

# Non-extensive Hadron Distributions at RHIC Energy

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- Transverse hadron spectra from a flowing but locally equilibrated QGP.
- Extensive and non-extensive hadron distributions.

# PLAN

- Confront

*Non-extensive*                      *200 GeV cm E AuAu RHIC*  
*Thermo*                      with                      *Transverse Spectra*

- Results

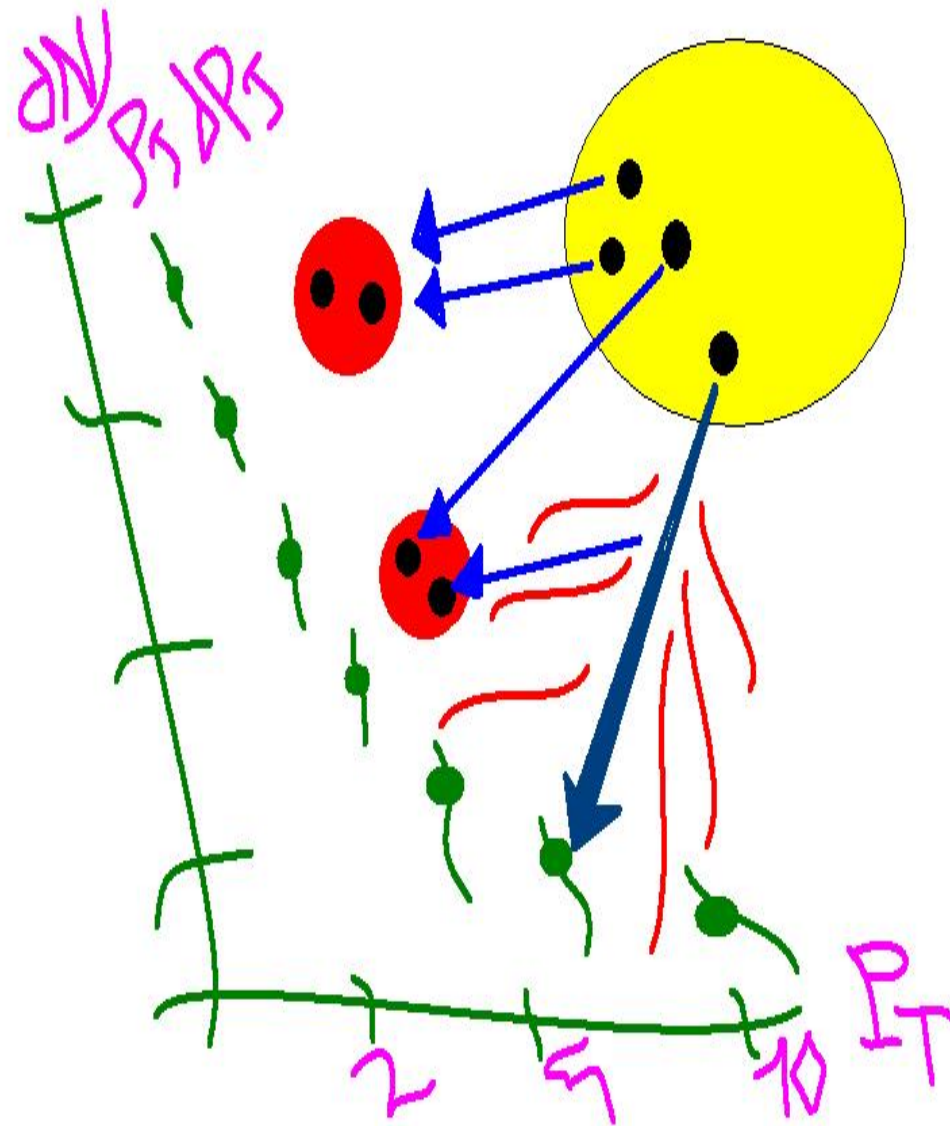
*If  $dN/p_T dp_T$  at RHIC is **thermal***

