Recent results on the QCD transition temperature and equation of state

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Outline

Lattice formulation

$$Z = \int dU d\Psi d\bar{\Psi} e^{-S_E}$$

S_E is the Euclidean action

Parameters:
gauge coupling g
quark masses m_i $(i = 1..N_f)$
(Chemical potentials μ_i)
Volume (V) and temperature (T)

Finite $T \leftrightarrow$ finite temporal lattice extension

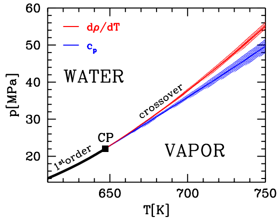
$$T = \frac{1}{N_t a}$$

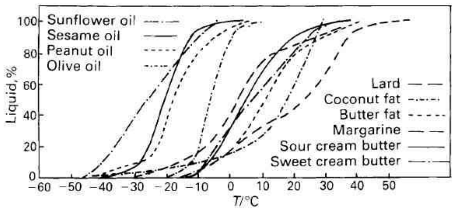
Continuum limit: $a \to 0 \Longleftrightarrow N_t \to \infty$

The transition temperature

Y. Aoki, Z. Fodor, S.D. Katz, K.K. Szabo, Phys. Lett. B. 643 (2006) 46

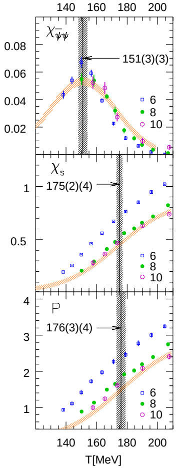
an analytic transition (cross-over) has no unique T_c:
example of water-steam transition





above the critical point c_p and $d\rho/dT$ give different T_cs.
QCD: chiral & quark number susceptibilities or Polyakov loop
they result in different T_c values $\Rightarrow$ physical difference

The transition temperature: results and scaling



Chiral susceptibility

T_c=151(3)(3) MeV

ΔT_c=28(5)(1) MeV

Quark number susceptibility

T_c=175(2)(4) MeV

ΔT_c=42(4)(1) MeV

Polyakov loop

T_c=176(2)(4) MeV

ΔT_c=38(5)(1) MeV

N_t=6,8,10 in the a^2 scaling region, N_t=8,10(12) are practically the same

Literature: discrepancies between T_c

Brookhaven-Bielefeld-Columbia-Riken Coll. (+MILC='hotQCD'):

M. Cheng et.al, Phys. Rev. D74 (2006) 054507

T_c from $\chi_{\bar{\psi}\psi}$ and Polyakov loop, from both quantities:

T_c=192(7)(4) MeV

Wuppertal-Budapest group (WB):

Y. Aoki, Z. Fodor, S.D. Katz, K.K. Szabo, Phys. Lett. B. 643 (2006) 46

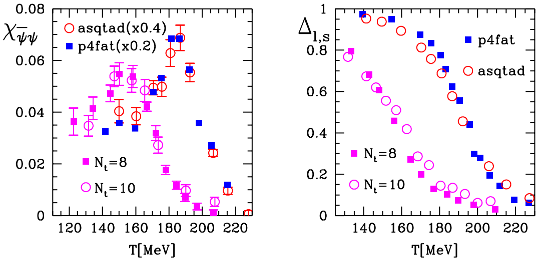
chiral susceptibility: T_c=151(3)(3) MeV
Polyakov and strange susceptibility: T_c=175(2)(4) MeV

'chiral T_c': ≈40 MeV; 'confinement T_c': ≈15 MeV difference

both groups give continuum extrapolated results with physical m_π

Literature: discrepancies between T dependences

Reason: shoulders, inflection points are difficult to define?
Answer: no, the whole temperature dependence is shifted



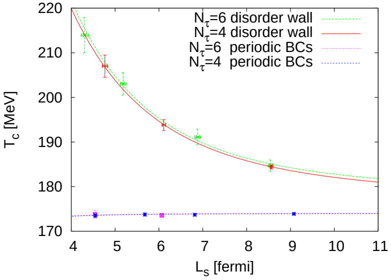
for chiral quantities ≈35 MeV; for confinement ≈15 MeV
this discrepancy would appear in all quantities (eos, fluctuations)

150 MeV transition temperature: isn't it a bit too small?
lattice works in V→ ∞, which gives much smaller T_c

T_c strongly depends on the geometry

nanotube-water doesn't freeze, even at hundred degrees below 0^oC

exploratory study: A. Bazavov and B. Berg, Phys.Rev. D76 (2007) 014502
use 'confined' spatial boundary conditions: more like experiments



large deviation (upto 30 MeV) from the infinite volume limit
if $V \rightarrow \infty$ is 150 MeV a 100 fm^3 system might have 170 MeV

