The QCD phase diagram for small chemical potentials

from lattice simulations

G. Endrődi[1], Z. Fodor[1,2], S.D. Katz[1], K.K. Szabó[2]

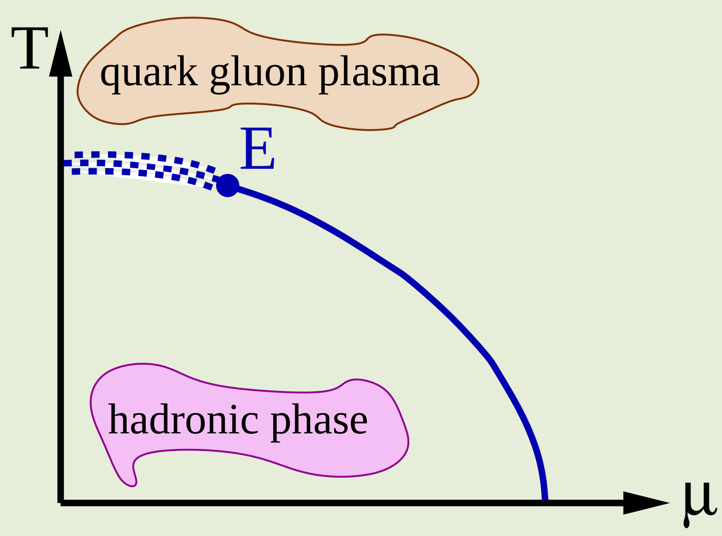
[1] Eötvös University, Dept. of Theoretical Physics
[2] University of Wuppertal, Dept. of Theoretical Physics

RHIC Winter School on Heavy Ion Physics
Budapest, 2. December 2009.

Contents

- Introduction, motivation
- Role of the curvature
- A cheap method to determine κ
- Results in the continuum limit
- Conclusions, outlook

Introduction, motivation



- $\mu = 0$ area is relevant for - the early Universe
  - high energy collisions
- $\mu_{RHIC} \approx 50$ MeV, $\mu_{SPS} \approx 250$ MeV
- QCD transition at $\mu = 0$ is found to be a crossover

  [Y. Aoki, GE, Z. Fodor, S.D. Katz, K.K. Szabó]
- Different observables give different values for T_c namely, $T_c(\bar{\psi}\psi) \approx 156$ MeV, $T_c(\chi_s) \approx 169$ MeV

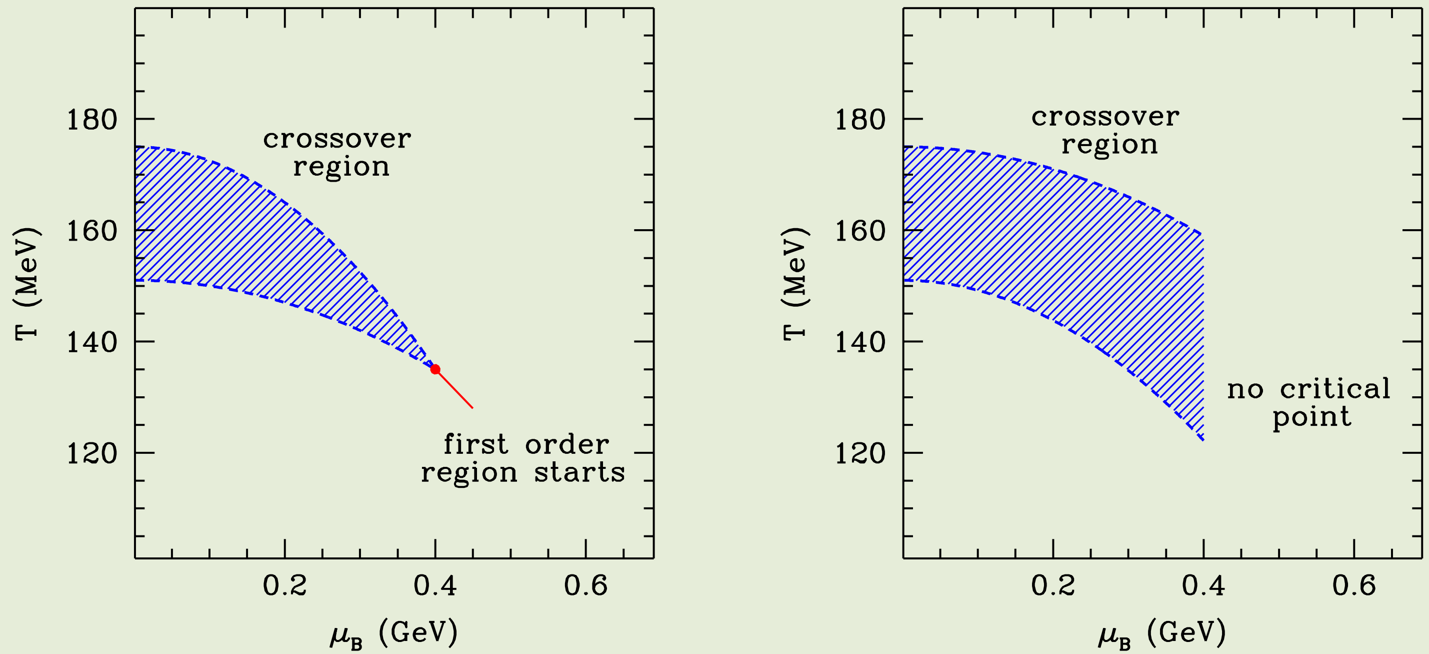
  [Y. Aoki, Sz. Borsányi, S. Durr, Z. Fodor, S.D. Katz, S. Krieg, K.K. Szabó]

