A potential including Heaviside function in 1+1 dimensional Landau hydrodynamics

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- Introduction
- Landau model and three solutions
- Data analyses of hadrons (RHIC π, K)
- Explanation of net-proton
- Summary

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Introduction

- 1+1 dimensional hydrodynamics proposed by Landau
  - Landau's solution (1953)
  - A boost non-invariant solution by Srivastava et al. (1993)
  - Our solution including Heviside function (2008, 9)
- Three solutions cannot explain the net-proton at RHIC
  - New approach (2009)
  - Preliminary results on net-proton

Perfect fluid

Quantities of fluid:

energy density e, pressure p, four velocity u^μ, entropy density s, speed of sound c_s

energy-momentum tensor $T^{\mu\nu} = (e+p)u^\mu u^\nu - pg^{\mu\nu}$

Conservation laws of energy-momentum and entropy

$$\frac{\partial T^{\mu\nu}}{\partial x^\mu} = 0, \quad \frac{\partial(su^\mu)}{\partial x^\mu} = 0$$

Thermodynamical relations and equation of state

$$de = T ds, \quad e + p = Ts, \quad p = c_s^2 e$$

(1+1) dimension

Rapidity of fluid y

$$(u^0,\ u^1) = (\cosh y,\ \sinh y)$$

1+1 dimensional conservation law of energy-momentum

$$\frac{\partial}{\partial t}(T \sinh y) + \frac{\partial}{\partial x}(T \cosh y) = 0$$

$$\left(\frac{\partial T}{\partial t} + c_s^2 \frac{\partial y}{\partial x}\right) \cosh y + \left(\frac{\partial T}{\partial x} + c_s^2 \frac{\partial y}{\partial t}\right) \sinh y = 0$$

Potential $\phi(t,\ x)$

$$d\phi = \frac{\partial \phi}{\partial t} dt + \frac{\partial \phi}{\partial x} dx = (-T \cosh y) dt + (T \sinh y) dx$$

Legendre transformation $\Longrightarrow$ Potential $\chi(T,\, y)$

$$\begin{aligned} d\chi &= d(\phi + Tt\cosh y - Tx\sinh y) \\ &= \frac{\partial \chi}{\partial T} dT + \frac{\partial \chi}{\partial y} dy \\ &= (t\cosh y - x\sinh y)dT + T(t\sinh y - x\cosh y)dy \end{aligned}$$

Solving about t and x

$$\begin{aligned} t &= \frac{\partial \chi}{\partial T}\cosh y - \frac{1}{T}\frac{\partial \chi}{\partial y}\sinh y \\ x &= \frac{\partial \chi}{\partial T}\sinh y - \frac{1}{T}\frac{\partial \chi}{\partial y}\cosh y \end{aligned}$$

1+1 dimensional conservation law of the entropy

$$\frac{\partial}{\partial t}(s\cosh y) + \frac{\partial}{\partial x}(s\sinh y) = 0$$

$(t,\ x) \rightarrow (T,\ y)$

$$\frac{1}{c_s^2 T}\left(\frac{\partial\chi}{\partial T} - \frac{1}{T}\frac{\partial^2\chi}{\partial y^2}\right) + \frac{\partial^2\chi}{\partial T^2} = 0$$

Introducing $\omega = -\ln(T/T_0)$ (T_0 initial temperature)

$$\frac{\partial^2\chi}{\partial y^2} = c_s^2\frac{\partial^2\chi}{\partial\omega^2} + (1-c_s^2)\frac{\partial\chi}{\partial\omega} \quad \text{(telegraph equation)}$$

Entropy distribution

$$\frac{dS}{dy} \sim \frac{\partial}{\partial\omega}\left(\chi + \frac{\partial\chi}{\partial\omega}\right)\Bigg|_{\omega=\omega_f}$$

with $\omega_f = -\ln(T_f/T_0)$ (T_f is final temperature)

Landau's solution

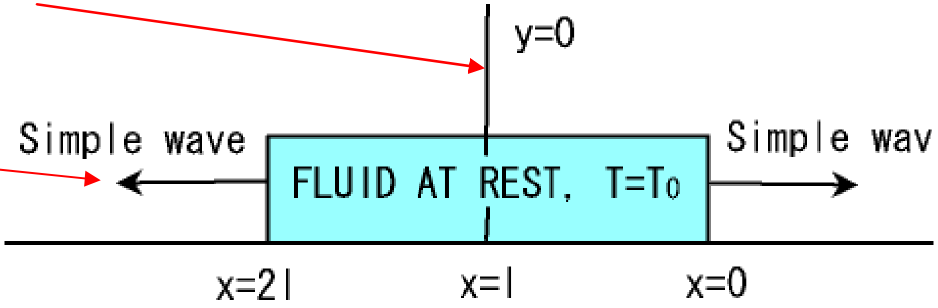
L. D. Landau, Izv. Akad. Nauk Ser. Fiz. **17**, 51 (1953)

Initial conditions

$$\left.\frac{\partial \chi}{\partial y}\right|_{y=0} = T_0 l e^{\omega} \ \ (y = 0 \text{ at } x = l)$$

$$\chi(y = \omega/c_s) = 0$$

with l is length of colliding matter



Solution

$$\chi \sim e^{-\omega} \int_{yc_s}^{\omega} e^{(1+\beta)\omega'} I_0\left(\beta\sqrt{\omega'^2 - c_s^2 y^2}\right) d\omega', \quad \beta = (1 - c_s^2)/2c_s^2$$

$(\omega^2 > y^2 c_s^2)$

Entropy distribution

$$\frac{dS}{dy} \sim e^{\beta\omega_f}\left[I_0(p) + \frac{\beta\omega_f}{p} I_1(p)\right], \quad p = \beta\sqrt{\omega_f^2 - c_s^2 y^2}$$

I_0 and I_1 are the modified Bessel functions of the 1st and 2nd order.

Solution by Srivastava *et al.*

D. K. Srivastava *et al.*, Annals Phys. **228**, 104 (1993)

Writing $\chi = \chi_1 \exp(\beta\omega)$

$$\frac{\partial^2 \chi_1}{\partial y^2} = c_s^2 \frac{\partial^2 \chi_1}{\partial \omega^2} - c_s^2 \beta^2 \chi_1 .$$

Using $q = \tanh^{-1}\left(\frac{c_s y}{\omega}\right)$, $p = \beta\sqrt{\omega^2 - c_s^2 y^2}$

$$\frac{1}{p^2}\frac{\partial^2 \chi_1}{\partial q^2} - \frac{\partial^2 \chi_1}{\partial p^2} - \frac{1}{p}\frac{\partial \chi_1}{\partial p} + \chi_1 = 0$$

Condition $\frac{\partial \chi_1}{\partial q} = 0$

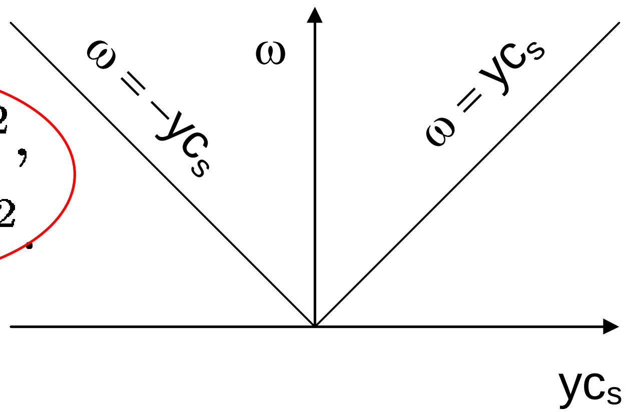
$$\cancel{\frac{1}{p^2}\frac{\partial^2 \chi_1}{\partial q^2}} - \frac{\partial^2 \chi_1}{\partial p^2} - \frac{1}{p}\frac{\partial \chi_1}{\partial p} + \chi_1 = 0 \ \text{(Bessel's equation)}$$