*from 0 to 2 GeV its Statistics is **Boltzmann***

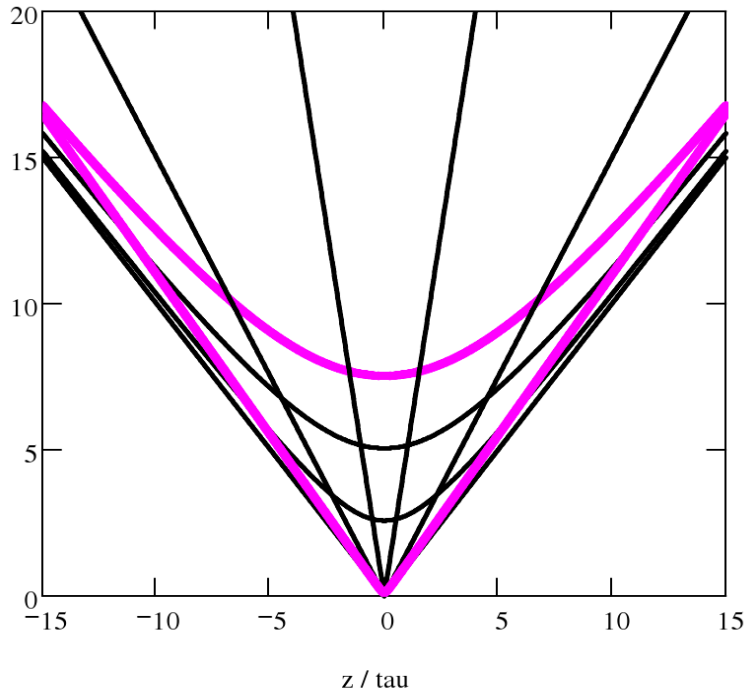
*from 0 to 5 GeV its Statistics is **Non-ext. Tsallis***

*from 0 to 20 GeV its Statistics is **Non-Ext. Hyper-Tsallis***

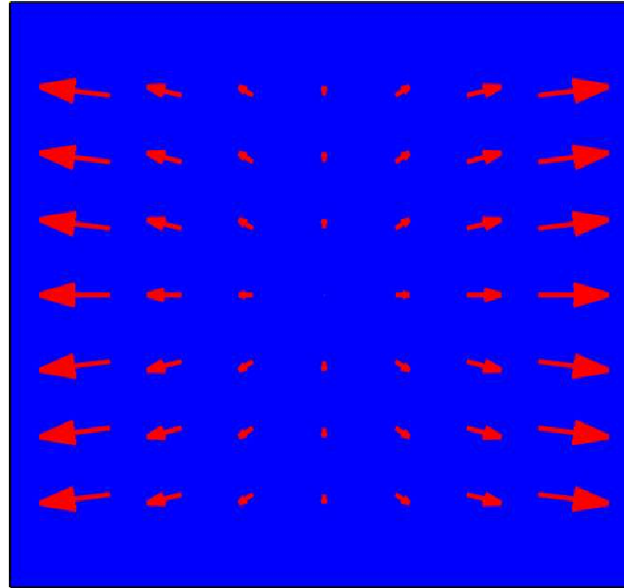
# Jets and recombination or non-extensivity?