Role of the curvature

- Explore the $\mu \neq 0$ region of the phase diagram
- At $\mu \neq 0$ sign problem emerges
  $\rightarrow$ importance sampling not possible
- Possible solutions:
  - reweighting $\mu = 0$ configurations
  - analytic continuation from imaginary μ
  - use Taylor-expansion in μ, around $\mu = 0$
    first term vanishes
    second term given by the curvature (κ)
- Aims:
  determine the curvature for different observables
  $\bar{\psi}\psi$ and χ_s
- Comparison: $N_t = 4$ and 6 results; the curvature is in the range of $\kappa = 0.003 \ldots 0.01$

[Bielefeld-Swansea; Philipsen, de Forcrand; D'Elia, Lombardo; Fodor, Katz]

Possible scenarios



- Does the crossover region shrink or expand?
- Analyze the width of the transition
  $\longrightarrow$ curvature can give insight
  $\longrightarrow$ recent study indicates a weakening [de Forcrand]
  $\longrightarrow$ non-monotonic behaviour is possible [Kapusta]
- Does a critical endpoint exist?
- $\mu \equiv \mu_B$

Curvature determination I.

- Equation of transition line is $T_c(\mu) = T_c\left(1 - \kappa\frac{\mu^2}{T_c^2}\right)$

$$\rightarrow \kappa = -T_c \left.\frac{\mathrm{d}T_c(\mu)}{\mathrm{d}(\mu^2)}\right|_{\mu=0}$$

- Determining $T_c(\mu)$ would be too expensive
- For an observable $\Phi(T,\mu)$ which satisfies:

$$\lim_{T\to 0} \Phi(T,\mu^2) = C_0, \quad \lim_{T\to\infty} \Phi(T,\mu^2) = C_\infty \quad \forall\mu$$

