A NEW NON-PERTURBATIVE ANALYTICAL EQUATION OF STATE FOR THE GLUON MATTER

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arXiv:0902.3901 [hep-ph]

PT QGP EoS

$$P = \frac{1}{3} f(\alpha_s, N_f, \mu_f/T) T^4 + \frac{1}{3}\phi_2(\alpha_s, \mu_f) T^2 + \frac{1}{3}\phi_4(\alpha_s, \mu_f) - B$$

$$f(\alpha_s, N_f, \mu_f/T) = f_0(N_f) - f_2(N_f)\alpha_s + f_3(N_f, \mu_f/T)\alpha_s^{3/2}$$

$$+ f_4(N_f, \mu_f/T, \ln\alpha_s)\alpha_s^2 + f_5(N_f)\alpha_s^{5/2} + O(\alpha_s^3 \ln\alpha_s)$$

$$\alpha_s = g^2/4\pi, \qquad m_f^{(0)} = 0, \qquad N_f = 0, 1, 2.$$

The 3d reduction exhibits serious problems

$$\int \frac{dk_0}{(2\pi)} \longrightarrow \frac{1}{\beta} \sum_{n=-\infty}^{\infty}, \qquad k_0 \longrightarrow (2n+1)\frac{\pi}{\beta}, \quad \beta = T^{-1}$$

I. Much more severe IR structure in 3d QCD
II. Coupling constant becomes dimensional

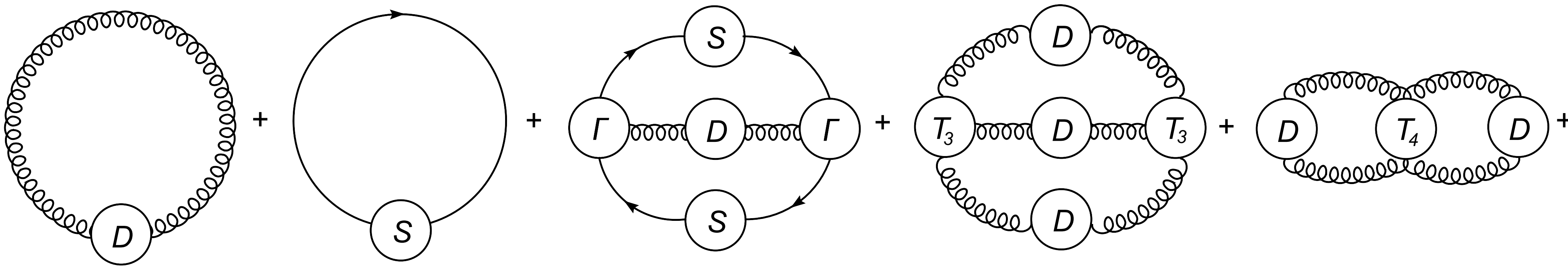
$$g = [M]^{\frac{4-d}{2}} g', \quad g^2 = [M], \quad T, \quad gT, \quad g^2 T$$

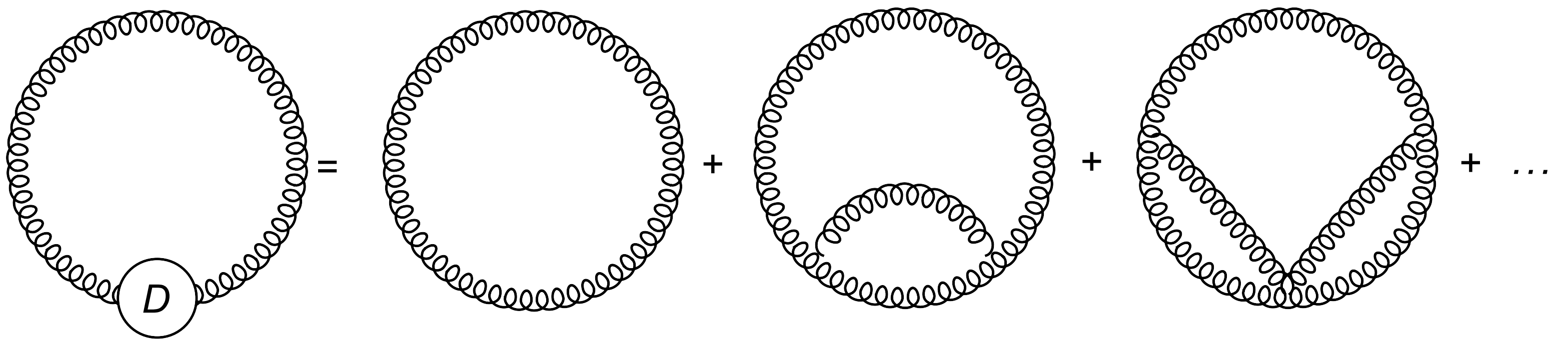
All this results in the non-analytical dependence on α_s, so thermal PT is not to be used.

