Zimanyi 2009 Winter School

Kolmogorov-Smirnov test and rapidity fluctuations in heavy ion collisions

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in collaboration with

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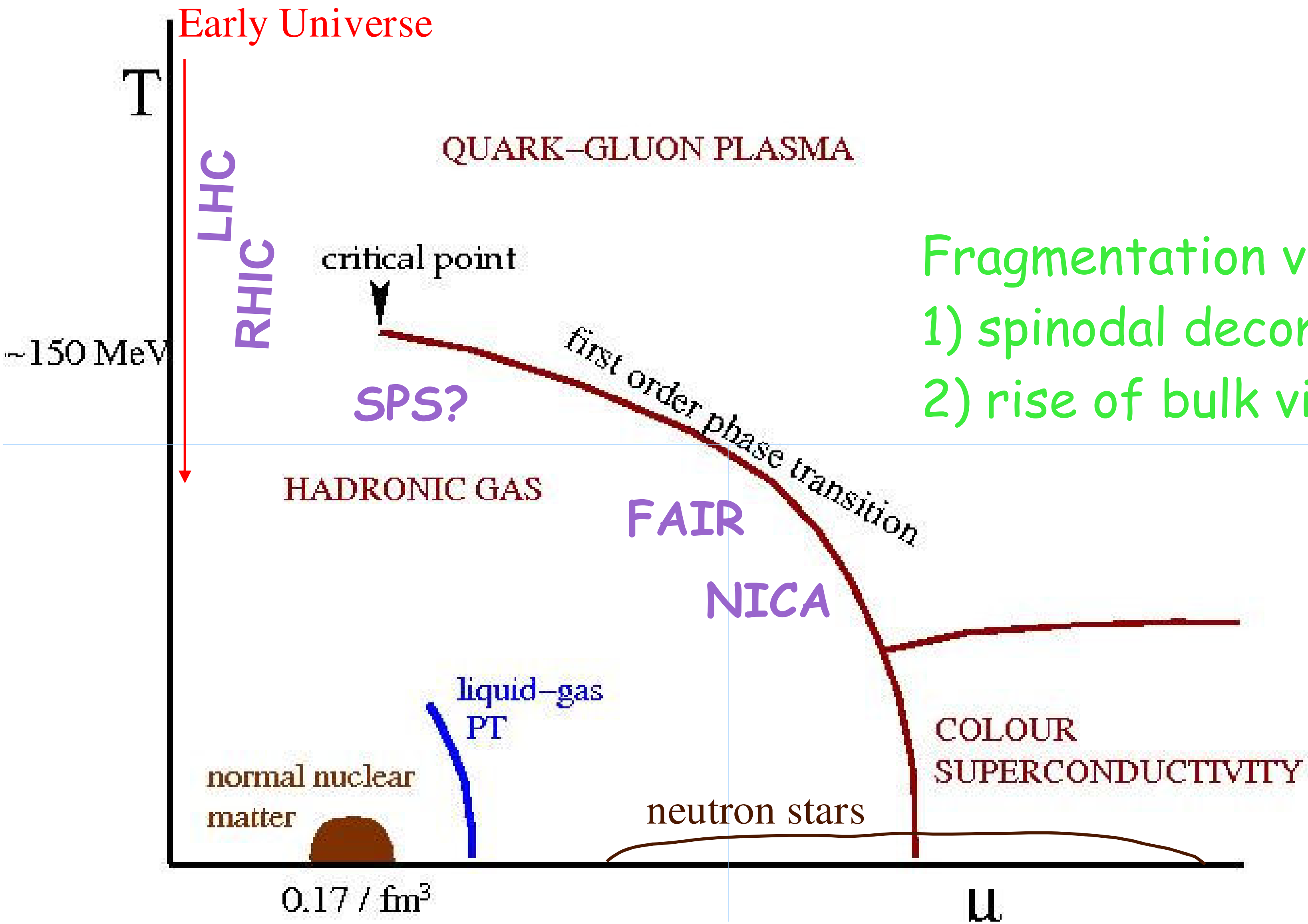
Outline:

- Event-by-event fluctuations of rapidity distributions

- Kolmogorov-Smirnov (KS) test and its application on simulated data

QCD phase diagram



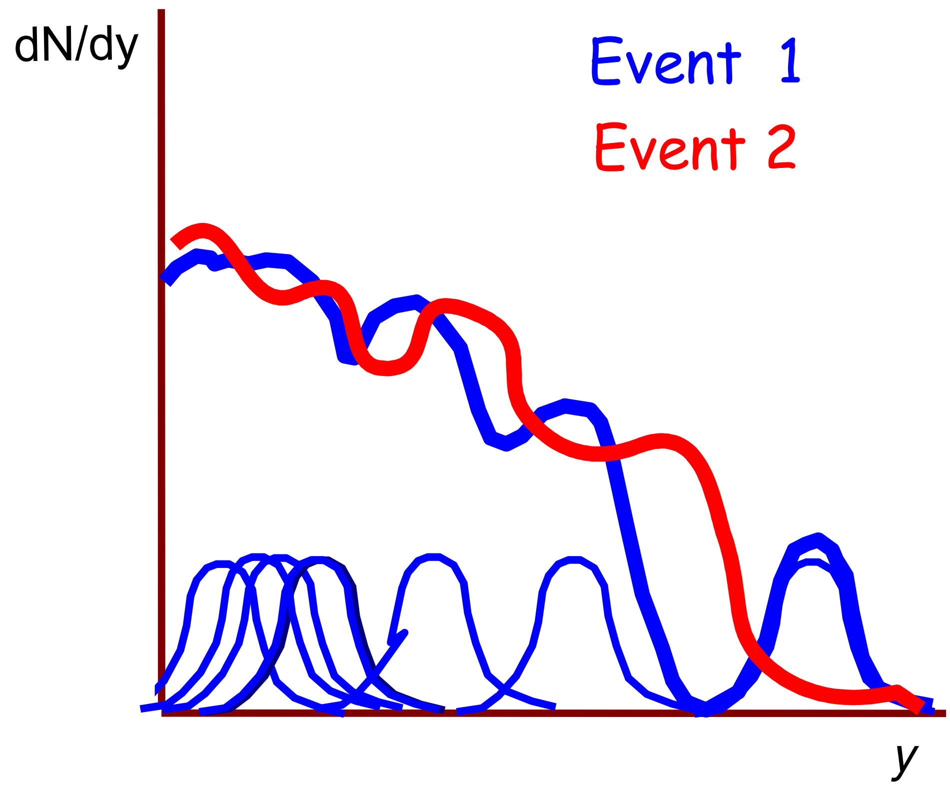
Fragmentation via

1) spinodal decomposition

2) rise of bulk viscosity

rapidity distribution in a single event

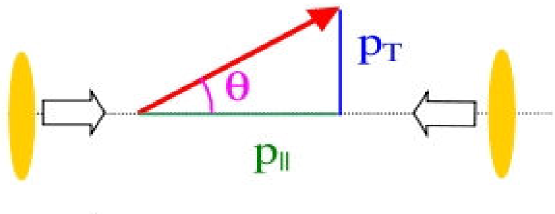
No droplets



Droplets

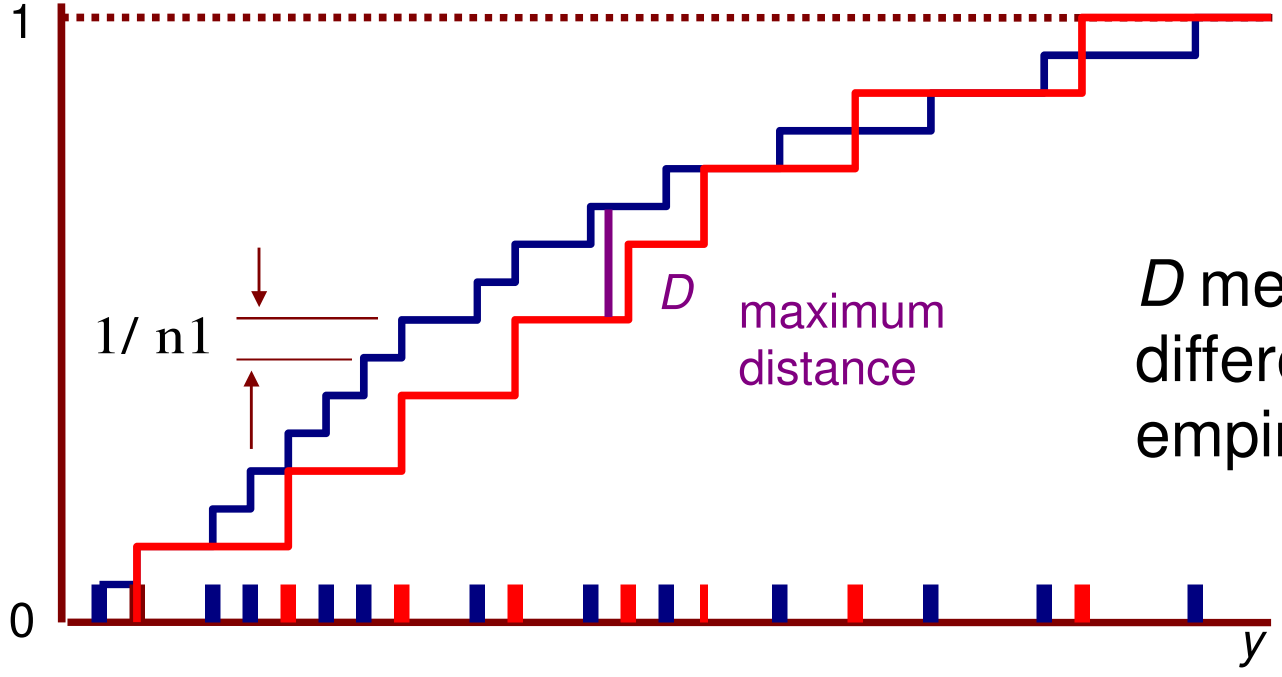
If we have droplets, each event will look differently