- Define a 'transition' temperature T_K where

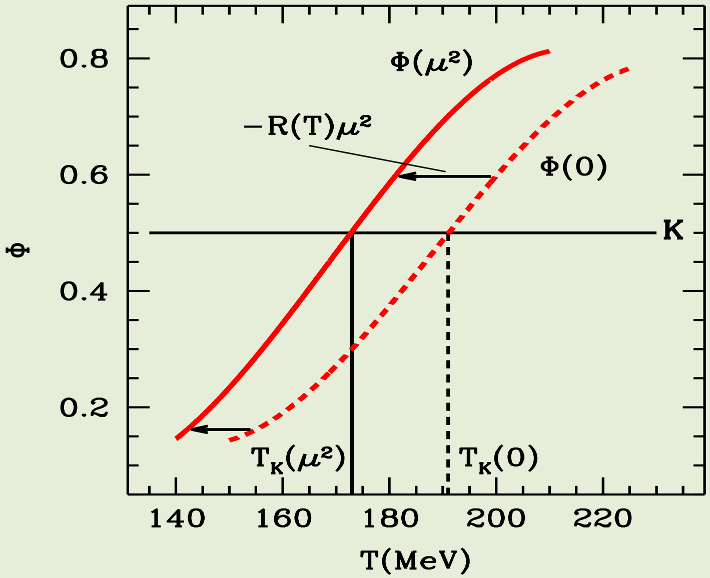
$$\Phi(T)|_{T=T_K} = K$$

with $C_0 < K < C_\infty$

- Set K according to the inflection point of $\Phi(T,0)$ so $T_K = T_c(\mu = 0)$

Curvature determination II.

- For $\Phi(T,\mu^2)$: $\mathrm{d}\Phi = \frac{\partial\Phi}{\partial T}\cdot \mathrm{d}T + \frac{\partial\Phi}{\partial\mu^2}\cdot \mathrm{d}\mu^2$



- along the $T_K(\mu)$ line $\mathrm{d}\Phi = 0$ by definition

$$\frac{\mathrm{d}T_c}{\mathrm{d}\mu^2} = \underbrace{-\left(\left.\frac{\partial\Phi}{\partial\mu^2}\right|_{T=T_K}\right)\Big/\left(\left.\frac{\partial\Phi}{\partial T}\right|_{T=T_K}\right)}_{R(T)}$$

- $\kappa(T) = -T_c\cdot R(T)$

- To leading order each point of Φ moves $-R(T)\cdot\mu^2$ to the left
- Also, $\kappa(T)$ gives curvature of the $\Phi =$ const. curve starting from T at $\mu = 0$
- Slope of $\kappa(T)$ related to width of transition:

$$\frac{1}{W}\frac{\partial W}{\partial(\mu^2)} = -\frac{1}{T_c}\left.\frac{\partial\kappa}{\partial T}\right|_{T=T_c}$$

Operators for $\frac{\partial\Phi}{\partial\mu^2}$

- Consider $\mathcal{Z} = \int \mathcal{D}U e^{-S_g(U)} \det M^{N_f/4}$

- $\frac{\partial \log \mathcal{Z}}{\partial \mu_{u,d}} = \langle n_{u,d} \rangle$; $\frac{\partial^2 \log \mathcal{Z}}{\partial \mu_{u,d}^2} = \langle \chi_{u,d} \rangle$

  $n_{u,d} = \frac{N_f}{4} \mathrm{Tr}\left(M^{-1}M'\right)$ and

  $\chi_{u,d} = n_{u,d}^2 + \frac{N_f}{4}\mathrm{Tr}\left(M^{-1}M'' - M^{-1}M'M^{-1}M'\right)$

  $(' \equiv \frac{\partial}{\partial \mu_{u,d}})$

- Observables Φ that don't depend on $\mu_{u,d}$ (L, χ_s):

$$\frac{\partial\langle\Phi\rangle}{\partial(\mu_{u,d}^2)} = \frac{1}{2}\frac{\partial^2\langle\Phi\rangle}{\partial\mu_{u,d}^2} = \langle\Phi\chi_{u,d}\rangle - \langle\Phi\rangle\langle\chi_{u,d}\rangle$$

- Observables Φ that depend on $\mu_{u,d}$ ($\bar\psi\psi$, $\chi_{\bar\psi\psi}$):

$$\frac{\partial\langle\Phi\rangle}{\partial(\mu_{u,d}^2)} = \frac{1}{2}\frac{\partial^2\langle\Phi\rangle}{\partial\mu_{u,d}^2} = \langle\Phi\chi_{u,d}\rangle - \langle\Phi\rangle\langle\chi_{u,d}\rangle + \langle 2\Phi' n_{u,d} + \Phi''\rangle$$

