Constraining gluon PDFs and TMDs with quarkonium production

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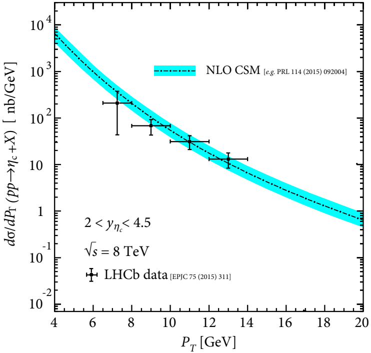

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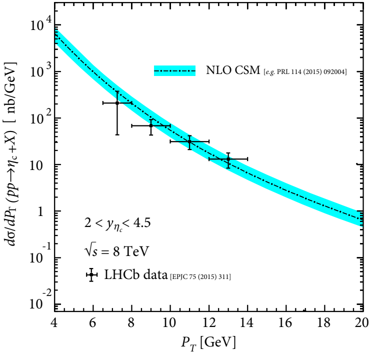

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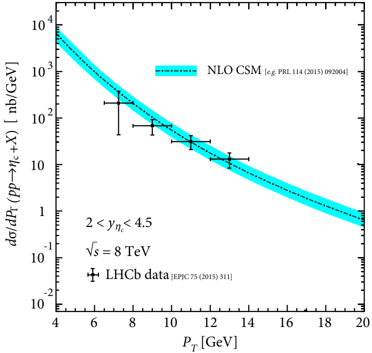

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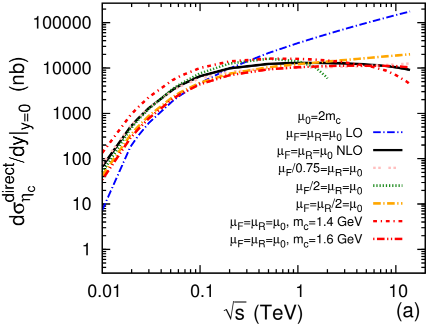
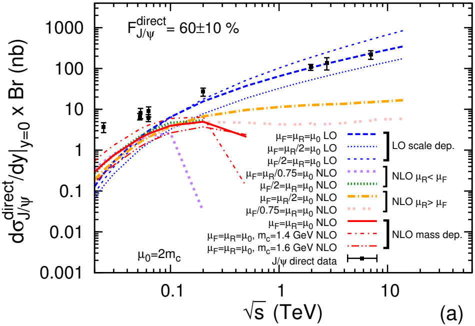
  $\rightarrow$ how is this related to PDFs and TMDs?





comparison of η_c (left) and J/ψ (right) differential cross-sections at NLO with different scale choices of μ_R and μ_F with CTEQ6M

[Y. Feng, J.-P. Lansberg, J.X. Wang, Eur.Phys.J. C75 (2015) no.7, 313]

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- *cross-sections* are *observable* quantities, hence physical probabilities $\mathcal{P}_{\mathrm{physical}}$ (σ, $\frac{d\sigma}{dy}$, ...) must be **positive**

negative cross-sections - sources?

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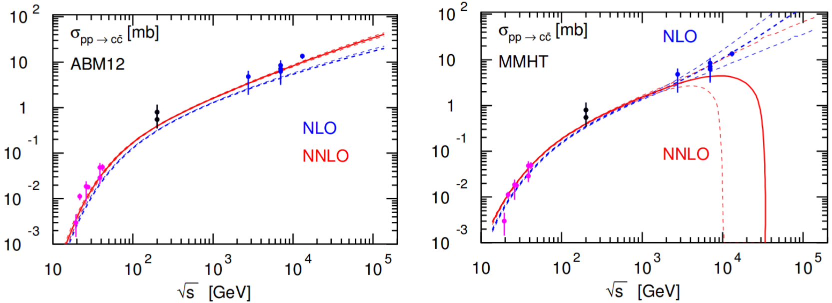

open $c\bar{c}$ production at NLO/N2LO, comparison with different PDFs (ABM12, MMHT) **[Accardi et al., Eur.Phys.J. C76 (2016) no.8, 471]**

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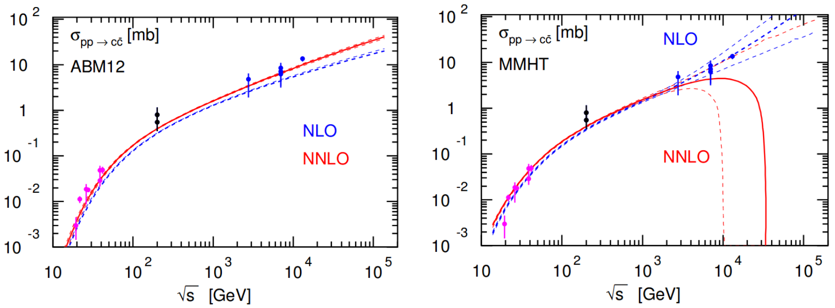