Initial condition Finiteness at $p = 0$

Solution

$$\chi \sim \begin{cases} e^{\beta\omega} I_0(\beta\sqrt{\omega^2 - c_s^2 y^2}) & \text{for } \omega^2 > c_s^2 y^2, \\ e^{\beta\omega} J_0(\beta\sqrt{c_s^2 y^2 - \omega^2}) & \text{for } \omega^2 < c_s^2 y^2. \end{cases}$$

Bessel function



Entropy distributions

$$\frac{dS}{dy} \sim \left[\beta(\beta+1)I_0(p) + \frac{\beta^2(p^2 + p^2(2\beta+1)\omega - \beta^2\omega^2)I_0'(p)}{p^3} + \frac{\beta^4\omega^2 I_0''(p)}{p^2}\right] \quad \text{for } \omega^2 > c_s^2 y^2$$

$$\frac{dS}{dy} \sim \left[\beta(\beta+1)J_0(\tilde{p}) + \frac{\beta^2(\tilde{p}^2 + \tilde{p}^2(2\beta+1)\omega - \beta^2\omega^2)J_0'(\tilde{p})}{\tilde{p}^3} + \frac{\beta^4\omega^2 J_0''(\tilde{p})}{\tilde{p}^2}\right] \quad \text{for } \omega^2 < c_s^2 y^2$$

with $\tilde{p} = \beta\sqrt{c_s^2 y^2 - \omega^2}$ and $\omega = \omega_f$

Our analytical solution with the Heaviside function

Mizoguchi, Miyazawa, Biyajima, Eur. Phys. J. A **40**, 99 (2009)

Method of Green functions

$$\frac{\partial^2\chi}{\partial\omega^2} + \left(\frac{1}{c_s^2} - 1\right)\frac{\partial\chi}{\partial\omega} - \frac{1}{c_s^2}\frac{\partial^2\chi}{\partial y^2} = \frac{1}{c_s^2}\delta(\omega-\omega_0)\delta(y-y_0). \quad (\omega_0,\ y_0 \to 0)$$

Writing $\chi = \chi_1 \exp(\beta\omega)$

General solution (Riemann's formula)

$$\begin{aligned}\chi_1(y,\omega) &= \frac{1}{2}\{g(y+\omega/c_s) + g(y-\omega/c_s)\} \\ &+\frac{1}{2}\int_{-\omega/c_s}^{\omega/c_s}\Big\{c_s G(z+y) I_0(\beta\sqrt{\omega^2 - c_s^2 z^2}) \\ &\qquad +\beta^2 c_s \omega g(z+y)\frac{I_1(\beta\sqrt{\omega^2-c_s^2z^2})}{\beta\sqrt{\omega^2-c_s^2z^2}}\Big\}dz\end{aligned}$$

with $g(y) = \chi_1(\omega = 0+,\, y),\ G(y) = \left.\frac{\partial\chi_1}{\partial\omega}\right|_{\omega=0+}$

(Cf. D. G. Duffy, “Green‘s functions with applications”,
Ivar Stakgold, "Boundary Value Problems of Mathematical Physics“)

Initial conditions on $(\omega,\ y)$ space

$$G(y) = \chi_1(\omega = 0+,\ y) = \delta(y)$$

(created localized point $((\omega,\ y) \approx (0,\ 0))$

$$g(y) = \left.\frac{\partial \chi_1}{\partial \omega}\right|_{\omega=0+} = 0$$

(collision in the central region)

Solution

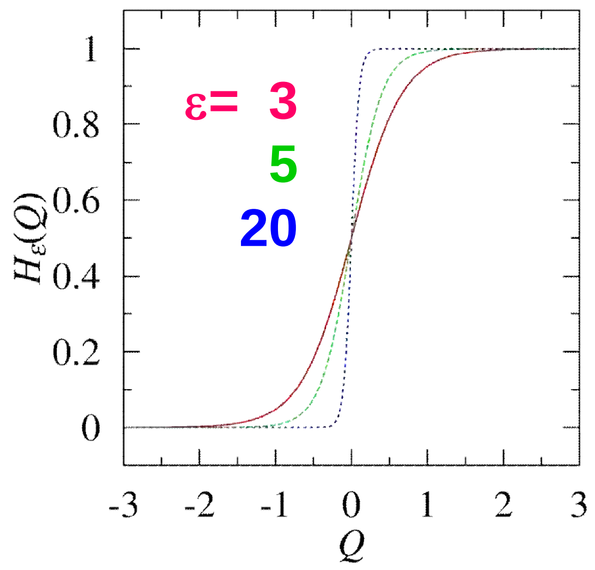
$$\chi \sim e^{\beta\omega} I_0\left(\beta\sqrt{\omega^2 - c_s^2 y^2}\right) H(Q = \omega - c_s|y|),$$

$H(Q)$ is the Heaviside function defined as

$$H(Q = \omega - c_s|y|) = \begin{cases} 1 & Q > 0 \\ 0 & Q < 0 \end{cases}$$

In concrete analyses, we use an approximate expression (sigmoid function):

$$H_\varepsilon(Q) = \frac{1}{e^{-\varepsilon Q} + 1}$$



Entropy distribution

$$\frac{dS}{dy} \sim \left[\beta(\beta+1)I_0(p) + \frac{\beta^2(p^2 + p^2(2\beta+1)\omega - \beta^2\omega^2)I_0'(p)}{p^3} + \frac{\beta^4\omega^2 I_0''(p)}{p^2}\right] H_\varepsilon(Q)$$
$$+ \left[(2\beta+1)I_0(p) + \frac{2\beta^2\omega I_0'(p)}{p}\right] H_\varepsilon'(Q) + I_0(p)H_\varepsilon''(Q)$$

with $p = \beta\sqrt{\omega^2 - c_s^2 y^2}$

When p is imaginary, the modified Bessel function is replaced by the Bessel functions.

Competition!!

arXiv.org > hep-ph > arXiv:0810.3550

High Energy Physics - Phenomenology

A potential including Heaviside function in 1+1 dimensional hydrodynamics by Landau

T. Mizoguchi, H. Miyazawa, M. Biyajima

(Submitted on 20 Oct 2008 (v1), last revised 27 Feb 2009 (this version, v2))

- **T. Mizoguchi and M. Biyajima, Genshikaku Kenkyu (in Japanese), Vol. 52 Suppl. 3 (Feb. 2008) 61.**
- **Our talk in Annual Meeting for Physical Society of Japan (Mar. 2008, Kinki Univ. (Osaka))**

arXiv.org > nucl-th > arXiv:0808.1073

Nuclear Theory

Entropy flow of a perfect fluid in (1+1) hydrodynamics

Guillaume Beuf, Robi Peschanski, Emmanuel N. Saridakis

(Submitted on 7 Aug 2008 (v1), last revised 19 Dec 2008 (this version, v2))

The same expression as our solution

Writing - Beuf, Peschanski, Saridakis, Phys.Rev.C78 (2008) 064909

$$\chi(\theta,y) = e^{-\left(\frac{g-1}{2}\right)\theta}\, Z(\theta,y) \tag{27}$$

and inserting it into (23), we acquire:

$$\partial_\theta^2 Z - g\,\partial_y^2 Z - \left(\tfrac{g-1}{2}\right)^2 Z = 0, \tag{28}$$

where we have used a compact notation for partial derivatives.