Observables

- Strange susceptibility $\chi_s = \frac{T}{V}\frac{\partial^2 \log \mathcal{Z}}{\partial \mu_s^2}$

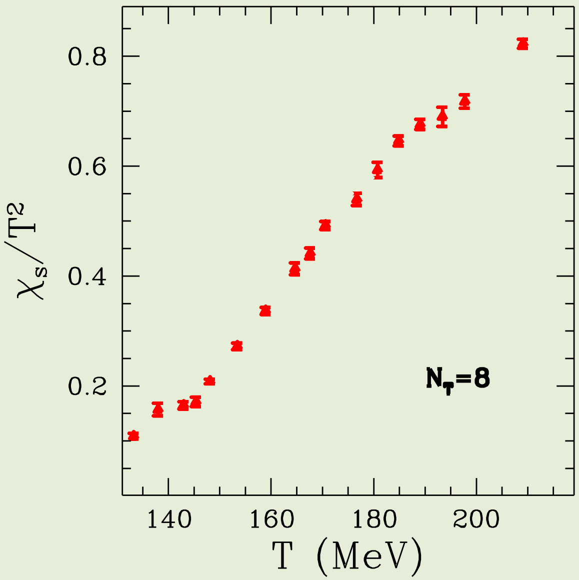
  no renormalization necessary, study combination χ_s/T^2
  here $C_0 = 0$, $C_\infty = 1$, both μ-independent

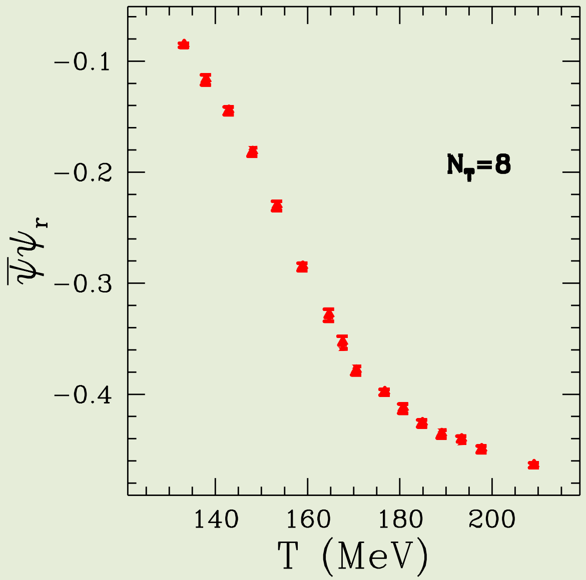
- Chiral condensate $\bar{\psi}\psi = \frac{T}{V}\frac{\partial \log \mathcal{Z}}{\partial m}$

  renormalization, subtraction of SB limit:

  $$\bar{\psi}\psi_r = \left[(\bar{\psi}\psi - \bar{\psi}\psi(T=0)) \cdot m - \alpha m^2 T^2\right] \cdot \frac{1}{m_\pi^4}$$

  here $C_0 = 0$, C_∞ both μ-independent





Simulation details

- Symanzik improved gauge and stout-link improved staggered fermionic lattice action
- Physical masses for $m_{u,d}$ and for m_s
- LCP determined by fixing m_K/f_K and m_K/m_π
- Scale set by f_K
- Lattice spacings used: $N_t = 4, 6, 8, 10$ ($a \approx 0.3 \ldots 0.12$fm)
- with aspect ratios $N_s/N_t = 4$ and 3
- Measurements carried out with 80 random vectors (measurements and config. production balanced)
- Derivatives Φ' and Φ'' calculated numerically using a purely imaginary chemical potential

How to extract results?

