Observables from a 3+1 dimensional reativistic solution

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arXiv.org/0909.4842 [nucl-th]

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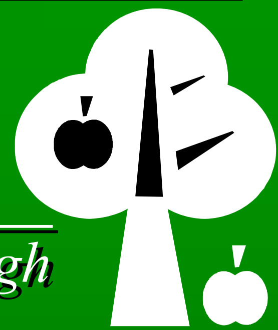
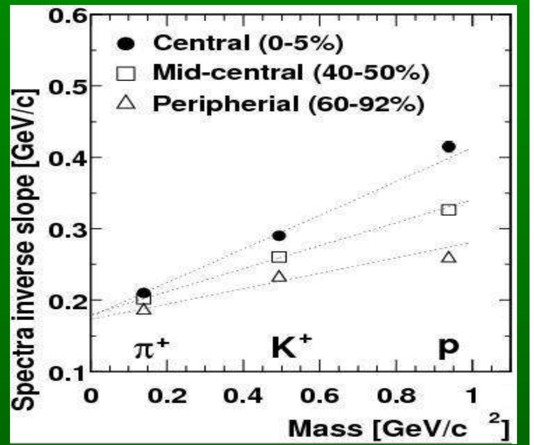
- Hydro models
- Observables
- Comparision to data

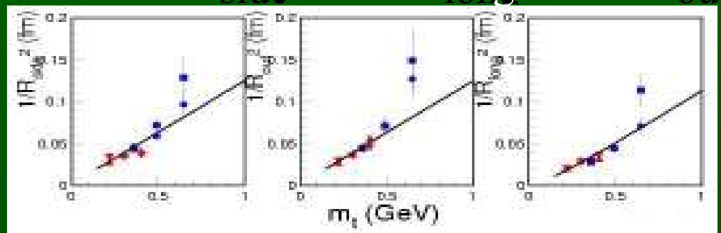
Hydrodynamic predictions

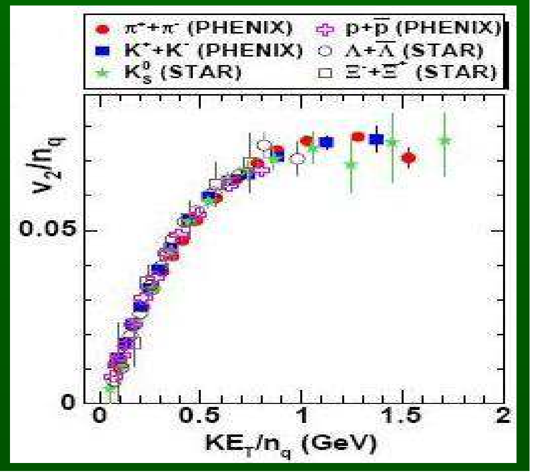
- Hydro predicts scaling (even viscous)
- What does a scaling mean?
  - See Hubble's law – or Newtonian gravity: $v=\sqrt{2gh}$
  - Cannot predict acceleration or height





- Collective, thermal behavior → Loss of information
- Spectra slopes: $T_{\text{eff}}=T_0+mu_t^2$
- Elliptic flow: $v_2=\dfrac{I_1(w)}{I_0(w)}\sim w\sim KE_T$
- HBT radii: $R_{\text{side}}^2\approx R_{\text{long}}^2\approx R_{\text{out}}^2\sim\dfrac{1}{m_t}$