It is convenient to replace the variables θ and y by α and β, defined by

$$\alpha \equiv -\theta + \frac{y}{\sqrt{g}} \quad \text{and} \quad \beta \equiv -\theta - \frac{y}{\sqrt{g}}, \tag{29}$$

such that equation (28) takes the form

$$\partial_\alpha\partial_\beta \bar{Z}(\alpha,\beta) - \tfrac{(g-1)^2}{16}\, \bar{Z}(\alpha,\beta) = 0\,. \tag{30}$$

We solve this equation following the Green's functions formalism, i.e. we look for distributions $\bar{G}(\alpha,\beta)$ such that

$$\partial_\alpha\partial_\beta \bar{G}(\alpha,\beta) - \tfrac{(g-1)^2}{16}\, \bar{G}(\alpha,\beta) = \delta(\alpha)\delta(\beta)\,. \tag{31}$$

The relevant solution of equation (31) is[4]

$$\bar{G}(\alpha,\beta) = \Theta(\alpha)\ \Theta(\beta)\ I_0\left(\frac{g-1}{2}\sqrt{\alpha\beta}\right), \tag{32}$$

with I_0 the modified Bessel function of the first kind and Θ the Heaviside function. Using the relation

$$\delta(\alpha)\,\delta(\beta) \equiv \delta\left(-\theta + \frac{y}{\sqrt{g}}\right)\ \delta\left(-\theta - \frac{y}{\sqrt{g}}\right) = \sqrt{g}\ \delta(\theta)\ \delta(y)\,, \tag{33}$$

we deduce from (32) the relevant Green's function of (28):

$$G(\theta,y) = \frac{1}{4\sqrt{g}}\,\bar{G}(\alpha,\beta) = \frac{1}{4\sqrt{g}}\,\Theta\left(-\theta + \frac{y}{\sqrt{g}}\right)\ \Theta\left(-\theta - \frac{y}{\sqrt{g}}\right)\ I_0\left(\frac{g-1}{2}\sqrt{\theta^2 - \frac{y^2}{g}}\right). \tag{34}$$

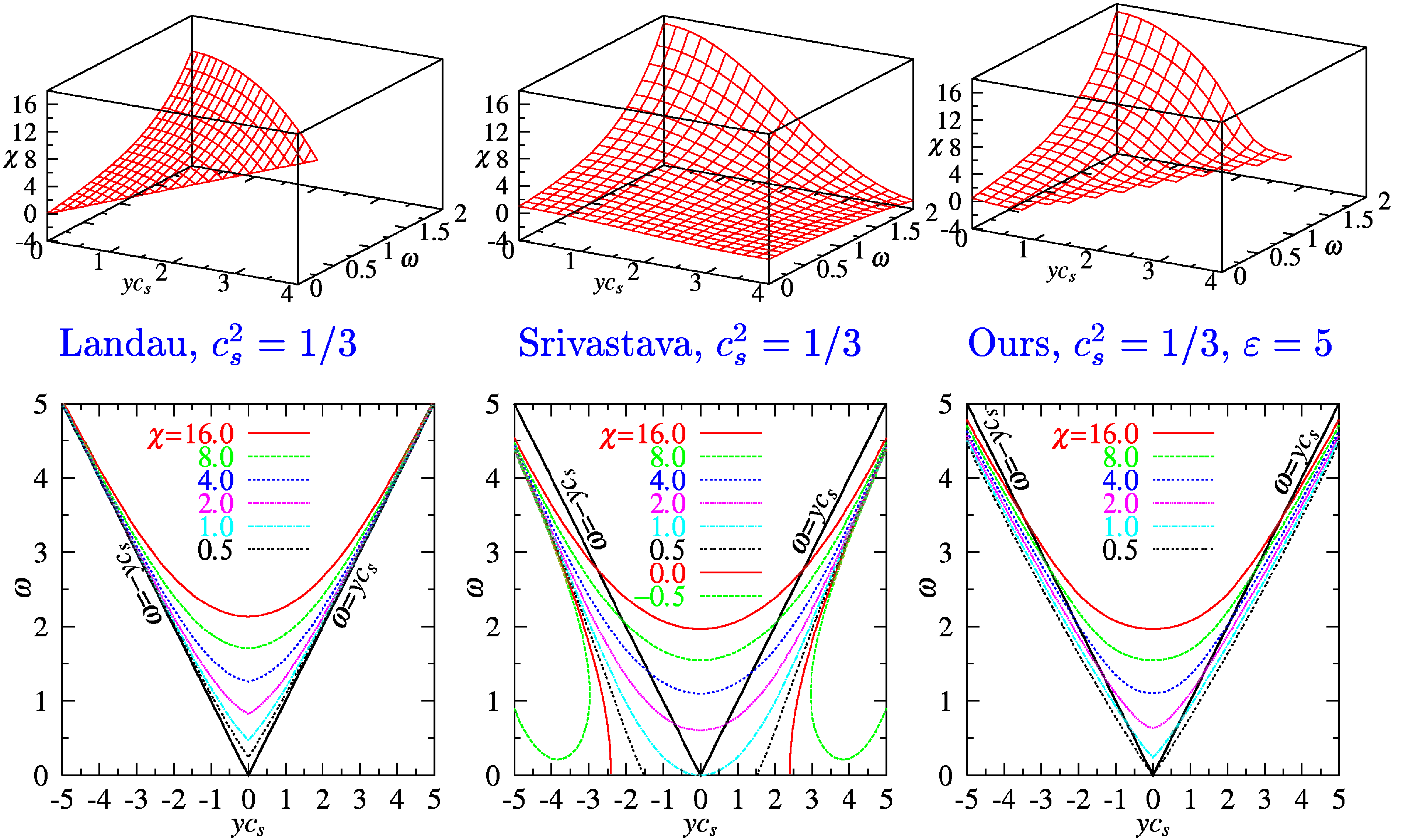
Comparison of three analytical solutions

Landau $\chi \sim e^{-\omega} \int_{yc_s}^{\omega} e^{(1+\beta)\omega'} I_0\left(\beta\sqrt{\omega'^2 - c_s^2 y^2}\right) d\omega'$

