Constraining gluon PDFs and TMDs with quarkonium production

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\[2 < y_{\eta_c} < 4.5\]

\[\sqrt{s} = 8 \text{ TeV}\]

\[\text{NLO CSM \cite{Butenschoen:2015bka}}\]

\[\text{LHCb data \cite{Aaij:2015gja}}\]
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$\eta_c$ data at LHCb - 2015

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[Y. Feng et al., arXiv:1901.09766 [hep-ph]]
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  → how is this related to PDFs and TMDs?
problem of negative cross-sections - $\eta_c$ and $J/\psi$ at NLO

comparison of $\eta_c$ (left) and $J/\psi$ (right) differential cross-sections at NLO with different scale choices of $\mu_R$ and $\mu_F$ with CTEQ6M

cross-sections & probabilities

\[ P_{\text{physical}} = \sum_i P_i \]  \hspace{1cm} (1)

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- **cross-sections** are **observable** quantities, hence physical probabilities \( P_{\text{physical}} (\sigma, \frac{d\sigma}{dy}, \ldots) \) must be **positive**
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  → therefore one cannot rule out the possibility of negative cross-sections with positive PDFs
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collinear factorisation - $\eta_c$ at NLO - hadronic cross-section

process

$$p + p \rightarrow \eta_c + X$$  \hspace{1cm} (2)

hadronic cross-section

$$\sigma_{pp} = \sum_{ij} \int dx_1 dx_2 \ f_i/p(x_1, \mu_F) f_j/p(x_2, \mu_F) \ \hat{\sigma}_{ij}(\mu_R, \mu_F, x_1, x_2, \hat{s} = s \ x_1 x_2)$$  \hspace{1cm} (3)

hadronic cross-section has dependence on the scales $(\mu_R, \mu_F, s)$

three channels contributing to $\eta_c$ production at NLO; left - $gg$ channel, middle - $q\bar{q}$ channel, right - $qg$ channel
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\( \eta_c \) at NLO - historical development

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appearance of negative cross-sections for quarkonia at high energies
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- behaviour of partonic cross-sections away from threshold
Most of the remarks which follow have already been made by G. Schuler in his ’94 review. Schuler at the time had available the full NLO corrections to $\eta$ production, as well as the leading small-$x$ behaviour of the $\chi$ cross sections. It is a pity that those remarks have passed almost unnoticed in the community!

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\( \bullet \) arrives to similar conclusions that steeper gluon PDF choices will give better results because real corrections become less relevant (see Schuler’s table) at high hadronic energies

\( \bullet \) confirms that partonic limit away from threshold has the general structure,

\[
\lim_{z \to 0} \hat{\sigma}_{gg} = 2C_A \frac{\alpha_s}{\pi} \hat{\sigma}_{\text{Born}} \left( \log \frac{M^2}{\mu_F^2} - C_J \right), \quad (4)
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\lim_{z \to 0} \hat{\sigma}_{qg} = C_F \frac{\alpha_s}{\pi} \hat{\sigma}_{\text{ Born}} \left( \log \frac{M^2}{\mu_F^2} - C_J \right), \quad (5)
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where \( C_J \) is a process-dependent quantity
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let’s make a comparison with \( \eta_b \), why do we not encounter negative cross-sections?
$\eta_c$ versus $\eta_b$

comparison of $\eta_b$ differential cross-section at NLO with different choices of $\mu_R$ and $\mu_F$ with CTEQ6M

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- \( \eta_b \) differential cross-section is much more stable than in case of \( \eta_c \). The NLO result is the same for both particles. With only the mass increasing from \( m_c \) to \( m_b \), we can describe three effects:
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  - evolution of the PDFs from the scale of $\eta_c$ to $\eta_b$. Evolution leads to steeper gluon PDFs, hence real corrections are further suppressed $\rightarrow$ essentially ensuring the positivity of the $\eta_b$ cross-section