Rapidity: $y = \frac{1}{2}\ln\left(\frac{E + p_{\parallel}}{E - p_{\parallel}}\right)$



The measure of difference between events

Kolmogorov–Smirnov test:
Are two empirical distributions generated from *the same underlying probability distribution?*



D measures the difference of two empirical distributions

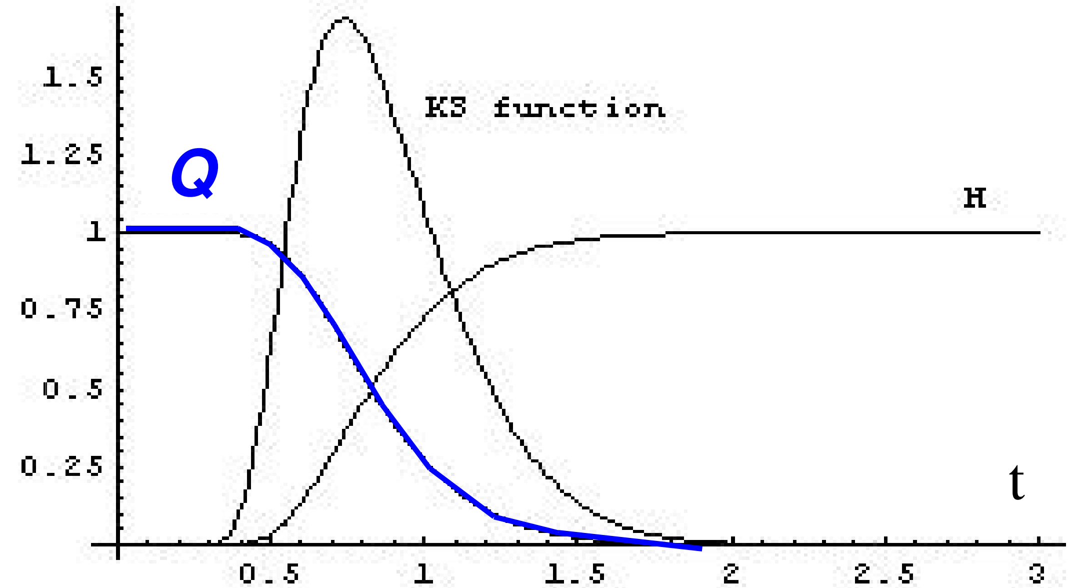
Kolmogorov-Smirnov: theorems

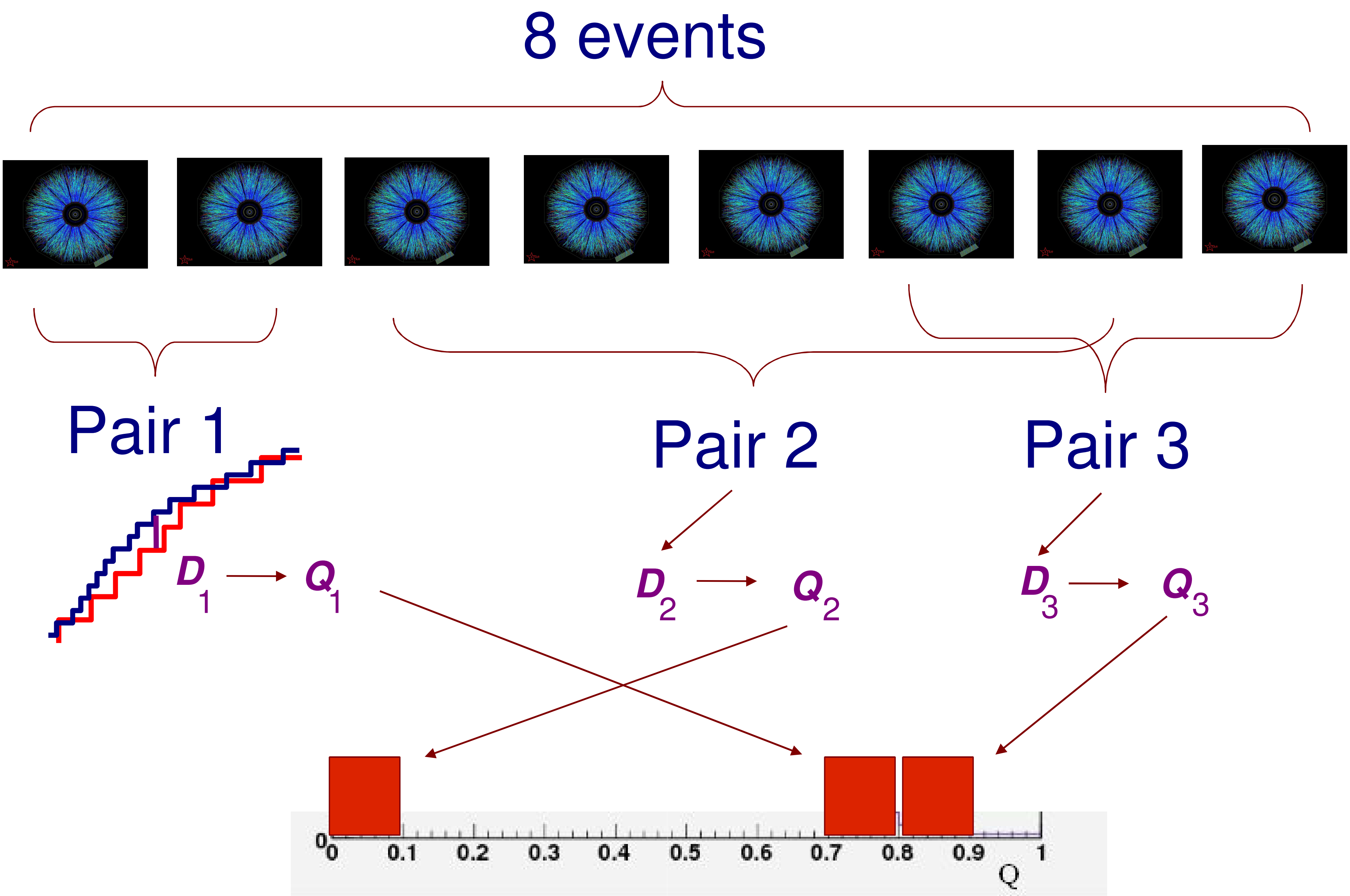
How are the *D's distributed?*

Smirnov (1944):
If we have two sets of data generated independently from the same underlying distribution, then D's are distributed according to

$$Q = \lim_{n_1, n_2 \to \infty} P(\sqrt{n} D > t) = 1 - \sum_{k=-\infty}^{\infty} (-1)^k \exp\left(-2k^2 t^2\right) \quad \text{with} \quad n = \frac{n_1 n_2}{n_1 + n_2}$$

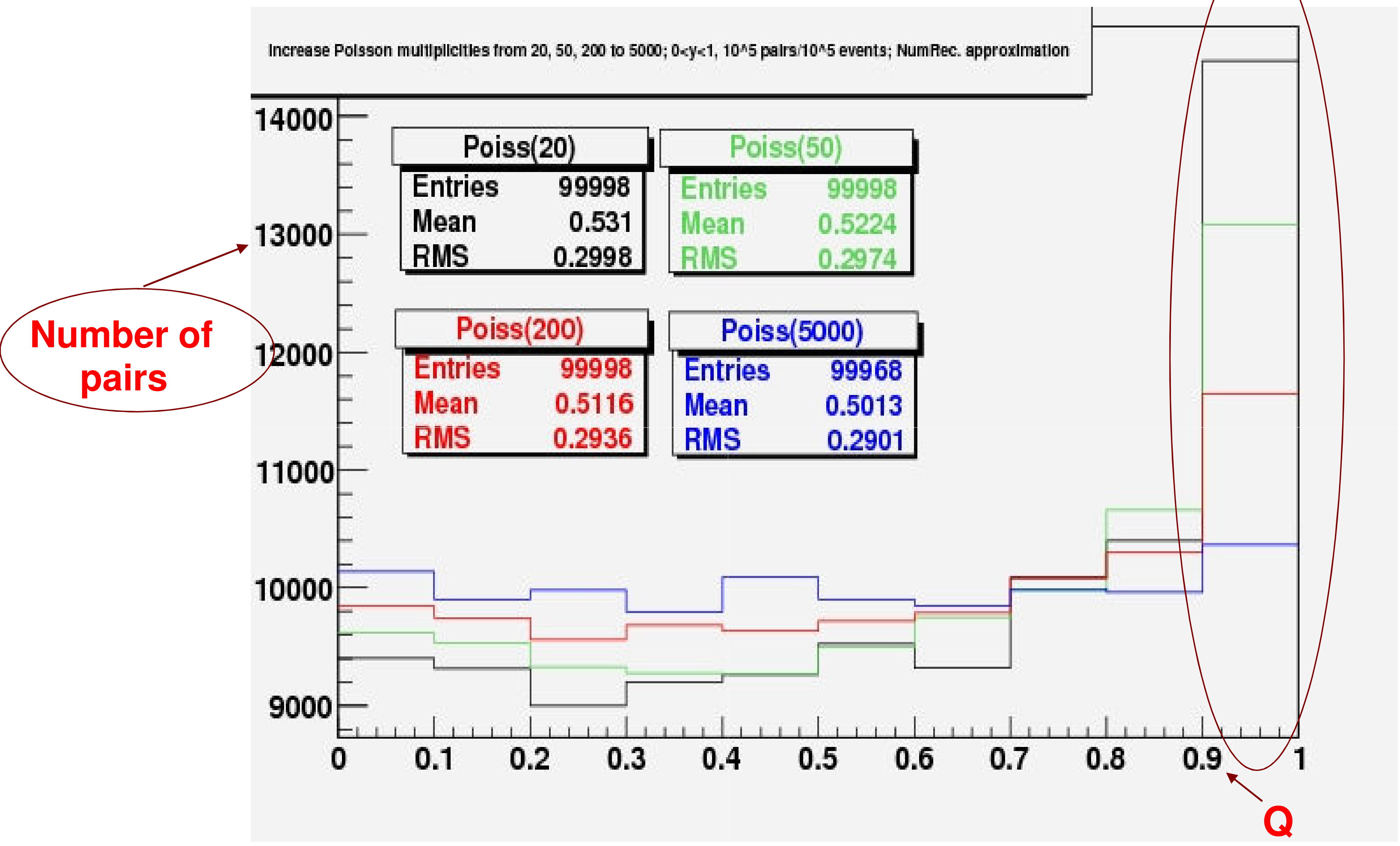
This is independent from the underlying distribution!