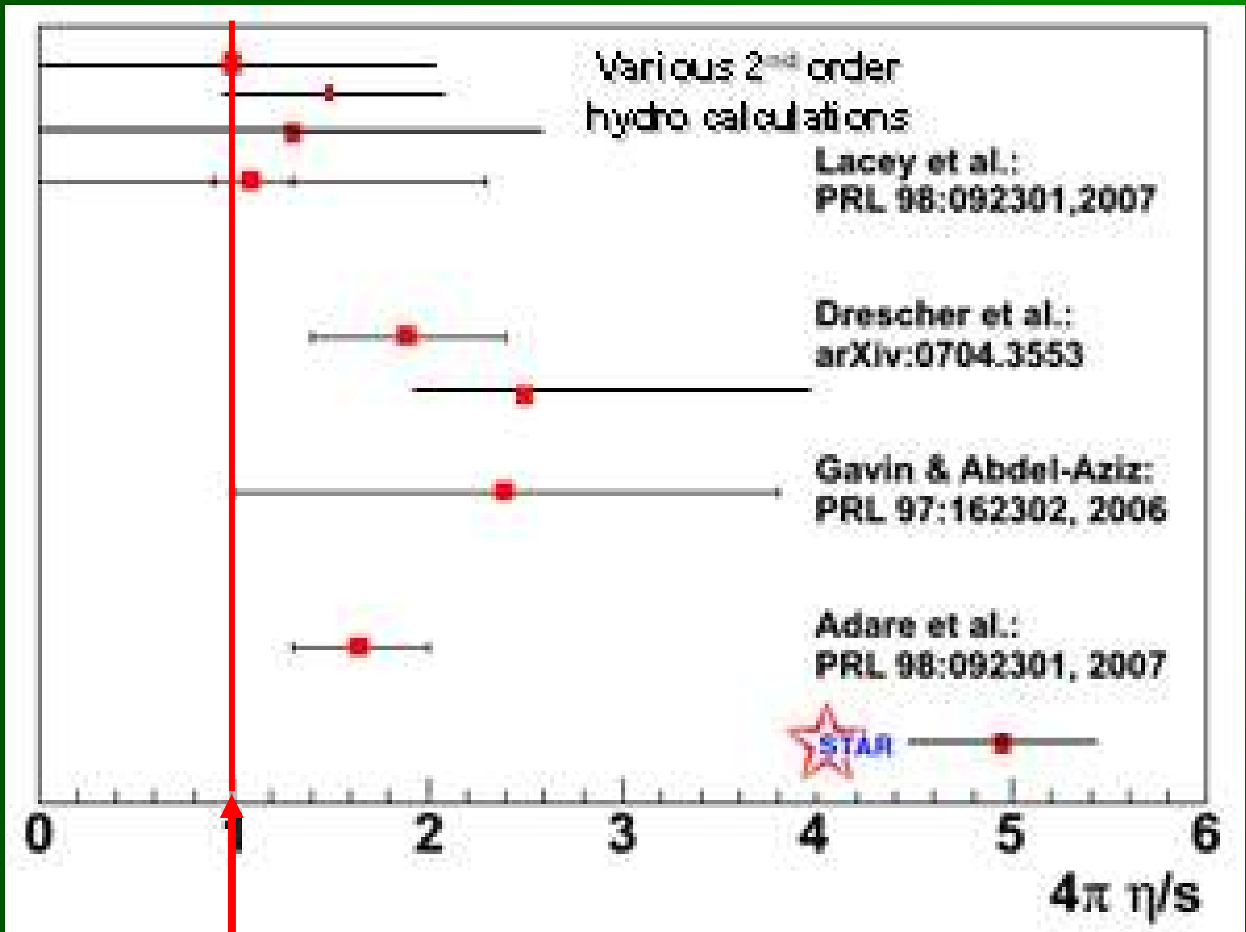




Why we use hydrodynamics?

- Hadron & parton cascade models
  - Noneqilibrium models
  - Don't describe most of particle creation at low p_t
- Flow + hadronic cascade and resonance corrections
  - Small correction at RHIC energies (arXiv:0903.1863)
- Most of particle creation according to hydro
  - Pion and kaon HBT radii m_t scaling (Acta Phys.Polon.Supp.1:521-524,2008)
- After ~10 fm/c noneqilibrium expanding hadron gas
  - Anomal diffusion, no correction for spectra or flow (Braz.J.Phys.37:1002-1013,2007)
- American Institute of Physics: success of hydro is the physics story 2005

Perfect hydro picture



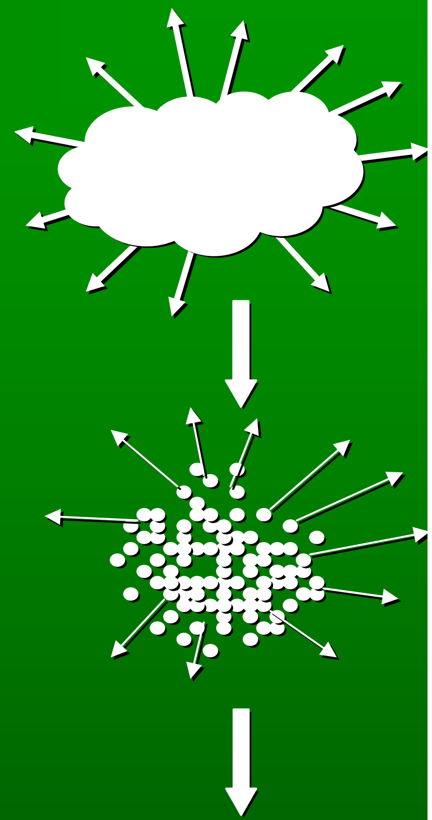
- No data point even near the kinematic viscosity of ^{4}He ($10/4\pi$)
- Close to AdS/CFT minimum, ($1/4\pi$)