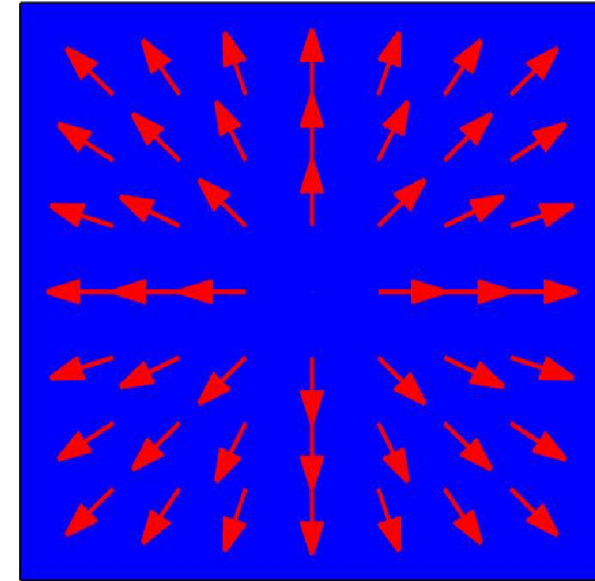
# The Flow Profile of the QGP



Hadronization hyper-  
surface in the  
longitudinal direction



Longitudinal flow  
profile



Transverse flow  
profile

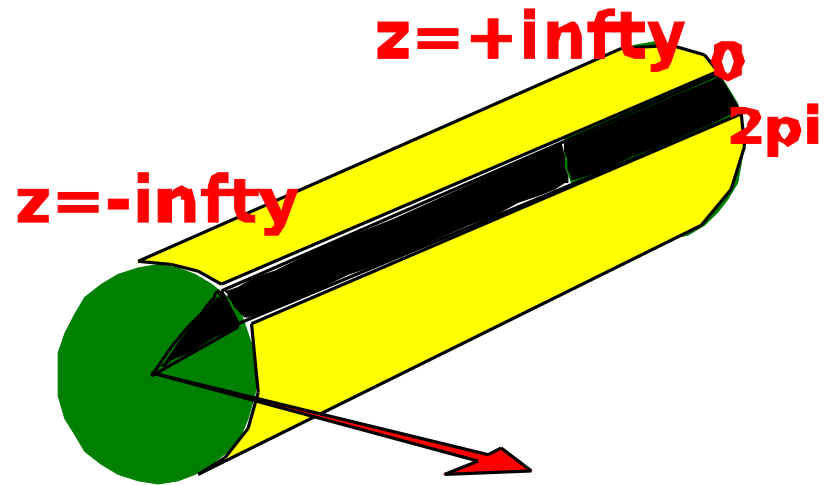
$$u^\mu = (\gamma \cosh \zeta, \gamma \sinh \zeta, \gamma v \cos \alpha, \gamma v \sin \alpha),$$

$$\zeta = \frac{1}{2} \ln \left( \frac{t-z}{t+z} \right)$$

# Transverse Hadron Spectrum From the Locally Equilibrated Plasma

$$\frac{dN}{p_T dp_T dy}_{y=0} = p^0 \frac{dN}{d^3 p} = \int p_\mu d\sigma^\mu F(\beta p_\mu u^\mu)$$

Boundaries:



$$\frac{dN}{p_T dp_T} = \int_0^\infty d\zeta \int_0^\pi d\phi \left( A^v m_T \cosh(\zeta) + A^s p_T \cos(\phi) \right) F(\beta p_\mu u^\mu)$$

$$A^v = \pi R^2 \tau \rightarrow A\xi, \quad A^s = \pi R \tau^2 \rightarrow A(1-\xi)$$

