On analytic solutions of 3+1 Relativistic Ideal Hydrodynamical Equations

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Outline

- Basics of Relativistic Ideal Hydrodynamics and its applicability to Heavy Ion Collisions
- Reduction from 3+1 problem to 1+1 problem by embedding
- Flow profile of the embedding solutions and their physical interpretations
- Possible connections with heavy ion collisions

Basics of relativistic hydrodynamics

Conservation equation $T^{mn}{}_{;n} = 0$ (1)

Constitutive equation $T^{mn} = (\epsilon + p)\, u^m u^n - p\, g^{mn} = w\, u^m u^n - p\, g^{mn}$ (2)

Equation of state p=p(ε) Assumption: local equilibrium (3)

If there is a conserved charge $(nu^m)_{;m} = 0$ (4)

Thermodynamical relations

$$\varepsilon + p = Ts$$

$$d\varepsilon = Tds, dp = sdT$$ (5)

(1), (2), (5) lead to conservation of entropy $(su^m)_{;m} = 0$

no disspative term
respect time reversion

Applicability of RIHD to HIC

Phenomenology of Heavy Ion Collisions

QGP produced at heavy ion collisions
is believed to be strongly coupled.

Lower bound for general strongly coupled gauge theory:

$$\frac{\eta}{s} \geq \frac{1}{4\pi}$$

Kovtun, Son, Starinets 2004

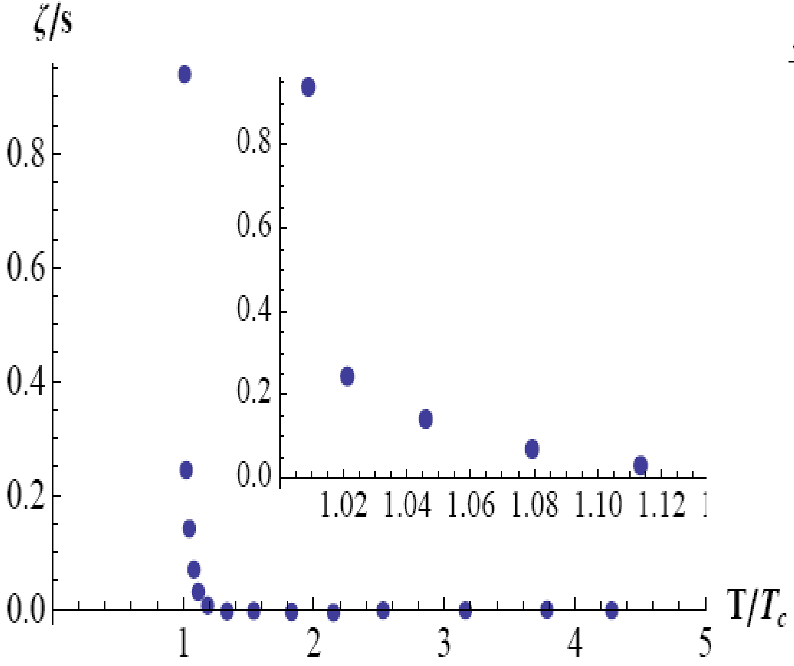
Even lower value for bulk viscosity at $T > 1.1 T_C$