Little vocabulary of hydrodynamics

- Exact/parametric solution
  - Solution of hydro equations analytically, without approximation
  - Usually has free parameters
- Hydro inspired parameterization
  - Distribution determined at freeze-out only, their time dependence is not considered
- Numerical solution
  - Solution of hydro equations numerically
  - Start from arbitrary initial state

How analytic hydro works

- Take hydro equations and EoS
- Find a solution
  - Will contain parameters (like Friedmann, Schwarzschild etc.)
  - Will use a possible set of initial conditions
- Use a freeze-out condition
  - Eg fixed proper time or fixed temperature
  - Generally a hyper-surface
- Calculate the hadron source function
- Calculate observables
  - E.g. spectra, flow, correlations
  - Straightforward calculation
- Hydrodynamics: Initial conditions ⊗ dynamical equations ⊗ freeze-out conditions



$N_1(p_t, \varphi, \eta)$

$HBT(p_t, \varphi, \eta)$

How analytic hydro works

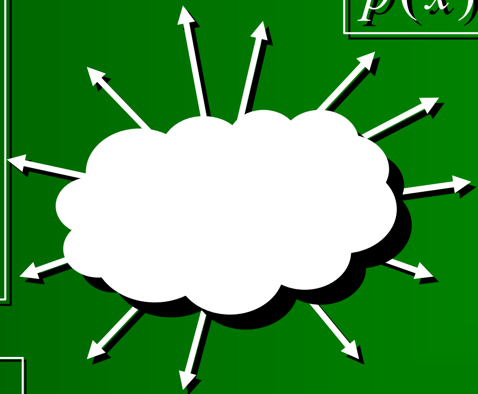
Hydro equations + EoS

$$(\partial_t + \nabla \vec{v})n = 0$$
$$(\partial_t + \nabla \vec{v})T = -p\nabla \vec{v}$$
$$(\partial_t + \vec{v}\nabla)\vec{v} = -(1/n)\nabla p$$
$$T = \tfrac{3}{2}\, p$$

PLB505:64-70,2001
hep-ph/0012127

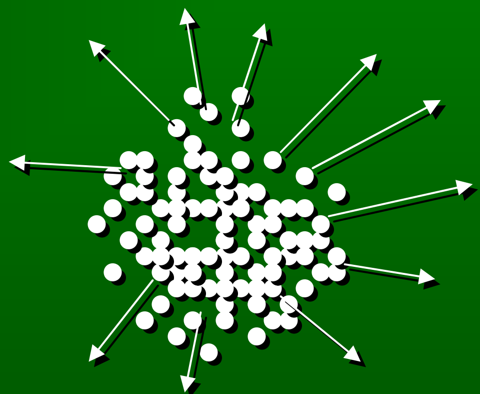
Self-similar solution:

$$p(x), n(x), T(x), \vec{v}(x)$$



Phase-space distribution
Boltzmann-equation

$$f(x,p) \sim n(x)\exp\left\{-\frac{(p-mv)^2}{kT}\right\}$$
$$(\partial_t + \vec{v}\nabla) f(x,p) = S(x,p)$$



PRC67:034904,2003
hep-ph/0108067

Source

S(x,p)

PRC54:1390-1403,1996
hep-ph/9509213

Observables

$N_1(p)$, $C_2(p_1,p_2)$, $v_2(p)$

Scheme works also backwords

Equations of relativistic hydro

- Assuming local thermal equilibrium
- For a perfect fluid: $T^{\mu\nu} = wu^\mu u^\nu - pg^{\mu\nu},\ w = \varepsilon + p \Rightarrow \partial_\nu T^{\mu\nu} = 0$
- Equations in four-vector form and nonrelativistic notation
  - Euler equation:

$$wu^\nu \partial_\nu u^\mu = (g^{\mu\eta} - u^\mu u^\eta)\partial_\eta p \qquad \frac{w}{1-v^2}\frac{d\mathbf{v}}{dt} = -\left(\nabla p + \mathbf{v}\frac{\partial p}{\partial t}\right)$$

  - Energy conservation:

$$w\partial_\nu u^\nu + u^\nu \partial_\nu \varepsilon = 0 \qquad \frac{1}{w}\frac{d\varepsilon}{dt} = -\nabla\mathbf{v} - \frac{1}{1-v^2}\frac{d}{dt}\frac{v^2}{2}$$