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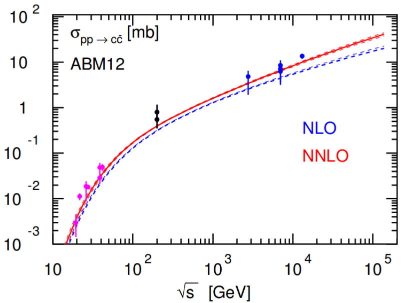

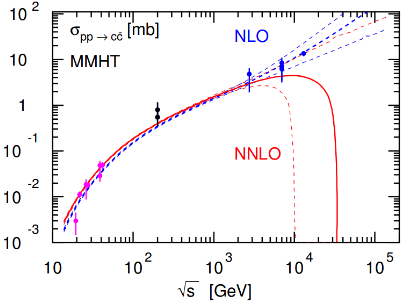

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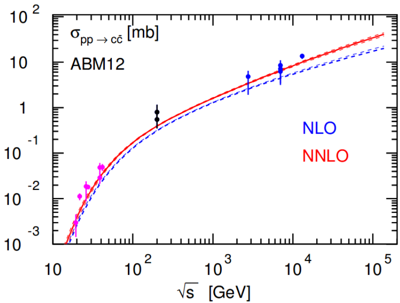

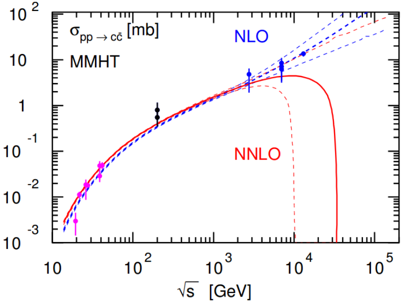

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- $\frac{d\sigma}{dy}$ does not exist at NNLO
- full scale analysis not yet performed
  $\rightarrow$ therefore one cannot rule out the possibility of negative cross-sections with positive PDFs

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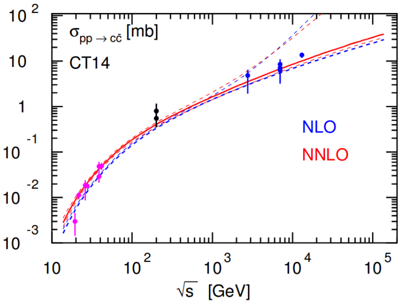

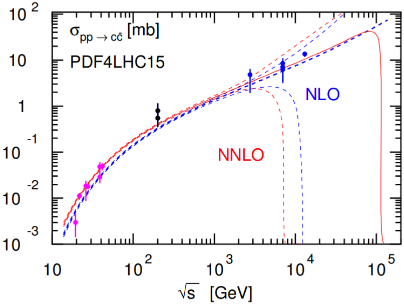

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  - or is it due to Parton Distribution Functions (PDFs)?

collinear factorisation - η_c at NLO - hadronic cross-section

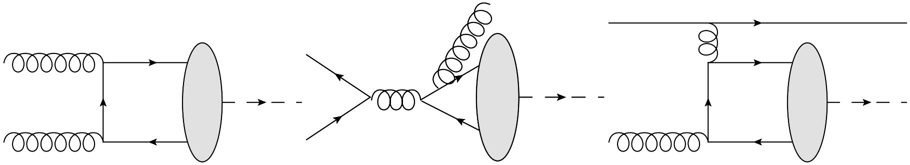
process

$$p + p \to \eta_c + X \tag{2}$$

hadronic cross-section

$$\sigma_{pp} = \sum_{ij} \int dx_1 dx_2 \; f_{i/p}(x_1, \mu_F) f_{j/p}(x_2, \mu_F) \; \hat{\sigma}_{ij}(\mu_R, \mu_F, x_1, x_2, \hat{s} = s\, x_1 x_2) \tag{3}$$

hadronic cross-section has dependence on the scales (μ_R, μ_F, s)



three channels contributing to η_c production at NLO; left - gg channel, middle - $q\bar{q}$ channel, right - qg channel

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- I confirm that everybody above was correct ;-)

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Mangano 1996 - Proceedings

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- arrives to similar conclusions that steeper gluon PDF choices will give better results because real corrections become less relevant (see Schuler's table) at high hadronic energies
- confirms that partonic limit away from threshold has the general structure,

$$\lim_{z\to 0} \hat{\sigma}_{gg} = 2C_A \frac{\alpha_s}{\pi} \hat{\sigma}_{\mathrm{Born}} \left(\log \frac{M^2}{\mu_F^2} - C_J \right), \tag{4}$$

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where C_J is a process-dependent quantity

