Effect of Freeze-out and Hadronization on Flow in ultrarelativistic HIC

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Introduction – the hydro description of ultrarelativistic HIC

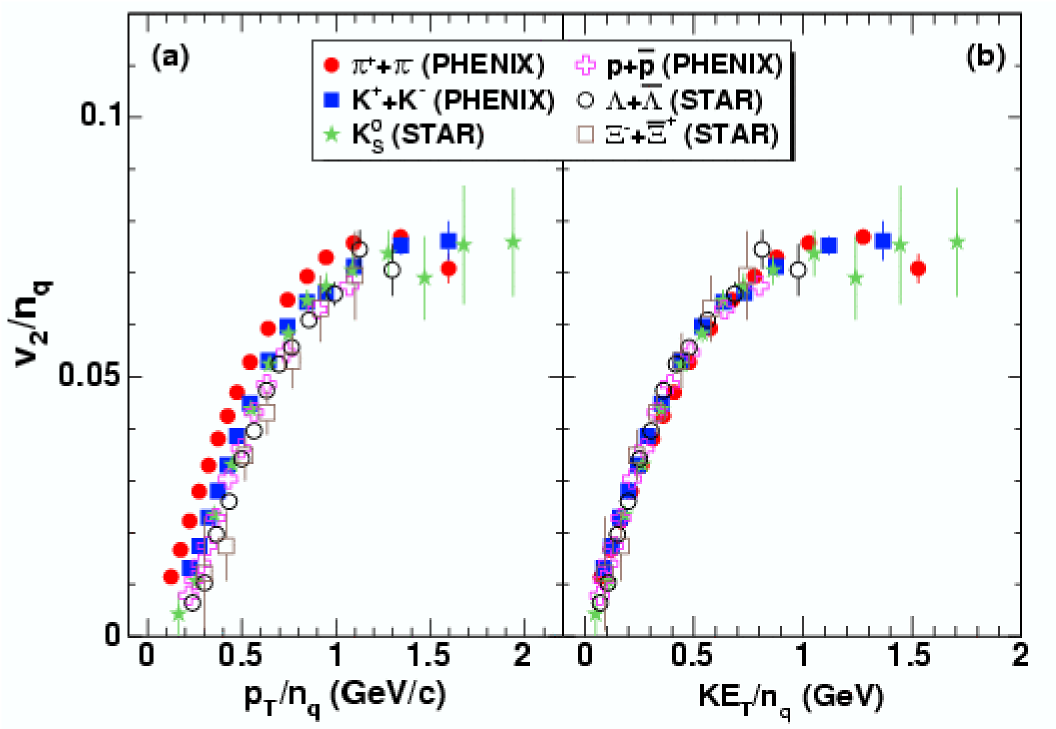
3 stages of the evolution in Ultrarelativistic Heavy Ion Collisions:

initial nonequilibrium evolution & QGP formation	e.g. fire streak model
equilibrated QGP	numerical hydro
hadronic phase & Freeze-out	(hadronic transport, “afterburner”)

Hadronization & Freeze-out:

- a very fast, nonequilibrium process
- first approximation: sudden and simultaneous FO & Hadronization

Constituent quark number scaling of $v_2(KE_T)$



Collective flow of hadrons can be described in terms of constituent quarks.

Does hadronization accelerate the fluid?

Boundary conditions on the hadronization surface: $[a] = a_{hadr} - a_{QGP}$

Baryon number: $[N^\mu d\hat{\sigma}_\mu] = 0$ $N^\mu = n u^\mu$ $u^\mu = \gamma(1, \boldsymbol{v})$

Energy & momentum: $[T^{\mu\nu} d\hat{\sigma}_\nu] = 0$ $T^{\mu\nu} = (e+P) u^\mu u^\nu - P g^{\mu\nu}$