$$\frac{\zeta}{s} \leq 0.04$$

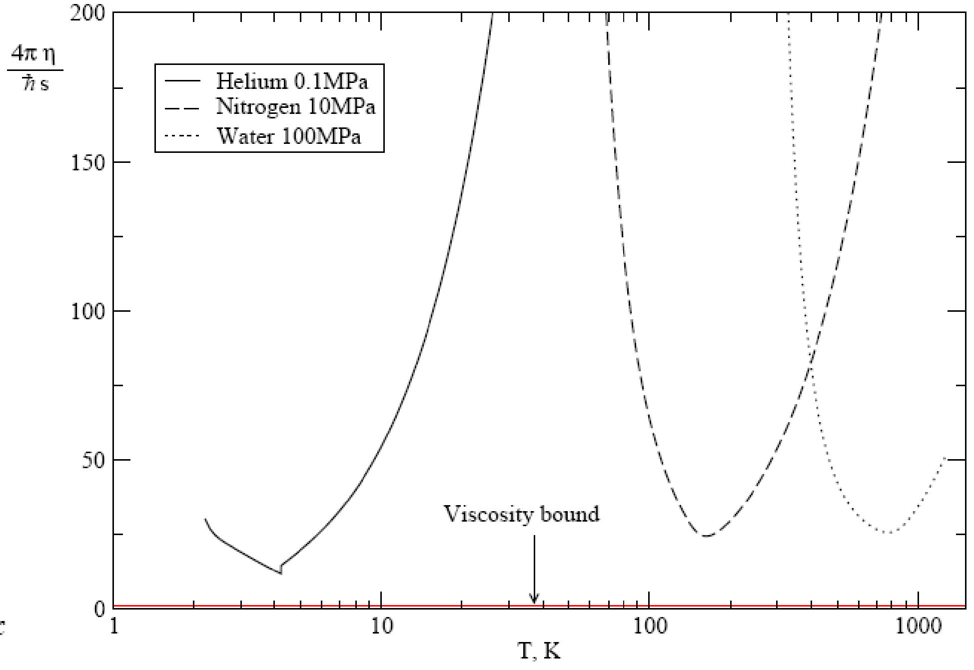
Kharzeev, Tuchin 2007

Relativistic Ideal Hydrodynamics applicable in wide region of temperature

Bulk and Shear viscosity



QCD with lattice data



Conjecture for strongly coupled matter

Some well known solutions to RIHD

Landau, Khalantnikov 1950s	1+1D rapidity distribution approximately gaussian
Hwa(1974)-Bjorken(1983)	1+1D boost invariant
Bialas, Janik, Peschanki 2007	1+1D interpolation between LK and HB
Biro 2000 Csörgő et al 2004 Nagy, Csörgő, Csanád 2008	generalization to 3+1D solution with spherical, cylindrical and ellipsoidal symmetries

2D Hubble embedding

Fluid flow $u^\mu \to \gamma(1, v_x, v_y, v_z)$ In flat coordinate (t,x,y,z)

Solving hydrodynamical equations with specific Hubble-like transverse flow:

$$v_x = \frac{x}{t} = \tanh\eta_\perp \cos\phi_\perp \ , \ v_y = \frac{y}{t} = \tanh\eta_\perp \sin\phi_\perp$$

$$t = \tau_\perp \cosh\eta_\perp \ , \ z = z \ ,$$
$$x = \tau_\perp \sinh\eta_\perp \cos\phi_\perp \ , \ y = \tau_\perp \sinh\eta_\perp \sin\phi_\perp$$

$$u^m = \bar{\gamma}(1, 0, 0, \bar{v}_z)$$

$$\bar{v}_z \neq v_z$$

energy, pressure and longtudinal flow independent of $\eta_\perp$ and $\phi_\perp$

Liao and Koch **0905.3406** [nucl-th]
SL, Liao **0909.2284** [nucl-th]

See also Jinfeng Liao's talk for more of embedding method

scaling ansatz

Equation of state $p = \nu(\epsilon + p)$

Speed of sound $c_s = \sqrt{\frac{\partial p}{\partial \epsilon}} = \sqrt{\frac{\nu}{1-\nu}} \leq 1,$ $0 < \nu \leq 1/2$

dimensionful $p(\tau_\perp, z) = \tau_\perp^{-\frac{a}{1-\nu}} g(\frac{z}{\tau_\perp})$

dimensionless $\bar{v}_z(\tau_\perp, z) = f(\frac{z}{\tau_\perp})$

scaling variable $\xi = \frac{z}{\tau_\perp}$

Symmetries of the EOM

$$f'(\xi) = -\frac{\nu(1-f(\xi)^2)(2-a+af(\xi)^2-2\xi f(\xi))}{(\nu-1)(f(\xi)-\xi)^2+\nu(\xi f(\xi)-1)^2}$$

