Simple explicit solutions of perfect fluid hydrodynamics and phase-space evolution

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Introduction

The role of hydrodynamics in heavy-ion physics:

- Experimentally: the matter created at RHIC is (almost) perfect fluid
- Hydrodynamics: successful in interpreting the observables (scaling properties, etc.) in the soft kinematic domain
- Can connect initial state – final state – equation of state (EoS)
- Equation of state: one of the primarily important questions

Hydrodynamic models: can follow dynamics, constrain EoS and/or initial and final state

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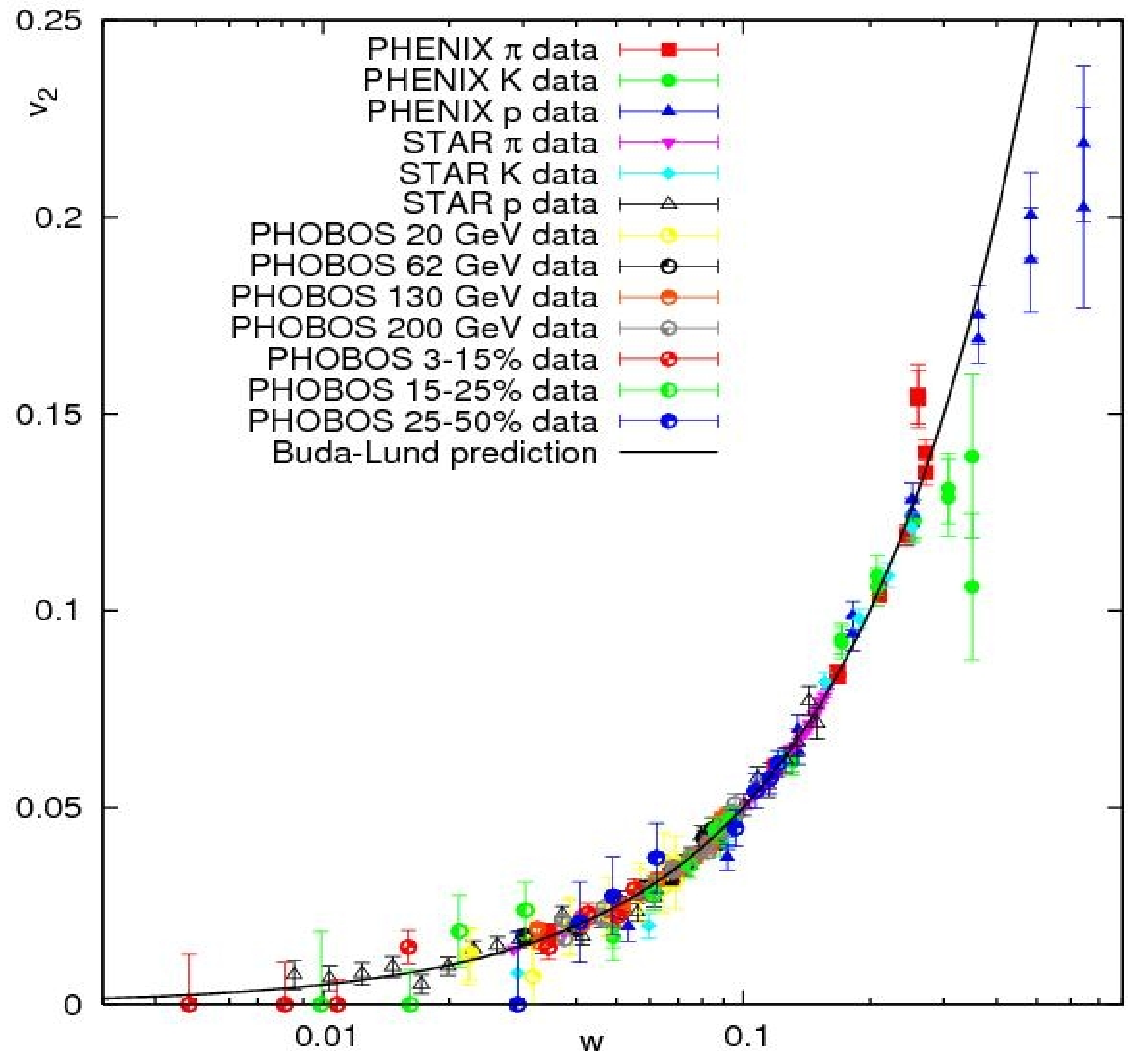
Perfect fluids

How perfect?