(Local) isochronous Freeze-out frame: $d\hat{\sigma}^\mu = (1, \boldsymbol{0})$

$$[n\gamma] = 0$$

$$[(e+P)\gamma^2 - P] = 0$$

Can be solved numerically

$$[(e+P)\gamma^2 \boldsymbol{v}] = 0$$

Acceleration, nonrelativitic limit

$v \ll 1 \quad \gamma \approx 1$ in the local isochronous frame of the fluid

(good approximation if close to Bjorken flow)

$$[n]=0, \quad [e]=0, \quad [(e+P)\mathbf{v}]=0$$

$$\frac{v_{hadr}}{v_{QGP}} = \frac{e+P_{QGP}}{e+P_{hadr}}$$

acceleration if $P_{hadr} < P_{QGP}$

Role of temperature

thermal smearing → anisotropy is reduced

Equation of State

QGP: Bag EOS $P = \frac{e}{3} - \frac{4}{3}B$ $e = e_0 + B$ $P = P_0 - B$

Post Freeze-out/Hadronization:

NCQ scaling → description in terms of constituent quarks & antiquarks

(anti)quarks are in hadrons → No. of d.o.f. is different from a gas of free quarks and antiquarks → everage degeneracy factor g_{av}

relativistic Boltzmann gas of massive quarks + antiquarks

the main effects:

appearance of quark mass

disappearance of Bag constant

disappearance of gluons

extra degeneracy factor

Equation of State II.

$$n(\mu,T)=\frac{g_{av}}{\pi^2}m^2T\sinh\left(\frac{mu}{T}\right)K_2\left(\frac{m}{T}\right) \qquad e(\mu,T)=3P\left(1+\frac{m}{3\mathrm{T}}\frac{K_1}{K_2}\right)$$

$$P(\mu,T)=\frac{g_{av}}{\pi^2}m^2T^2\cosh\left(\frac{mu}{T}\right)K_2\left(\frac{m}{T}\right) \qquad K_i=K_i\left(\frac{m}{T}\right)$$

e,n,**v**: obtained numerically from the boundary conditions

EOS can be solved for μ and T

thermal distribution for (anti)quarks (Juttner):

$$f_{q/\bar{q}}(x,p)=\frac{g_{av}}{(2\pi)^3}\exp\left(\frac{-p^\rho u_\rho\pm\mu}{T}\right)$$

Flow observables can be obtained using the Cooper-Frye formula.

$$E\frac{d^3N}{d\boldsymbol{p}^3}=\int f(x,p)p^\mu d\sigma_\mu$$

Entropy

Ideal gas: s/N = const. → entropy production = particle (pion) production

Coalescence: N is reduced by a factor of 2-3

→ entropy is decreasing

Phase transition: EOS of state is different from ideal gas

relevant degrees of freedom change

Bíró, Zimányi '07:

isentropycally expanding fireball

with lattice QCD EOS

effective particle number: $N_{eff} = V\, P(\mu,T)/T$

N_{eff} is reduced by a factor of 1/3 as we go below T_C

Entropy II

Increasing entropy condition: $[S^{\mu} d\hat{\sigma}_{\mu}] \geqslant 0$ $s^{\mu} = s u^{\mu}$