$$g'(\xi) = -\frac{g}{1-\nu}\times$$

$$\frac{-2f(\xi)+af(\xi)+2\nu f(\xi)-2a\nu f(\xi)+2\xi-a\xi-2\nu\xi+a\nu\xi+a\nu\xi f(\xi)^2}{(\nu-1)(f(\xi)-\xi)^2+\nu(\xi f(\xi)-1)^2}$$

$$\xi \to -\xi,\ f \to -f,\ g \to g \quad \Leftrightarrow \tau_\perp \to \tau_\perp, z \to -z, v_z \to -v_z, p \to p$$

The solutions should preserve parity

Solving the equations

$$f'(\xi) = -\frac{\nu(1-f(\xi)^2)(2-a+af(\xi)^2-2\xi f(\xi))}{(\nu-1)(f(\xi)-\xi)^2+\nu(\xi f(\xi)-1)^2}$$

$$t(f) \equiv \frac{f\,\xi(f)-1}{f-\xi(f)}$$

$$\left[(1-f^2)(a\,f-2t+a\,t)\right] \times \frac{dt}{df}$$
$$-\left[(\kappa^2-a)f+(\kappa^2+2-a)t+(a-1)f\,t^2+(a-3)t^3\right]=0$$

Linear ansatz $t(f) = \alpha\,f + \beta$

Nonlinear ansatz $t(f) = \alpha\,f + \beta + \dfrac{\rho f + \lambda}{1-f^2}$

SL, Liao **0909.2284** [nucl-th]

Solutions for general ν(EOS)

$$\bar{v}_z = 0,\ p \propto \tau_\perp^{-2/(1-\nu)}$$

2D Hubble flow(analog of Hwa-Bjorken flow)

$$\bar{v}_z = \xi,\ p \propto \tau_\perp^{-3/(1-\nu)} (1-\xi^2)^{\frac{-3}{2(1-\nu)}}$$