- Nonrelativistically, the equations are much simpler in the perfect fluid case, relativistically, even the viscous equations are not clear
- Perfect fluid picture works well in most of the cases (eg. Prediction for the scaling of elliptic flow)

Absence of physical scale:

- Naturally, there are scaling solutions



Exact solutions of hydrodynamics

Principles:

- local thermal equilibrium & energy and momentum conservation (no physical scale -> scaling behavior can be explained)
- Stress-energy-momentum tensor: $T_{\mu\nu} = w u_\mu u_\nu - p g_{\mu\nu}$

Models based on exact solutions:

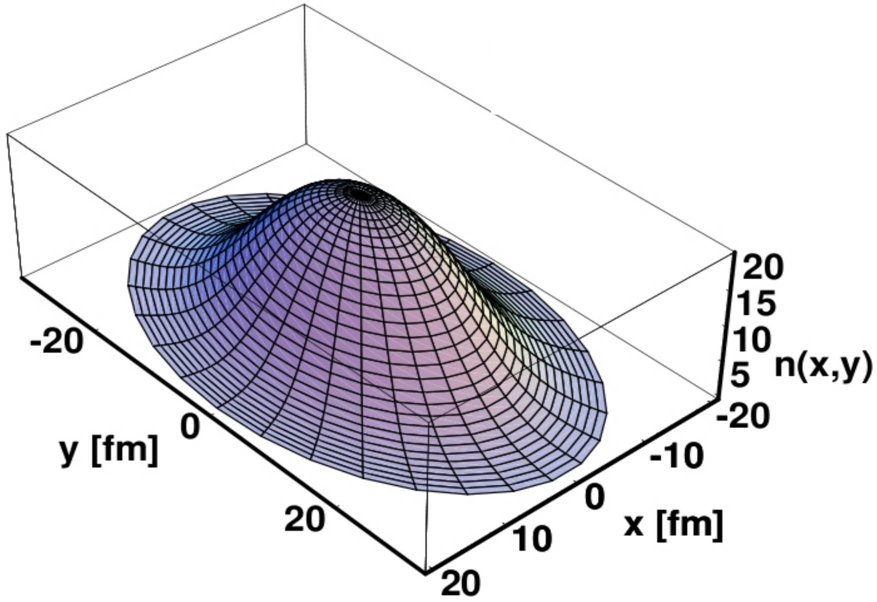
- Different from hydrodynamic parametrizations, as well as numeric methods: one has to apply an exact solution of the equations
- Challenge, only approximate correspondence with reality
- But: can map a manifold of initial conditions:
- Scaling (self-similar) solutions:
  -> The observed scaling behavior can be explained in a natural way

Kinetic theory and hydrodynamics

Nonrelativistic case:

- A simple explicit solution exhibits a peculiar feature: the solution is accelerating, but the corresponding phase-space distribution is a solution to the collisionless Maxwell-Boltzmann equation!