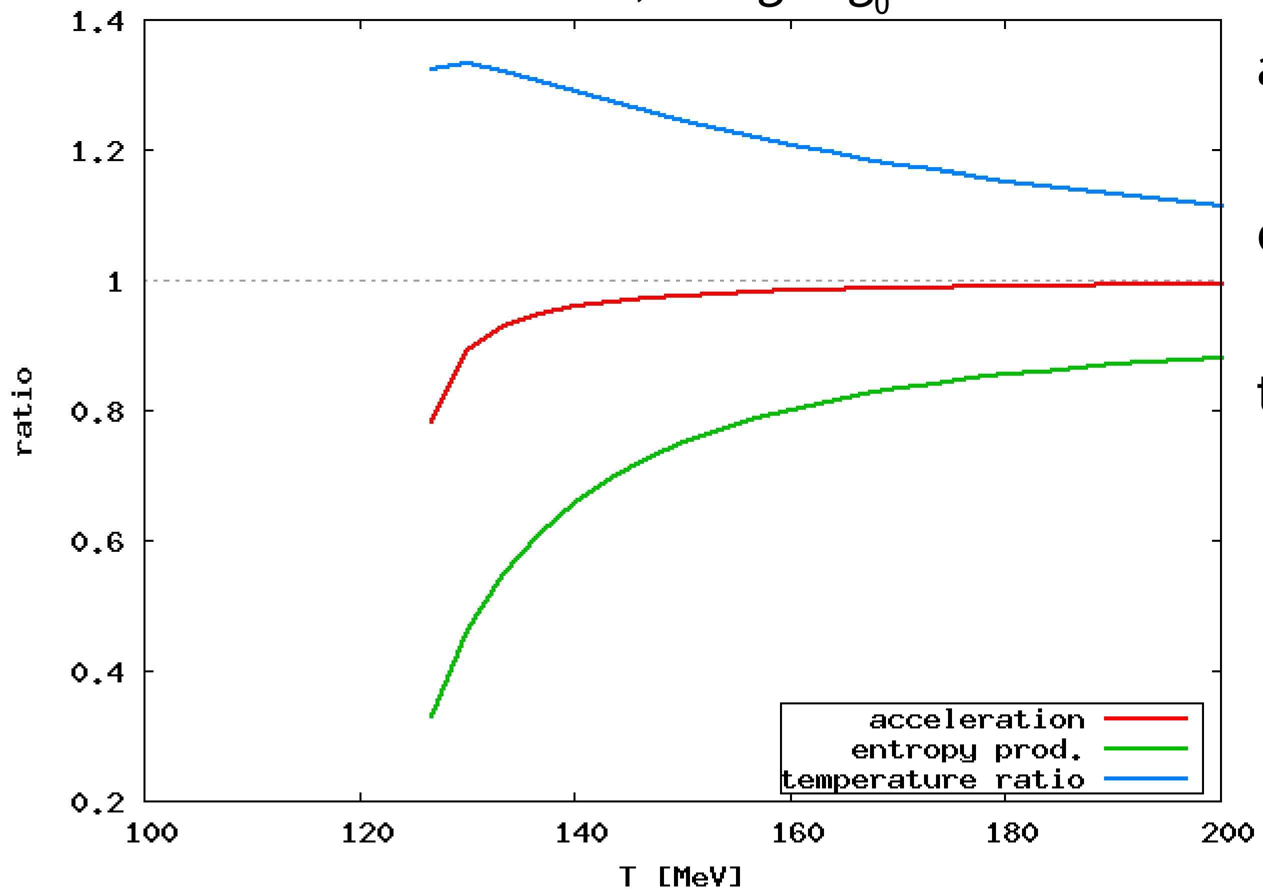
Isochronous Freeze-out frame: $[s\gamma] = 0$

In the constituent quark + antiquark model:

$$s = \left(\frac{\partial P}{\partial T}\right)_{\mu} = \frac{g_{av}}{\pi^2} m^2 \left[\cosh\left(\frac{\mu}{T}\right)(4TK_2 + mK_1) - \mu \sinh\left(\frac{\mu}{T}\right) K_2\right]$$

Numerical results for acceleration, entropy prod.