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In order to discriminate between the PDF choices we will use two different scale configurations:
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  - $\mu_R = \mu_F = 2m_c = 3\text{GeV}$ - default scale choice
  - $\mu_R = m_c = 1.5\text{GeV}$, $\mu_F = 2m_c = 3\text{GeV}$
    - lower renormalisation choice leads to larger $\alpha_s \rightarrow$ real emission contributions become more important; the objective is to see the impact of the PDFs on the real corrections
K-factor at $y = 0$ - $\mu_R = \mu_F = 2m_c = 3\text{GeV}$

K-factor of $\eta_c$ production at $y=0$ with $n_f=3$, $\mu_r=\mu_f=2m_c=3\text{GeV}$

K-factor at $y=0$ as a function of energy and with different PDF choices. Default scale choice used $\mu_R = \mu_F = 2m_c = 3\text{GeV}$. 

Melih A. Ozcelik (IPNO)
$K$-factor at $y = 0$ - $\mu_R = m_c = 1.5\text{GeV}, \mu_F = 2m_c = 3\text{GeV}$

**K-factor of $\eta_c$ production at $y=0$ with $n_f=3$, $\mu_r=m_c$, $\mu_f=2m_c$**

![Graph of K-factor of $\eta_c$ production](image)

**K-factor at $y=0$ as a function of energy and with different PDF choices.**

Alternative scale choice used $\mu_R = m_c = 1.5\text{GeV}, \mu_F = 2m_c = 3\text{GeV}$. 

MRS(G), $g(x) \sim 1/x^{1.30037}$

MRS(A'), $g(x) \sim 1/x^{1.14215}$
Uncertainty of $K$-factor at $y = 0$ - 100 Replicas of NNPDF31_nlo_as_0118

- use standard NNPDF31_nlo_as_0118 set and run over 100 Replicas
Uncertainty of $K$-factor at $y = 0$ - 100 Replicas of NNPDF31\_nlo\_as\_0118

- use standard NNPDF31\_nlo\_as\_0118 set and run over 100 Replicas
- difference between NNPDF31\_nlo\_as\_0118 and NNPDF31sx\_nlo\_as\_0118 is that the latter one with small $x$ extension has been probed at a minimally lower scale $Q$ such that in the Replica generation the $2.7\text{GeV}^2$ bin has been taken into account which turns out to be crucial
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- expect very large $K$-factor uncertainty associated to NNPDF31_nlo_as_0118 PDF choice
- we will try with two different scale choices as before, set $y = 0$ and use $\sqrt{s} = 115$ GeV, 7 TeV and 14 TeV
Figure: Strong variation of $K$-factor over replica number of NNPDF31_nlo_as_0118 ($y=0$, $\sqrt{s} = 7$ TeV, default/alternative scale choice)

default ($\mu_R = \mu_F = 2m_c = 3$GeV): $\rightarrow K = 0.2 \pm 0.2$

alternative ($\mu_R = m_c = 1.5$GeV, $\mu_F = 2m_c = 3$GeV): $\rightarrow K = -0.8 \pm 0.3$
**Figure:** Strong variation of \( K \)-factor over replica number of NNPDF31\_nlo\_as\_0118 (\( y=0, \sqrt{s} = 14 \) TeV, default/alternative scale choice)

default (\( \mu_R = \mu_F = 2m_c = 3 \)GeV): \( \rightarrow K = -0.1 \pm 0.4 \)

alternative (\( \mu_R = m_c = 1.5 \)GeV, \( \mu_F = 2m_c = 3 \)GeV): \( \rightarrow K = -1.1 \pm 0.5 \)
### K-factor - default scale - summary so far

<table>
<thead>
<tr>
<th>PDF choice</th>
<th>$\sqrt{s} = 7$ TeV</th>
<th>$\sqrt{s} = 14$ TeV</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>y = 0</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MRS(G)</td>
<td>1.26</td>
<td>1.21</td>
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<tr>
<td>MRS(A')</td>
<td>0.70</td>
<td>0.61</td>
</tr>
<tr>
<td>NNPDF31sx_nlonllx_as_0118</td>
<td>0.68</td>
<td>0.59</td>
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<tr>
<td><strong>y = 1</strong></td>
<td></td>
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</tr>
<tr>
<td>CT14nlo_NF3</td>
<td>0.54</td>
<td>0.44</td>
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<tr>
<td>NNPDF31sx_nlo_as_0118</td>
<td>0.51</td>
<td>0.37</td>
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<tr>
<td><strong>y = 2</strong></td>
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<tr>
<td>NNPDF31_nlo_as_0118</td>
<td>$0.2 \pm 0.2$</td>
<td>$-0.1 \pm 0.4$</td>
</tr>
</tbody>
</table>