P. Csizmadia, T. Csörgő, B. Lukács, PLB 443, 21 (2003)



Properties:

- Self-similar velocity profile
- Constant temperature in space
- Gaussian density profile
- accelerating expansion

Nonrelativistic case

Hydrodynamical equations:

- Euler: $$nm_0\left(\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}\nabla)\,\mathbf{v}\right)=-\nabla p$$
- Energy conservation: $$\frac{\partial \varepsilon}{\partial t}+\nabla\left(\varepsilon\mathbf{v}\right)=0,$$
- Particle number conservation: $$\frac{\partial n}{\partial t}+\nabla\left(n\mathbf{v}\right)=0.$$

- Collisionless Maxwell-Boltzmann: $$\frac{\partial}{\partial t}f\left(\mathbf{r},\mathbf{p}\right)+\frac{\mathbf{p}}{m_0}\frac{\partial}{\partial \mathbf{r}}f\left(\mathbf{r},\mathbf{p}\right)=0.$$
- Phase-space distribution: $$f\left(t,\mathbf{r},\mathbf{p}\right)=\exp\left(\frac{\mu}{T}-\frac{\left(\mathbf{p}-m_0\mathbf{v}\right)^2}{2m_0T}\right)$$
- Equation of State (obligatory): $$\varepsilon=\kappa p \quad , \quad p=nT.$$
- Combining these, one can prove “uniqueness” of the mentioned solution: it is (essentially) the only one with this feature

Relativistic case

Fundamental equations for perfect fluids:

- Euler equation: $wu^\nu \partial_\nu u^\mu = \left(g^{\mu\rho} - u^\mu u^\rho\right) \partial_\rho p.$
- Energy conservation: $w\partial_\mu u^\mu = -u^\mu \partial_\mu \varepsilon.$
- Entropy conservation: $\partial_\mu \left(\sigma u^\mu\right) = 0.$
- Equation of State: connection between pressure, energy density and temperature

$$\varepsilon = \kappa p \quad , \quad p = nT.$$

Nonlinear, coupled ...

- Exact solutions are interesting in themselves
- Question: are there such relativistic solutions in which local thermal equilibrium can be maintained with free streaming of particles?

Relativistic collisionlessness

Fundamental equations

- Phase-space distribution: $$f\left(x,p\right)=\exp\left(\frac{\mu}{T}-\frac{p_\mu u^\mu}{T}\right)$$

- Collisionless Maxwell-Boltzmann: $p^\mu \partial_\mu f = 0 \quad (\forall p)$

- It turns out that the individual particles must be massless (in other words, ultrarelativistic)

Combining these:

- One arrives at a set of *linear* equations (!)

- Easy general solutions found:

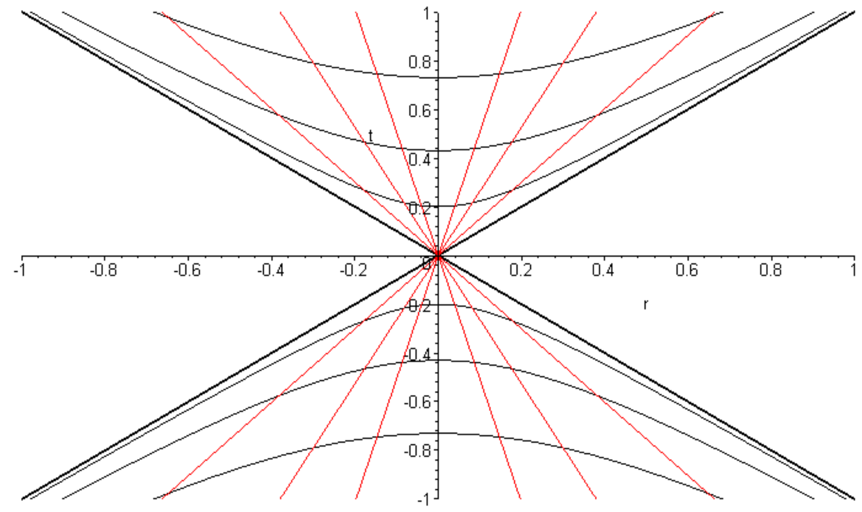
  - Thus one can explore all collisionless solutions