$v = 0.99\,c, \qquad g = g_0$

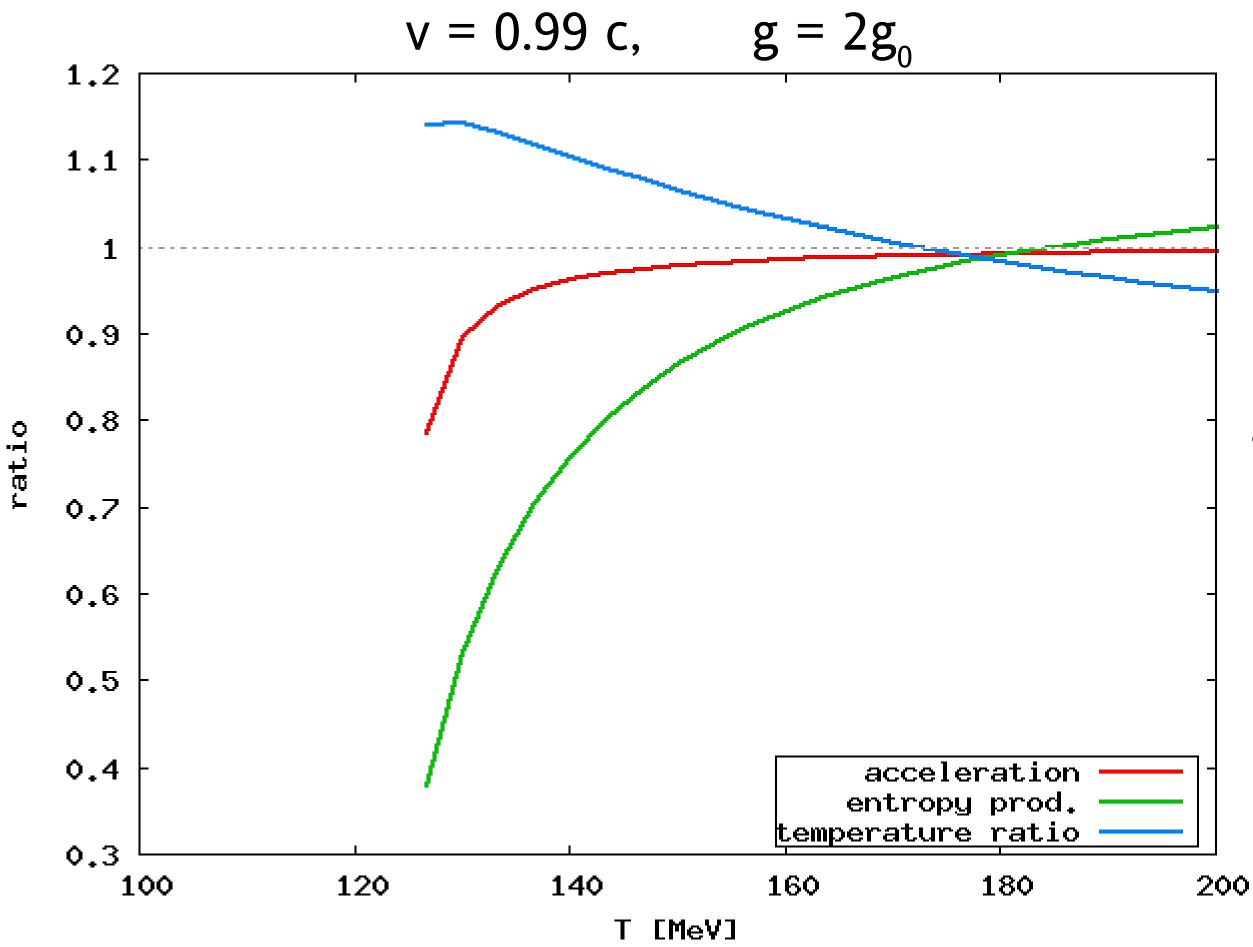


acceleration = v_{hadr}/v_{QGP}

entropy prod = $(\gamma\ s)_{hadr}/(\gamma\ s)_{QGP}$

temperature ratio = T_{hadr}/T_{QGP}

Numerical results for acceleration, entropy prod.