√s = 7 TeV

√s = 14 TeV
can we improve the $K$-factor for NNPDF31_nlo_as_0118 PDF set by applying constraints?
can we improve the $K$-factor for NNPDF31_nlo_as_0118 PDF set by applying constraints?

- strategy is to discard all replicas that gave unphysical $d\sigma/dy < 0$ in a given set of results. We will assign weight 0 to each such replica.
can we improve the $K$-factor for NNPDF31_nlo_as_0118 PDF set by applying constraints?

strategy is to discard all replicas that gave unphysical $d\sigma/dy < 0$ in a given set of results. We will assign weight 0 to each such replica

we will use the results for $y = 0$ and $\sqrt{s} = 14$ TeV with default scale choice
discarding all Replicas that yielded unphysical $d\sigma/dy < 0$
→ around half of the Replicas remained.

<table>
<thead>
<tr>
<th>result set</th>
<th>before re-weighting</th>
<th>after re-weighting</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{s} = 7$ TeV &amp; $y = 0$</td>
<td>0.2 ± 0.2</td>
<td>0.4 ± 0.1</td>
</tr>
<tr>
<td>$\sqrt{s} = 7$ TeV &amp; $y = 1$</td>
<td>0.2 ± 0.4</td>
<td>0.4 ± 0.1</td>
</tr>
<tr>
<td>$\sqrt{s} = 7$ TeV &amp; $y = 2$</td>
<td>0.2 ± 1.1</td>
<td>0.5 ± 0.1</td>
</tr>
<tr>
<td>$\sqrt{s} = 14$ TeV &amp; $y = 0$</td>
<td>−0.1 ± 0.4</td>
<td>0.3 ± 0.1</td>
</tr>
</tbody>
</table>
## $K$-factor - default scale - updated summary

<table>
<thead>
<tr>
<th>PDF choice</th>
<th>$\sqrt{s} = 7$ TeV</th>
<th>$\sqrt{s} = 14$ TeV</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>$y = 0$</td>
<td>$y = 1$</td>
</tr>
<tr>
<td>MRS(G)</td>
<td>1.26</td>
<td>1.27</td>
</tr>
<tr>
<td>MRS(A’)</td>
<td>0.70</td>
<td>0.72</td>
</tr>
<tr>
<td>NNPDF31sx_nlonllx_as_0118</td>
<td>0.68</td>
<td>0.71</td>
</tr>
<tr>
<td>CT14nlo_NF3</td>
<td>0.54</td>
<td>0.57</td>
</tr>
<tr>
<td>NNPDF31sx_nlo_as_0118</td>
<td>0.51</td>
<td>0.52</td>
</tr>
<tr>
<td><strong>NNPDF31_nlo_as_0118 (re-weighted)</strong></td>
<td><strong>0.4 ± 0.1</strong></td>
<td><strong>0.4 ± 0.1</strong></td>
</tr>
<tr>
<td><strong>NNPDF31_nlo_as_0118 (equal weight)</strong></td>
<td><strong>0.2 ± 0.2</strong></td>
<td><strong>0.2 ± 0.4</strong></td>
</tr>
</tbody>
</table>
next step & further constraints on PDFs from Quarkonium Physics

- re-weighted cross-sections are not compatible with sx and NLLx
next step & further constraints on PDFs from Quarkonium Physics