  - Charge conservation:

$$\partial_\mu(nu^\mu) = 0 \qquad \frac{d}{dt}\ln\frac{n}{\sqrt{1-v^2}} = -\nabla\mathbf{v}$$

$$u^\mu \partial_\mu = \frac{d}{d\tau}$$ comoving proper-time derivative

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v}\nabla$$ comoving derivative

Famous solutions

- Landau's solution (1D, developed for p+p):
  - Accelerating, implicit, complicated, 1D
  - L.D. Landau, Izv. Acad. Nauk SSSR 81 (1953) 51
  - I.M. Khalatnikov, Zhur. Eksp.Teor.Fiz. 27 (1954) 529
  - L.D.Landau and S.Z.Belenkij, Usp. Fiz. Nauk 56 (1955) 309
- Hwa-Bjorken solution:
  - Non-accelerating, explicit, simple, 1D, boost-invariant
  - R.C. Hwa, Phys. Rev. D10, 2260 (1974)
  - J.D. Bjorken, Phys. Rev. D27, 40(1983)
- Others
  - Chiu, Sudarshan and Wang
  - Baym, Friman, Blaizot, Soyeur and Czyz
  - Srivastava, Alam, Chakrabarty, Raha and Sinha

Nonrelativistic solutions

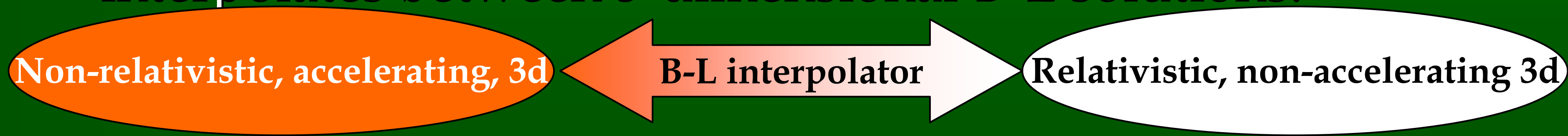
Solution	Symmetry	Density prof.	EoS	Observables
Csizmadia et al. Phys. Lett. B443:21-25, 1998	Sphere	Gaussian	$\varepsilon = \frac{3}{2} nT$	Calculated
Csörgő Central Eur.J.Phys.2: 556-565,2004	Sphere	Arbitrary	$\varepsilon = \frac{3}{2} nT$	Not calculated
Akkelin et al. Phys.Rev. C67,2003	Ellipsoid	Gaussian (T=T(t))	$\varepsilon = \kappa(T) nT$	Calculated
Csörgő Acta Phys.Polon. B37:483-494,2006	Ellipsoid	Arbitrary (T=T(r,t))	$\varepsilon = \kappa nT$	Not calculated
Csörgő, Zimányi Heavy Ion Phys.17:281-293,2003	Ellipsoid	Gaussian	$\varepsilon = \kappa_e nT - B$	Calculated

Relativistic solutions

Solution	Basic prop's	EoS	Observables
Csörgő, Nagy, Csanád Phys.Lett.B 663:306-311, 2008 Phys.Rev.C77:024908,2008	Ellipsoidal, 1D accelerating	$\varepsilon - B = \kappa(p + B)$	dn/dy, ε
Landau Izv. Acad. Nauk SSSR 81 (1953) 51	Cylindr., 1D, accelerating	$\varepsilon = \kappa n T$	none
Hwa-Björken R.C. Hwa, PRD10, 2260,1974 J.D. Bjorken, PRD27, 40(1983)	Cylindr., 1D, non-accelerating	$\varepsilon = \kappa n T$	dn/dy, ε
Bialas et al. A. Bialas, R. A. Janik, and R. B. Peschanski, Phys. Rev. C76, 054901 (2007).	1D, betweend Landau and Hwa-Björken	$\varepsilon = \kappa n T$	dn/dy
Csörgő, Csernai, Hama, Kodama Heavy Ion Phys. A 21, 73 (2004))	Ellipsoidal, 3D, non-accelerating	$\varepsilon = \kappa n T$	This work does the calculation

Where we are

- Revival of interest, new solutions
  - Sinyukov, Karpenko, nucl-th/0505041
  - Pratt, nucl-th/0612010
  - Bialas et al.: Phys.Rev.C76:054901,2007
  - Borsch, Zhdanov: SIGMA 3:116,2007
  - Nagy et al.: J.Phys.G35:104128,2008 and arXiv/0909.4285
  - Liao et al.: arXiv/09092284 and Phys.Rev.C80:034904,2009
  - Mizoguchi et al.: Eur.Phys.J.A40:99-108,2009
  - Beuf et al.:Phys.Rev.C78:064909,2008 (dS/dy as well!)
- Need for solutions that are:
  - accelerating + relativistic+ 3 dimensional
  - explicit + simple + compatible with the data
- Buda-Lund type of solutions: each, but not simultaneously
- Buda-Lund interpolator: hydro inspired source function, interpolates between 3-dimensional B-L solutions:



The solution we investigate

- The hydro fields are these:
  - ν(s) arbitrary, but realistic to choose Gaussian $\nu(s) = e^{-bs/2}$ b<0 is realistic
- Ellipsoidal symmetry

$$n = n_0 \left(\frac{\tau_0}{\tau}\right)^3 \nu(s)$$

$$T = T_0 \left(\frac{\tau_0}{\tau}\right)^{(3/\kappa)} \frac{1}{\nu(s)}$$

$$p = p_0 \left(\frac{\tau_0}{\tau}\right)^{\left(3+\frac{3}{\kappa}\right)}$$

$$s = \frac{x^2}{X(t)^2} + \frac{y^2}{Y(t)^2} + \frac{z^2}{Z(t)^2}$$

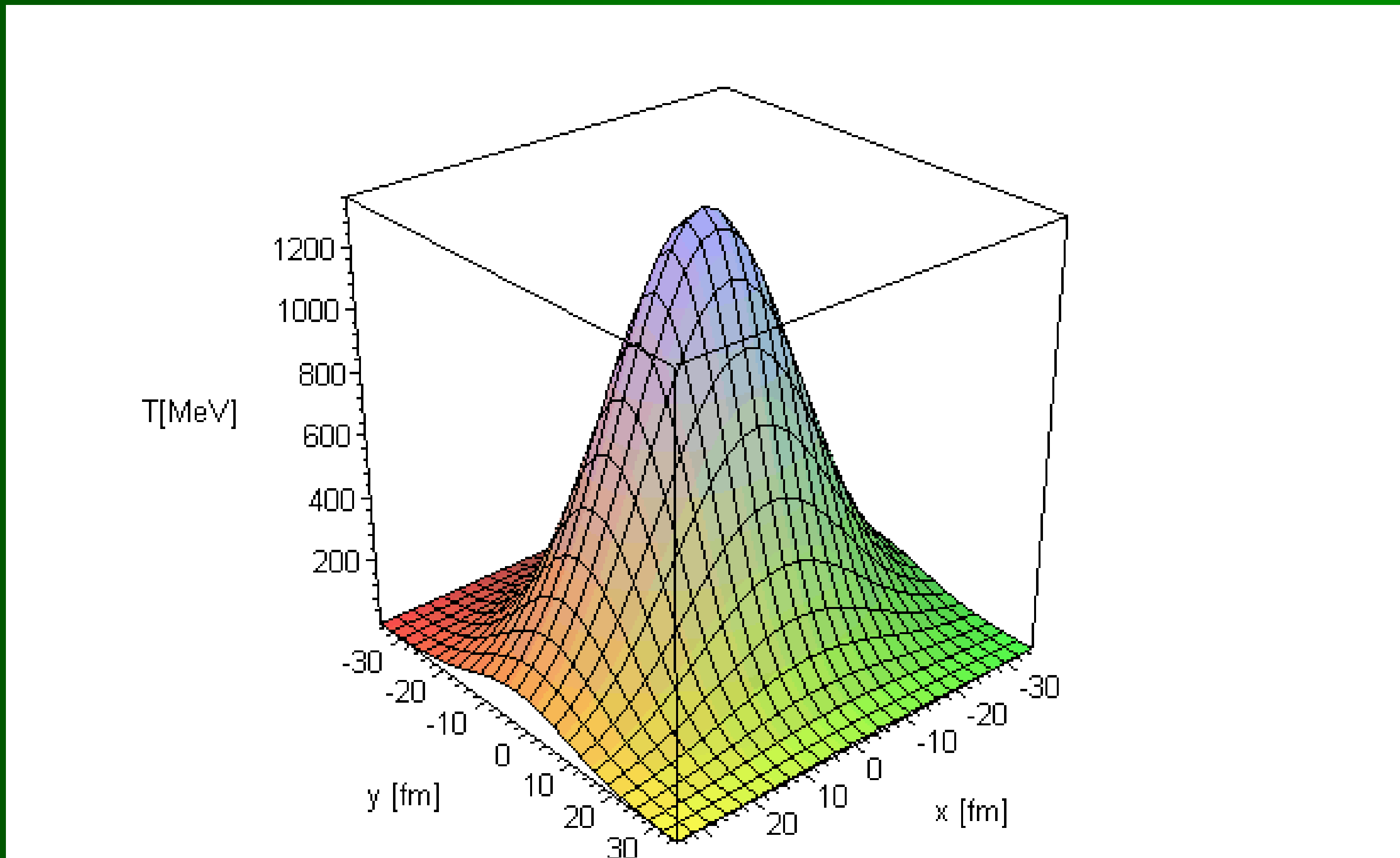
$$u^\mu = \gamma \left(1, \frac{\dot{X}(t)}{X(t)}x, \frac{\dot{Y}(t)}{Y(t)}y, \frac{\dot{Z}(t)}{Z(t)}z\right)$$