- Determine $\kappa(T)$ over a temperature-interval
  study $\kappa(T)|_{T=T_c}$ $\rightarrow$ curvature of $T_c(\mu)$ line
  study $\left.\frac{\partial\kappa}{\partial T}\right|_{T=T_c}$ $\rightarrow$ change in width of transition
- Expand around T_c ($t \equiv \frac{T-T_c}{T_c}$):

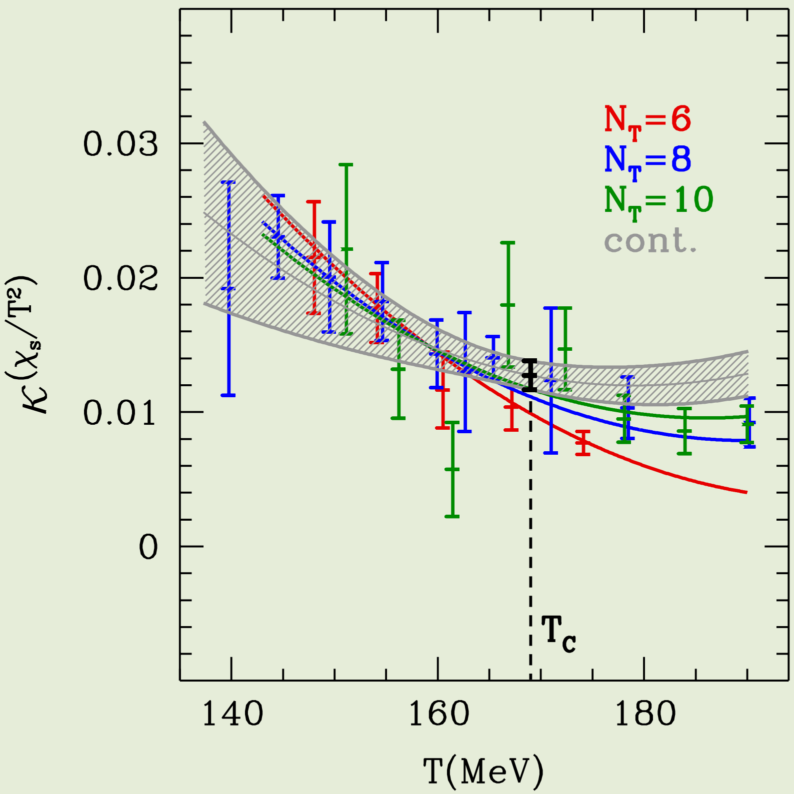
$$\kappa(T) = \kappa(T_c) + b_1 \cdot t + b_2 \cdot t^2$$

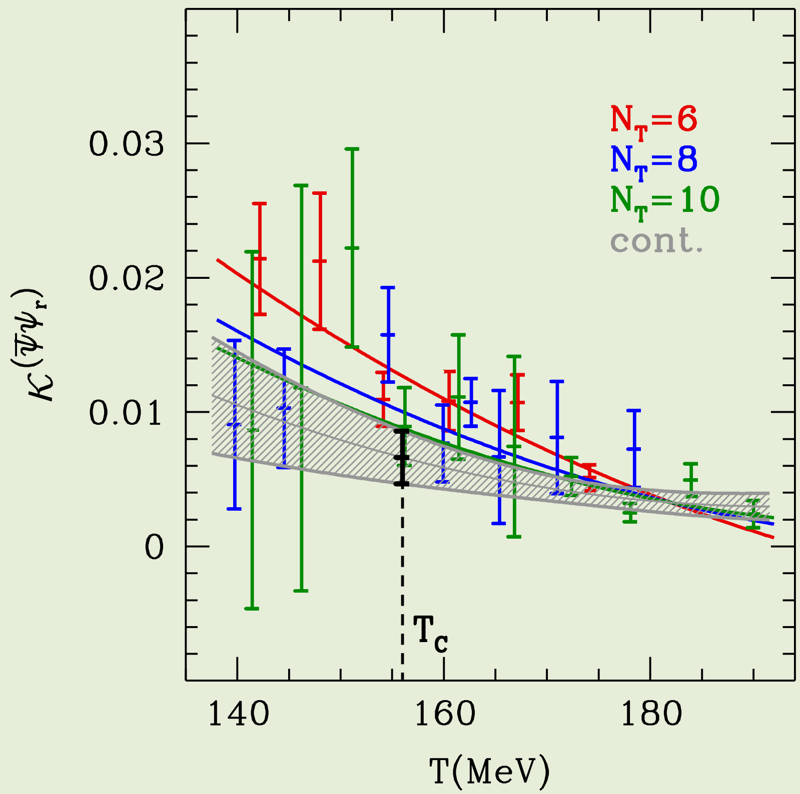
- Fit different a (different N_t) data together:

$$\kappa(T; N_t) = \kappa(T_c; \text{cont}) + b_1 \cdot t + b_2 \cdot t^2 + c_1/N_t^2 \quad + c_2 \cdot t/N_t^2$$