VACUUM ENERGY DENSITY (VED)

The CJT effective potential approach for composite operators

J.M. Cornwall, R. Jackiw, E. Tomboulis, Phys. Rev. D **10** (1974) 2428





The gluon effective potential to leading order (log-loop level $\sim \hbar$) is

$$V(D) = \frac{i}{2} \int \frac{d^4 q}{(2\pi)^4} \mathrm{Tr} \left\{ \ln(D_0^{-1} D) - (D_0^{-1} D) + 1 \right\}$$

where the normalization is $V(D_0) = 0$.

$$iD_{\mu\nu}(q) = \left\{T_{\mu\nu}(q)d(-q^2,\xi) + \xi L_{\mu\nu}(q)\right\}\frac{1}{q^2}$$

with ξ - gauge fixing parameter and

$$T_{\mu\nu}(q) = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} = g_{\mu\nu} - L_{\mu\nu}(q)$$

$iD_{\mu\nu}(q) \to iD^0_{\mu\nu}(q)$ when $d(-q^2,\xi) = 1$.

$$Tr\,\ln(D_0^{-1}D) = 8\times 4\ln det(D_0^{-1}D) = 32\ln\left\{\frac{3}{4}d(-q^2,\xi) + \frac{1}{4}\right\}$$

and going over to Euclidean space, one obtains $[\epsilon_g = V(D),\;\; a = (3/4) - 2\ln 2]$.

$$\epsilon_g = -16\int \frac{d^4q}{(2\pi)^4}\left\{\ln[1 + 3\alpha_s(q^2)] - \frac{3}{4}\alpha_s(q^2) + a\right\}$$

$d(q^2;\xi) \equiv \alpha_s(q^2)$ – eff. ("running") charge

Colorless, Transversal, $16 = 8\times 2$

THE BAG CONSTANT

$$B = VED^{PT} - VED = VED^{PT} - [VED - VED^{PT} + VED^{PT}] =$$

$$VED^{PT} - [VED^{TNP} + VED^{PT}] = -VED^{TNP} > 0$$

$$B_{YM} = 16 \int^{q^2_{eff}} \frac{d^4 q}{(2\pi)^4} \left\{ \ln[1 + 3\alpha_s^{TNP}(q^2)] - \frac{3}{4}\alpha_s^{TNP}(q^2) \right\}$$

q^2_{eff} separates the NP from the PT regions

G.G. Barnaföldi, V. Gogokhia, arXiv:0708.0163

The truly NP (TNP) gluon effective charge

$$\alpha_s^{TNP}(q^2;\Delta^2) = \alpha_s(q^2,\Delta^2) - \alpha_s(q^2,\Delta^2=0)$$
$$= \alpha_s(q^2,\Delta^2) - \alpha_s^{PT}(q^2)$$

Δ^2 is the mass gap responsible for the NP dynamics in the QCD ground state (the so-called Jaffe-Witten (JW) mass gap).

$$\alpha_s(q^2;\Delta^2) = \alpha_s(q^2,\Delta^2) - \alpha_s^{PT}(q^2) + \alpha_s^{PT}(q^2)$$
$$= \alpha_s^{TNP}(q^2,\Delta^2) + \alpha_s^{PT}(q^2)$$

$$P_g = \epsilon_g + B_{YM} =$$

$$B_{YM} - 16 \int \frac{d^4q}{(2\pi)^4} \left\{ \ln[1 + 3\alpha_s(q^2)] - \frac{3}{4}\alpha_s(q^2) + a \right\}$$

$$\alpha_s(q^2) = \alpha_s^{TNP}(q^2) + \alpha_s^{PT}(q^2) =$$

$$\alpha_s^{TPT}(q^2) + 1 + \alpha_s^{AF}(q^2)$$

$$\alpha_s^{AF}(q^2) = \frac{\alpha_s}{1 + \alpha_s b \ln(q^2/\Lambda^2_{QCD})}, \quad b = (11/4\pi), \ \alpha_s \equiv \alpha_s(M_z) = 0.1187$$

$$P_g = P_{NP} + P_{PT}$$

$$P_{NP} = B_{YM} + P_{YM}$$

$$P_{YM} = -16 \int \frac{d^4q}{(2\pi)^4} \left\{ \ln[1 + \frac{3}{4}\alpha_s^{TNP}(q^2)] - \frac{3}{4}\alpha_s^{TNP}(q^2) \right\}$$

$$P_{PT} = -16 \int \frac{d^4q}{(2\pi)^4} \left\{ \ln\left[1 + \frac{3\alpha_s^{AF}(q^2)}{4 + 3\alpha_s^{TNP}(q^2)}\right] - \frac{3}{4}\alpha_s^{AF}(q^2) \right\}$$

$$P_{NP} = B_{YM} + P_{YM}$$

$$B_{YM} = 16 \int^{q^2_{eff}} \frac{d^4q}{(2\pi)^4} \left\{ \ln[1 + 3\alpha_s^{TNP}(q^2)] - \frac{3}{4}\alpha_s^{TNP}(q^2) \right\}$$

$$P_{YM} = -16 \int \frac{d^4q}{(2\pi)^4} \left\{ \ln[1 + \frac{3}{4}\alpha_s^{TNP}(q^2)] - \frac{3}{4}\alpha_s^{TNP}(q^2) \right\}$$

Confining Ansatz for $\alpha_s^{TNP}(q^2)$

$$\alpha_s^{TNP}(q^2) \Longrightarrow \alpha_s^{INP}(q^2) = \frac{\Delta^2}{q^2}$$

Well justified

Wilson criteria –Area Law,
Linear rising potential between heavy quarks
Explicitly gauge-invariant

Exactly defined, $\alpha_s^{TNP}(q^2) = 0$ at $\Delta^2 = 0$
Analytic (i.e., exact) summation over the
Matsubara frequncies (see below)

Uniquely defined

$$\alpha_s^{TNP}(q^2) \to \frac{\Delta^2}{q^2} \times f(q^2;\Delta^2)$$

$$f(q^2;\Delta^2) = f(0) + \frac{q^2}{M^2} f'(q^2;\Delta^2) + \ldots$$

$$\alpha_s^{TNP}(q^2) = \frac{\Delta^2}{q^2} f(0) + \frac{\Delta^2}{M^2} f'(q^2;\Delta^2) + \ldots$$

$$\alpha_s(q^2) = \alpha_s^{TNP}(q^2) + \alpha_s^{PT}(q^2) = \frac{\Delta^2}{q^2} + \alpha_s^{PT}(q^2)$$