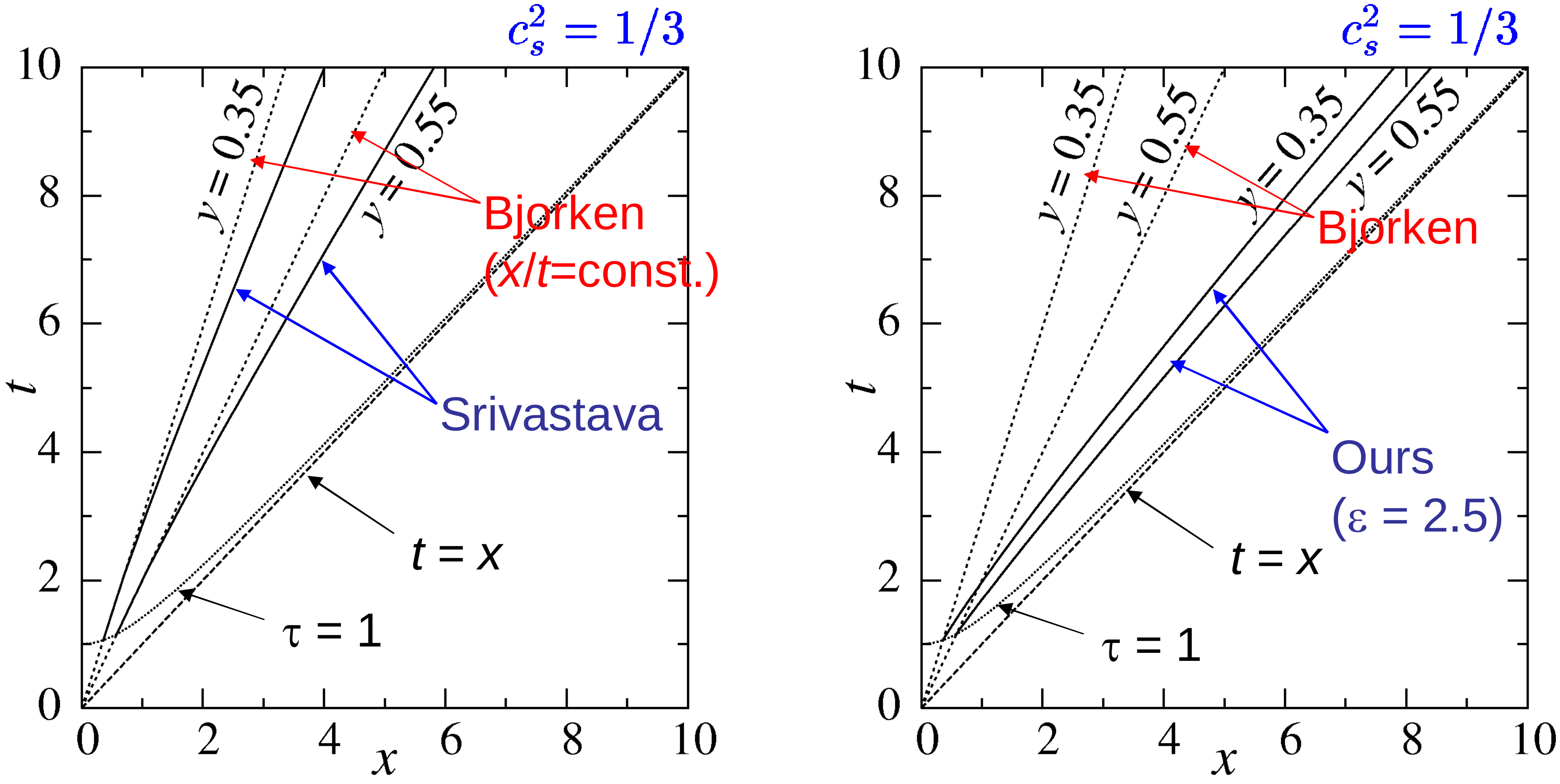
Srivastava $\chi \sim \begin{cases} e^{\beta\omega} I_0(\beta\sqrt{\omega^2 - c_s^2 y^2}) & \text{for } \omega^2 > c_s^2 y^2 \\ e^{\beta\omega} J_0(\beta\sqrt{c_s^2 y^2 - \omega^2}) & \text{for } \omega^2 < c_s^2 y^2 \end{cases}$

Ours $\chi \sim e^{\beta\omega} I_0\left(\beta\sqrt{\omega^2 - c_s^2 y^2}\right) H_\varepsilon(\omega - c_s|y|)$

Potencial χ(y, ω) and Contour map

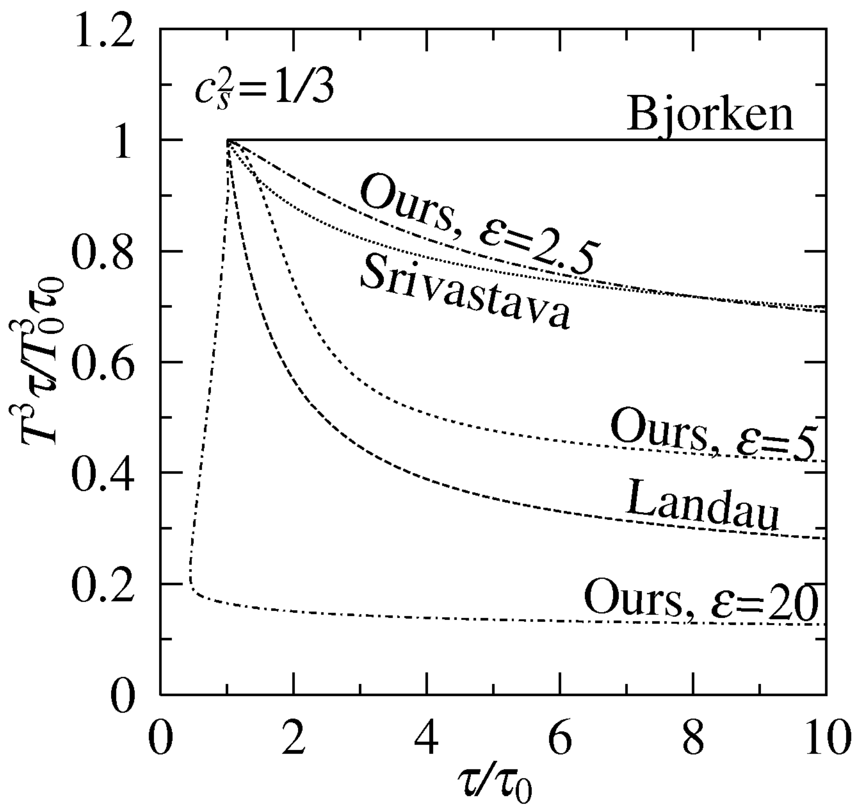


Trajectory of $t = \sqrt{\tau^2 + x^2}$



Bjorken: boost invariant solution.
(Cf. J.D. Bjorken, Phys. Rev. D27 (1983) 140.)

Cooling law at y = 0



τ_0: proper time at (y, ω) = (0, 0).

Bjorken: $T^3\tau/T_0^3\,\tau_0$= const. (c_s^2 = 1/3)

Rapidity distribution of hadrons

Quantities of hadrons:

hadron rapidity y_h, transverse momentum p_T, transverse mass $m_T = \sqrt{p_T^2 + m_p^2}$, baryon chemical potential μ_B, temperature T_f