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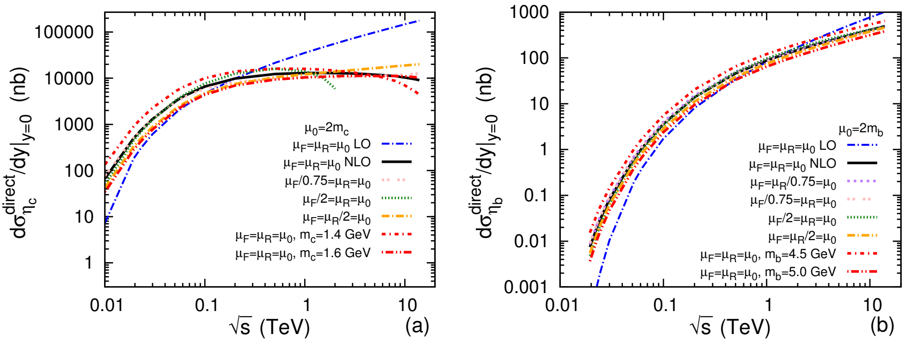
- let's make a comparison with η_b, why do we not encounter negative cross-sections?



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  - the rescaling of strong coupling constant α_s; higher scales mean lower coupling $\rightarrow$ QCD corrections become weaker, hence the NLO cross-section will be closer to LO

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  - the rescaling of strong coupling constant α_s; higher scales mean lower coupling $\rightarrow$ QCD corrections become weaker, hence the NLO cross-section will be closer to LO
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    $\rightarrow$ **essentially ensuring the positivity of the η_b cross-section**
  - note however that the NLO result start to deviate from LO at large $\sqrt{s}$

negative cross-sections - sources?

- What are potential sources for negative cross-sections?:
  - ~~is it due to failure of theoretical models (NRQCD etc.) for quarkonia?~~ $\rightarrow$ No, it is a more general problem; see open $c\bar{c}$ production
  - ~~is it due to truncation of fixed-order calculation? Do we need to go to higher orders (N2LO, N3LO, ...) to solve the issue of negative cross-sections?~~ $\rightarrow$ No, the situation at higher orders will be worse; see open $c\bar{c}$ production
  - do we even need to include k_T-Resummation? (see TMD side)
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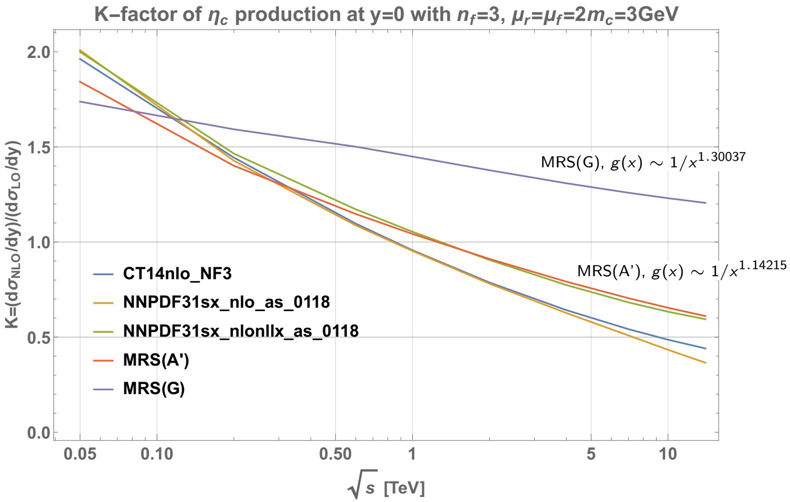
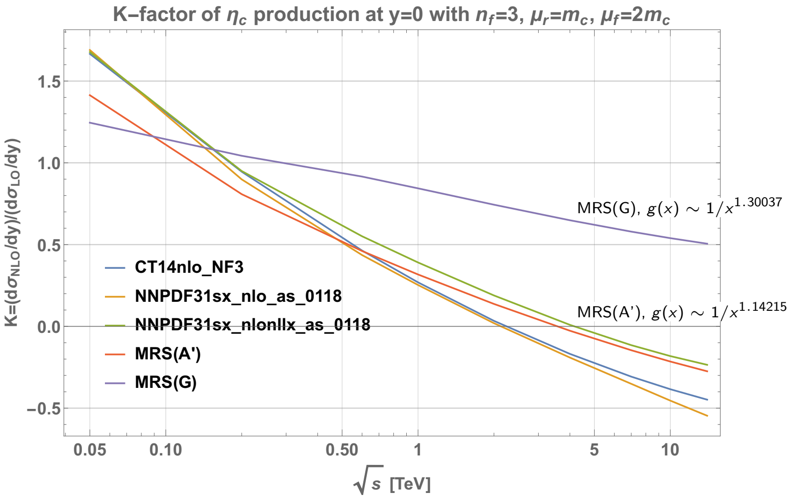
  - $\mu_R = m_c = 1.5\text{GeV}$, $\mu_F = 2m_c = 3\text{GeV}$
    - lower renormalisation choice leads to larger $\alpha_s \rightarrow$ real emission contributions become more important; the objective is to see the impact of the PDFs on the real corrections

K-factor at $y = 0$ - $\mu_R = \mu_F = 2m_c = 3\text{GeV}$



K-factor at y=0 as a function of energy and with different PDF choices. Default scale choice used $\mu_R = \mu_F = 2m_c = 3\text{GeV}$.