For events generated from the same distribution, Q's will be distributed ***uniformly*.**

Rapidities uniform on (0,1), Poisson multiplicities 20 - 1000



Statistical fluctuations ~ √(Number of pairs per bin)

Droplet generator DRAGON

MC generator of (momenta and positions of) particles emitted from a fragmented fireball

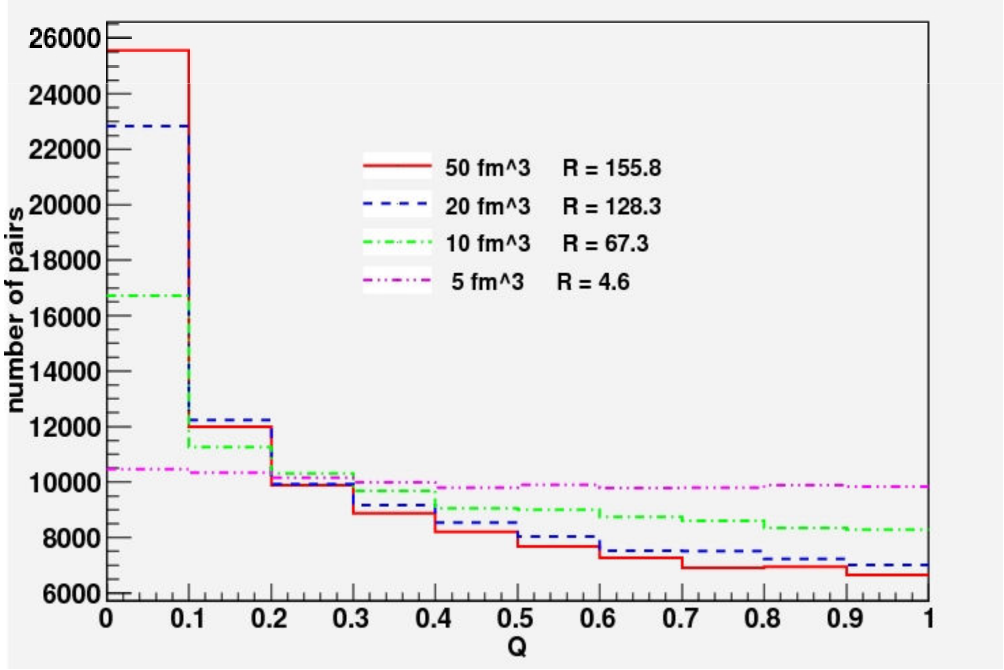
B. Tomášik, arXiv:0806.4770 [nucl-th], published in Computer Physics Communications.

- some particles are emitted from droplets (clusters)
- droplets are generated from a blast-wave source (tunable parameters)
- droplets decay exponentially in time (tunable time, T)
- tunable size of droplets
- no overlap of droplets
- also directly emitted particles (tunable amount)
- chemical composition: equilibrium with tunable params.

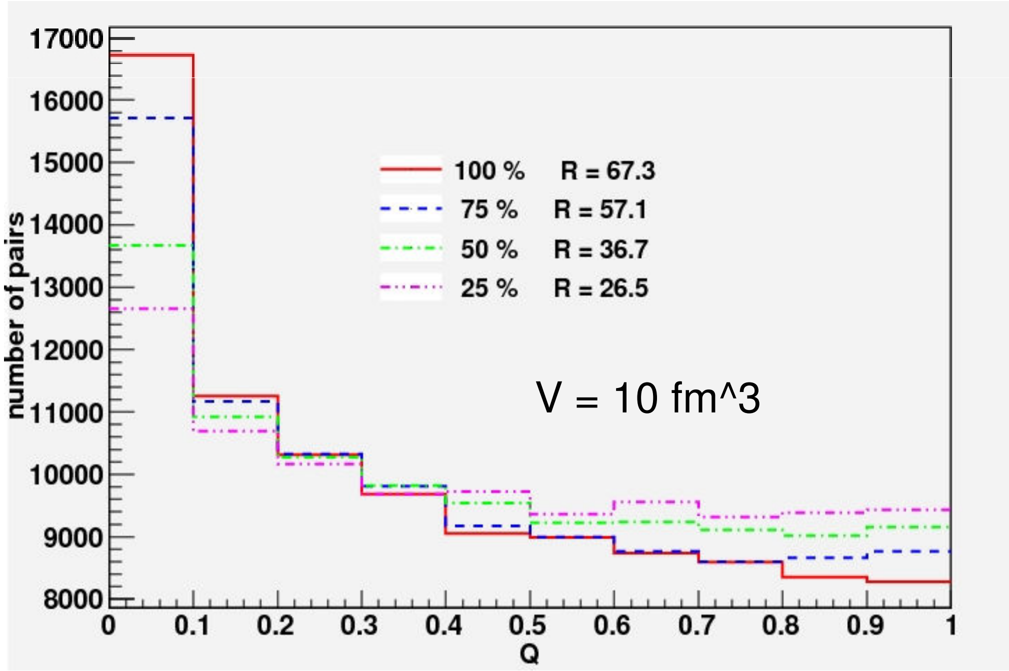
rapidity KS test, y=(-0.5, 0.5), RHIC

1[st] bin: $$R = \frac{N_0 - \frac{N_{\rm tot}}{B}}{\sigma_0} = \frac{N_0 - \frac{N_{\rm tot}}{B}}{\sqrt{\frac{N_{\rm tot}}{B}}}$$

Droplets of different size



Percentage of particles from droplets

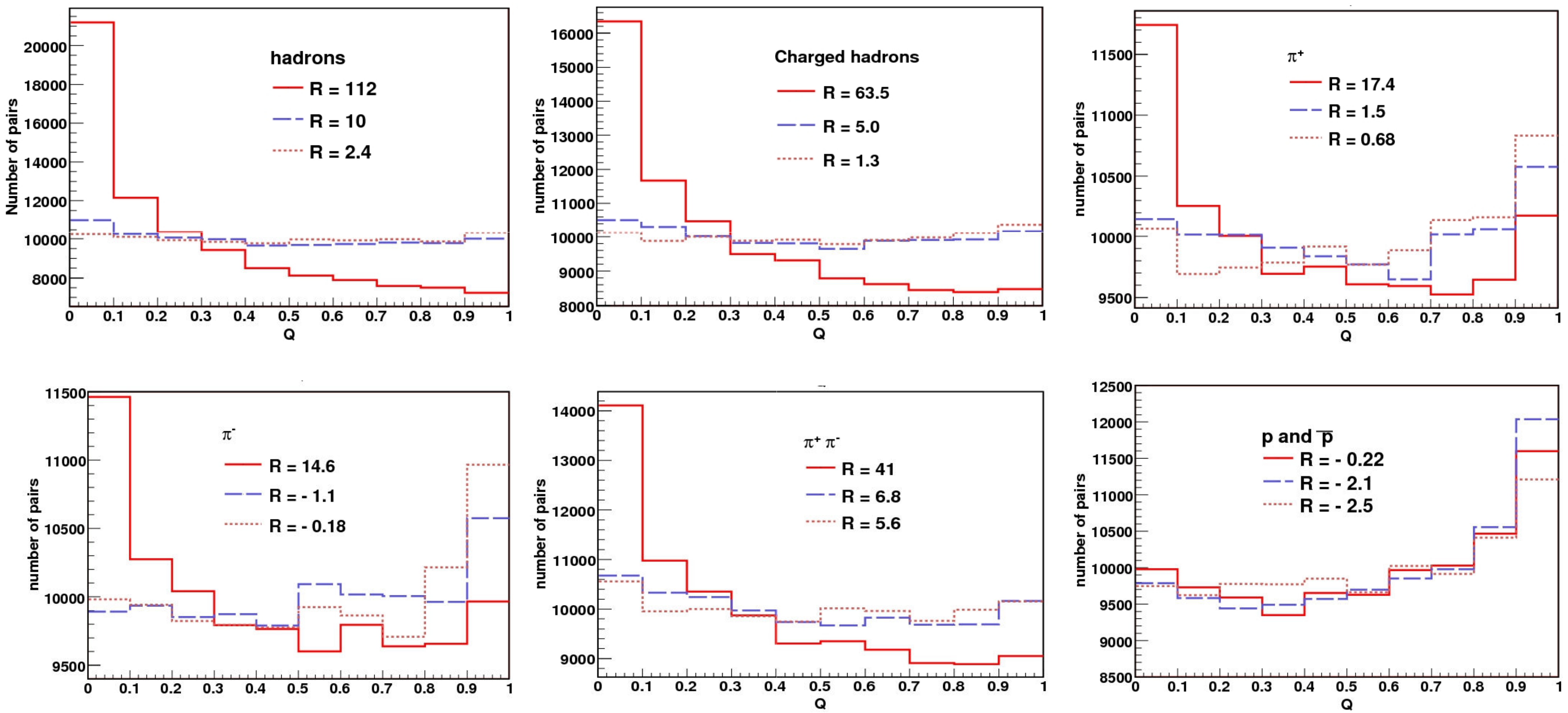


Study of different particle species

rapidity KS test, y=(-0.5, 0.5)   V = 10 fm^3

RHIC Au+Au collisions at $\sqrt{s} = 200\ AGeV$ — droplets

FAIR Au+Au at $\sqrt{s} = 7.6\ AGeV$ — no droplets



I. Melo, B. Tomasik, et.al. Phys.Rev.C80:024904,2009.

Energy and momentum conservation

(preliminary)