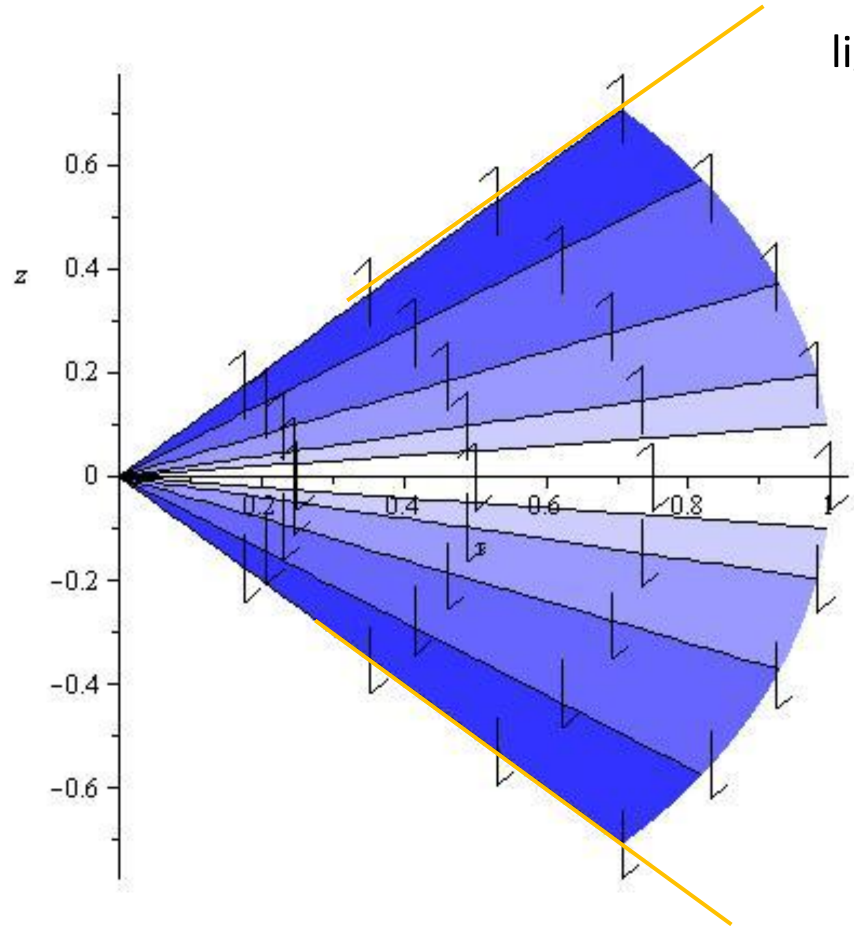
3D Hubble flow(spherical) $|\xi|<1$

$$\bar{v}_z = 1/\xi,\ p \propto \tau_\perp^{-1/\nu} (\xi^2 - 1)^{\frac{(3\nu-1)}{2\nu(1-\nu)}}$$

Anti-Hubble flow $|\xi|>1$

$$\xi = z / \tau_\perp$$

3D Hubble flow(spherical)