K-factor at $y = 0$ - $\mu_R = m_c = 1.5\text{GeV}, \mu_F = 2m_c = 3\text{GeV}$



K-factor at y=0 as a function of energy and with different PDF choices. Alternative scale choice used $\mu_R = m_c = 1.5\text{GeV}$, $\mu_F = 2m_c = 3\text{GeV}$.

Uncertainty of K-factor at $y = 0$ - 100 Replicas of NNPDF31_nlo_as_0118

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- expect very large K-factor uncertainty associated to NNPDF31_nlo_as_0118 PDF choice
- we will try with two different scale choices as before, set $y = 0$ and use $\sqrt{s} = 115\ \mathrm{GeV}, 7\ \mathrm{TeV}$ and $14\ \mathrm{TeV}$

K-factor results - $y = 0$ & $\sqrt{s} = 7$ TeV - 100 Replicas

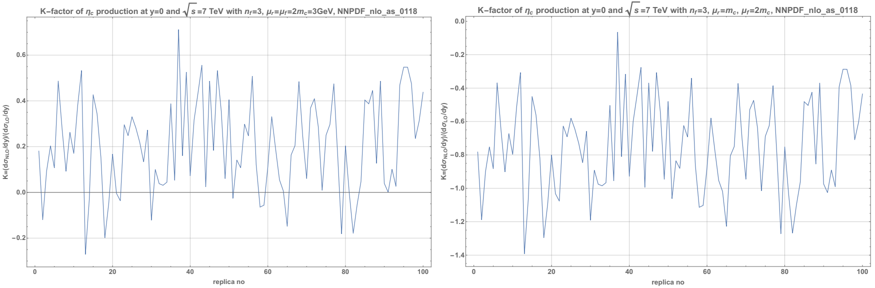

Figure: Strong variation of K-factor over replica number of NNPDF31_nlo_as_0118 (y=0, $\sqrt{s} = 7$ TeV, default/alternative scale choice)

default ($\mu_R = \mu_F = 2m_c = 3\text{GeV}$): $\rightarrow K = 0.2 \pm 0.2$
alternative ($\mu_R = m_c = 1.5\text{GeV}$, $\mu_F = 2m_c = 3\text{GeV}$): $\rightarrow K = -0.8 \pm 0.3$

K-factor results - $y = 0$ & $\sqrt{s} = 14$ TeV - 100 Replicas

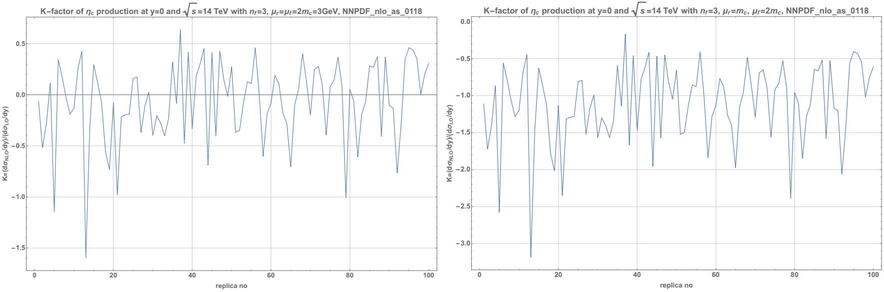

Figure: Strong variation of K-factor over replica number of NNPDF31_nlo_as_0118 (y=0, $\sqrt{s} = 14$ TeV, default/alternative scale choice)

default ($\mu_R = \mu_F = 2m_c = 3$GeV): $\rightarrow K = -0.1 \pm 0.4$
alternative ($\mu_R = m_c = 1.5$GeV, $\mu_F = 2m_c = 3$GeV): $\rightarrow K = -1.1 \pm 0.5$

K-factor - default scale - summary so far

	$\sqrt{s} = 7$ TeV			$\sqrt{s} = 14$ TeV
PDF choice	$y = 0$	$y = 1$	$y = 2$	$y = 0$
MRS(G)	1.26	1.27	1.29	1.21
MRS(A')	0.70	0.72	0.76	0.61
NNPDF31sx_nlonllx _as_0118	0.68	0.71	0.80	0.59
CT14nlo_NF3	0.54	0.57	0.65	0.44
NNPDF31sx_nlo _as_0118	0.51	0.52	0.61	0.37
NNPDF31_nlo _as_0118	0.2 ± 0.2	0.2 ± 0.4	0.2 ± 1.1	-0.1 ± 0.4

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- we will use the results for $y = 0$ and $\sqrt{s} = 14$ TeV with default scale choice

Improve K-factor with constraints in Quarkonia

- discarding all Replicas that yielded unphysical $d\sigma/dy < 0$
  $\rightarrow$ around half of the Replicas remained.