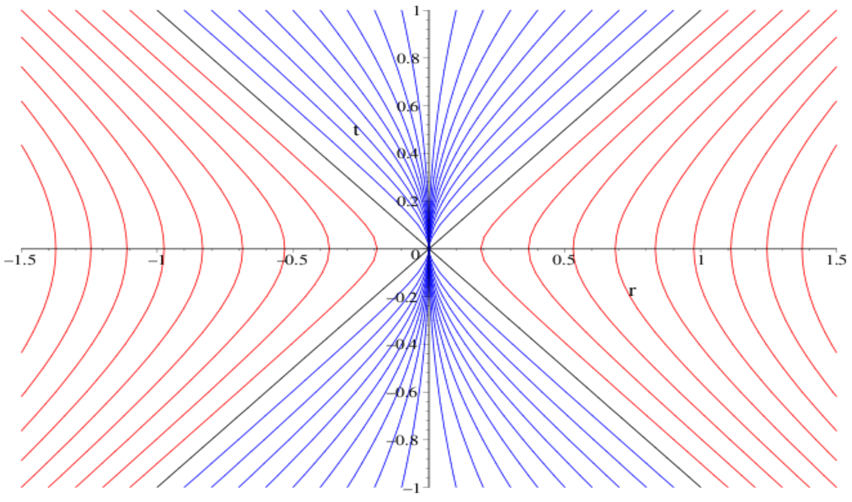
  - Even new ones, not known before

Collisionless solutions

Collisionless solutions:

- Known ones:
  - Hwa-Bjorken solution in 1+1D, or Hubble-like solution in 1+3D: simple, well-known: the new result is the collisionlessness
  - Accelerating solution: known since ~3 years, has applications in energy density estimation, collisionless nature is also a new result





Collisionless solutions

The accelerating solution:

- Known formulas: $v = \frac{2tr}{t^2 + r^2}$ $\quad p = p_0 \frac{\tau_0^2}{\tau^2}$

- Generalization, still collisionless (new solution):

$$v = \frac{2tr}{t^2 + r^2 + \mu} \qquad p = \frac{p_0 \tau_0^2}{\sqrt{(\tau^2 + \mu)^2 + 4\mu\tau^2 \sinh^2 \eta}}$$

- Important property: pressure is finite in pseudo-rapidity!

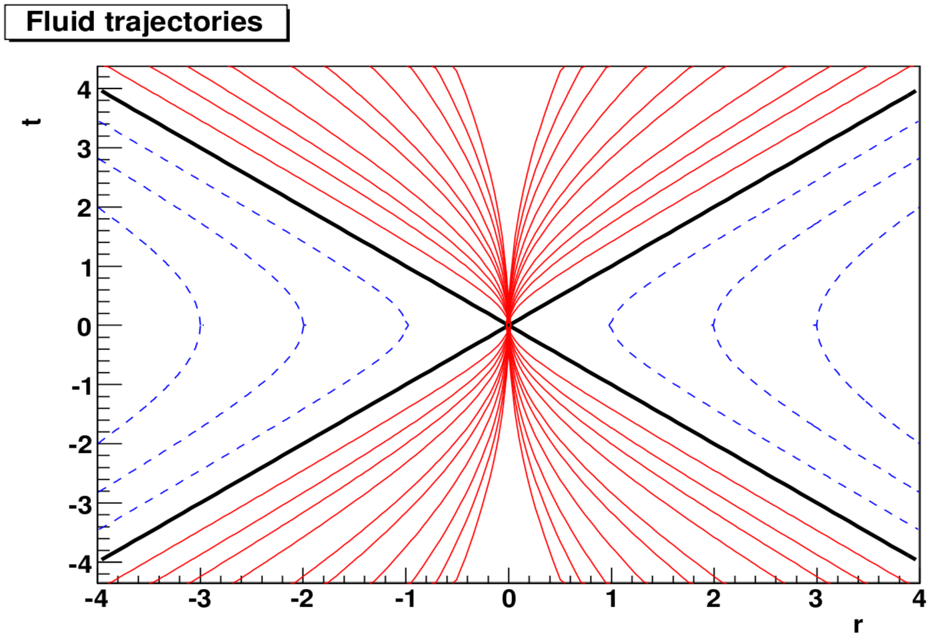
Further generalizations:

- Rotating solutions (with nonzero curl)

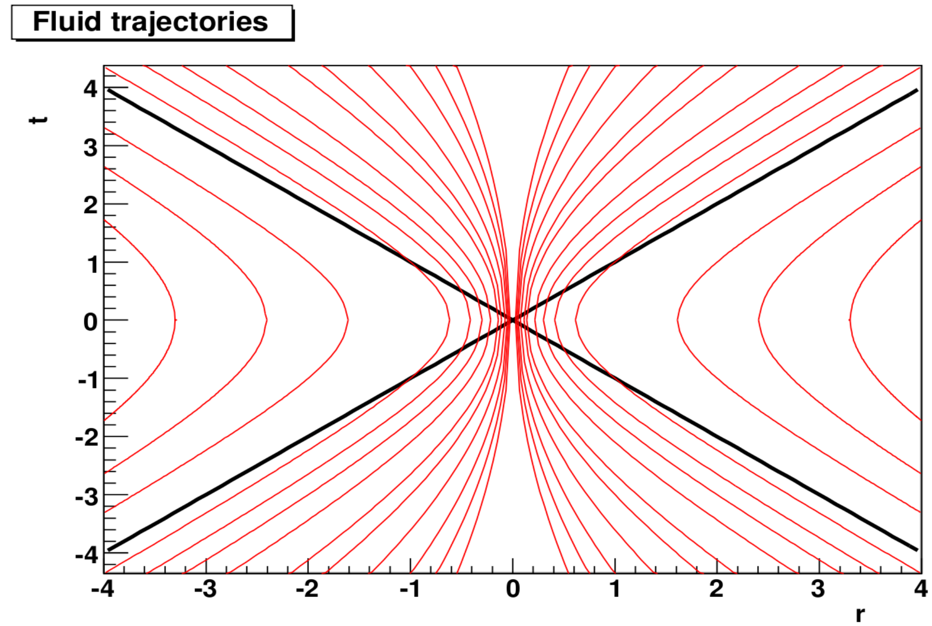
- Solutions with a preferred spatial direction

    - Still collisionless, but allow for more general temperature profile

New explicit solutions



Trajectories of the already known solution, with $\mu=0$



Trajectories of the new solution, with finite μ

- Important: finite total energy on a τ=const. hypersurface

A special case: 1+1 dimensions

If κ=1, equations become simpler

- Potential function is appearing, and for it we get a wave equation, even in the general hydrodynamic case
- Exact solution is possible for any initial condition
- Hard to generalize for realistic EoS
  - Rather a theoretically interesting topic

Special case:

$$v = \tanh \lambda\eta, \qquad p = p_0 \left(\frac{\tau_0}{\tau}\right)^{\lambda d \frac{\kappa+1}{\kappa}} \left(\cosh \frac{\eta}{2}\right)^{-(d-1)\phi_\lambda}$$

For a slight constraint on the temperature,

the general solution is collisionless

Summary

A special problem investigated:

- Do the hydordynamical and the collisionless Maxwell-Boltzmann equations have simultaneous solutions?
- In the non-relativistic case: there was a result, uniqueness proven
- In the relativistic case: already known solutions are shown to be enjoy this property, and new ones found

Interpretation of the collisionlessness:

- The hydrodynamical solutions presented here do not require collisions between individual particles for maintaining thermalization
- In other words: if one specifies the initial condition, then free streaming of the particles maintains local thermal equilibrium
- Does *not* mean that if this flow is realized, it contains no collisions

Summary

New solutions:

- Generalized accelerating flow (with initial “pointlike center”) to that with (variable size) initial “fireball”
- Derived rotating solutions (with initial “rotating” fireball), and solutions with preferred spatial directions (initial “fireball with non-homogeneous motion”)
- Proved uniqueness for these solutions
- Due to more generality, and finite total energy, the new solutions may be a good candidate for applications in the description of the initial accelerating period of heavy-ion collisions

Thank you for your attention!