Generalization to non-zero temperatures

In the imaginary time formalism these expressions can be easily generalized to non-zero temperatures T according to the prescription (there is already Euclidean signature)

$$\int \frac{dq_0}{(2\pi)} \to T \sum_{n=-\infty}^{+\infty}, \qquad q^2 = \mathbf{q}^2 + q_0^2 = \mathbf{q}^2 + \omega_n^2 = \omega^2 + \omega_n^2$$

$$\omega_n = 2n\pi T$$

i.e., each integral over q_0 of a loop momentum is to be replaced by the sum over Matsubara frequencies labelled by n, which obviously assumes the replacement $q_0 \to \omega_n = 2n\pi T$ for bosons (gluons). In frequency-momentum space

$$\alpha_s^{TNP}(q^2) = \alpha_s^{TNP}(\mathbf{q}^2, \omega_n^2) = \frac{\Delta^2}{\mathbf{q}^2 + \omega_n^2} = \frac{\Delta^2}{\omega^2 + \omega_n^2},$$

$$T^{-1} = \beta, \quad \omega = \sqrt{\mathbf{q}^2},$$

The derivation of $B_{YM}(T)$

$$B_{YM}(T) = 16 \int \frac{d^3q}{(2\pi)^3}\, T \sum_{n=-\infty}^{+\infty} \left[\ln[1 + 3\alpha_s^{TNP}(\mathbf{q}^2, \omega_n^2)] - \frac{3}{4}\alpha_s^{TNP}(\mathbf{q}^2, \omega_n^2) \right]$$

$$B_{YM}(T) = 16 \int \frac{d^3q}{(2\pi)^3}\, T \sum_{n=-\infty}^{+\infty} \left[\ln[\omega'^2 + \omega_n^2] - \ln[\omega^2 + \omega_n^2] - \frac{3}{4}\frac{\Delta^2}{\omega^2 + \omega_n^2} \right]$$

$$\omega' = \sqrt{\mathbf{q}^2 + 3\Delta^2} = \sqrt{\omega^2 + m'^2_{eff}}$$

$$m'_{eff} = \sqrt{3}\Delta.$$

$$\sum_{n=-\infty}^{+\infty} \frac{1}{\mathbf{q}^2 + \omega_n^2} = \sum_{n=-\infty}^{\infty} \frac{1}{\omega^2 + (2\pi T)^2 n^2} = (2\pi/\beta)^{-2} \sum_{n=-\infty}^{+\infty} \frac{1}{n^2 + (\beta\omega/2\pi)^2}$$

$$= (2\pi/\beta)^{-2}(2\pi^2/\beta\omega) \left(1 + \frac{2}{e^{\beta\omega} - 1}\right) = \frac{\beta}{2\omega} \left(1 + \frac{2}{e^{\beta\omega} - 1}\right).$$

The summation of the thermal logarithms

$$\sum_{n=-\infty}^{+\infty} \ln[3\Delta^2 + \mathbf{q}^2 + \omega_n^2] = \ln \omega'^2 + 2 \sum_{n=1}^{\infty} \ln(2\pi/\beta)^2[n^2 + (\beta\omega'/2\pi)^2]$$

$$\sum_{n=-\infty}^{+\infty} \ln[\mathbf{q}^2 + \omega_n^2] = \ln \omega^2 + 2 \sum_{n=1}^{\infty} \ln(2\pi/\beta)^2[n^2 + (\beta\omega/2\pi)^2].$$

$$L(\omega') = \sum_{n=1}^{\infty} \ln[n^2 + (\beta\omega'/2\pi)^2] = \sum_{n=1}^{\infty} \ln n^2 + \sum_{n=1}^{\infty} \ln\left[1 - \frac{x'^2}{n^2\pi^2}\right]$$

$$L(\omega) = \sum_{n=1}^{\infty} \ln[n^2 + (\beta\omega/2\pi)^2] = \sum_{n=1}^{\infty} \ln n^2 + \sum_{n=1}^{\infty} \ln\left[1 - \frac{x^2}{n^2\pi^2}\right].$$

$$x'^2 = -\left(\frac{\beta\omega'}{2}\right)^2, \quad x^2 = -\left(\frac{\beta\omega}{2}\right)^2.$$

$$L(\omega') - L(\omega) = \sum_{n=1}^{\infty} \ln\left[1 - \frac{x'^2}{n^2\pi^2}\right] - \sum_{n=1}^{\infty} \ln\left[1 - \frac{x^2}{n^2\pi^2}\right]$$

$$= \ln\sin x' - \frac{1}{2}\ln x'^2 - \ln\sin x + \frac{1}{2}\ln x^2,$$

$$L(\omega') - L(\omega) = -\frac{1}{2}\ln\left(\frac{x'^2}{x^2}\right) + \ln\left(\frac{\sin x'}{\sin x}\right).$$

$$x' = \pm i\left(\frac{\beta\omega'}{2}\right), \quad x = \pm i\left(\frac{\beta\omega}{2}\right),$$

$$L(\omega') - L(\omega) = -\frac{1}{2}\ln\left(\frac{\omega'^2}{\omega^2}\right) + \frac{1}{2}\beta(\omega' - \omega) + \ln\left(\frac{1 - e^{-\beta\omega'}}{1 - e^{-\beta\omega}}\right).$$