result set	*before* re-weighting	*after* re-weighting
$\sqrt{s} = 7$ TeV & $y = 0$	0.2 ± 0.2	$\mathbf{0.4} \pm \mathbf{0.1}$
$\sqrt{s} = 7$ TeV & $y = 1$	0.2 ± 0.4	$\mathbf{0.4} \pm \mathbf{0.1}$
$\sqrt{s} = 7$ TeV & $y = 2$	0.2 ± 1.1	$\mathbf{0.5} \pm \mathbf{0.1}$
$\sqrt{s} = 14$ TeV & $y = 0$	-0.1 ± 0.4	$\mathbf{0.3} \pm \mathbf{0.1}$

K-factor - default scale - updated summary

	$\sqrt{s} = 7$ TeV			$\sqrt{s} = 14$ TeV
PDF choice	$y = 0$	$y = 1$	$y = 2$	$y = 0$
MRS(G)	1.26	1.27	1.29	1.21
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NNPDF31sx_nlo _as_0118	0.51	0.52	0.61	0.37
NNPDF31_nlo _as_0118 (re-weighted)	0.4 ± 0.1	0.4 ± 0.1	0.5 ± 0.1	0.3 ± 0.1
NNPDF31_nlo _as_0118 *(equal weight)*	0.2 ± 0.2	0.2 ± 0.4	0.2 ± 1.1	-0.1 ± 0.4

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TMD - Transverse Momentum Distribution

$$\sigma \propto H \times \mathcal{C}[f_1^g f_1^g] \tag{6}$$

$$\begin{aligned}
\mathcal{C}[f_1^g f_1^g] &= \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{b}_T\cdot\vec{q}_T} \tilde{f}_1^g\left(x_1, \vec{b}_T; \zeta, \mu\right) \tilde{f}_1^g\left(x_2, \vec{b}_T; \zeta, \mu\right) \\
\tilde{f}_1^{g/A}\left(x, \vec{b}_T; \zeta, \mu\right) &= \sum_{j=q,\bar{q},g} \int_x^1 \frac{d\tilde{x}}{\tilde{x}} \tilde{C}_{g/j}\left(\tilde{x}, \vec{b}_T; \zeta, \mu\right) f_{j/A}(x/\tilde{x}; \mu) \\
\tilde{C}_{g/g} &= \delta(1-x) + \frac{\alpha_s}{2\pi}\left[C_A\delta(1-x)\left(-\frac{1}{2}L_T^2 + L_T \ln\frac{\mu^2}{\zeta} - \frac{\pi^2}{12}\right)\right. \\
&\quad \left. -L_T\left(P_{g/g} - \delta(1-x)\frac{\beta_0}{2}\right)\right] \\
\tilde{C}_{g/q} &= \frac{\alpha_s}{2\pi}\left[-L_T P_{g/q} + C_F x\right] \\
L_T &= \ln\frac{\mu^2 b_T^2}{4e^{-2\gamma_E}}
\end{aligned} \tag{7}$$

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Backup

shape of rapidity differential at LO - CT14nlo_NF3

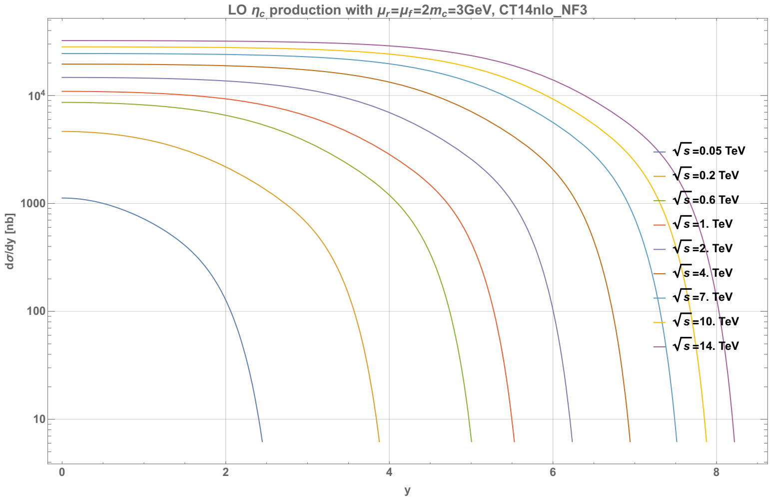


Figure: rapidity differential cross-section at LO for different energies, default scale choice, CT14nlo_NF3

shape of rapidity differential at NLO - CT14nlo_NF3, def.