- N_t-dependent quadratic term not necessary to describe data
- Good fit qualities: $\chi^2/d.o.f \approx 0.8$

Results





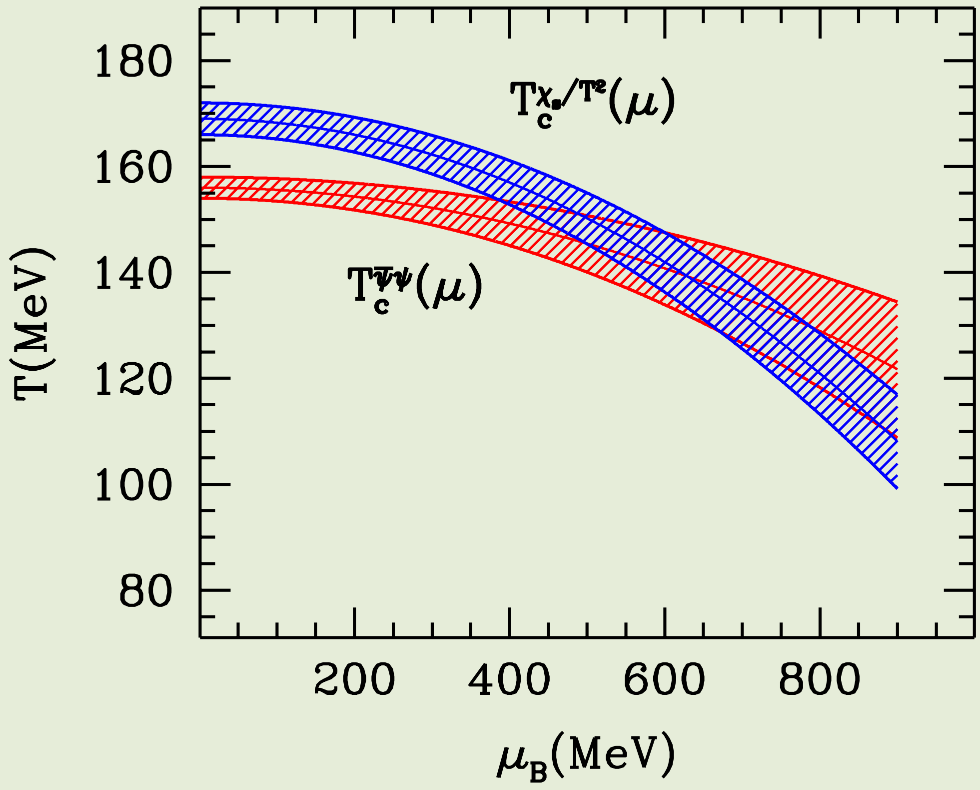
$$\kappa^{(\chi_s/T^2)} = 0.0127(11) \qquad \kappa^{(\bar\psi\psi_r)} = 0.0066(19)$$

$$\frac{1}{W}\frac{\partial W}{\partial(\mu^2)}^{(\chi_s/T^2)} = 0.025(19) \qquad \frac{1}{W}\frac{\partial W}{\partial(\mu^2)}^{(\bar\psi\psi_r)} = 0.031(18)$$