The temperature dependence of B_{YM}

$$B_{YM}(T) = -\frac{6}{\pi^2}\Delta^2 B^{(1)}_{YM}(T) - \frac{16}{\pi^2} T \left[B^{(2)}_{YM}(T) - B^{(3)}_{YM}(T) \right]$$

$$B^{(1)}_{YM}(T) = \int_0^{\omega_{eff}} d\omega \frac{\omega}{e^{\beta\omega} - 1}, \qquad \beta^{-1} = T$$

$$B^{(2)}_{YM}(T) = \int_0^{\omega_{eff}} d\omega \; \omega^2 \ln\left(1 - e^{-\beta\omega}\right)$$

$$B^{(3)}_{YM}(T) = \int_0^{\omega_{eff}} d\omega \; \omega^2 \ln\left(1 - e^{-\beta\omega'}\right), \qquad \omega' = \sqrt{\omega^2 + 3\Delta^2}$$

The temperature dependence of P_{YM}

$$P_{YM}(T) = \frac{6}{\pi^2}\Delta^2 P_{YM}^{(1)}(T) + \frac{16}{\pi^2} T \left[P_{YM}^{(2)}(T) - P_{YM}^{(3)}(T) \right]$$

$$P_{YM}^{(1)}(T) = \int_0^\infty d\omega \frac{\omega}{e^{\beta\omega} - 1} = \frac{\pi^2}{6} T^2$$

$$P_{YM}^{(2)}(T) = \int_0^\infty d\omega \ \omega^2 \ln\left(1 - e^{-\beta\omega}\right)$$

$$P_{YM}^{(3)}(T) = \int_0^\infty d\omega \ \omega^2 \ln\left(1 - e^{-\beta\bar{\omega}}\right), \qquad \bar{\omega} = \sqrt{\omega^2 + (3/4)\Delta^2}$$

THE GLUON MATTER EoS

$$P_{GM}(T) = B_{YM}(T) + P_{YM}(T) + P_{PT}(T) = P_{NP}(T) + P_{PT}(T)$$

$$P_{NP}(T) = \frac{6}{\pi^2}\Delta^2 P_1(T) + \frac{16}{\pi^2} T[P_2(T) + P_3(T) - P_4(T)]$$

$$P_1(T) = \int_{\omega_{eff}}^{\infty} d\omega \frac{\omega}{e^{\beta\omega} - 1}$$

$$P_2(T) = \int_{\omega_{eff}}^{\infty} d\omega \; \omega^2 \ln\left(1 - e^{-\beta\omega}\right)$$

$$P_3(T) = \int_0^{\omega_{eff}} d\omega \ \omega^2 \ln\left(1 - e^{-\beta\omega'}\right)$$

$$P_4(T) = \int_0^{\infty} d\omega \ \omega^2 \ln\left(1 - e^{-\beta\bar{\omega}}\right)$$

$$\omega' = \sqrt{\omega^2 + 3\Delta^2} = \sqrt{\omega^2 + m'^2_{eff}}$$

$$\bar{\omega} = \sqrt{\omega^2 + \frac{3}{4}\Delta^2} = \sqrt{\omega^2 + \bar{m}^2_{eff}}$$

In the formal PT limit ($\Delta^2 = 0$) from these relations it follows that $\bar{\omega} = \omega' = \omega$. And always $\beta = T^{-1}$.

The temperature dependence of P_{PT}

$$P_{PT}(T) = -\frac{8}{\pi^2} \int_0^{\infty} d\omega \ \omega^2 \ T$$

$$\times \sum_{n=-\infty}^{+\infty} \left[\ln \left[1 + \frac{3\alpha_s^{AF}(\omega^2, \omega_n^2)}{4 + 3\alpha_s^{TNP}(\omega^2, \omega_n^2)} \right] - \frac{3}{4}\alpha_s^{AF}(\omega^2, \omega_n^2) \right],$$

$$\alpha_s^{AF}(q^2) = \alpha_s^{AF}(\mathbf{q}^2, \omega_n^2) = \alpha_s^{AF}(\omega^2, \omega_n^2) = \frac{\alpha_s}{1 + \alpha_s b \ln[(\omega^2 + \omega_n^2)/\Lambda_{YM}^2]}.$$

$$P_{GM}(T) \equiv P(T) \Longrightarrow P_{NP}(T)$$

Main thermodynamical quantities

The energy density

$$\epsilon(T) = T\left(\frac{\partial P(T)}{\partial T}\right) - P(T) = Ts(T) - P(T)$$

The entropy

$$s(T) = \frac{\partial P(T)}{\partial T}$$

The heat capacity

$$c_V(T) = \frac{\partial \epsilon(T)}{\partial T} = T\left(\frac{\partial s(T)}{\partial T}\right)$$

The velocity of sound

$$c_s^2(T) = \frac{\partial P(T)}{\partial \epsilon(T)} = \frac{s(T)}{c_V(T)}$$

Conformity

$$C(T) = \frac{P(T)}{\epsilon(T)}$$

The trace anomaly relation

$$\epsilon(T) - 3P(T)$$

The gluon condensate

$$< G^2 >_T = < G^2 >_{T=0} - [\epsilon(T) - 3P(T)]$$

SB limit

$$\frac{3P_{SB}(T)}{T^4} = \frac{\epsilon_{SB}(T)}{T^4} = \frac{3s_{SB}(T)}{4T^3} = \frac{c_{V(SB)}(T)}{4T^3} =$$

$$\frac{24}{45}\pi^2 = 5.26, \quad T \to \infty \ (\beta \to 0),$$

$$C_{SB}(T) = c^2_{s(SB)}(T) = \frac{1}{3}, \quad T \to \infty \ (\beta \to 0),$$