Rapidity distribution function of hadrons

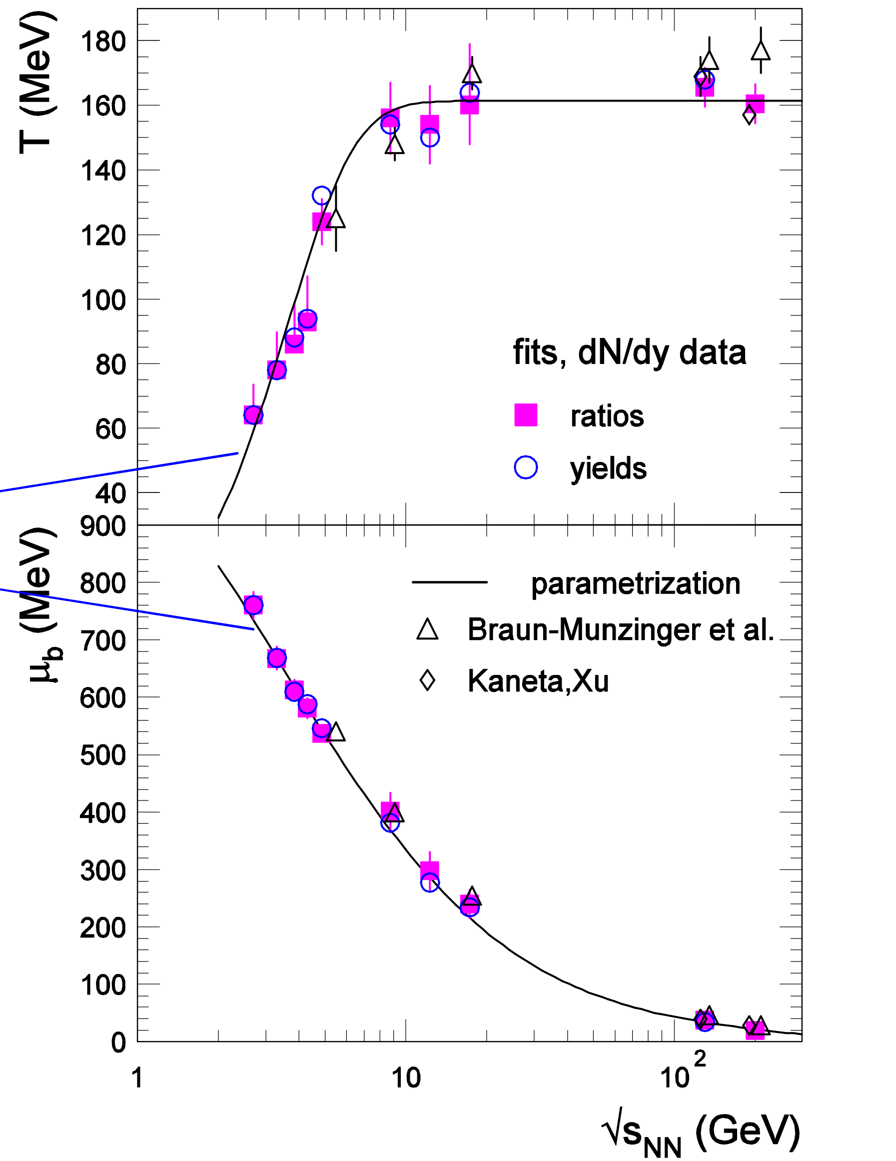
$$\frac{dN}{dy_h} = c\int_{-y_0}^{y_0} dy \int_0^{\infty} dp_T\, p_T \times \frac{m_T \cosh(y_h - y)}{\exp[\{m_T \cosh(y_h - y) - \mu_B\}/T_f] + \delta}\frac{dS}{dy}$$

For mesons, $\delta = -1$ and $\mu_B = 0$, for baryons $\delta = 1$

c is normalization factor.

Data analyses

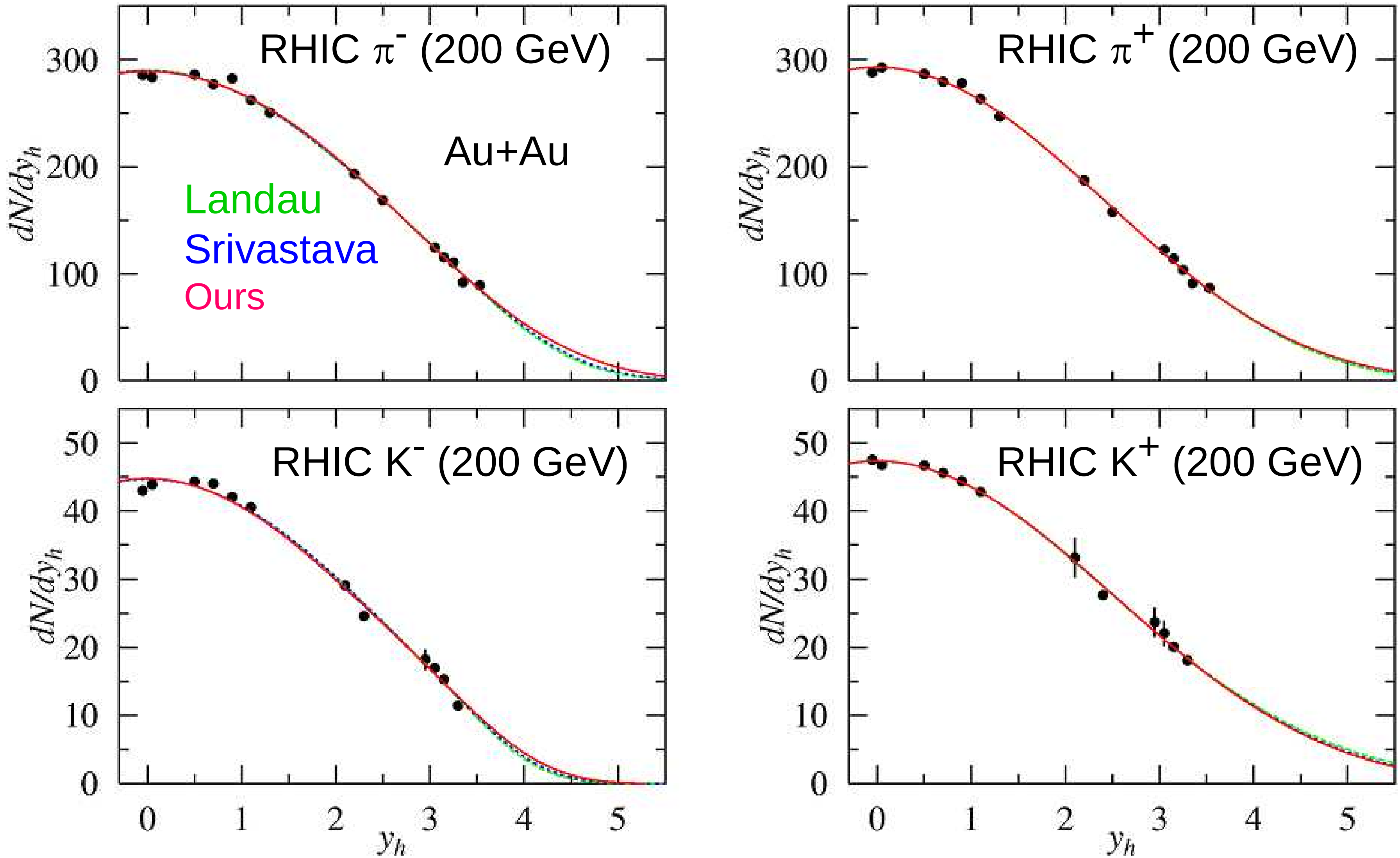
- Parameter fitting by least-squares (CERN MINUIT is used)
- Values to input : T_f, μ_B
- Free parameters: ω_f, c_s^2(<=1/3), c, ε



Temperature and baryon chemical Potential (cf. Andronic et al., Nucl. Phys. A772 (2006) 167)

Analyses of charged π and K data

Mizoguchi, Miyazawa, Biyajima, Eur. Phys. J. A **40**, 99 (2009)



Comparison of parameter ω_f

$$\omega_f = -\ln(T_f/T_0)$$

data	solution		
	Landau	Srivastava	Ours
π^- (200 GeV)	1.77±0.03	1.09±0.15	1.55±0.15
π^+ (200 GeV)	1.84±0.02	1.73±0.02	1.86±0.02
K^- (200 GeV)	1.58±0.05	0.90±0.20	1.40±0.14
K^+ (200 GeV)	2.49±0.02	2.24±0.04	2.28±0.04
π^- (62.4 GeV)	1.61±0.01	0.48±0.02	0.86±0.02
π^+ (62.4 GeV)	1.49±0.16	0.43±0.02	0.77±0.03
K^- (62.4 GeV)	1.22±0.23	0.41±0.13	0.68±0.09
K^+ (62.4 GeV)	1.40±0.08	0.66±0.12	1.16±0.18

ω_f (Srivastava) < ω_f (Ours) < ω_f (Landau)

We cannot determine which temperature is right.