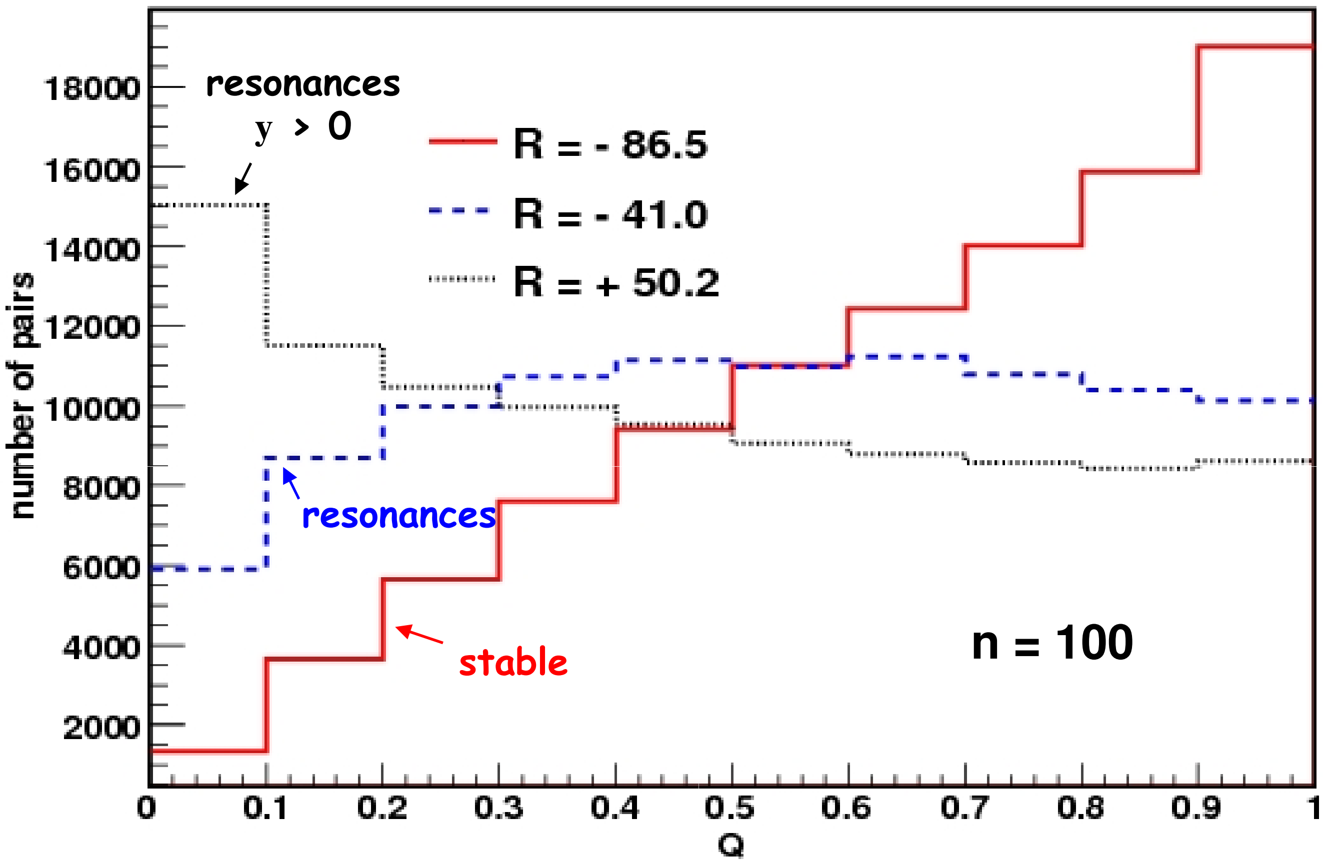


Fig. 3. Q-histograms demonstrating the 4-momentum conservation: for stable particles (solid red line), for resonances decaying into two daughters (dashed blue line), and decayed resonances with a cut $y > 0$ (dotted green line).

Conclusions

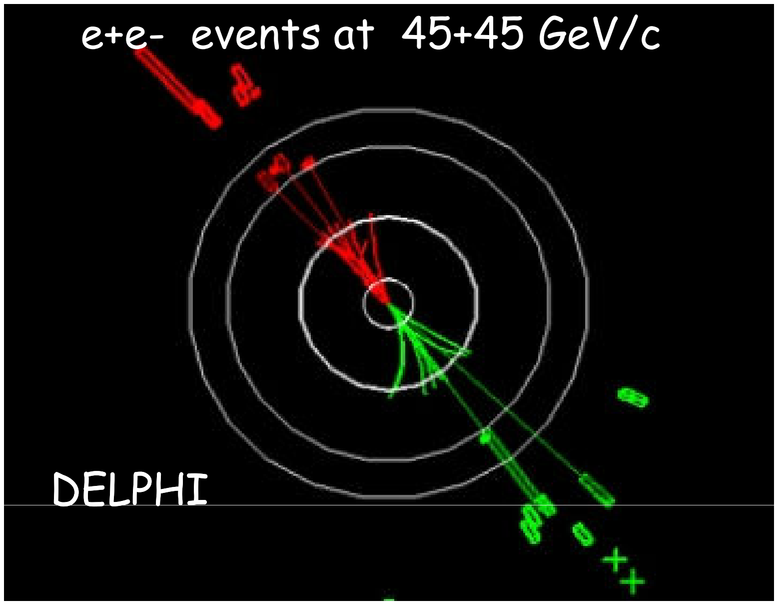
KS test is a sensitive tool in the search for fluctuations

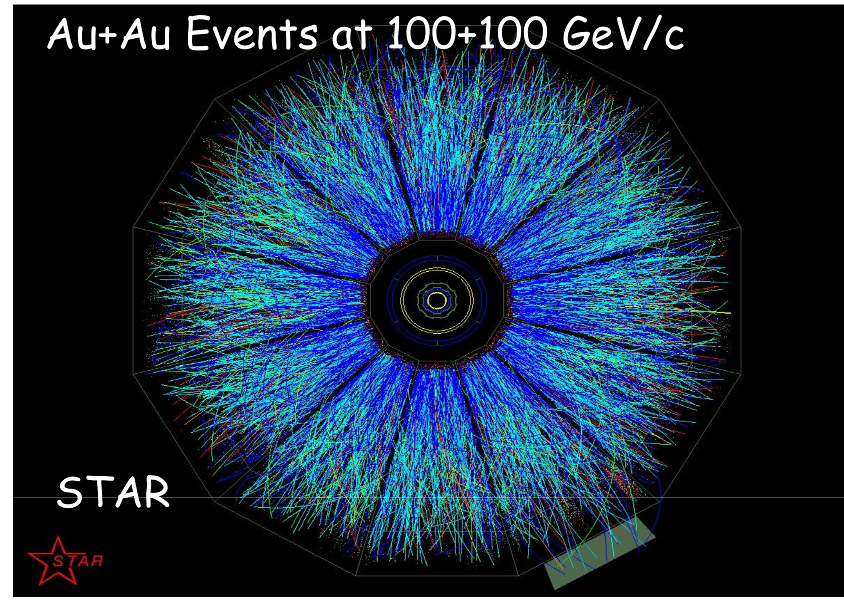
Identify sources of fluctuations other than droplets