Possible reasons for the discrepancy

“Non-lattice artefact/formulation” related reasons

a. bug in the codes

b. systematic errors are largely underestimated

“Lattice artefact/formulation” related reasons

a. the pion mass is not small enough:
'hotQCD' 230MeV ⇒ shift of 5 MeV, WB: 135 MeV pseudogoldstone

b. not small enough lattice spacings: new 'hotQCD'/WB upto N_t=8/12

c. actually it is not QCD, what we are studying
(most large scale thermodynamics studies use staggered fermions)

Discretization errors in the transition region

we always have discretization errors: nothing wrong with it as long as

a. result: close enough to the continuum value (error subdominant)
b. we are in the scaling regime (a^2 in staggered)

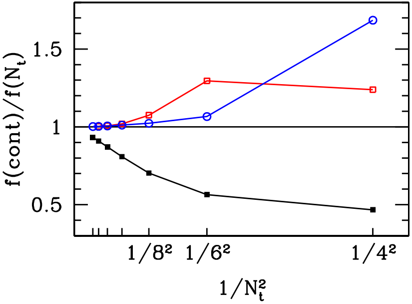
various types of discretization errors $\Rightarrow$ we improve on them (costs)

we are speaking about the transition temperature region
interplay between hadronic and quark-gluon plasma physics
smooth cross-over: one of them takes over the other around T_c

both regimes (low T and high T) are equally important
improving for one: $T \gg T_c$, doesn't mean improving for the other: $T < T_c$

Examples for improvements, consequences

how fast can we reach the continuum pressure at T=∞?



p4 action is essentially designed for this quantity T$\gg T_c$

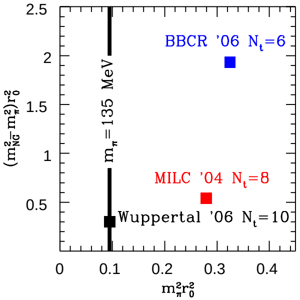
asqtad designed mostly for T=0 physics (but good at high T, too)

stout-smeared one-link converges slower but in the a^2 scaling regime (e.g. extrapolation from N_t=8,10 provides a result within about 1%)

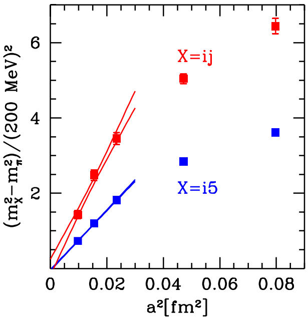
Chiral symmetry breaking and pions

transition temperature for remnant of the chiral transition:
balance between the chirally broken and chirally symmetric sectors
chiral symmetry breaking: 3 pions are the pseudo-Goldstone bosons

staggered QCD: 1 pseudo-Goldstone instead of 3 (taste violation)
staggered lattice artefact $\Rightarrow$ disappears in the continuum limit
WB: stout-smeared improvement is designed to reduce this artefact



Scaling for the pion splitting



scaling regime is reached if a^2 scaling is observed

asymptotic scaling starts only for N_t>8 (a$\lesssim$0.15 fm): two messages

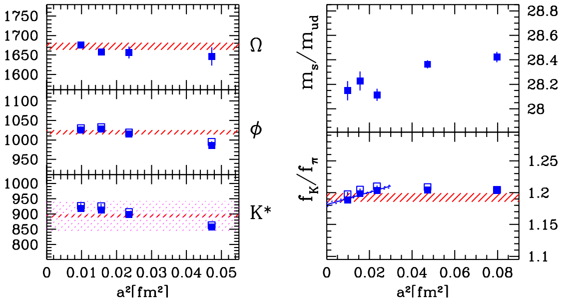
a. N_t=8,10 extrapolation gives 'p' on the $\approx$1% level: good balance

b. stout-smeared improvement is designed to reduce this artefact

most other actions need even smaller 'a' to reach scaling

Setting the scale (T=0 results)

Y.Aoki et al. [Wuppertal-Budapest Collaboration] arXiv:0903.4155

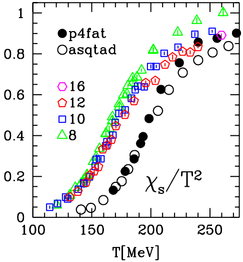


independently which quantity is taken (we used physical masses)