$$\epsilon_{SB}(T) - 3P_{SB}(T) = 0, \quad T \to \infty \ (\beta \to 0),$$

The scale-setting scheme

$$q^2_{eff} = \mathbf{q}^2_{eff} + \omega_c^2 = \omega^2_{eff} + \omega_c^2, \quad \omega_c = 2\pi n_c T_c$$

$$\omega_{eff} = \sqrt{q^2_{eff} - \omega_c^2}, \quad \omega_{eff} \leq q_{eff}$$

$$\omega_{eff} = \sqrt{q^2_{eff}} = 1\ GeV$$

$$\Delta^2 = 0.4564\ GeV^2, \qquad \Delta = 0.6756\ GeV$$

$$\omega' = \sqrt{\omega^2 + m'^2_{eff}}, \quad m'_{eff} = \sqrt{3}\Delta = 1.17\ GeV.$$

$$\bar{\omega} = \sqrt{\omega^2 + \bar{m}^2_{eff}}, \quad \bar{m}_{eff} = (\sqrt{3}/2)\Delta = 0.585\ GeV,$$

$$< G^2 >_{T=0} = 0.1052\ GeV^4$$

The confinement dynamics is nontrivially taken into account directly through the mass gap, and through the thermalization of the Bag constant itself.

Numerical results and figures

The most remarkable feature of our numerical calculations with the NP pressure is that at $T = T_c = 266.5\ MeV$, the NP pressure, entropy and energy densities satisfy a SB-type relations, namely

$$\frac{3P_{NP}(T_c)}{T_c^4} = \frac{\epsilon_{NP}(T_c)}{T_c^4} = \frac{3s_{NP}(T_c)}{4T_c^3} = \frac{12}{45}\pi^2 = 2.63,$$

$$C_{NP}(T_c) = \frac{1}{3}, \quad \epsilon_{NP}(T_c) - 3P_{NP}(T_c) = 0,$$

$$\frac{c_{V(NP)}(T_c)}{4T_c^3} = 1.91, \quad c^2_{s(NP)}(T_c) = 0.45.$$

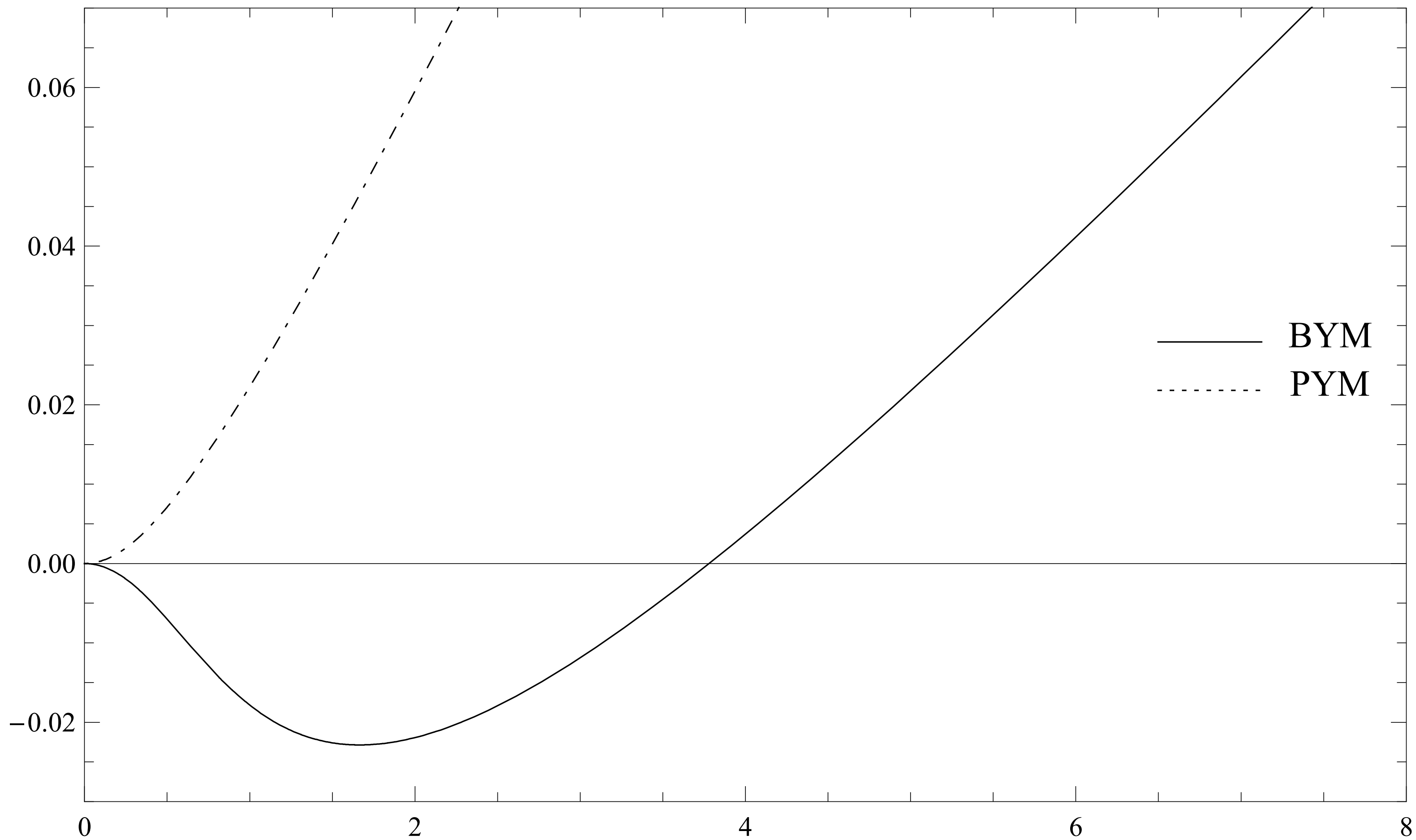


Figure 1: The Bag constant and the NP YM part as a functions of T/T_c. The Bag constant at $T/T_c = 3.75$ is zero, so that up to this value the Bag constant is responsible for the NP vacuum contributions to the pressure.