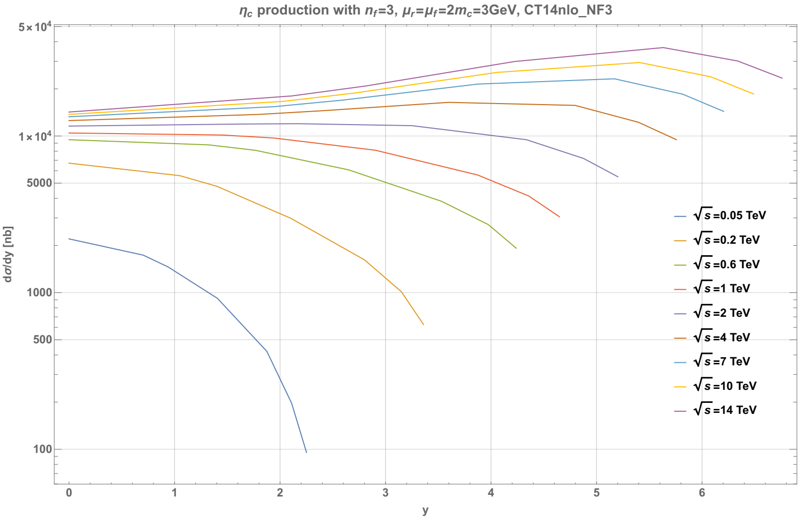


Figure: rapidity differential cross-section at NLO for different energies, default scale choice, CT14nlo_NF3

shape of rapidity differential at NLO - CT14nlo_NF3, alt.

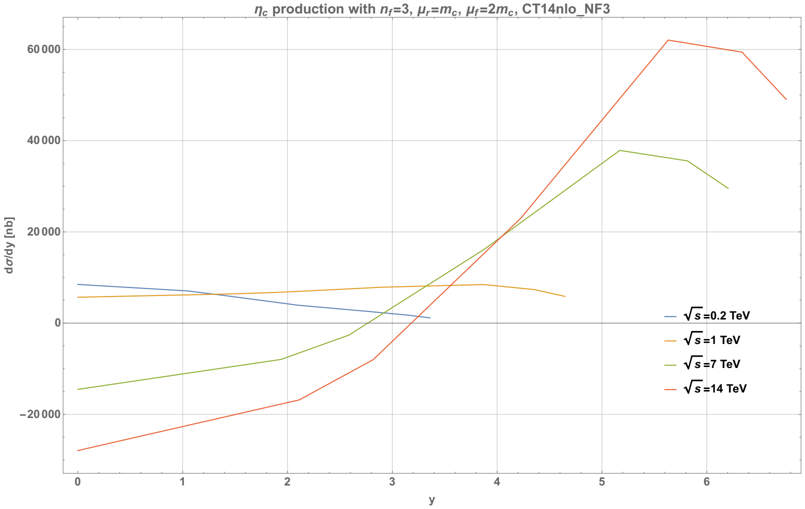


Figure: rapidity differential cross-section at NLO for different energies, alternative scale choice, CT14nlo_NF3

shape of rapidity differential at NLO - NNPDF31sx_nlo_as_0118

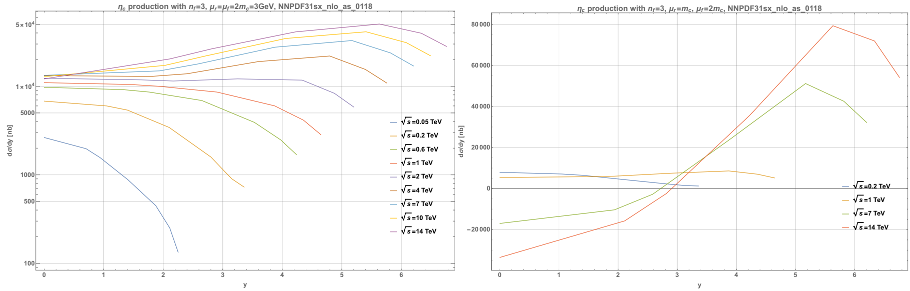


Figure: rapidity differential cross-section at NLO for different energies, default/alternative scale choice, NNPDF31sx_nlo_as_0118

shape of rapidity differential at NLO - NNPDF31sx_nlonllx_as_0118

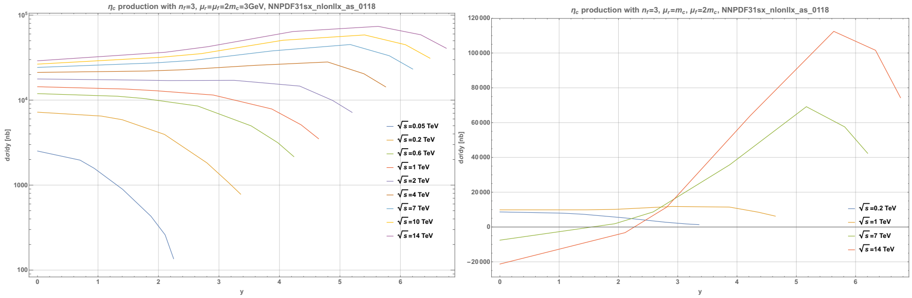


Figure: rapidity differential cross-section at NLO for different energies, default/alternative scale choice, NNPDF31sx_nlonllx_as_0118