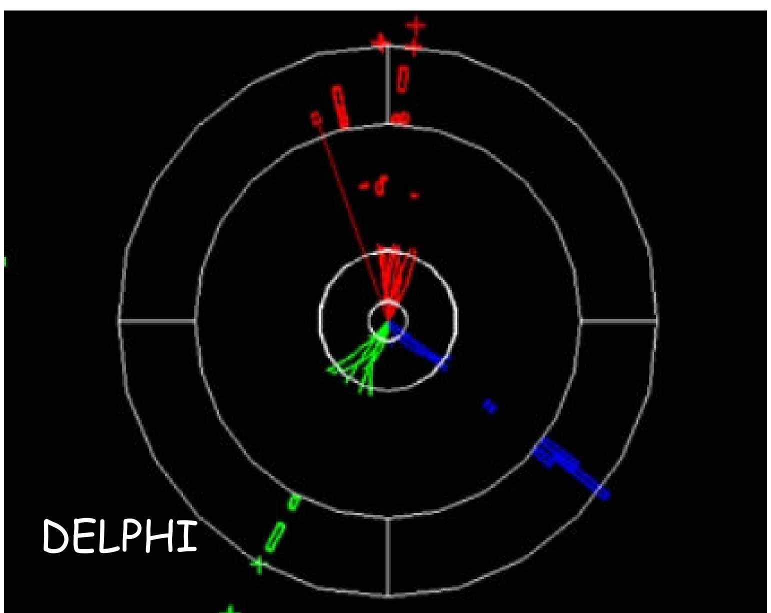
Back-up slides

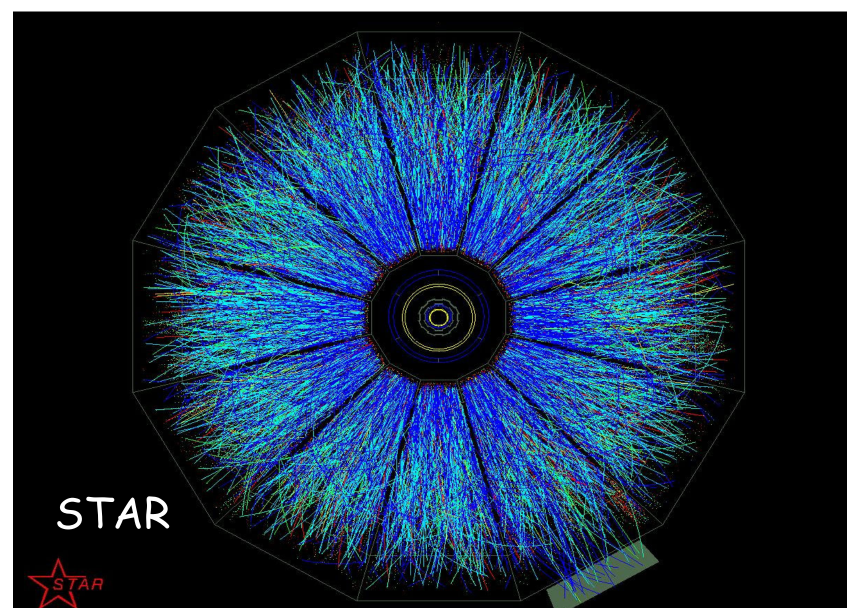
e+e-, LEP

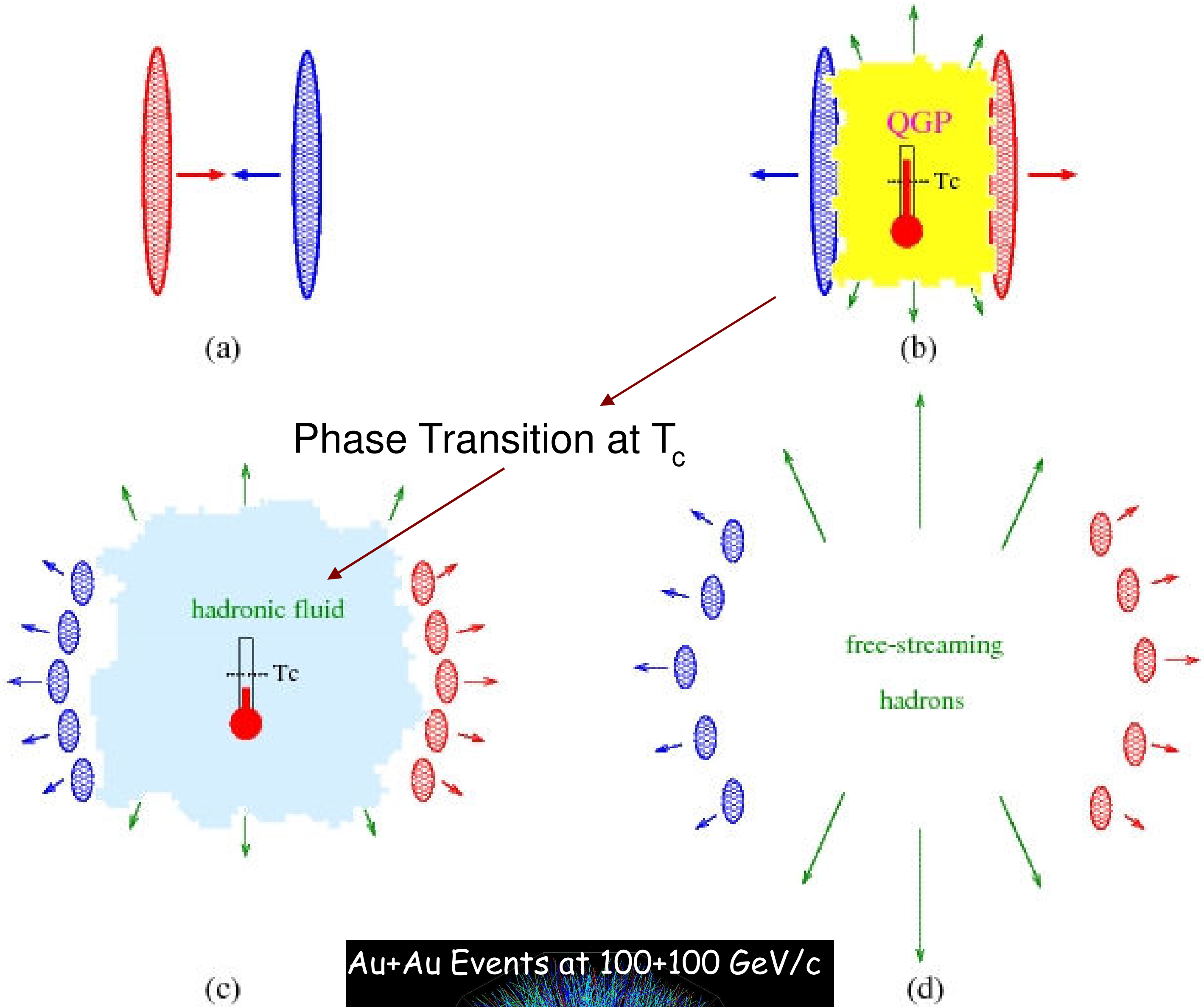
Heavy ion, RHIC

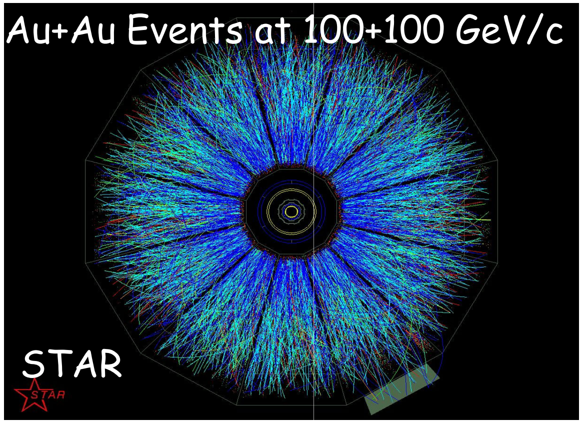








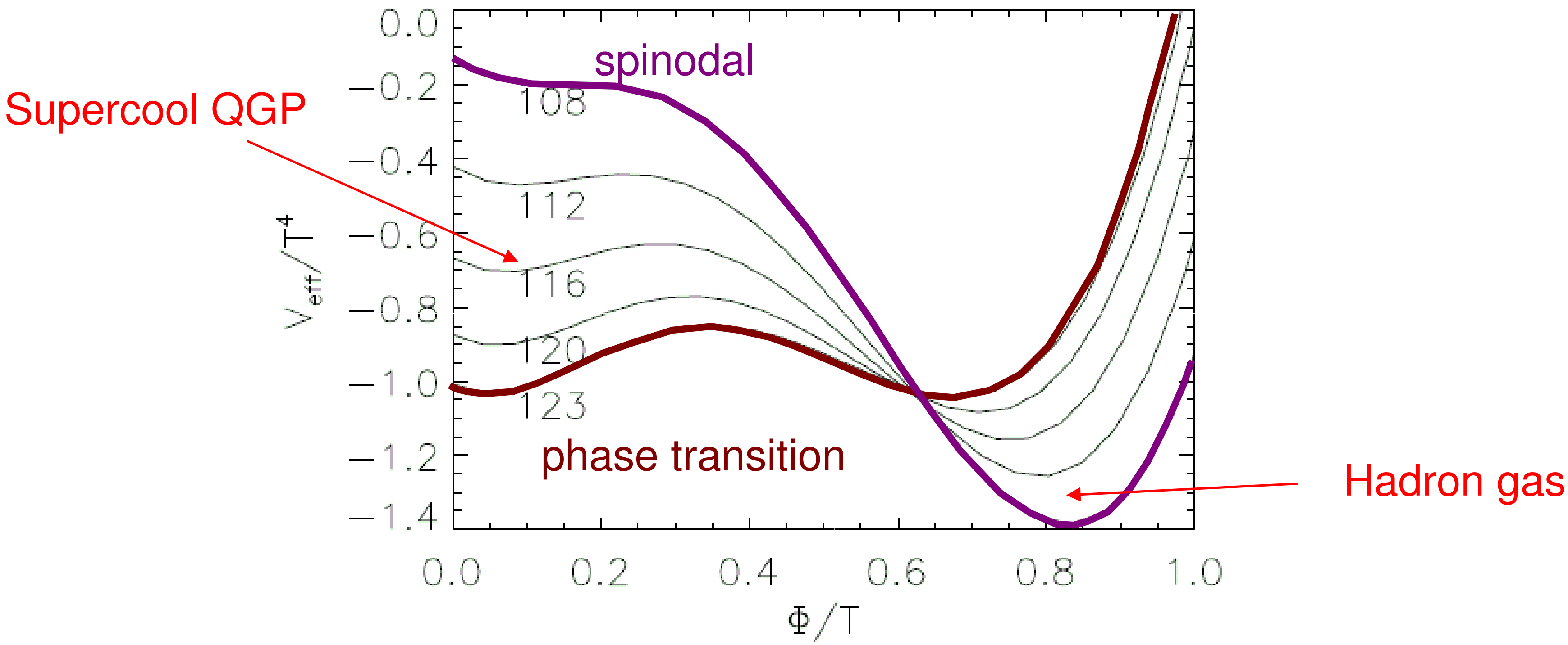

QGP
Tc
(a)
(b)
Phase Transition at T_c
hadronic fluid
Tc
free-streaming
hadrons
(c)
(d)