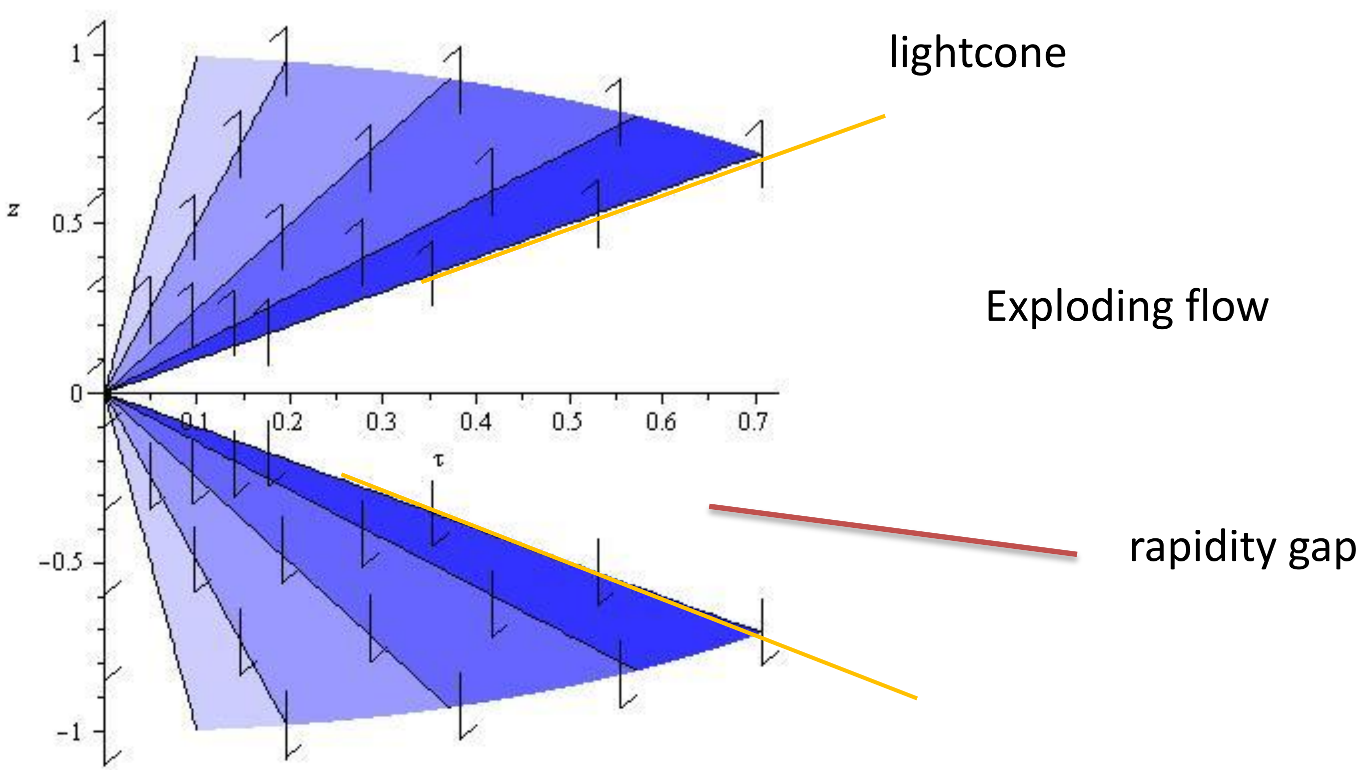
lightcone

Exploding flow

$v_x = x/t$, $v_y = y/t$, $v_z = z/t$

arrows indicate direction of the flow
darkness of the color indicate the flow magnitude

Anti-Hubble flow



Solutions for general ν(EOS)

$$\bar{v}_z = f(\frac{z}{\tau_\perp})$$

$$p = \text{constant} \times \tau_\perp^{-3} \frac{f^{\frac{1-3\nu}{1-\nu}}}{(1-f^2)^{\frac{2-3\nu}{1-\nu}}}$$

domain 1 and domain 3, domain 2 and domain 4 are related by parity!

$$\text{domain 1: } f = \frac{z/\tau_\perp + \sqrt{z^2/\tau_\perp^2 - 4\nu(1-\nu)}}{2(1-\nu)}$$

$$\text{with } 2\sqrt{\nu(1-\nu)} < \frac{z}{\tau_\perp} < 1$$

$$\text{domain 2: } f = \frac{z/\tau_\perp + \sqrt{z^2/\tau_\perp^2 - 4\nu(1-\nu)}}{2(1-\nu)}$$

$$\text{with } \frac{z}{\tau_\perp} < -1$$

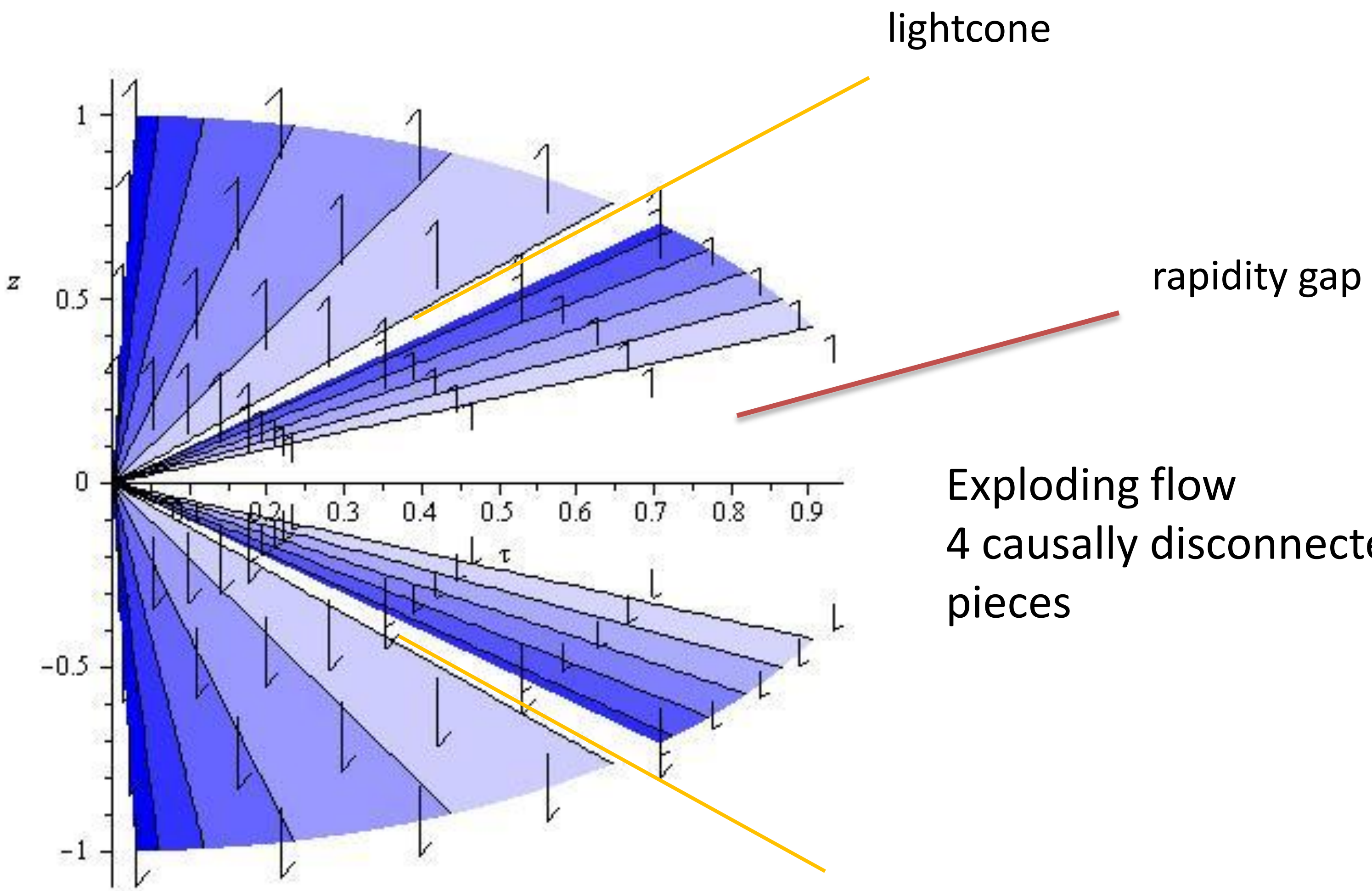
$$\text{domain 3: } f = \frac{z/\tau_\perp - \sqrt{z^2/\tau_\perp^2 - 4\nu(1-\nu)}}{2(1-\nu)}$$