Quarkonia - three different models

- Colour-Evaporation Model
  - quark and anti-quark colours are summed up at amplitude squared level (evaporation)
  - no spin-projection
- Colour-Octet Model
  - quark and anti-quark pair are in color-octet state
  - heavy quark spins projected on final bound state
  - higher Fock states in NRQCD, higher v-order
- **Colour-Singlet Model**
  - quark and anti-quark pair are in color-singlet state
  - heavy quark spins projected on final bound state
  - leading Fock state in NRQCD

gluon-gluon channel

$$
\begin{aligned}
\hat{\sigma}_{gg}(s,\hat{s},\mu_R,\mu_F) &= \frac{\alpha_s^2(\mu_R)\pi^2}{96m_c^5}|R(0)|^2\delta(1-z) \\
&+ \frac{\alpha_s^3(\mu_R)\pi}{1152m_c^5}|R(0)|^2\left[\left(-44+7\pi^2+54\log\left(\frac{\mu_R^2}{\mu_F^2}\right)\right.\right. \\
&+72\log\left(1-\frac{4m_c^2}{s}\right)\left(\log\left(1-\frac{4m_c^2}{s}\right)-\log\left(\frac{\mu_F^2}{4m_c^2}\right)\right)\Bigg)\delta(1-z) \\
&+6\left(24\left(\frac{\log(1-z)}{1-z}\right)_\rho(1-(1-z)z)^2\right. \\
&+12\left(\frac{1}{1-z}\right)_\rho\frac{\log(z)}{(1-z)(1+z)^3}\left(1-z^2\left(5+z\left(2+z+3z^3+2z^4\right)\right)\right) \\
&-\left(\frac{1}{1-z}\right)_\rho\frac{1}{(1+z)^2}\left(12+z^2\left(23+z\left(24+2z+11z^3\right)\right)\right) \\
&\left.\left.+12\left(1+z^3\right)^2\log\left(\frac{z\mu_F^2}{4m_c^2}\right)\right)\right], \text{ where } z=4m_c^2/\hat{s} \text{ and } \rho=4m_c^2/s
\end{aligned}
\tag{8}
$$

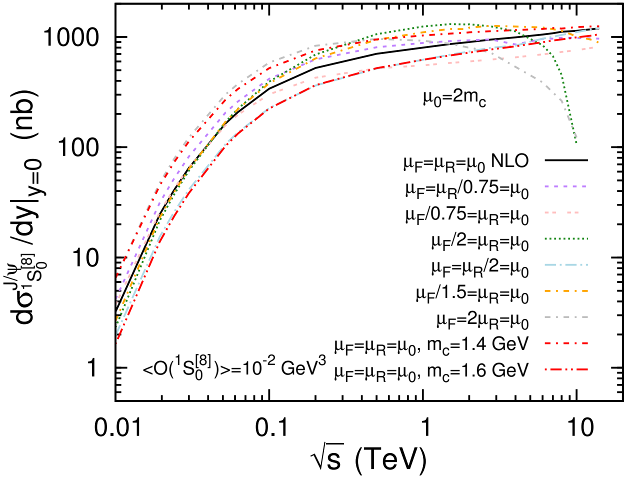
quark-antiquark channel

$$\hat{\sigma}_{q\bar{q}}(\hat{s},\mu_R) = \frac{16\alpha_s^3(\mu_R)\pi}{81m_c}|R(0)|^2\frac{(\hat{s}-4m_c^2)}{\hat{s}^3} \tag{9}$$

quark-gluon channel

$$\begin{aligned}\hat{\sigma}_{qg}(\hat{s},\mu_R,\mu_F) = \frac{\alpha_s^3(\mu_R)\pi}{72m_c^5\hat{s}^2}|R(0)|^2 & \left(8m_c^4+4m_c^2\hat{s}-\hat{s}^2\right. \\ & +2\left(8m_c^4-4m_c^2\hat{s}+\hat{s}^2\right)\log\left(1-\frac{4m_c^2}{\hat{s}}\right) \\ & +\hat{s}\left(-4m_c^2+\hat{s}\right)\log\left(\frac{4m_c^2}{\hat{s}}\right) \\ & \left.-\left(8m_c^4-4m_c^2\hat{s}+\hat{s}^2\right)\log\left(\frac{\mu_F^2}{\hat{s}}\right)\right)\end{aligned} \tag{10}$$

problem of negative cross-sections - J/ψ, ${}^1S_0^{[8]}$ at NLO



comparison of J/ψ ${}^1S_0^{[8]}$ differential cross-section at NLO with different choices of μ_R and μ_F with CTEQ6M **[Y. Feng, J.-P. Lansberg, J.X. Wang, Eur.Phys.J. C75 (2015)]**

Schuler 1994 - structure of partonic cross-section

- let's define $z = M^2/\hat{s}$ and $\tau_0 = M^2/s$
- LO partonic cross-section and virtual corrections ($2 \to 1$ process) have $\delta(1-z)$ function while real corrections ($2 \to 2$) are complicated functions of z
- negative contributions come from real corrections which have interference terms
- idea is to use simple toy-models for gluon PDFs and convolute with partonic cross-section; different z-terms will contribute differently at hadronic level