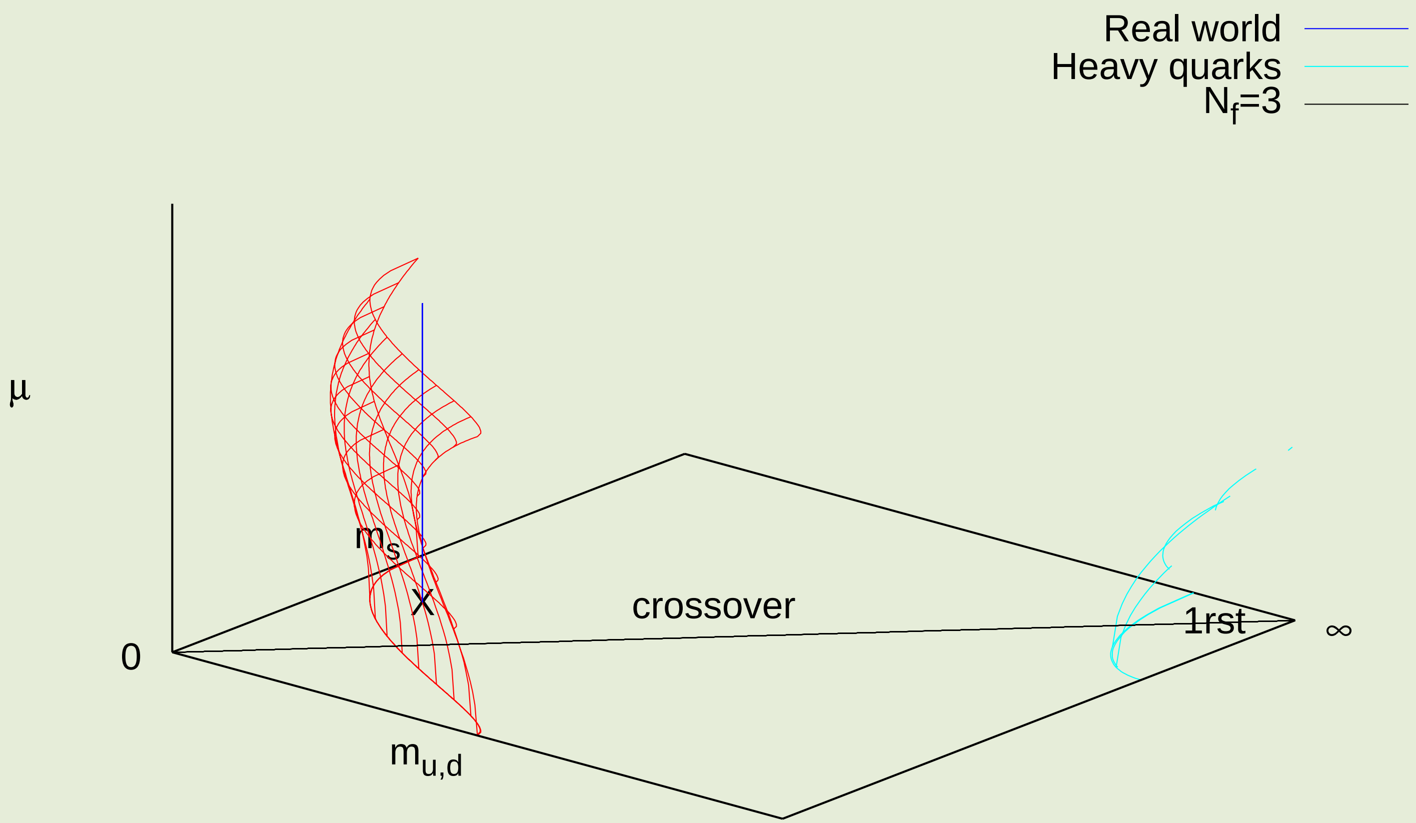
Conclusions

- Taylor-expansion in μ to determine the curvature
- Crossover nature of the transition is visible
- In leading order, transition curves of $\bar{\psi}\psi_r$ and χ_s/T^2 converge to each other
  - Higher orders? Third observable?
- Both quantities get smoother as μ increases
- Leading order in μ indicates a weakening of the transition
  - no evidence for a critical endpoint
  - non-monotonic behaviour?
- Full reweighting is needed to study to existence of the critical point