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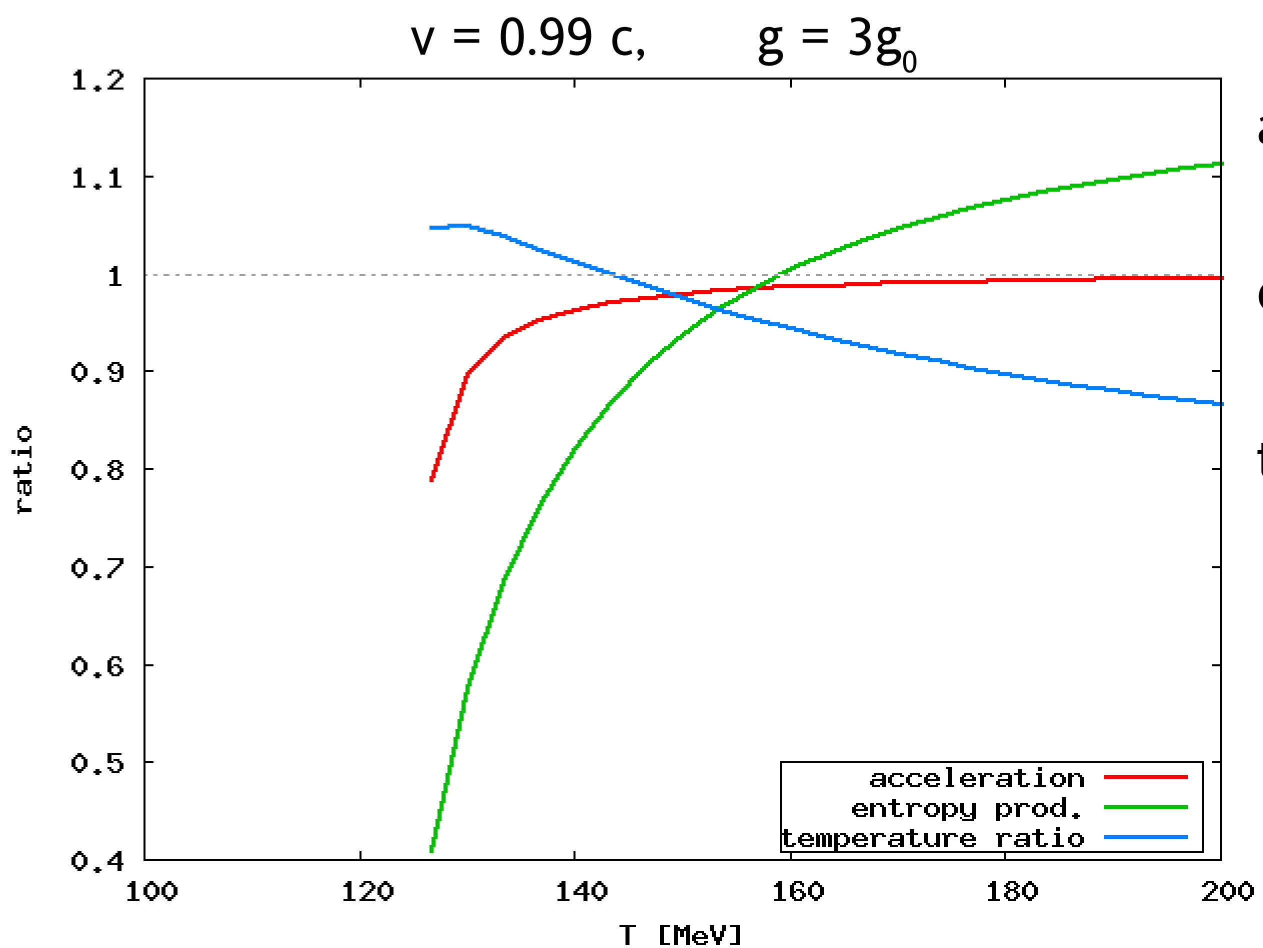

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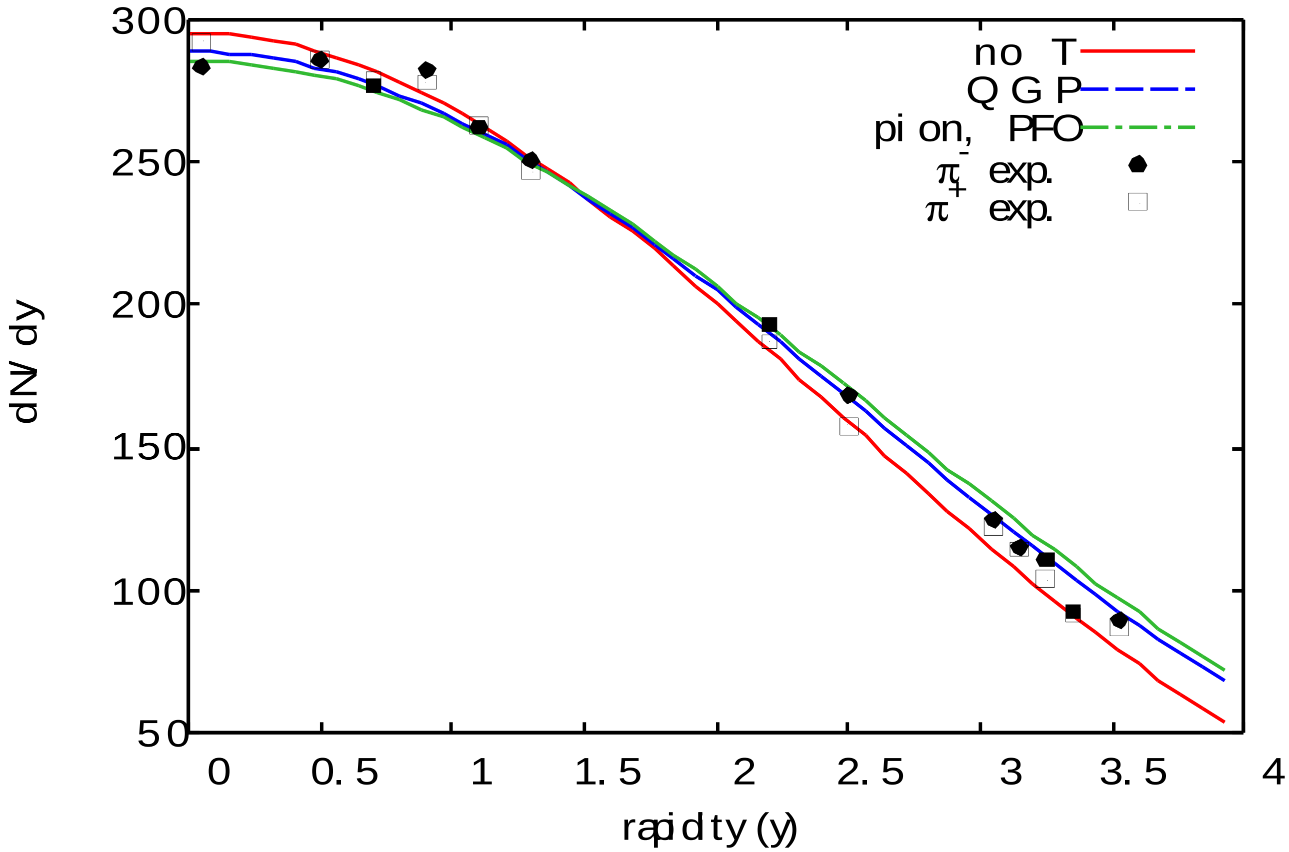
Effect of thermal smearing – Landau model