  - (thermodynamic quantities const. on the s=const. ellipsoid)
- Directional Hubble-flow
  - v=Hr or H=v/r, the Hubble-constants: $\frac{\dot{X}(t)}{X(t)}, \frac{\dot{Y}(t)}{Y(t)}, \frac{\dot{Z}(t)}{Z(t)}$
  - $\dot{X}(t), \dot{Y}(t), \dot{Z}(t) : const.; \left(\dot{X}, \dot{Y}\right) \Leftrightarrow \left(u_t, \varepsilon\right)$

(T. Csörgő, L. P. Csernai, Y. Hama és T. Kodama, Heavy Ion Phys. A 21, 73 (2004))

Temperature dependence

Transverse temperature profile as a function of time ($3.5\ fm/c < \tau < 7\ fm/c$):



The source function

- Source function: probability of a particle created at x with p
- Maxwell-Boltzmann distribution + extra terms

$$S(x,p)d^4x = \mathcal{N} n(x) \exp\left[-\frac{p_\mu u^\mu(x)}{T(x)}\right] \frac{p_\mu u^\mu}{u^0} H(\tau) d^4x$$

  - $\mathcal{N}$ normalization
  - $H(\tau)d\tau$ freeze-out distribution
    if sudden: $H(\tau) = \delta(\tau - \tau_0)$
  - $\frac{p_\mu u^\mu}{u^0} d^3x$ Cooper-Fry prefactor (flux term)
  - Validity: $\tau_0 > R_{HBT}$, $m_t > T_0$

Single particle spectrum

- Source function: spatial origin and momentum
- Momentum distribution

  → integrate on spatial coordinates: $$N_1(p) = \int_{R^4} S(x,p)\, d^4x$$

- Second order Gaussian approximation around emission maximum
- After integration:

$$N_1(p) = \bar{N} \cdot \bar{E} \cdot \bar{V} \cdot \exp\left[\frac{p^2}{2ET_0} - \frac{E}{T_0} - \frac{p_x^2}{2ET_x} - \frac{p_y^2}{2ET_y} - \frac{p_z^2}{2ET_z} \right]$$

- Directional slope parameter:

$$T_x = T_0 + \frac{ET_0 \dot{X}_0^2}{b(T_0 - E)}$$

Transverse momentum spectrum

- Go to mid-rapidity (y=0)
- Integrate on transverse angle ϕ

$$N_1(p_t) = \bar{N}\,\bar{V}\left(m_t - \frac{p_t^2(T_{\text{eff}} - T_0)}{m_t T_{\text{eff}}}\right)\exp\left[-\frac{p_t^2}{2m_t T_{\text{eff}}} + \frac{p_t^2}{2m_t T_0} - \frac{m_t}{T_0}\right]$$

– The effective temperature is from the slopes:

$$T_x = T_0 + m_t \dot{X}^2 \frac{T_0}{b(T_0 - E)},\quad T_y = T_0 + m_t \dot{Y}^2 \frac{T_0}{b(T_0 - E)},$$

$$\frac{1}{T_{\text{eff}}} = \frac{1}{2}\left(\frac{1}{T_x} + \frac{1}{T_y}\right)$$

The elliptic flow

- The elliptic flow can be calculated as:

$$v_2 = \frac{\int_0^{2\pi} d\phi N_1(p_t,\phi)\cos(2\phi)}{\int_0^{2\pi} d\phi N_1(p_t,\phi)}$$

- Result (similar to other models): $v_2 = \frac{I_1(w)}{I_0(w)}$
- $I_n(w)$: modified Bessel functions

$$I_n(w) = \frac{1}{2\pi}\int_0^{2\pi} \mathrm{e}^{w\cos(2\phi)}\cos(2n\phi)d\phi$$

- Where w is:

$$w = \frac{p_t^2}{4m_t}\left(\frac{1}{T_y} - \frac{1}{T_x}\right) \sim E_K \frac{\varepsilon}{T_{eff}}$$