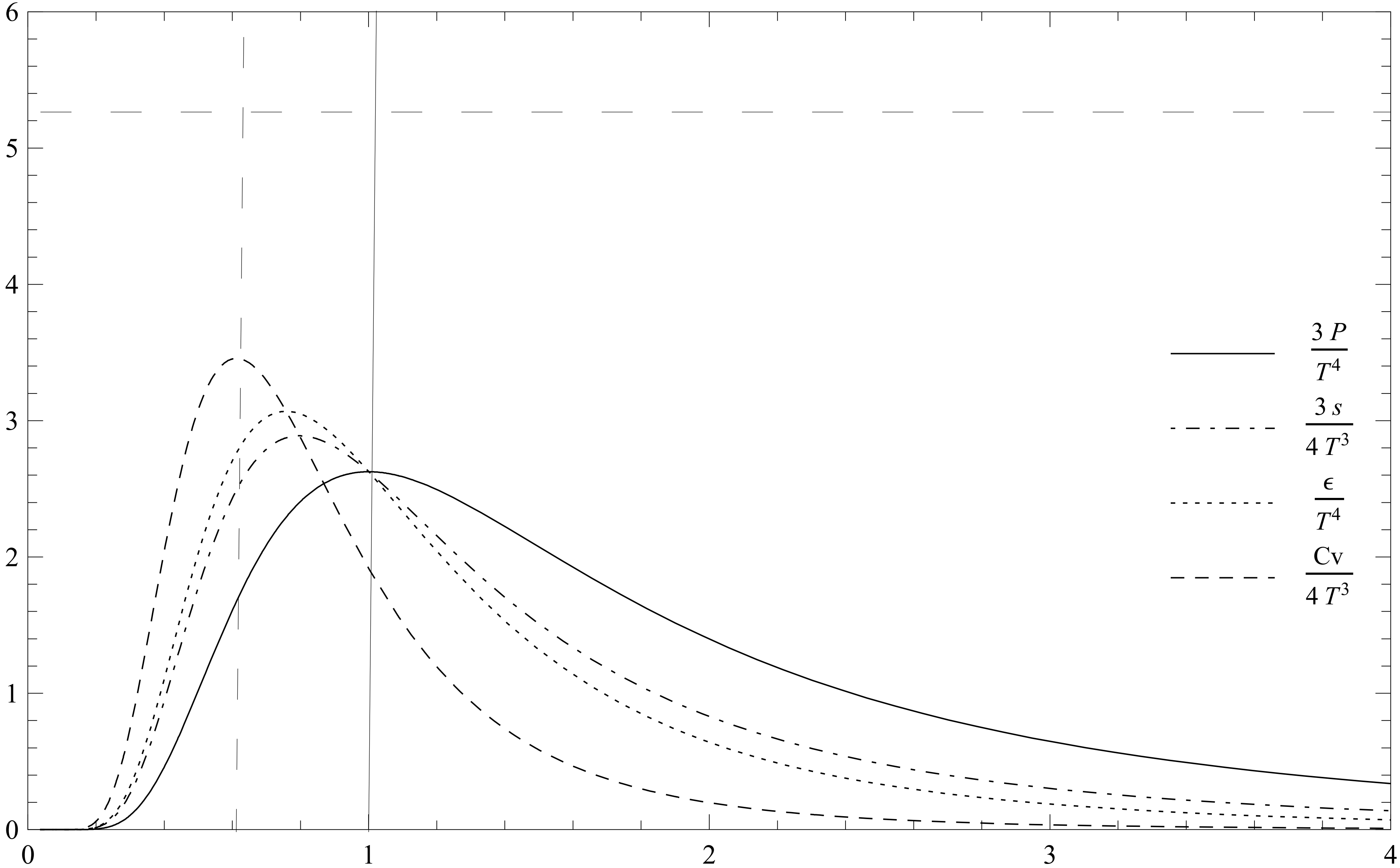


Figure 2: The NP pressure has a maximum at $T_c = 266.5\ MeV$.