Attention!

We cannot determine the value uniquely except for Landau's solution.

$$\frac{dN}{dy_h} = c \int_{-y_0}^{y_0} dy \int_0^\infty dp_T\, p_T \frac{m_T \cosh(y_h - y)}{\exp[\{m_T \cosh(y_h - y) - \mu_B\}/T_f] + \delta} \frac{dS}{dy}$$

Dependence of parameters on integration cutoff y_0

(a) $y_0 = \omega/\sqrt{c_s^2} + \delta$ ($\delta = 2$), (b) $y_0 = \ln(\sqrt{s_{NN}}/m_p)$

data	solution	y_0	ω_f	c_s^2	c	ε
π^-	Srivastava	(a)	1.09±0.15	0.25±0.02	1140±659	—
(200	Srivastava	(b)	0.87±0.68	0.27±0.07	2030±2910	—
GeV)	Ours	(a)	1.55±0.15	0.28±0.05	1120±1190	1.71±0.44
	Ours	(b)	1.55±0.09	0.28±0.03	1150±690	1.70±0.32
K^-	Srivastava	(a)	0.90±0.20	0.24±0.03	657±500	—
(200	Srivastava	(b)	0.69±1.59	0.26±0.26	1100±5110	—
GeV)	Ours	(a)	1.40±0.14	0.17±0.01	50.9±18.6	6.15±7.66
	Ours	(b)	1.37±0.16	0.17±0.01	58.9±28.3	7.31±6.82

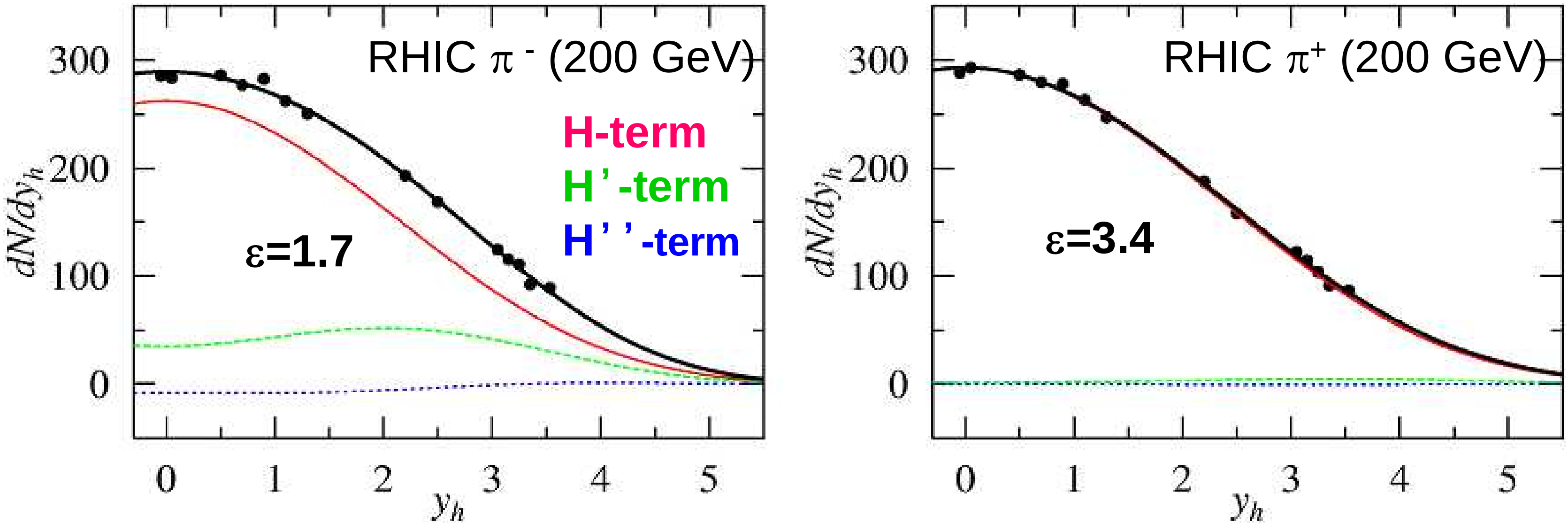
- Solution of Slivastava *et al*. depends on y_0
- Our solution doesn't depends on y_0

Contribution of the derivative term of *H*(*Q*)

Entropy distribution of our solution

$$\frac{dS}{dy} \sim \left[\beta(\beta+1)I_0(p) + \frac{\beta^2(p^2+p^2(2\beta+1)\omega-\beta^2\omega^2)I_0'(p)}{p^3} + \frac{\beta^4\omega^2 I_0''(p)}{p^2}\right]H_\varepsilon(Q)$$
$$+\left[(2\beta+1)I_0(p) + \frac{2\beta^2\omega I_0'(p)}{p}\right]H_\varepsilon'(Q) + I_0(p)H_\varepsilon''(Q).$$

Distributions of the terms containing $H_\varepsilon(Q)$, $H_\varepsilon'(Q)$, and $H_\varepsilon''(Q)$

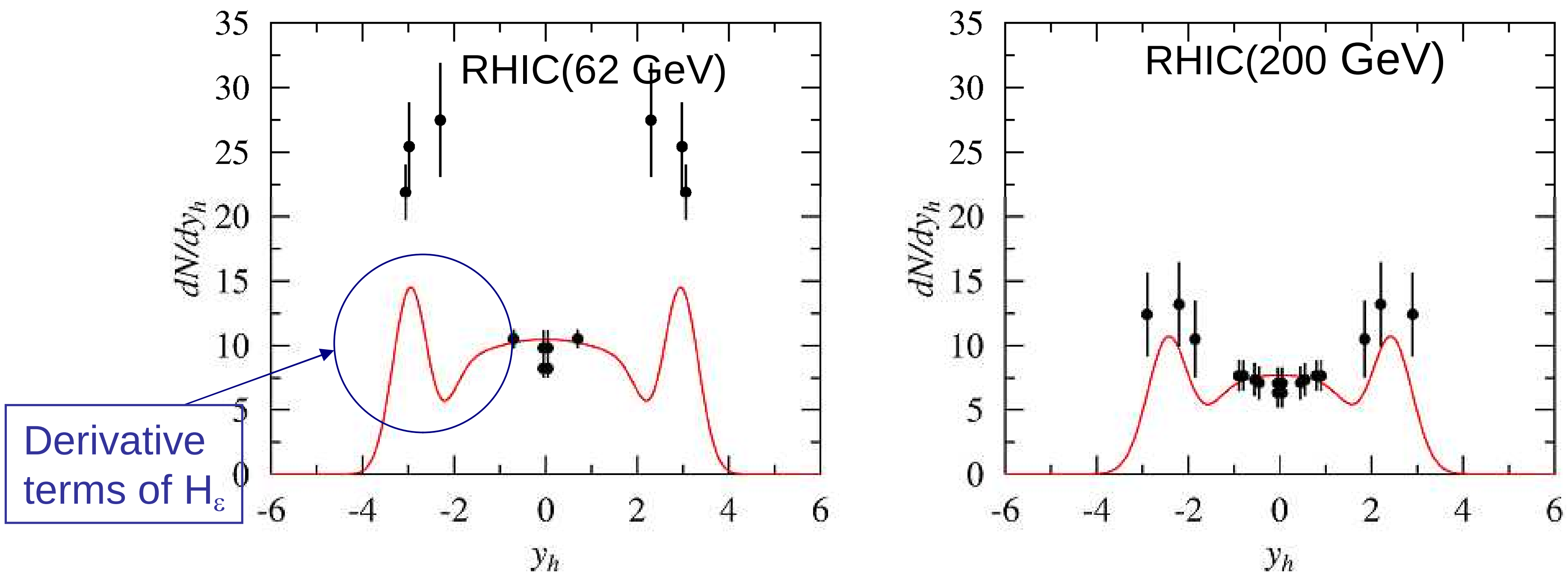


If ε is large, contributions of the derivative terms are small.