- re-weighted cross-sections are not compatible with $sx$ and NLLx
  → re-do exercise with $sx$ and NLLx
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→ work on-going
\[ \sigma \propto H \times C[f_1^g f_1^g] \]  

\[ C[f_1^g f_1^g] = \int \frac{d^2 \vec{b}_T}{(2\pi)^2} e^{i \vec{b}_T \cdot \vec{q}_T} \tilde{f}_1^g \left(x_1, \vec{b}_T; \zeta, \mu\right) \tilde{f}_1^g \left(x_2, \vec{b}_T; \zeta, \mu\right) \]  

\[ \tilde{f}_1^g/A \left(x, \vec{b}_T; \zeta, \mu\right) = \sum_{j=q,\bar{q},g} \int_{x}^{1} \frac{d\tilde{x}}{\tilde{x}} \tilde{C}_{g/j} \left(\tilde{x}, \vec{b}_T; \zeta, \mu\right) f_{j/A} \left(x/\tilde{x}; \mu\right) \]  

\[ \tilde{C}_{g/g} = \delta(1-x) + \frac{\alpha_s}{2\pi} \left[ C_A \delta(1-x) \left(-\frac{1}{2}L_T^2 + L_T \ln \frac{\mu^2}{\zeta} - \frac{\pi^2}{12}\right) \right. \]  

\[ - L_T \left( P_{g/g} - \delta(1-x) \frac{\beta_0}{2} \right) \]  

\[ \tilde{C}_{g/q} = \frac{\alpha_s}{2\pi} \left[-L_T P_{g/q} + C_F x \right] \]  

\[ L_T = \ln \frac{\mu^2 b_T^2}{4e^{-2\gamma_E}} \]  

(7)
TMD vs. collinear factorisation

- TMD factorisation is more universal than collinear factorisation
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  - leading-order process plus virtual corrections are factorised into hard part $H$ (*process-dependent*)
TMD vs. collinear factorisation

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  - leading-order process plus virtual corrections are factorised into hard part $H$ (process-dependent)
  - real and mixed real-virtual corrections are included inside the TMDPDFs (process-independent)

\[ \frac{d\sigma}{dy} > 0, \quad \text{always (universal property)!} \]

However, we encounter at $\eta_{cs}$ scales, that

\[ C[f_1g_1f_1g_1] < 0 \]

\[ \rightarrow \]

constrain PDFs such that

\[ C[f_1g_1f_1g_1] > 0 \text{ at } \eta_{cs} \text{ scales} \]

re-weighting PDFs with similar criteria if the re-weighted Replicas obtained by imposing $\frac{d\sigma}{dy} > 0$ (+ good shape behaviour) in collinear factorisation more or less coincide with $C[f_1g_1f_1g_1]$, this would mean that we are on the right track to use quarkonium as quantitative gluon probes

\[ \rightarrow \text{work ongoing} \]
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  → work on-going
Backup
Figure: rapidity differential cross-section at LO for different energies, default scale choice, CT14nlo_NF3
Figure: rapidity differential cross-section at NLO for different energies, default scale choice, CT14nlo_NF3
Figure: rapidity differential cross-section at NLO for different energies, alternative scale choice, CT14nlo_NF3
shape of rapidity differential at NLO - NNPDF31sx_nlo_as_0118

**Figure:** rapidity differential cross-section at NLO for different energies, default/alternative scale choice, NNPDF31sx_nlo_as_0118
Figure: rapidity differential cross-section at NLO for different energies, default/alternative scale choice, NNPDF31sx_nlonllx_as_0118
Quarkonia - three different models

- Colour-Evaporation Model
  - quark and anti-quark colours are summed up at amplitude squared level (evaporation)
  - no spin-projection

- Colour-Octet Model
  - quark and anti-quark pair are in color-octet state
  - heavy quark spins projected on final bound state
  - higher Fock states in NRQCD, higher $\nu$-order

- Colour-Singlet Model
  - quark and anti-quark pair are in color-singlet state
  - heavy quark spins projected on final bound state
  - leading Fock state in NRQCD
gluon-gluon channel

\[ \hat{\sigma}_{gg}(s, \hat{s}, \mu_R, \mu_F) = \frac{\alpha_s^2(\mu_R) \pi^2}{96 m_c^5} |R(0)|^2 \delta(1 - z) \]

\[ + \frac{\alpha_s^3(\mu_R) \pi}{1152 m_c^5} |R(0)|^2 \left[ \left( -44 + 7\pi^2 + 54 \log \left( \frac{\mu_R^2}{\mu_F^2} \right) \right) \right. \]