$\pi^{\pm}$ rapidity spectra, Au+Au, 200 GeV, BRAHMS



Inclusion of thermal effects slightly improves the agreement with data.

Problems to be faced in a more realistic treatment

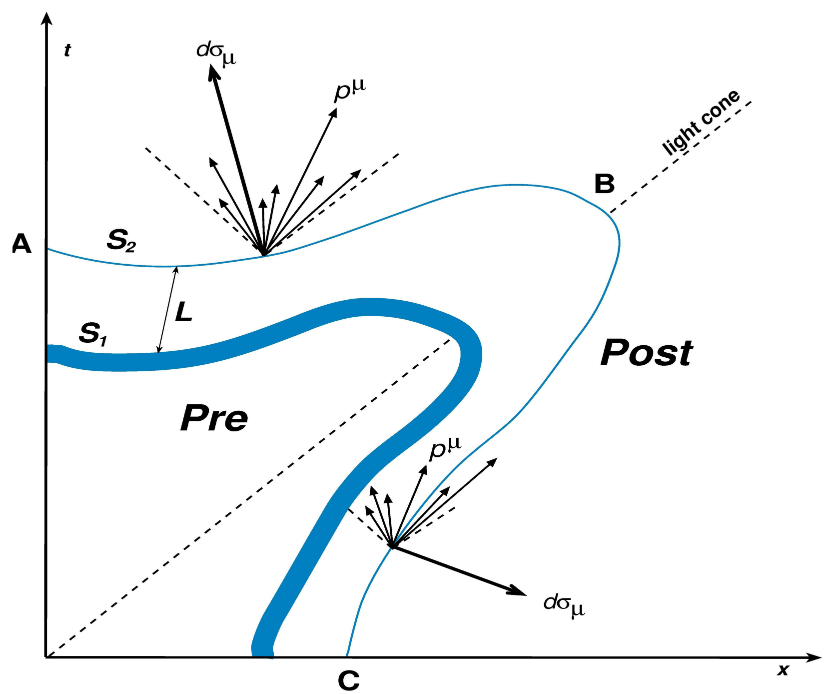
The distribution on the PFO side is not necessarily thermal.