Preliminary works

Analyses of net-proton data

Parameter fitting by means of our solution



- Our solution cannot explain the characteristic peak of RHIC net-proton data.
- Thus we consider another approach.

Remember the famous book!!

Morse and Feshbach, ``Methods of Theoretical Physics'',
Chapter 7 (1953)

General solution of telegraph equation

$$\begin{aligned}
\chi(y,\,\omega) \;=\; & \frac{1}{2}e^{-\beta\omega}\{g(y+\omega/c_s)+g(y-\omega/c_s)\} \\
& +\frac{c_s}{2}e^{-\beta\omega}\int_{-\omega/c_s}^{\omega/c_s} g(z+y)\left\{\beta I_0\left(\beta\sqrt{\omega^2-c_s^2z^2}\right)\right. \\
& \qquad\qquad \left. +\frac{\partial}{\partial\omega}I_0\left(\beta\sqrt{\omega^2-c_s^2z^2}\right)\right\}dz \\
& +\frac{c_s}{2}e^{-\beta\omega}\int_{-\omega/c_s}^{\omega/c_s} G(z+y)I_0\left(\beta\sqrt{\omega^2-c_s^2z^2}\right)dz.
\end{aligned}$$

with $g(y)=\chi(y,\ \omega=0)$, $G(y)=\left.\frac{\partial\chi}{\partial\omega}\right|_{\omega=0}$

See also, Masoliver, Weiss, ``Finite-velocity diffusion '',
Eur. J. Phys. 17 (1996)190

Initial condition for net-proton $g(y) = \delta(y)$, $G(y) = 0$

(cf. for meson (π, K): $g(y) = 0$, $G(y) = \delta(y)$)

Solution $(p = \beta\sqrt{\omega^2 - c_s^2 y^2})$

$$\begin{aligned}\chi &= (1/2)e^{-\beta\omega}\{\underbrace{[\delta(y-\omega/c_s) + \delta(y+\omega/c_s)]}_{f_\delta(\omega)} \\ &\quad + \underbrace{c_s\beta[I_0(p) + \beta\omega I_1(p)/p]}_{f_I(\omega)} H(\omega - c_s|y|)\} \\ &= (1/2)e^{-\beta\omega}\{f_\delta(\omega) + f_I(\omega)H(\omega - c_s|y|)\}\end{aligned}$$

Entropy distribution

$$\begin{aligned}\frac{dS}{dy} \sim\ & \beta(\beta+1)[f_\delta(\omega) + f_I(\omega)\, H_\varepsilon(\omega - c_s|y|)] \\ & -(2\beta+1)[f_\delta'(\omega) + f_I'(\omega)\, H_\varepsilon(\omega - c_s|y|) \\ & + f_I(\omega)\, H_\varepsilon'(\omega - c_s|y|) + f_\delta''(\omega) + f_I''(\omega)\, H_\varepsilon(\omega - c_s|y|) \\ & + 2f_I'(\omega)\, H_\varepsilon'(\omega - c_s|y|) + f_I(\omega)\, H_\varepsilon''(\omega - c_s|y|)\end{aligned}$$

Approximate expression of delta function:

$$\delta(x) \approx \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{x^2}{2\sigma^2}\right] \quad \text{(Gaussian)} \ [f_\delta(\omega) \to f_G(\omega)]$$

Assuming contribution of the derivative terms of $H_\varepsilon(Q)$ is small, we put

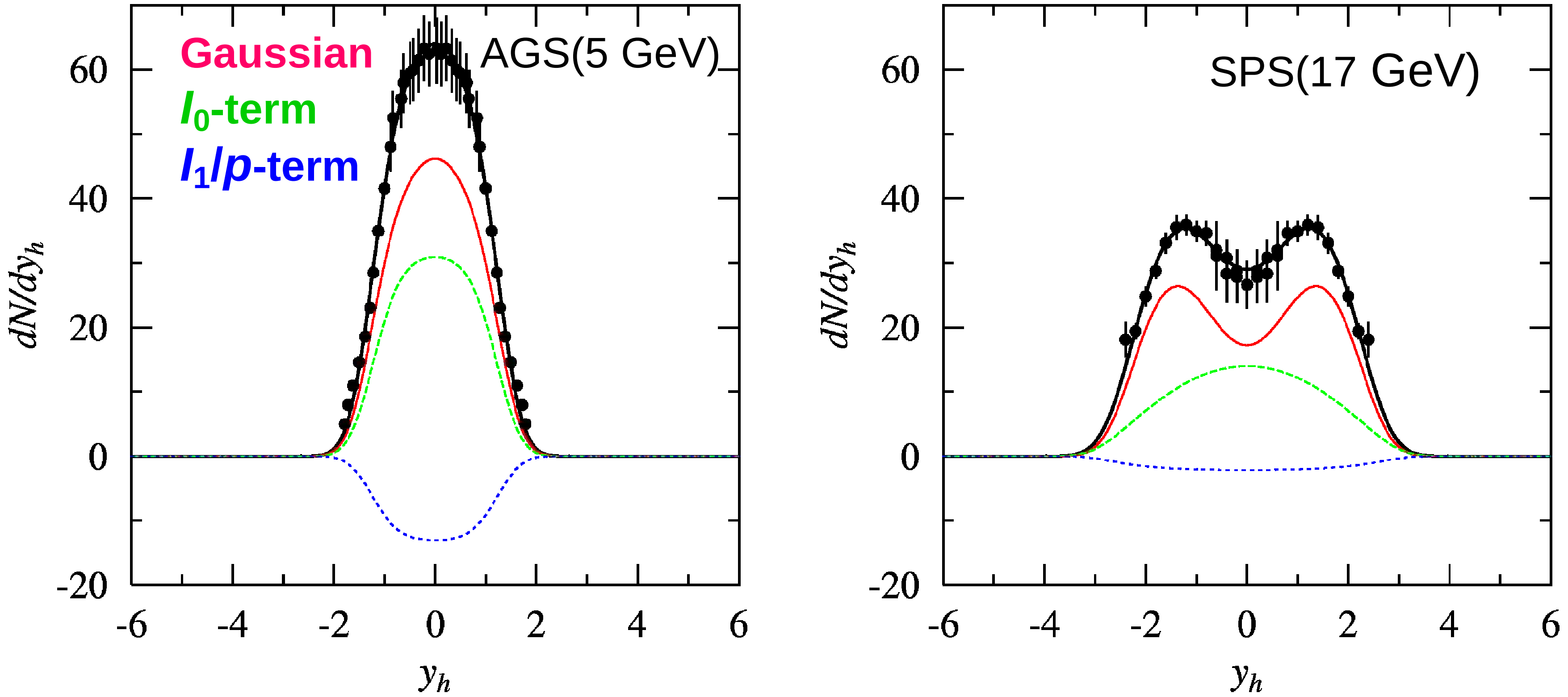
$$H'_\varepsilon(Q) = H''_\varepsilon(Q) = 0$$