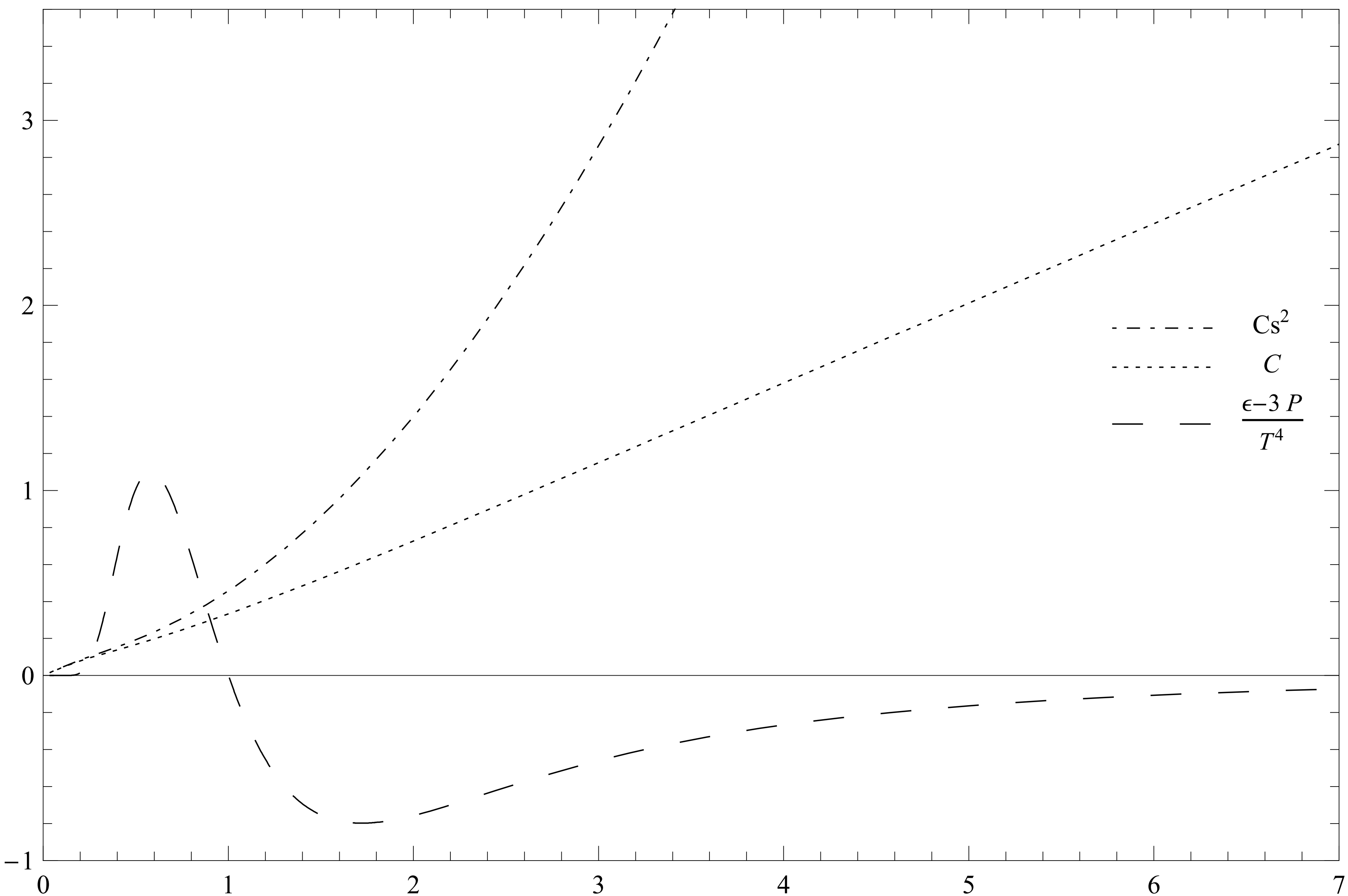


Figure 3: The NP velocity of sound, conformity and the trace anomaly relation as a functions of T/T_c.