⇒ one obtains the same '*a*' and T, result is safe

T>0 results: strange susceptibility

Y.Aoki et al. [Wuppertal-Budapest Collaboration] arXiv:0903.4155



'hotQCD' results are on N_t=8, WB results are on N_t=8,10,12,(16)

'hotQCD': results with two different actions are almost the same

WB: for large T one extrapolates according to the known a^2 behaviour

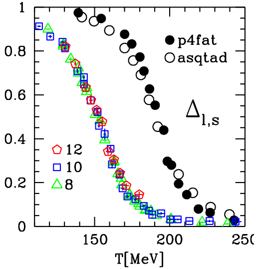
WB: no change in the lattice results compared to our 2006 paper

note, that the experimental value of f_K decreased by 3% since 2006

about 20 MeV difference between the results

T>0 results: chiral condensate

Y.Aoki et al. [Budapest-Wuppertal Collaboration] arXiv:0903.4155



'hotQCD' results are on N_t=8, WB results are on N_t=8,10,12

'hotQCD': results with two different actions are almost the same

WB: no lattice spacing dependence observed for N_t=8,10,12

WB: no change in the lattice results compared to our 2006 paper

note, that the experimental value of f_K decreased by 3% since 2006

about 35 MeV difference between the results

transition temperatures for various observables

	$\chi_{\bar{\psi}\psi}/T^4$	$\chi_{\bar{\psi}\psi}/T^2$	$\chi_{\bar{\psi}\psi}$	$\Delta_{l,s}$	L	χ_s
WB'09	146(2)(3)	152(3)(3)	157(3)(3)	155(2)(3)	170(4)(3)	169(3)(3)
WB'06	151(3)(3)	-	-	-	176(3)(4)	175(2)(4)
BBCR	-	192(4)(7)	-	-	192(4)(7)	-

renormalized chiral susceptibility, renormalized chiral condensate
Polyakov loop and strange quark number susceptibility

no change compare to our 2006 data (errors are reduced)
note, that the experimental value of f_K decreased by 3% since 2006
Particle Data Group now gives f_K=155.5(2)(8)(2) MeV (error 0.5%)

r_0 is not directly measurable:

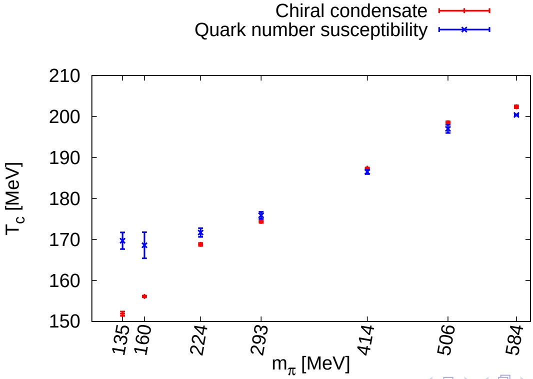
ETM:0.444(4) fm, QCDSF:0.467(6) fm,
HPQCD&UKQCD:0.469(7) fm, PACS-CS:0.492(6)(+7) fm

Effect of the pion splitting

At finite a the average pion mass can be significantly higher than the Goldstone mass.

Average pion mass of HotQCD on $N_t = 8$ is $\sim$400MeV

This can change T_c:



Renormalization of the pressure 1.

towards the continuum limit ($a \to 0$):

$p \propto \log Z \propto a^{-4}$

$\to$ renormalization is necessary.

Logical choice: $p_R(T) = p(T) - p(T = 0)$

This gives automatically $p_R(T = 0) = 0$

Disadvantage: required $T = 0$ simulations can be very expensive

Back of an envelope estimate:

$T_c \approx 150$–200 MeV, $m_\pi = 135$ MeV and try to reach $T = 20 \cdot T_c$ for $N_t = 8$

$N_s > 4/m_\pi \approx 6/T_c = 6 \cdot 20/T = 6 \cdot 20 \cdot N_t \approx 1000 \Rightarrow$

completely out of reach

Renormalization of the pressure 2.