Then we write

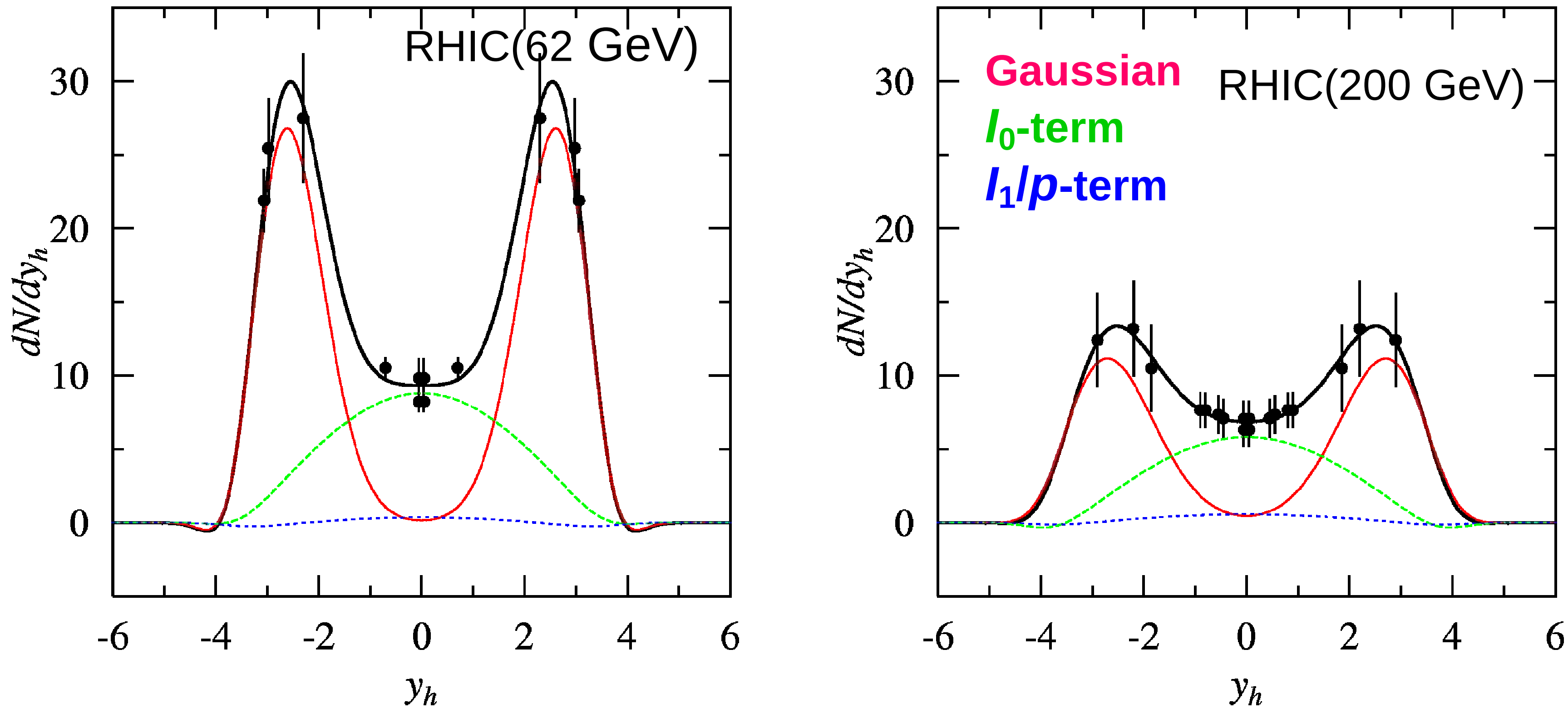
$$\begin{aligned}\frac{dS}{dy} \sim \ & \beta(\beta+1)[f_G(\omega) + f_I(\omega)\, H_\varepsilon(\omega - c_s|y|)] \\ & -(2\beta+1)[f'_G(\omega) + f'_I(\omega)\, H_\varepsilon(\omega - c_s|y|)] + f''_G(\omega) \\ & +f''_I(\omega)\, H_\varepsilon(\omega - c_s|y|)\end{aligned}$$

Free parameters: ω_f, c_s^2 $(\leq 1/3)$, c, ε, σ^2

Analyses of net-proton data at AGS and SPS



Analyses of net-proton data at RHIC



Parameters and initial Temperature

$N = \int_{-\infty}^{\infty} (dN/dy_h) dy_h$

data	ω_f	c_s^2	σ^2	ε	N
AGS (5 GeV) $p-\bar{p}$	0.77	1/3	2.65	10.1	148
SPS (17 GeV) $p-\bar{p}$	1.61	1/3	1.22	3.2	153
RHIC (62.4 GeV) $p-\bar{p}$	2.19	1/3	0.69	11.6	124
RHIC (200 GeV) $p-\bar{p}$	2.36	1/3	1.19	10.2	73

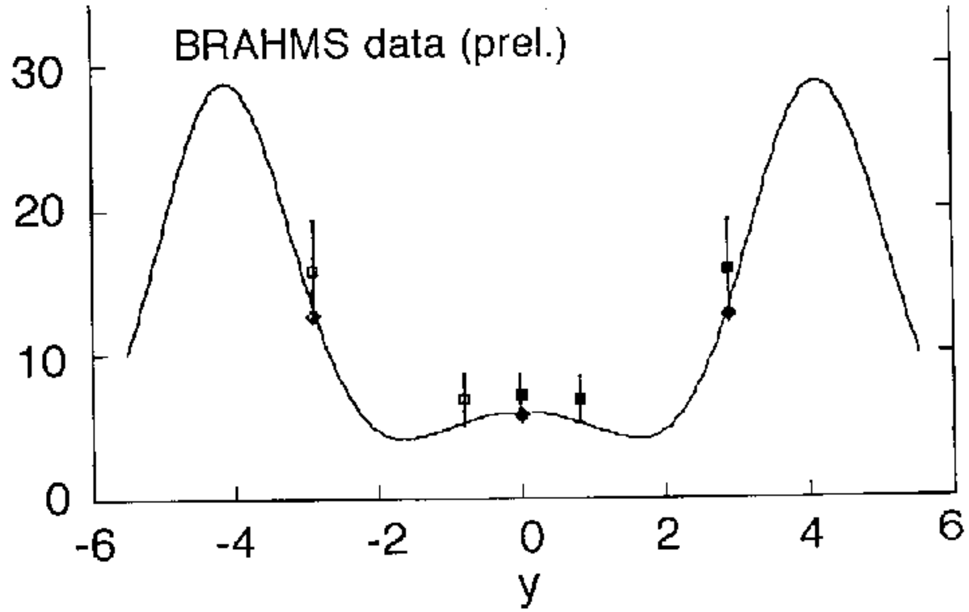
upper limit

Existence of missing proton !!

Estimation of initial Temperature

data	ω_f	T_f (input)	$T_0 = T_f e^{\omega_f}$ (computed)
AGS (5 GeV) $p-\bar{p}$	0.77	0.124	0.27
SPS (17 GeV) $p-\bar{p}$	1.61	0.160	0.80
RHIC (62.4 GeV) $p-\bar{p}$	2.19	0.1605	1.43
RHIC (200 GeV) $p-\bar{p}$	2.36	0.1605	1.69

G. Wolschin, Phys. Lett. B 569 (2003) 67.



For fitting our solution to with this form, it is necessary to impose another conditions.

Summary

- We consider the (1+1)-dimensional hydrodynamics, and derive the solution including the Heaviside function.
- Our solution explains the data π and K distributions fairly well.

Preliminary work

- Since our solution doesn't explain the characteristic peak at large *y* of net-proton distribution, we have considered a new analytic solution.
- The new solution explains the data of net-proton fairly well, except for data at 200 GeV.