$$\text{with } -1 < \frac{z}{\tau_\perp} < -2\sqrt{\nu(1-\nu)}$$

$$\text{domain 4: } f = \frac{z/\tau_\perp - \sqrt{z^2/\tau_\perp^2 - 4\nu(1-\nu)}}{2(1-\nu)}$$

$$\text{with } \frac{z}{\tau_\perp} > 1$$

Solution with 4 domains



Exploding flow
4 causally disconnected pieces

Solutions for specific ν(EOS)

$$\bar{v}_z = f(\frac{z}{\tau_\perp})$$

$$p = \text{constant} \times \tau_\perp^{-6} \frac{1 + 1/f}{(1/f - 1)^3}$$

$$\text{with } \nu = \frac{1}{2}$$

$$\text{domain 1: } f = \frac{1}{2 - z/\tau_\perp}$$

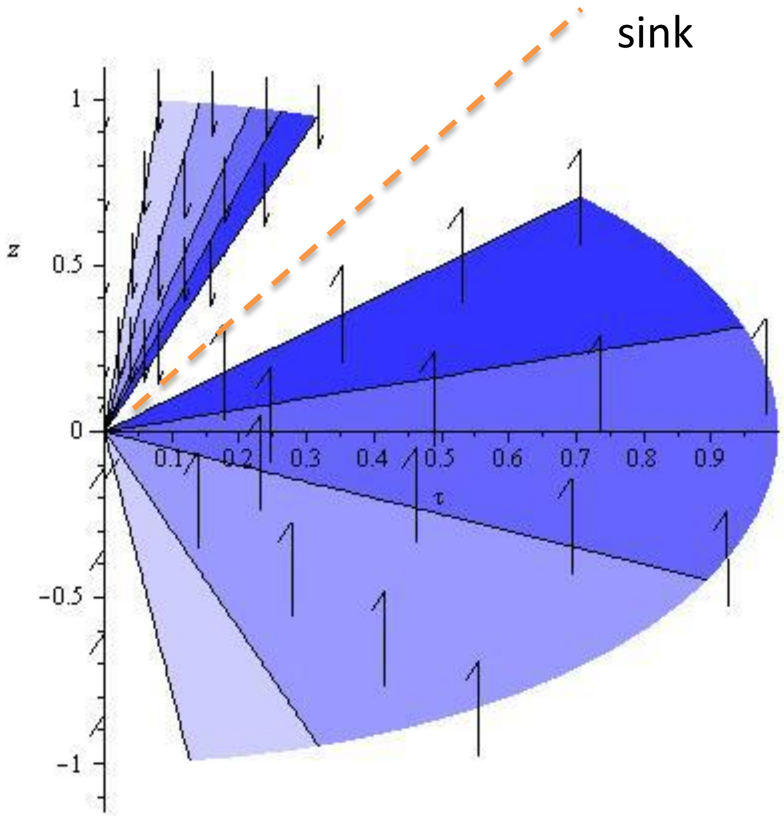
$$\text{with } \frac{z}{\tau_\perp} > 3$$