Use $T > 0$ for renormalization instead
Let

$$\bar{p}(T) = p(T) - p(T/2)$$

Then

$$p_R(T) = p(T) - p(T/2) \; + \; p(T/2) - p(T/4) \; + \; \cdots = \bar{p}(T) + \bar{p}(T/2) + \bar{p}(T/4) + \ldots$$

Since $\bar{p}(T) \sim T^4$, only a few terms are needed

Only finite T simulations are needed with N_t (T) and $2N_t$ $(T/2)$
Typical scale $\sim g^2 T \sim T \log a$

Since $Ta = 1/Nt$ is fixed, the required lattice size increases only logarithmically with $T \propto 1/a$

Extremely large temperatures can be reached

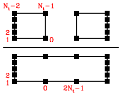
Direct approach

Consider $\bar{p}$:

$$\bar{p} = \frac{1}{N_t N_s^3} \log Z(N_t) - \frac{1}{2N_t N_s^3} \log Z(2N_t) = \frac{1}{2N_t N_s^3} \log \left(\frac{Z(N_t)^2}{Z(2N_t)} \right)$$

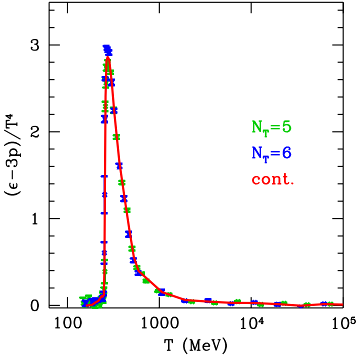
The ratio of partition functions can be drawn as

$$\frac{Z^2(N_t)}{Z(2N_t)} =$$

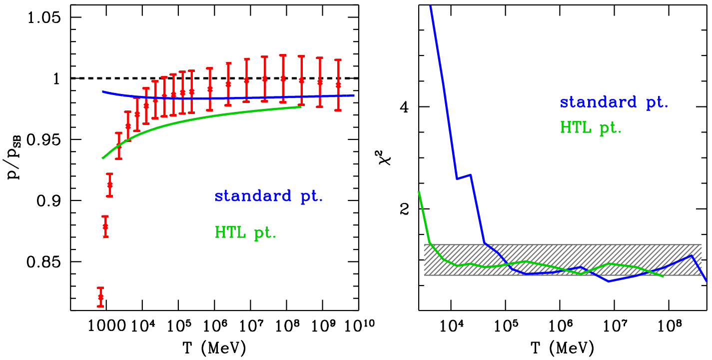


Preliminary results for the pure gauge theory

Symanzik improved gauge action
Interaction measure $((\epsilon - 3p)/T^4)$
$N_t = 5, 6$ data and continuum limit



Comparison with perturbation theory

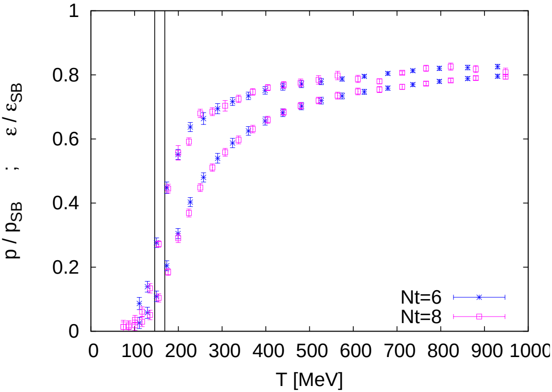


At high T compare our data to perturbation theory

- standard perturbation theory [Arnold, Zhai, '94]
- hard thermal loop (HTL) perturbation theory [Andersen et al. '02]

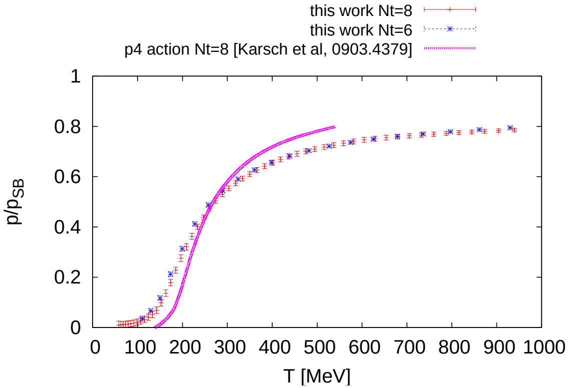
Correlated χ^2 quantifies the agreement

Preliminary results with dynamical fermions



good scaling for $N_t = 6, 8$
different T_c-s indicated by vertical lines
energy density would give $T_c \approx 170$ MeV

Comparison of the pressure



$\approx$ 20 MeV discrepancy with HotQCD results around T_c

our results indicate a smoother transition than those of HotQCD

Summary

- New $T = 0$ results: unambiguous scale setting
- New $N_t = 12$ results on T_c consistent with previous ones
- Discrepancy between T_c values remains
- EoS determined on $N_t = 8$ lattices