\[ + 72 \log \left( 1 - \frac{4m_c^2}{s} \right) \left( \log \left( 1 - \frac{4m_c^2}{s} \right) - \log \left( \frac{\mu_F^2}{4m_c^2} \right) \right) \delta(1 - z) \]

\[ + 6 \left( 24 \left( \frac{\log (1 - z)}{1 - z} \right) \right) \rho \left( 1 - (1 - z) z \right)^2 \]

\[ + 12 \left( \frac{1}{1 - z} \right) \rho \frac{\log (z)}{(1 - z)(1 + z)^3} \left( 1 - z^2 \left( 5 + z \left( 2 + z + 3z^3 + 2z^4 \right) \right) \right) \]

\[ - \left( \frac{1}{1 - z} \right) \rho \frac{1}{(1 + z)^2} \left( 12 + z^2 \left( 23 + z \left( 24 + 2z + 11z^3 \right) \right) \right) \]

\[ + 12 \left( 1 + z^3 \right)^2 \log \left( \frac{z \mu_F^2}{4m_c^2} \right) \right] \]

, where \( z = 4m_c^2/\hat{s} \) and \( \rho = 4m_c^2/s \)
quark-antiquark channel

\[ \hat{\sigma}_{q\bar{q}}(\hat{s}, \mu_R) = \frac{16\alpha_s^3(\mu_R)\pi}{81 m_c}\left| R(0) \right|^2 \frac{\hat{s} - 4m_c^2}{\hat{s}^3} \]  \hspace{1cm} (9)

quark-gluon channel

\[ \hat{\sigma}_{qg}(\hat{s}, \mu_R, \mu_F) = \frac{\alpha_s^3(\mu_R)\pi}{72 m_c^5 \hat{s}^2}\left| R(0) \right|^2 \left( 8m_c^4 + 4m_c^2\hat{s} - \hat{s}^2 \right. \]
\[ + 2 \left( 8m_c^4 - 4m_c^2\hat{s} + \hat{s}^2 \right) \log \left( 1 - \frac{4m_c^2}{\hat{s}} \right) \]
\[ + \hat{s} \left( -4m_c^2 + \hat{s} \right) \log \left( \frac{4m_c^2}{\hat{s}} \right) \]
\[ - \left( 8m_c^4 - 4m_c^2\hat{s} + \hat{s}^2 \right) \log \left( \frac{\mu_F^2}{\hat{s}} \right) \]  \hspace{1cm} (10)
problem of negative cross-sections - $J/\psi, {^1S_0}^{[8]}$ at NLO

comparison of $J/\psi {^1S_0}^{[8]}$ differential cross-section at NLO with different choices of $\mu_R$ and $\mu_F$ with CTEQ6M [Y. Feng, J.-P. Lansberg, J.X. Wang, Eur.Phys.J. C75 (2015)]
let’s define $z = \frac{M^2}{\hat{s}}$ and $\tau_0 = \frac{M^2}{s}$

LO partonic cross-section and virtual corrections ($2 \rightarrow 1$ process) have $\delta(1 - z)$ function while real corrections ($2 \rightarrow 2$) are complicated functions of $z$

negative contributions come from real corrections which have interference terms

idea is to use simple toy-models for gluon PDFs and convolute with partonic cross-section; different $z$-terms will contribute differently at hadronic level
Asymptotic ($\tau_0 = M^2/s \to 0$) behaviour of the proton-proton or proton-antiproton cross section for various forms of the gluon-gluon subprocess ($z = M^2/\hat{s} = \tau_0/\tau$) and two extreme choices of the gluon distribution function. Taken from G. Schuler, Review, 1994