Two-particle correlation radii

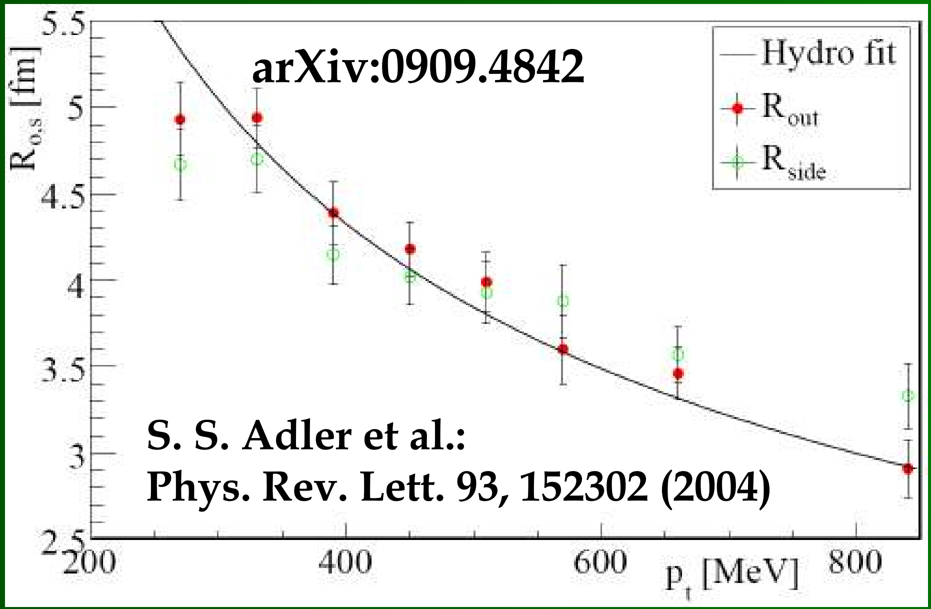
- Definition: $$C_2(p_1,p_2)=\frac{N_2(p_1,p_2)}{N_1(p_1)N_1(p_1)}$$
- From the source function: $$C_2(q,K)=1+\lambda\left|\frac{\tilde{S}(q,K)}{\tilde{S}(0,K)}\right|^2$$
- Changing coordinates

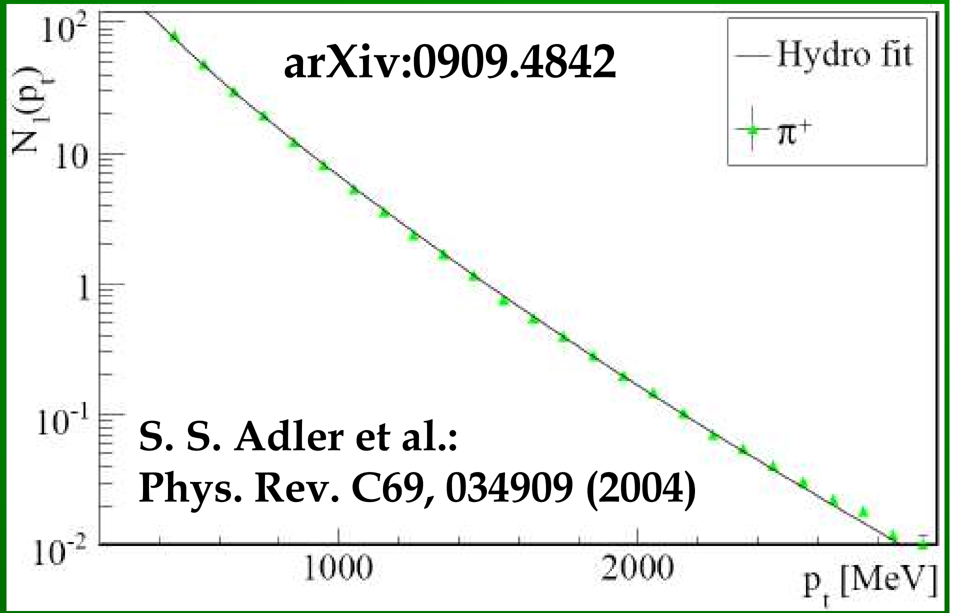
$$q=p_1-p_2,\ K=0.5\left(p_1+p_2\right)\Rightarrow C_2(q,K)=1+\lambda\exp\left(-R_x^2q_x^2-R_y^2q_y^2-R_z^2q_z^2\right)$$

