1,1,1)}{\partial x_1} \\ \frac{\partial I_1(2,1,1,1)}{\partial x_1} \end{bmatrix} = \begin{bmatrix} \blacksquare & 0 & 0 & 0 & 0 & 0 \\ 0 & \blacksquare & 0 & 0 & 0 & 0 \\ \blacklozenge & \blacklozenge & \blacksquare & 0 & 0 & 0 \\ 0 & \blacklozenge & 0 & \blacksquare & 0 & 0 \\ \blacklozenge & \blacklozenge & \blacklozenge & \blacklozenge & \blacksquare & \blacksquare \\ \blacklozenge & \blacklozenge & \blacklozenge & \blacklozenge & \blacksquare & \blacksquare \end{bmatrix} \begin{bmatrix} I_1(1,1,0,0) \\ I_1(0,1,1,0) \\ I_1(1,1,1,0) \\ I_1(1,0,1,1) \\ I_1(1,1,1,1) \\ I_1(2,1,1,1) \end{bmatrix}$$

Construct differential equations in x_1 and solve (`Fuchsia`)

[Gituliar, Magerya, Comput. Phys. Commun. 219 (2017) 329-338]

$$I_1(0,1,1,0) \to \pi^{3-2\epsilon} x_3^{1-\epsilon} (x_1 x_2)^{\epsilon} \frac{\Gamma[-\epsilon]}{\sin[2\pi\epsilon]\Gamma[1-3\epsilon]}$$

Computation of $x_3 \to 0$ limit of master integrals using method of regions (boundary conditions)

Results for bare graphs!

Cross-checks

- Full computation of bare graphs done using light-cone and covariant Feynman gauge ✓
- Master integrals satisfy differential equation in x_2 ✓
- Master integrals all checked numerically at 10 random points using FIESTA ✓
- Individual graphs have poles in ϵ up to ϵ^{-3}, as well as rapidity divergences. ϵ^{-3} pole + rapidity divergences cancel after summing over graphs, ϵ^{-2} pole is as predicted by renormalisation group equation ✓
- Splitting functions $P^{(1)}_{a_1 a_2, a_0}$ satisfy constraints related to number and momentum sum rules:

[Gaunt, Stirling JHEP 1003 (2010) 005
Diehl, Plößl, Schafer, arXiv:1811.00289]

$$\int_0^{1-x_1} dx_2 \left[P_{a_1 q, a_0}(x_1, x_2) - P_{a_1 \bar{q}, a_0}(x_1, x_2)\right] = \left(\delta_{a_1 \bar{q}} - \delta_{a_1 q} - \delta_{a_0 \bar{q}} + \delta_{a_0 q}\right) P_{a_1 a_0}(x_1)\,,$$

$$\sum_{a_2} \int_0^{1-x_1} dx_2\, x_2\, P_{a_1 a_2, a_0}(x_1, x_2) = (1 - x_1)\, P_{a_1 a_0}(x_1) \qquad ✓$$

Small x_1, x_2 limit

Interesting processes/regions for studying DPS typically involve small x values (higher density of partons→greater chance of DPS, plus smaller Q such that power suppression is reduced).

→ Interesting to study matching coefficients and splitting functions in limits of small x_i. For example, small x_1, x_2 limit of $P^{(1)}_{gg,g}(x_1, x_2)$:

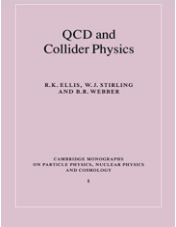
$$P^{(1)}_{gg,g}(x_1,x_2) \longrightarrow \frac{C_A^2\left((1-6u+6u^2)+\left(8-\frac{2}{u}-4u+4u^2\right)\log[1-u]+\{u\leftrightarrow 1-u\}\right)}{x^2}$$

$$x \equiv x_1 + x_2$$
$$u \equiv x_1/(x_1+x_2)$$

Same $1/x^2$ behaviour for other splitting functions, and V kernels

$V^{(1)}(x_1,x_2) \sim 1/x^2 \Rightarrow F(x_1,x_2,\boldsymbol{y}) \sim \alpha_s^{n+2}\log^{n+1}(x)/x$ (for NLO splitting)
i.e. NLL in small x logarithms!

[$V^{(1)}(x_1,x_2) \sim \log(x)/x^2 \Rightarrow F(x_1,x_2,\boldsymbol{y}) \sim \alpha_s^{n+2}\log^{n+2}(x)/x$, i.e. LL]



Similar cf usual splitting functions, where $P^{(1)}(x) \sim 1/x$ and not $\log(x)/x$.

Modelling the DPDs

Construct model of DPDs, with 'intrinsic' and 'splitting' components:

$$F^{ij}(x_1, x_2, y, \mu) = F^{ij}_{\rm spl}(x_1, x_2, y, \mu) + F^{ij}_{\rm int}(x_1, x_2, y, \mu)$$

Smooth transverse y profile, radius ~ R_p

'Usual' product of PDFs

Initialise at low scale μ_0 = 1 GeV

$$F_{a_1 a_2,\rm int}(x_1, x_2, \boldsymbol{y}; \mu_0, \mu_0) = \frac{1}{4\pi h_{a_1 a_2}} \exp\left[-\frac{y^2}{4h_{a_1 a_2}}\right] f_{a_1}(x_1;\mu_0)\, f_{a_2}(x_2;\mu_0)$$
$$\times\, (1-x_1-x_2)^2\,(1-x_1)^{-2}\,(1-x_2)^{-2}$$

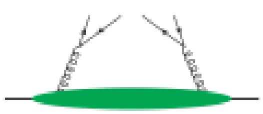
Factor to suppress DPD near phase space limit $x_1 + x_2 = 1$

Initialise at low scale μ_y ~ 1/y

Gaussian suppression at large y

$$F_{a_1 a_2,\rm spl}(x_1, x_2, \boldsymbol{y}; \mu_y, \mu_y)$$

Perturbative splitting expression



$$= \frac{1}{\pi y^2} \exp\left[-\frac{y^2}{4h_{a_1 a_2}}\right] \frac{f_{a_0}(x_1+x_2;\mu_y)}{x_1+x_2} \frac{\alpha_s(\mu_y)}{2\pi} P_{a_0\to a_1 a_2}\left(\frac{x_1}{x_1+x_2}\right)$$

Evolve both to scale μ using homogeneous double DGLAP $\frac{d}{d\log\mu_i} F(x_i, \boldsymbol{y}; \mu_i) = P \otimes_{x_i} F$