$$\text{domain 2: } f = \frac{1}{2 - z/\tau_\perp}$$

$$\text{with } \frac{z}{\tau_\perp} < 1$$

Also its partiy partner

Flow with ν=1/2



Impolding flow with a moving “sink” at ξ=2

Solutions for specific ν(EOS)

$$\bar{v}_z = f(\frac{z}{\tau_\perp})$$

$$p = \text{constant} \times \tau_\perp^{-7/2}(1/f^2 - 1)^{-7/4}$$

$$\text{with } \nu = \frac{1}{7}$$

$$\text{domain 1: } f = f_1 \equiv \frac{2\tau_\perp}{3z}\left[1 - \zeta(z/\tau_\perp) - \bar{\zeta}(z/\tau_\perp)\right]$$

$$\text{with } \frac{z}{\tau_\perp} > 0 \text{ or } \frac{z}{\tau_\perp} < 0$$

$$\text{domain 2: } f = f_2 \equiv \frac{2\tau_\perp}{3z}\left[1 + \frac{1+i\sqrt{3}}{2}\zeta(z/\tau_\perp) + \frac{1-i\sqrt{3}}{2}\bar{\zeta}(z/\tau_\perp)\right]$$

$$\text{with } 0 < \frac{z}{\tau_\perp} < 1 \text{ or } -1 < \frac{z}{\tau_\perp} < 0$$

$$\zeta(\xi) = \left[-1 + \frac{27}{16}\xi^2 + i\sqrt{1 - (1 - \frac{27}{16}\xi^2)^2}\right]^{1/3}$$

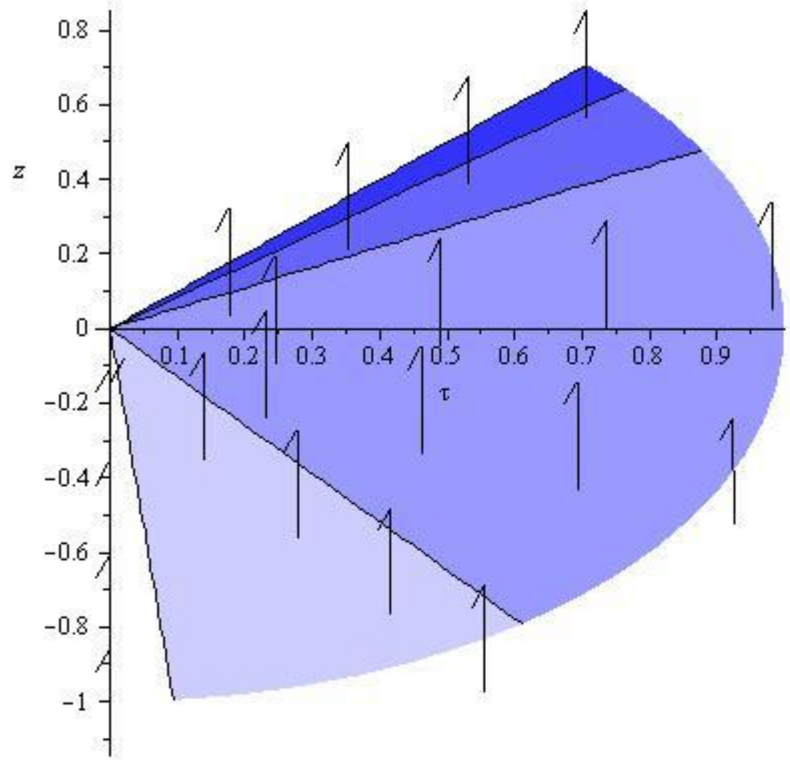
$$\bar{\zeta}(\xi) = \left[-1 + \frac{27}{16}\xi^2 - i\sqrt{1 - (1 - \frac{27}{16}\xi^2)^2}\right]^{1/3}$$