Illustration

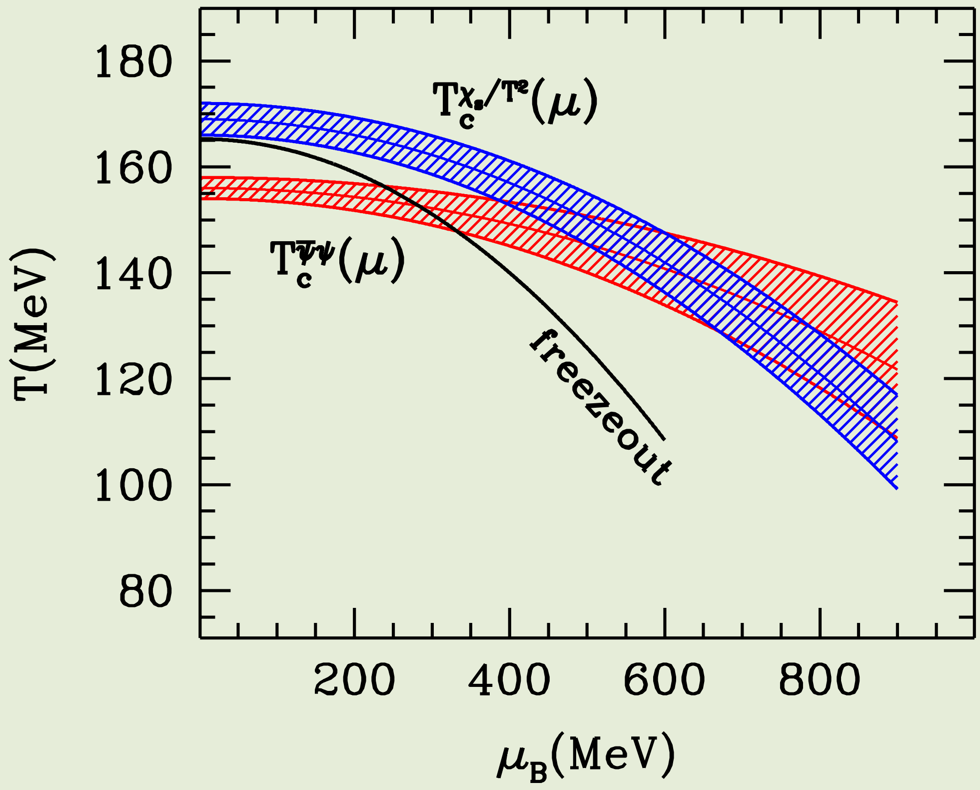


Illustration



Taken from [Kapusta, de Forcrand]

Illustration



Freezeout curve from [Cleymans, Redlich]

On the applicability of the method

- Is Φ μ-independent at $T = 0$ and at $T \to \infty$?
- χ_s/T^2 and $\bar{\psi}\psi_r$ both fulfill this, because:
  - imaginary $\mu \equiv$ boundary condition, which is irrelevant at $T = 0$
    this independence can be analytically continued to real μ also
  - μ enters $\log \mathcal{Z}$ only through the fugacity $e^{\mu/T}$, so at $T \to \infty$ inclusion of a small μ has no effect
- $\Phi(T_c) =$ const. is a good definition for T_c for both quantities

On the renormalization of $\bar{\psi}\psi$

- Another usual renormalization is:

$$\Delta_\ell = \left[\bar{u}u - \frac{2m_{ud}}{m_s}\bar{s}s\right] \Big/ \left[\left(\bar{u}u - \frac{2m_{ud}}{m_s}\bar{s}s\right)\Big|_{T=0}\right]$$

  - only divergences proportional to $m_u^2 + m_s^2$ cancel
  - advantage: Δ_ℓ goes $1 \ldots 0$ as $T \in [0, \infty)$

- $\bar{\psi}\psi_r$ contains no divergent terms
  - to approach finite limits as $T \to 0$ and $T \to \infty$, the SB contribution has to be subtracted
  - the term $\alpha m^2 T^2$ appears with $\alpha = -1/6$ (in the continuum)

  $\to$ its subtraction does not effect $\bar{\psi}\psi_r$ around T_c, where $\alpha m^2 T^2 / m_\pi^4 \sim \mathcal{O}(10^{-4})$