Schuler 1994 - two toymodels - table partonic vs. hadronic

	$xg(x) \to 1$	$xg(x) \to 1/\sqrt{x}$
$\hat{\sigma}_{gg}(z, M^2)$	$\sigma_{pp}(\tau_0, M^2) \overset{\tau_0 \to 0}{\longrightarrow}$	
$\delta(1-z)$	$\ln\left(\frac{1}{\tau_0}\right)$	$\frac{1}{\sqrt{\tau_0}} \ln\left(\frac{1}{\tau_0}\right)$
z^k	$\frac{1}{k} \ln\left(\frac{1}{\tau_0}\right)$	$\frac{2}{(2k+1)\sqrt{\tau_0}} \ln\left(\frac{1}{\tau_0}\right)$
1	$\frac{1}{2} \ln^2\left(\frac{1}{\tau_0}\right)$	$\frac{2}{\sqrt{\tau_0}} \ln\left(\frac{1}{\tau_0}\right)$
$\ln^k\left(\frac{1}{z}\right)$	$\frac{1}{(k+1)(k+2)} \ln^{k+2}\left(\frac{1}{\tau_0}\right)$	$\frac{k!\,2^{k+1}}{\sqrt{\tau_0}} \ln\left(\frac{1}{\tau_0}\right)$

Asymptotic ($\tau_0 = M^2/s \to 0$) behaviour of the proton-proton or proton-antiproton cross section for various forms of the gluon-gluon subprocess ($z = M^2/\hat{s} = \tau_0/\tau$) and two extreme choices of the gluon distribution function. Taken from G. Schuler, Review, 1994

toymodel $g(x) = 1/x$: real corrections dominate at high energies;
toymodel $g(x) = 1/x^{1.5}$: all contributions have same energy scaling

high energy behaviour - η_c - partonic cross-section

partonic cross-section away from threshold, $z \to 0$

$$\lim_{z\to 0} \hat{\sigma}_{gg} = \frac{\alpha_s^3(\mu)\pi}{16 m_c^5} |R(0)|^2 \left(\log\left(\frac{4m_c^2}{\mu_F^2}\right) - 1 \right), \tag{11}$$

$$\lim_{z\to 0} \hat{\sigma}_{q\bar{q}} = 0, \tag{12}$$

$$\lim_{z\to 0} \hat{\sigma}_{qg} = \frac{\alpha_s^3(\mu)\pi}{72 m_c^5} |R(0)|^2 \left(\log\left(\frac{4m_c^2}{\mu_F^2}\right) - 1 \right) \tag{13}$$

high energy behaviour - η_c - partonic cross-section

partonic cross-section away from threshold, $z \to 0$

- $\mu_F = m_c$

$$\lim_{z\to 0} \hat{\sigma}_{gg} = \frac{\alpha_s^3(\mu)\pi}{16 m_c^5} |R(0)|^2 \left(\log(4) - 1\right) = 0.2 * \hat{\sigma}_{gg,LO}, \tag{14}$$

- $\mu_F = 2m_c$

$$\lim_{z\to 0} \hat{\sigma}_{gg} = \frac{\alpha_s^3(\mu)\pi}{16 m_c^5} |R(0)|^2 (-1) = -0.5 * \hat{\sigma}_{gg,LO}, \tag{15}$$

hadronic cross-section - dependence on μ_F

- toy model 1 PDF with $f_{g/p}(x) = 1/x$
  - dependence of hadronic cross-section on μ_F
  - for $\mu_F > m_c$, hadronic cross-section is negative
  - for $\mu_F < m_c$, hadronic cross-section is positive
- toy model 2 PDF with $f_{g/p}(x) = 1/x^{1.5}$
  - weak dependence of hadronic cross-section on μ_F
  - cross-section always positive (independent of choice of μ_F)
- similar behaviour for qg channel at high energies because of same asymptotic limit as in gg channel apart from global factor

high energy behaviour - partonic cross-section

- for non-steep PDF choices, the high-energy hadronic limit is governed by the high-energy partonic limit $\rightarrow$ strong dependence on factorisation scale μ_F
- some values for C_J:
  - $C_J = 1$ for pseudo-scalar quarkonia $\eta_{c/b/t}$
  - $C_J = 43/27$ for $\chi_{c/b,0}$
  - $C_J = 53/36$ for $\chi_{c/b,2}$
  - $C_J = 11/12 + \log z$ for Higgs (in infinite-top quark mass limit)
- as an aside note, ratio between qg and gg channel in high-energy partonic limit is process-independent (same for Quarkonia and Higgs Physics)

$$\lim_{z\to 0} \frac{\hat{\sigma}_{qg}}{\hat{\sigma}_{gg}} = \frac{C_F}{2C_A} = \frac{2}{9} \tag{16}$$