Au+Au Events at 100+100 GeV/c
STAR

RHIC

Spinodal fragmentation at phase transition

Example: linear sigma model coupled to quarks
[O. Scavenius *et al.*, Phys. Rev. D **63** (2001) 116003]

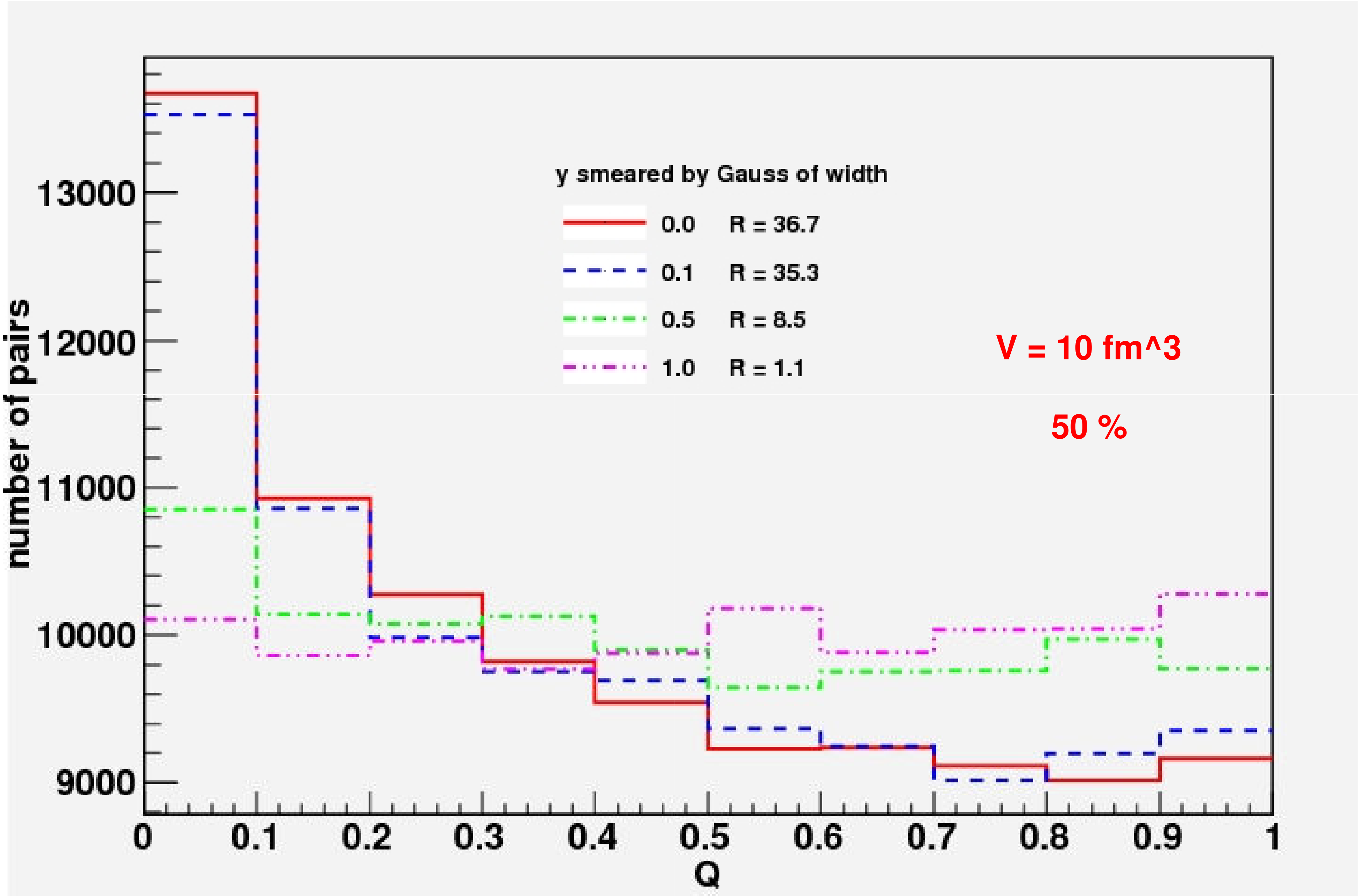


What is fast?

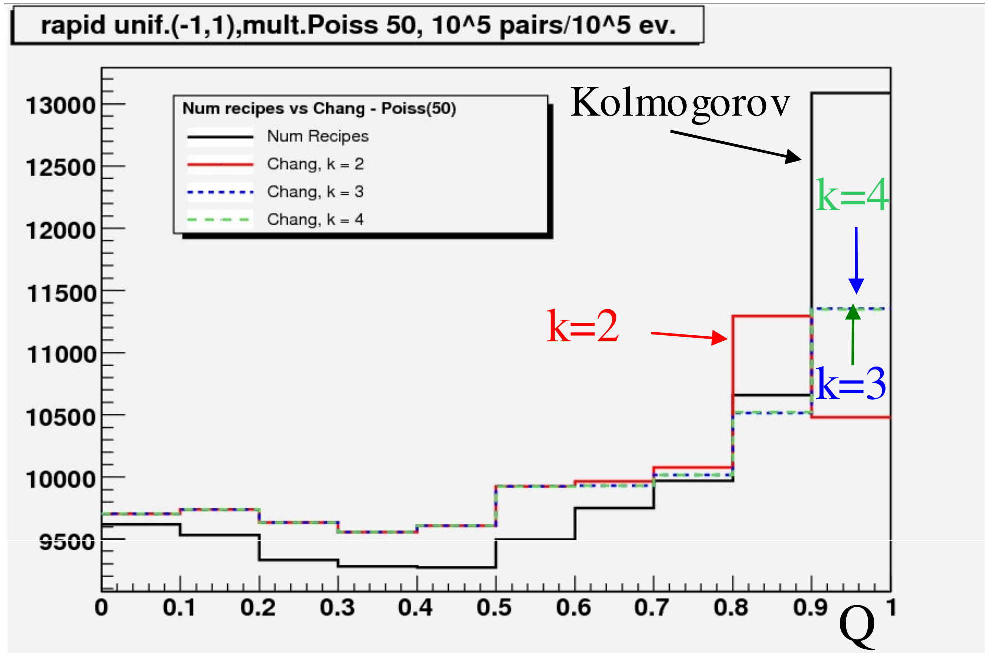
bubble nucleation rate < expansion rate

$\Gamma = T^4 \exp(- F/T)$ $\qquad$ H

Effect of finite experimental resolution

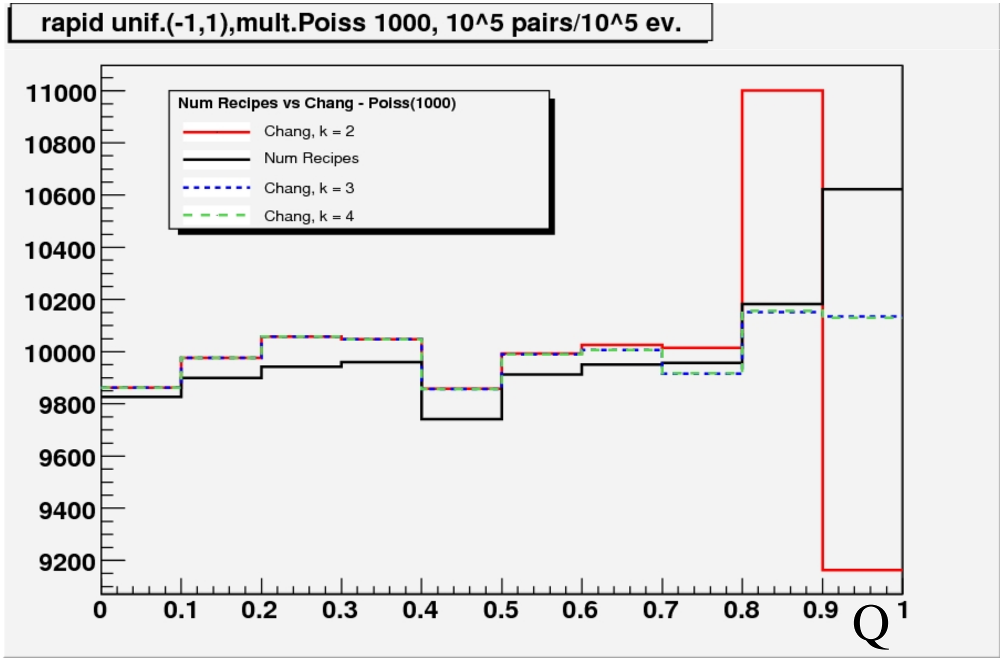


Comparison of Kolmogorov formula with improved one for finite multiplicity n (Li-Chien, 1956)