- Result: $$R_i=\frac{T_0\tau_0(T_i-T_0)}{m_tT_i}$$
- The usual scaling (same for kaons!): $$R_i^2\sim\frac{1}{m_t}$$
- Bertsch-Pratt coordinates: $$R_{out}=R_{side}=0.5\left(R_x^2+R_y^2\right)$$
- Freeze-out: τ = const. $\leftrightarrow$ $\Delta\tau = 0 \rightarrow R_{out} = R_{side}$

Single pion spectum with HBT radii

- 0-30% centrality, Au+Au, PHENIX

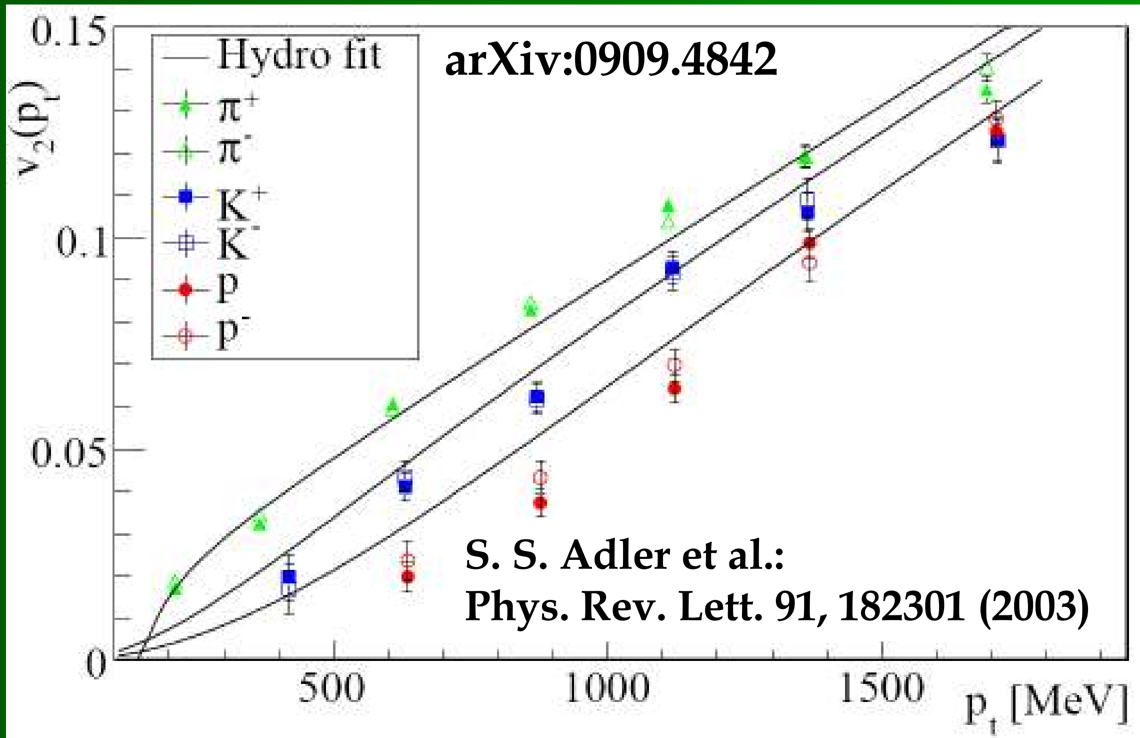




T_0	197 ± 2 MeV	central freeze-out temp.
ε	0.85 ± 0.01	momentum space ecc.
u_t^2/b	-0.95 ± 0.07 (b<0)	transv. flow/temp. grad
τ_0	7.6 ± 0.1	freeze-out proper time
χ^2	215 (29 with theory error)	

Elliptic flow

- 0-92% centrality, Au+Au, PHENIX (R.P. method technique)



- T_0 204 ± 7 MeV f.o. temperature
- ε 0.34 ± 0.01 eccentricity
- u_t^2/b -0.34 ± 0.07 (b<0) transv. flow/temp. grad
- χ^2 256 (68 with theory error)

Summary of the fit results

- Freeze-out temperature around 200 MeV
- Flow strongly depends on centrality
- Momentum space eccentricity: 0.3-0.9
  - This is flow assymmetry
- Average transverse flow and temp. gradient:
  - Strongly coupled, ratio around 0.3-1.0 (with $b<0$)
- Confidence levels very low
- With estimated 3% theory error: acceptable

Summary

- Revival of interest in perfect hydro
- Our model: a 3+1d relativistic model without acceleration
- Calculated particle source → N_1, v_2, HBT
- Compared to PHENIX data
- Describes data (but low conf. level)

Thank you for your attention