## Boltzmann-Gibbs Hadron Distribution:

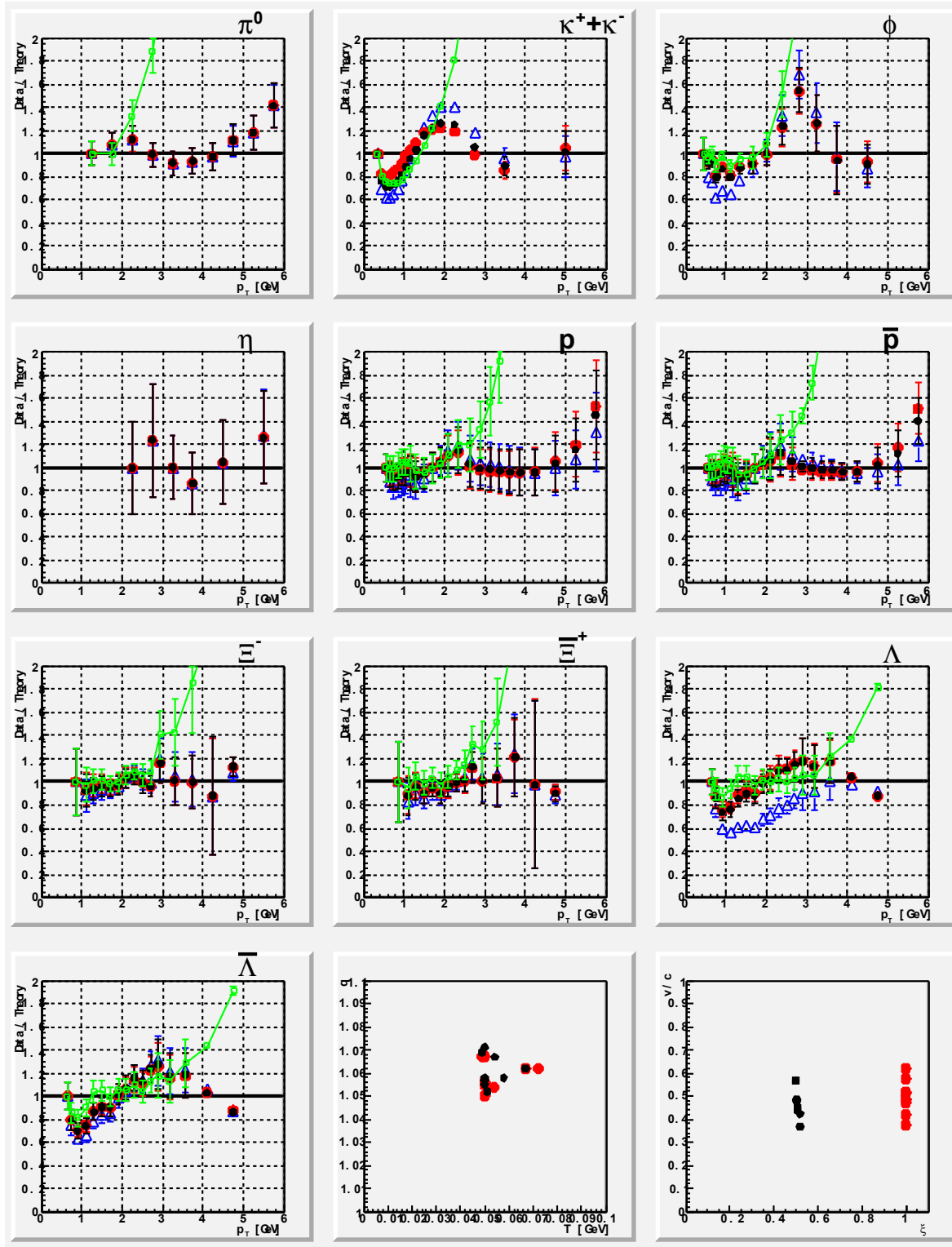
$$\frac{dN}{p_T dp_T} = A \xi m_T K_1(\beta \gamma m_T) I_0(\beta \gamma v p_T) \\ + A (1 - \xi) K_0(\beta \gamma m_T) I_1(\beta \gamma v p_T) \\ \rightarrow e^{-\beta \gamma (m_T - v p_T)}$$

## Tsallis Hadron Distribution:

$$\frac{dN}{p_T dp_T} = \frac{A^{vol} G_0(p_T) m_T + A^{surf} G_2(p_T) p_T}{(1 + (q-1) \beta \gamma (m_T - v p_T))^{1/(q-1)}} \sim \frac{1}{p_T^{1/(q-1)-1}}$$

$S^{1/2}=200$  GeV  
RHIC AuAu

Data / Theory



## **How to go on? Maximum entropy principle**

**If we can derive a quasi-energy composition rule:**

$$E_{12} = h(E_1, E_2) \rightarrow L(E_{12}) = L(E_1) + L(E_2), \quad L(E) = \int^E \frac{d\tilde{E}}{h'_2(\tilde{E}, 0)}$$

**The maximum entropy with fixed mean quasi-energy**

$$\max = - \int f \ln f - \beta \left( \int L(E) f(E) - L(E_0) / N \right)$$

**gives**

$$f(E) = \frac{\exp(-\beta L(E))}{Z(\beta)}$$



# $h(x,y)$ as the effect of the environment

**Fokker-Planck approach:**

$$\partial_t f = \partial_p \left( G(E) E' + \partial_p D(E) \right) f$$

**With stationary solution**

$$f(E) = \frac{A}{D(E)} \exp \left( -\beta \int^E \frac{d\tilde{E} G(\tilde{E})}{D(\tilde{E})} \right)$$