e.g.: hypersurface with spacelike normal

$\rightarrow$ cut-Juttner, cancelling Juttner distribution

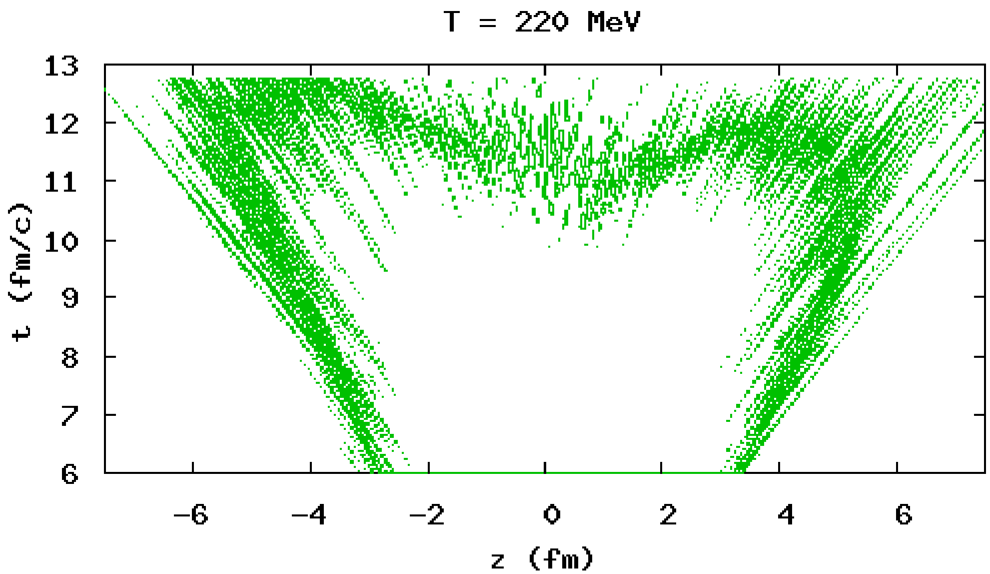
Freeze-out in finite time: hypersurface $\rightarrow$ finite layer

(approximate solution of Boltzmann eq.) Molnár, Csernai, Magas,... '03



Freeze-out hypersurface in numerical hydro

e.g. isotherm (T = 220 MeV):



Conclusions

NCQ scaling → flow can be studied in terms of constituent quarks

FO & Hadronization can affect flow observables in two ways:
increase/decrease the collective flow velocity
change in temperature – thermal smearing

The effect is not very large

It depends very much on the specific hadronization model

Backup slides

Hadronization via recombination

Momentum distribution of mesons in the recombination model:

$$\frac{d^3N}{dp^3} \propto \int \prod_{i=1}^{2} d^3x_i \, d^3p_i \, f_q(x_1, p_1) f_q(x_2, p_2) W_M(p, p1, p2, x1, x2)$$

meson Wigner function:

momentum conservation

$$W_M(p, p_1, p_2, x_1, x_2) = \Phi_M(x_1 - x_2, p_1 - p_2) \delta(p_T - p_{T1} - p_{T2})$$

comoving quark and antiquark: $\Phi_M \propto \delta^3(x_1 - x_2) \delta^3(p_1 - p_2)$

For the momentum distribution of mesons we get:

$$\frac{d^3N_M}{p_T \, d\,p_T \, dy \, d\phi} \propto \int d^3x \, f_q\left(x, p_t/2\right)^2$$

similar expression for baryons, $2 \rightarrow 3$

Hadronization via recombination II.

calculation of flow moments:

$$v_n(p_T) = \frac{\int dy\, d\phi\, \cos n\phi\, \frac{d^3N}{p_T\, dp_T\, dy\, d\phi}}{\int dy\, d\phi\, \frac{d^3N}{p_T\, dp_T\, dy\, d\phi}}$$

elliptic flow of mesons:

$$v_{2,M}(p_T) = \frac{2v_{2,q}(p_T/2)}{1+2v_{2,q}^2(p_T/2)} \qquad \frac{v_{2,M}(p_T)}{2} = v_{2,q}(p_T/2)$$

baryons:

$$v_{2,B}(p_T) = \frac{3v_{2,q}(p_T/3)+3v_{2,q}^3(p_T/3)}{1+6v_{2,q}^2(p_T/3)} \qquad \frac{v_{2,B}(p_T)}{3} = v_{2,q}(p_T/3)$$

in a medium p_T is not necessarily conserved, $KE_T = m_T - m$ might be conserved

→ scaling in the variable KE_T J. Jia & C. Zhang '07