Kolmogorov

$$d = \sqrt{n}D = \sqrt{\frac{n_1 n_2}{n_1 + n_2}}D$$



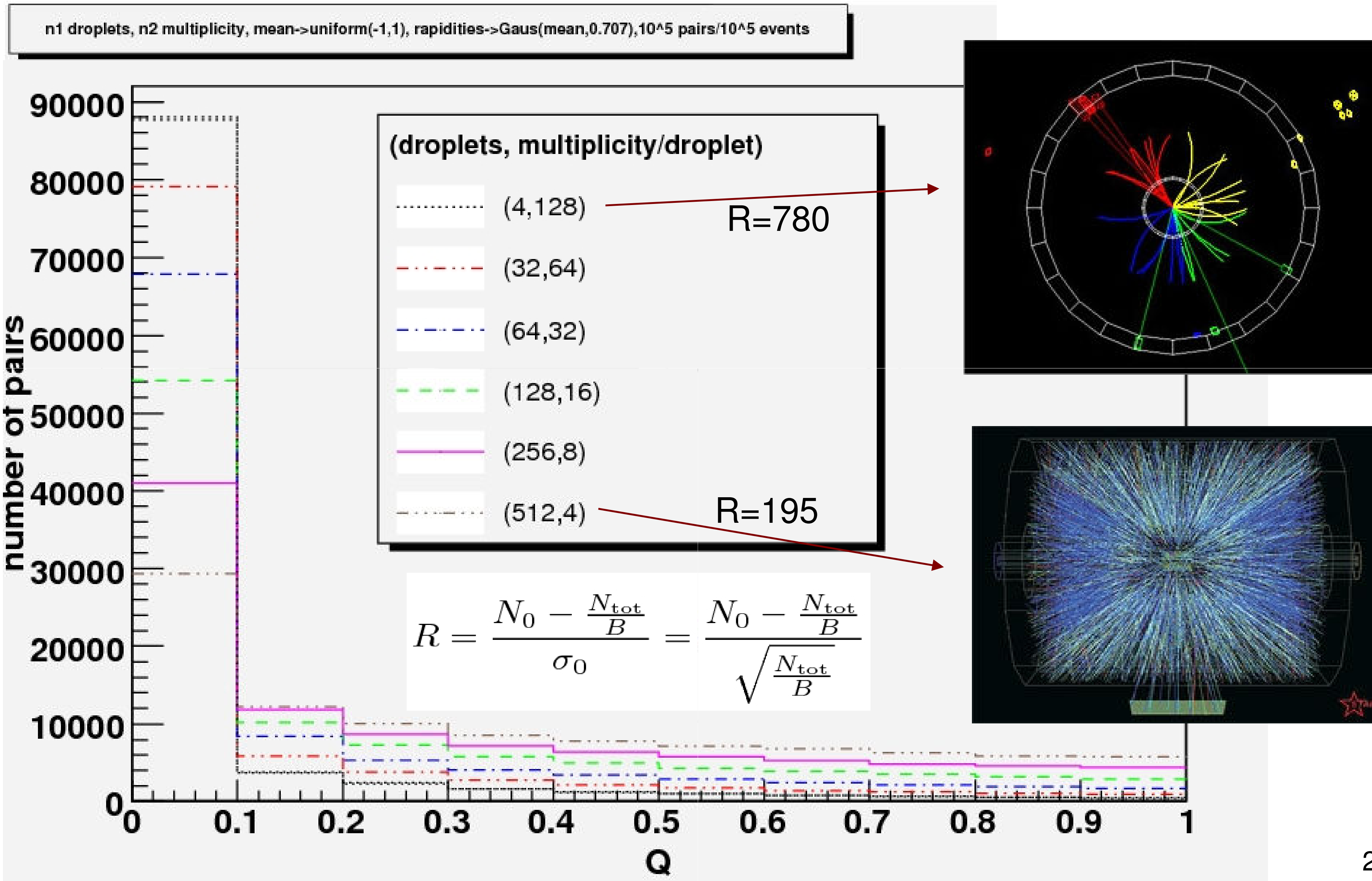
Li-Chien

$$Q(d) = K_0(d) + K_1(d) + K_2(d) + K_3(d) + \ldots.$$

$$K_1(d) = \frac{4d}{3\sqrt{n}} \sum_{k=1}^{\infty} (-1)^k k^2 \exp\left(-2k^2 d^2\right)$$

$$K_2(d) = \frac{1}{9n} \sum_{k=1}^{\infty} (-1)^k \left(k^2 - \frac{1}{2}\left(1 - (-1)^k\right) - 4k^2 d^2 \left(k^2 - \frac{1}{2}\left(1-(-1)^k\right) + 3 \right) + 8k^4 d^4 \right) \times \exp\left(-2k^2 d^2\right)$$

Toy droplets: decreasing number of droplets



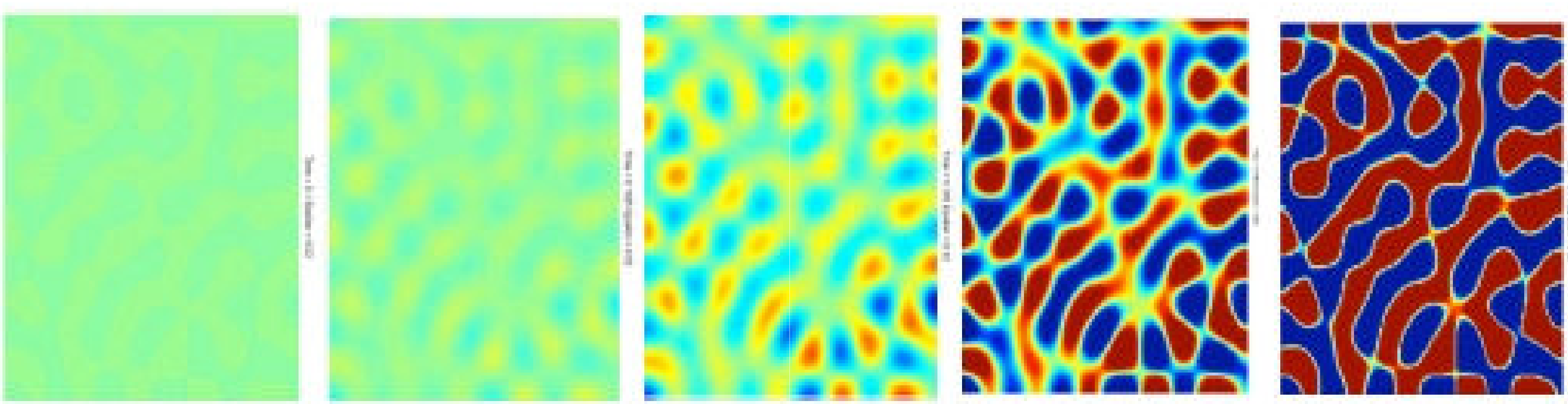
Spinodal decomposition - from Wikipedia, the free encyclopedia

Spinodal decomposition is a process by which a mixture of two materials can separate into distinct regions with different material concentrations. [This differs from *nucleation* in that spinodal phase separation occurs throughout the material, not just at nucleation sites.]

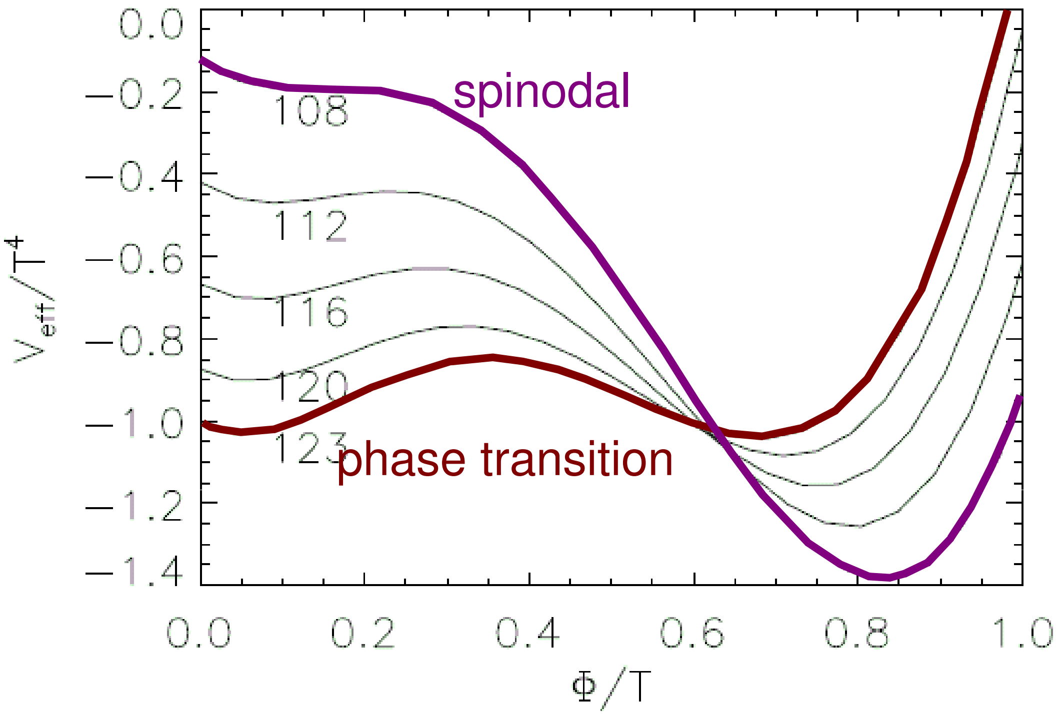
Phase separation may occur whenever a material finds itself in the *thermodynamically unstable region* of the phase diagram. The boundary of this unstable region (the *binodal*) is signaled by a common tangent of the *thermodynamic potential*. Inside the binodal boundary, the *spinodal region* is entered when the curvature of the potential turns negative. The binodal and spinodal meet at the *critical point*. It is when a material is brought into the spinodal phase region that spinodal decomposition can occur.

To reach the spinodal region of the phase diagram, the system must be brought through the binodal region where nucleation may occur. For spinodal decomposition to be realized, a very fast transition (a *quench*) is required to evolve the system from the stable region through the meta-stable nucleation region and well into the mechanically unstable spinodal phase region.