**Hence the connection of  $h$  and  $D$ ,  $G$ :**

$$G(E) = \frac{\beta D(E)}{h'_2(E,0)} - D'(E)$$

## Boltzmann-Gibbs case:

$$h(E_1, E_2) = E_1 + E_2 \rightarrow L(E) = E$$

$$f = \exp(-\beta E) / Z$$

## Tsallis case:

$$h(E_1, E_2) = E_1 + E_2 + a E_1 E_2 \rightarrow L(E) = \frac{1}{a} \ln(1 + aE)$$

$$f = (1 + aE)^{-\beta/a} / Z$$

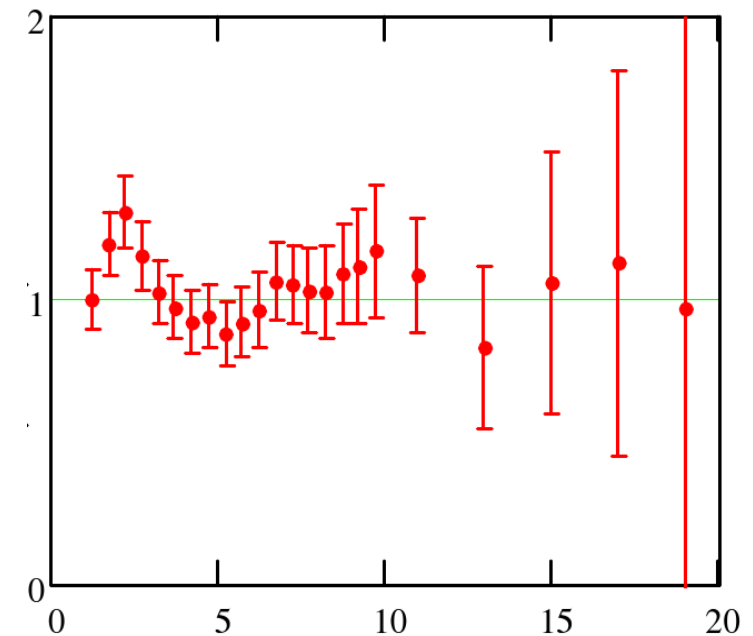
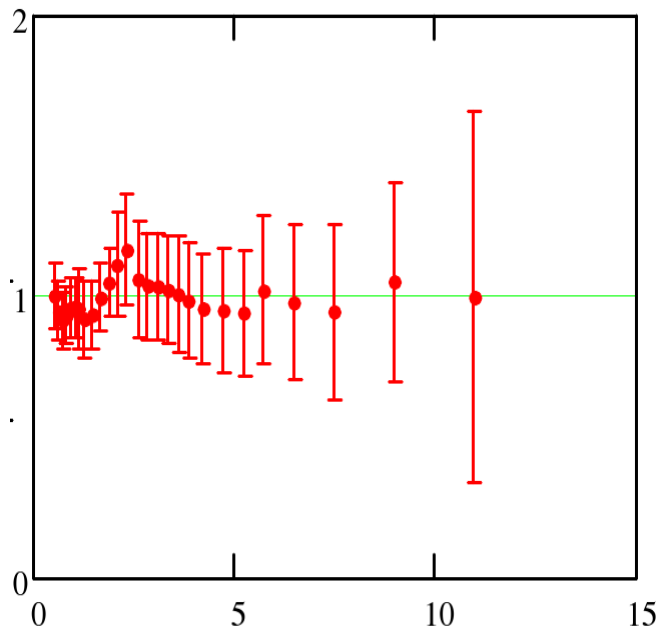
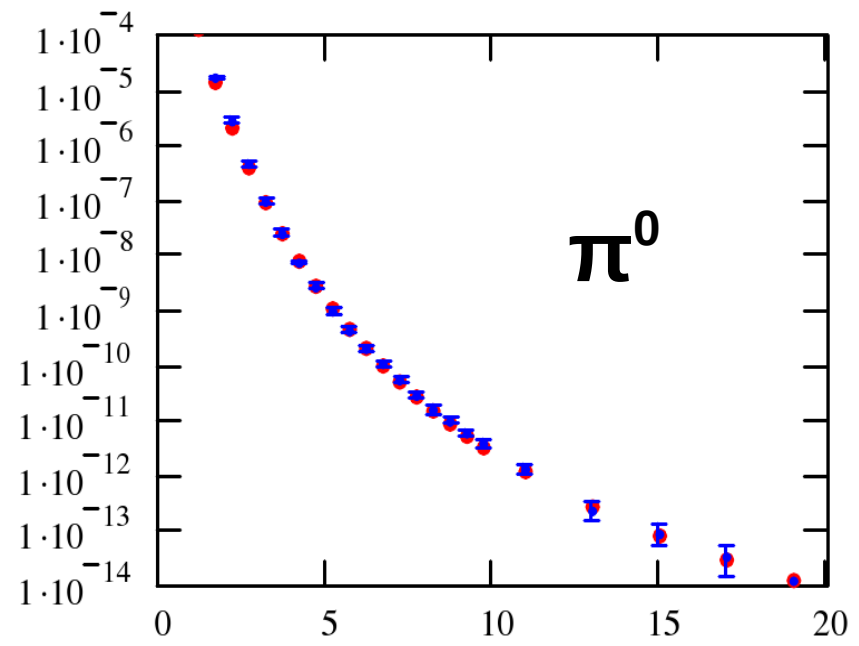
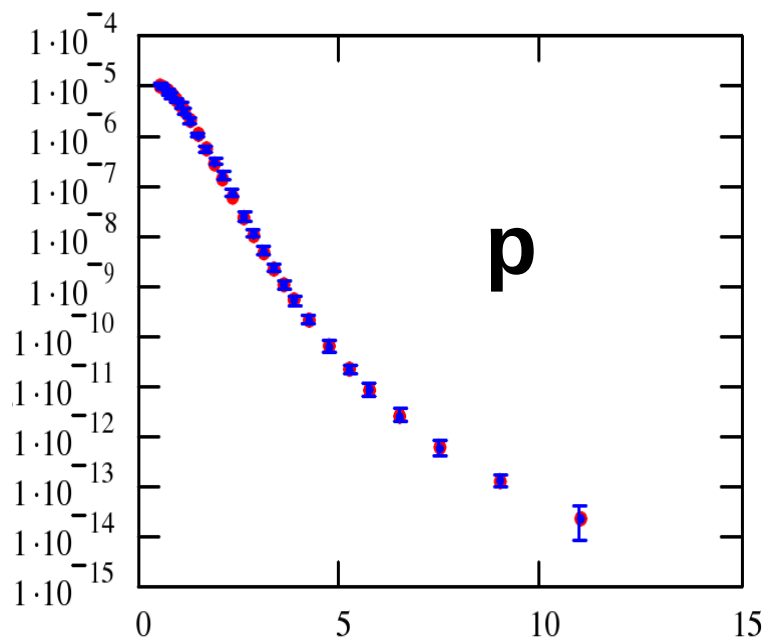
## Hyper-Tsallis case:

$$h(E_1, E_2) = E_1 + E_2 + a E_1 E_2 + b/2 (E_1^2 E_2 + E_2^2 E_1)$$

$$L(E) = \frac{1}{\alpha} \ln \left( \frac{2 + (a - \alpha) E}{2 + (a + \alpha) E} \right) \quad f = \left( 1 - \frac{2\alpha E}{2 + (a + \alpha) E} \right)^{\beta/\alpha} / Z$$

$$\alpha = \sqrt{a^2 - 2b}$$

# Hyper-Tsallis Fittings



# Conclusion

- Either by a *stochastic transport* or by a *non-extensive thermo model* the  $s=(200 \text{ GeV})^2$  RHIC AuAu transverse spectra can be described well.

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