Flow with ν=1/7

$$f = \begin{cases} f_1 & z/\tau_\perp < 0 \\ f_2 & 0 < z/\tau_\perp < 1 \end{cases}$$

$$f = \begin{cases} f_2 & -1 < z/\tau_\perp < 0 \\ f_1 & z/\tau_\perp > 0 \end{cases}$$

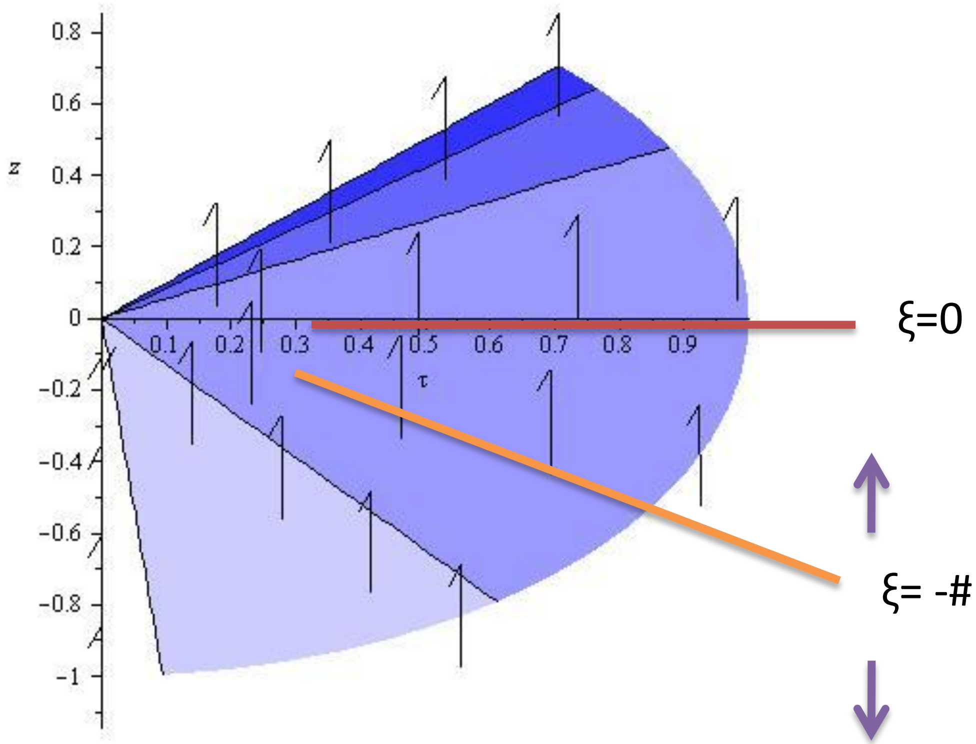
Form a parity pair



One-way shock wave

Flow reaches speed of light at ξ=1

Connection to Heavy Ion Collisions



Flow direction is observer dependent

One-way shock wave viewed from observer at ξ=0
Explosion viewed from observer at ξ= -#

A change of reference frame from ξ=0 to ξ= -# may be close to the situation of fireball explosion

Summary

- We have found several longitudinal flow profiles based on prescribed transverse flow(embedding)
- Connections to HIC may be established by applying longitudinal boost to certain solutions
- Extension from cylindrical symmetry to ellipsoidal symmetry can be used to gain insight to elliptic flow

Thank you!