In the spinodal phase region, the thermodynamics favors spontaneous separation of the components. But large regions will change their concentrations only slowly due to the amount of material that must be moved, and small regions will shrink away due to the energy cost of the *interface* between the two different component materials. Thus domains of a characteristic *spinodal length* scale will be favored and since the growth is exponential, such domain sizes will come to dominate the morphology in the course of the associated *spinodal time*.



Spinodal fragmentation at phase transition



Example: linear sigma model coupled to quarks
[O. Scavenius *et al.,* Phys. Rev. D **63** (2001) 116003]

$$L = q[i\,\gamma^\mu \partial_\mu - g\,(\sigma + i\,\gamma_5\,\boldsymbol{\tau}.\boldsymbol{\pi})]\,q + \tfrac{1}{2}\,(\partial_\mu \sigma\, \partial^\mu \sigma + \partial_\mu \boldsymbol{\pi} . \partial^\mu \boldsymbol{\pi}) - U(\sigma, \boldsymbol{\pi})$$

where

$$U(\sigma, \boldsymbol{\pi}) = \lambda^2/4\,(\sigma^2 + \boldsymbol{\pi}^2 - v^2)^2 - h_q\,\sigma$$

$$q = (u,d)$$

$$\boldsymbol{\pi} = (\pi_1, \pi_2, \pi_3)$$

$$\Phi = (\sigma, \boldsymbol{\pi})$$

$$V_{eff} = U(\sigma, \boldsymbol{\pi}) + V_q$$

$$V_q(\Phi) = -d_q T \int \frac{d^3k}{(2\pi)^3} \log\left(1 + e^{-E/T}\right) \quad .$$

What is fast?

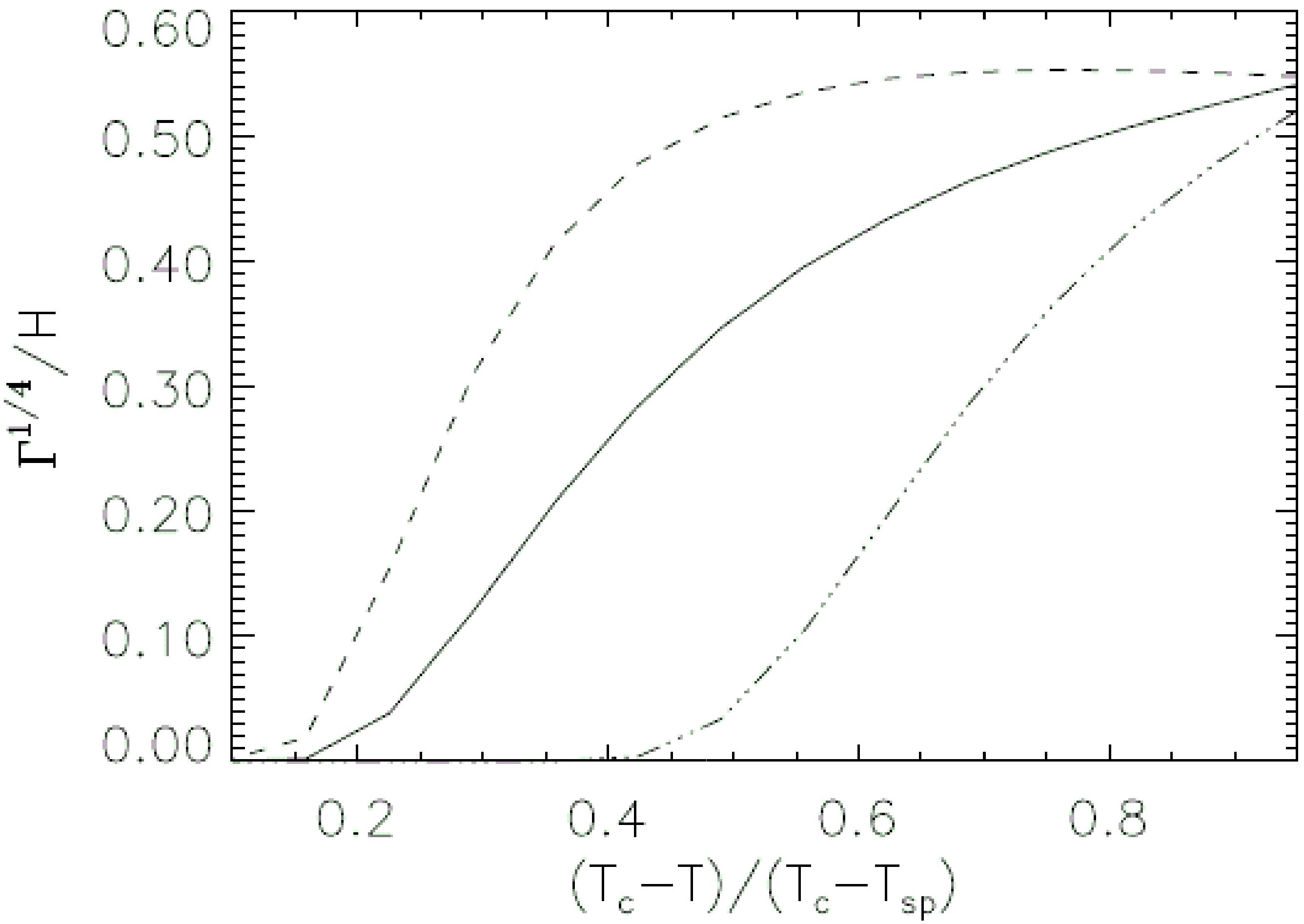
bubble nucleation rate < expansion rate

$\Gamma = T^4 \exp(- F/T)$ $\qquad\qquad$ H

Supercooling down to spinodal

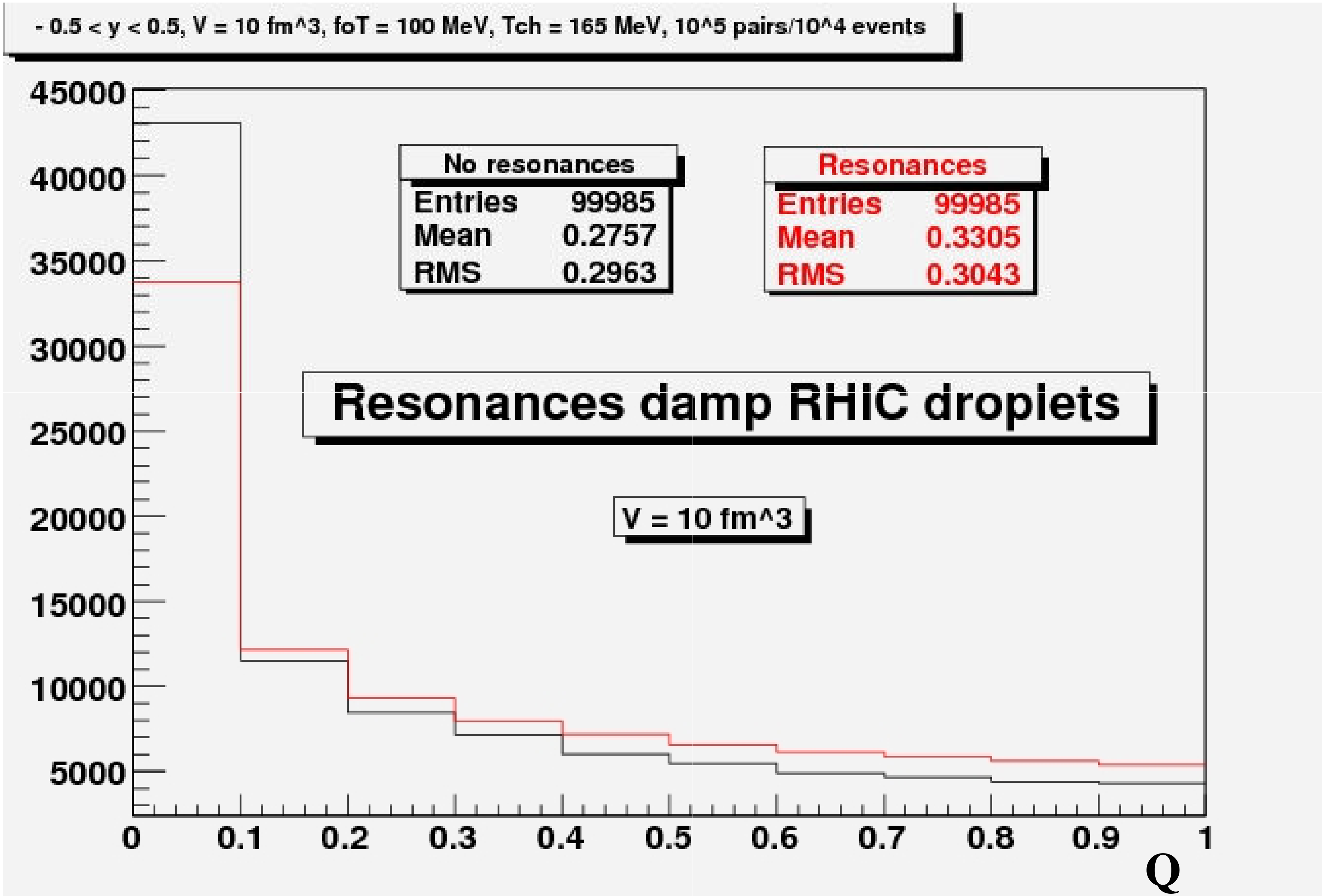
O. Scavenius *et al., Phys. Rev. D* ***63 (2001) 116003:***

compare nucleation rate with Hubble constant (1D)



likely to reach spinodal

Introducing resonances into generator



Rapidity cuts

V = 10 fm^3