<table>
<thead>
<tr>
<th>$\hat{\sigma}_{gg}(z, M^2)$</th>
<th>$xg(x) \to 1$</th>
<th>$xg(x) \to 1/\sqrt{x}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta(1 - z)$</td>
<td>$\ln\left(\frac{1}{\tau_0}\right)$</td>
<td>$\frac{1}{\sqrt{\tau_0}}\ln\left(\frac{1}{\tau_0}\right)$</td>
</tr>
<tr>
<td>$z^k$</td>
<td>$\frac{1}{k}\ln\left(\frac{1}{\tau_0}\right)$</td>
<td>$\frac{2}{(2k+1)\sqrt{\tau_0}}\ln\left(\frac{1}{\tau_0}\right)$</td>
</tr>
<tr>
<td>$1$</td>
<td>$\frac{1}{2}\ln^2\left(\frac{1}{\tau_0}\right)$</td>
<td>$\frac{2}{\sqrt{\tau_0}}\ln\left(\frac{1}{\tau_0}\right)$</td>
</tr>
<tr>
<td>$\ln^k\left(\frac{1}{z}\right)$</td>
<td>$\frac{1}{(k+1)(k+2)}\ln^{k+2}\left(\frac{1}{\tau_0}\right)$</td>
<td>$\frac{k!2^{k+1}}{\sqrt{\tau_0}}\ln\left(\frac{1}{\tau_0}\right)$</td>
</tr>
</tbody>
</table>

toymodel $g(x) = 1/x$: real corrections dominate at high energies;
toymodel $g(x) = 1/x^{1.5}$: all contributions have same energy scaling
partonic cross-section away from threshold, \( z \to 0 \)

\[
\lim_{z \to 0} \hat{\sigma}_{gg} = \frac{\alpha_s^3(\mu) \pi}{16 m_c^5} |R(0)|^2 \left( \log \left( \frac{4 m_c^2}{\mu_F^2} \right) - 1 \right), \quad (11)
\]

\[
\lim_{z \to 0} \hat{\sigma}_{q\bar{q}} = 0, \quad (12)
\]

\[
\lim_{z \to 0} \hat{\sigma}_{qg} = \frac{\alpha_s^3(\mu) \pi}{72 m_c^5} |R(0)|^2 \left( \log \left( \frac{4 m_c^2}{\mu_F^2} \right) - 1 \right). \quad (13)
\]
partonic cross-section away from threshold, $z \to 0$

- $\mu_F = m_c$

\[
\lim_{z \to 0} \hat{\sigma}_{gg} = \frac{\alpha_s^3(\mu)\pi}{16m_c^5} |R(0)|^2 (\log(4) - 1) = 0.2 \times \hat{\sigma}_{gg,LO}, \tag{14}
\]

- $\mu_F = 2m_c$

\[
\lim_{z \to 0} \hat{\sigma}_{gg} = \frac{\alpha_s^3(\mu)\pi}{16m_c^5} |R(0)|^2 (-1) = -0.5 \times \hat{\sigma}_{gg,LO}, \tag{15}
\]
hadronic cross-section - dependence on $\mu_F$

- toy model 1 PDF with $f_{g/p}(x) = 1/x$
  - dependence of hadronic cross-section on $\mu_F$
  - for $\mu_F > m_c$, hadronic cross-section is negative
  - for $\mu_F < m_c$, hadronic cross-section is positive
- toy model 2 PDF with $f_{g/p}(x) = 1/x^{1.5}$
  - weak dependence of hadronic cross-section on $\mu_F$
  - cross-section always positive (independent of choice of $\mu_F$)
- similar behaviour for $qg$ channel at high energies because of same asymptotic limit as in $gg$ channel apart from global factor
for non-steep PDF choices, the high-energy hadronic limit is governed by the high-energy partonic limit → strong dependence on factorisation scale $\mu_F$

some values for $C_J$:
- $C_J = 1$ for pseudo-scalar quarkonia $\eta_{c/b/t}$
- $C_J = 43/27$ for $\chi_{c/b,0}$
- $C_J = 53/36$ for $\chi_{c/b,2}$
- $C_J = 11/12 + \log z$ for Higgs (in infinite-top quark mass limit)

as an aside note, ratio between $qg$ and $gg$ channel in high-energy partonic limit is process-independent (same for Quarkonia and Higgs Physics)

$$\lim_{z \to 0} \frac{\hat{\sigma}_{qg}}{\hat{\sigma}_{gg}} = \frac{C_F}{2C_A} = \frac{2}{9}$$ (16)