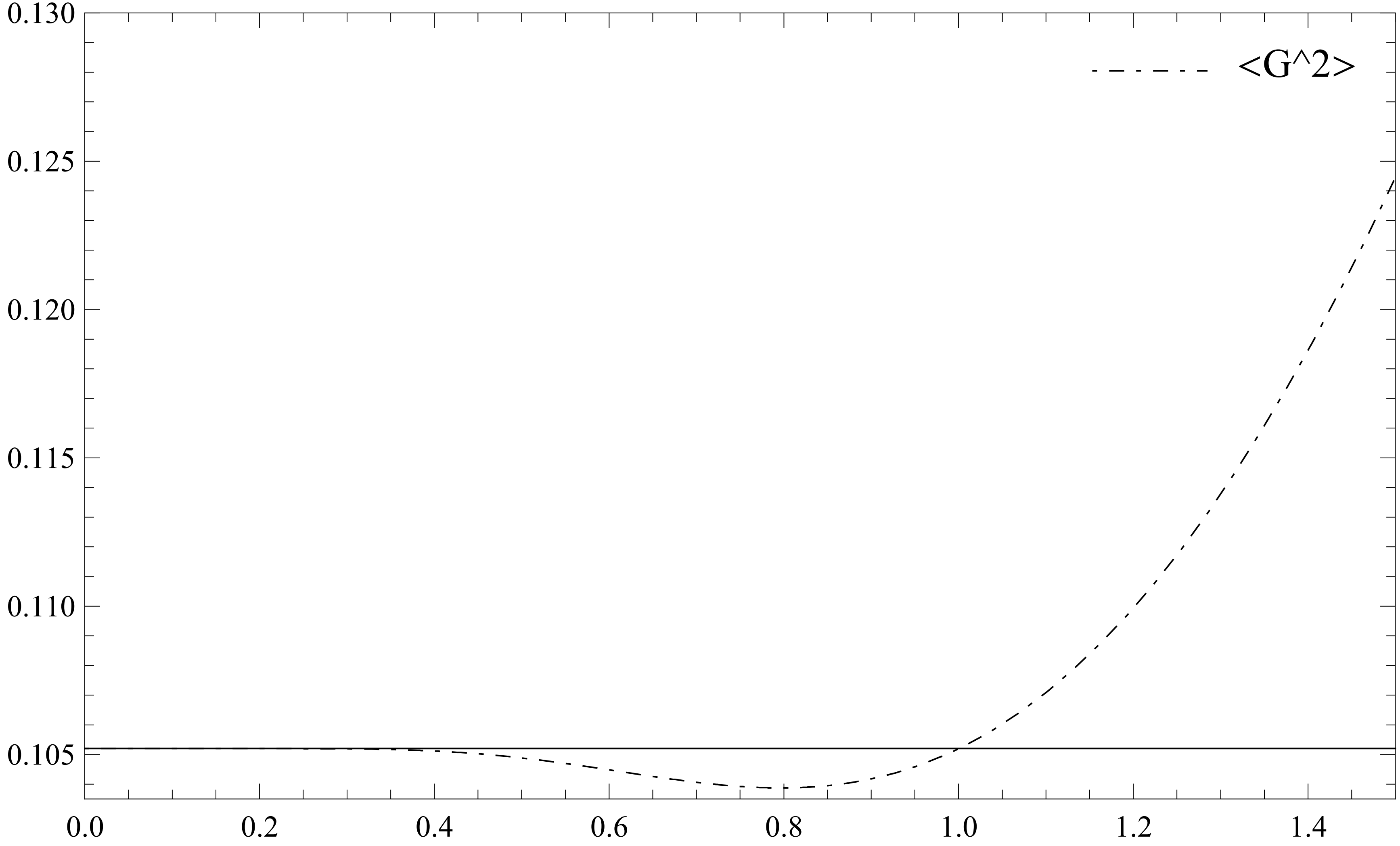


Figure 4: The NP Gluon condensate as a function of T/T_c. Here the solid line is its value at zero temperature $< G^2 >_{T=0}$. It shows little temperature dependence below T_c and grows rapidly above T_c.

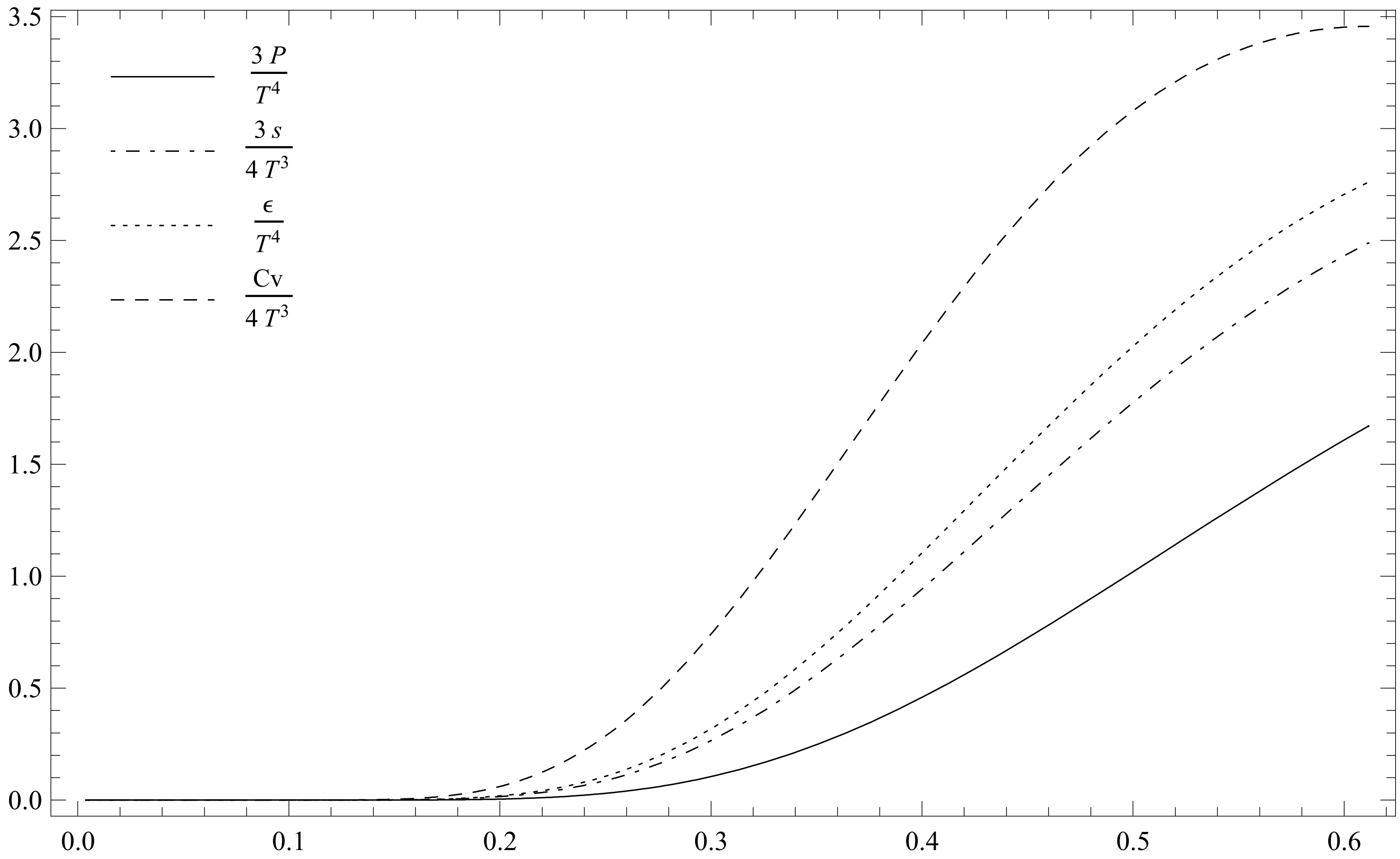


Figure 5: The NP GM pressure, entropy and energy densities, heat capacity as a functions of T/T_c in the low-temperatures region shown up to $0.6T_c$.

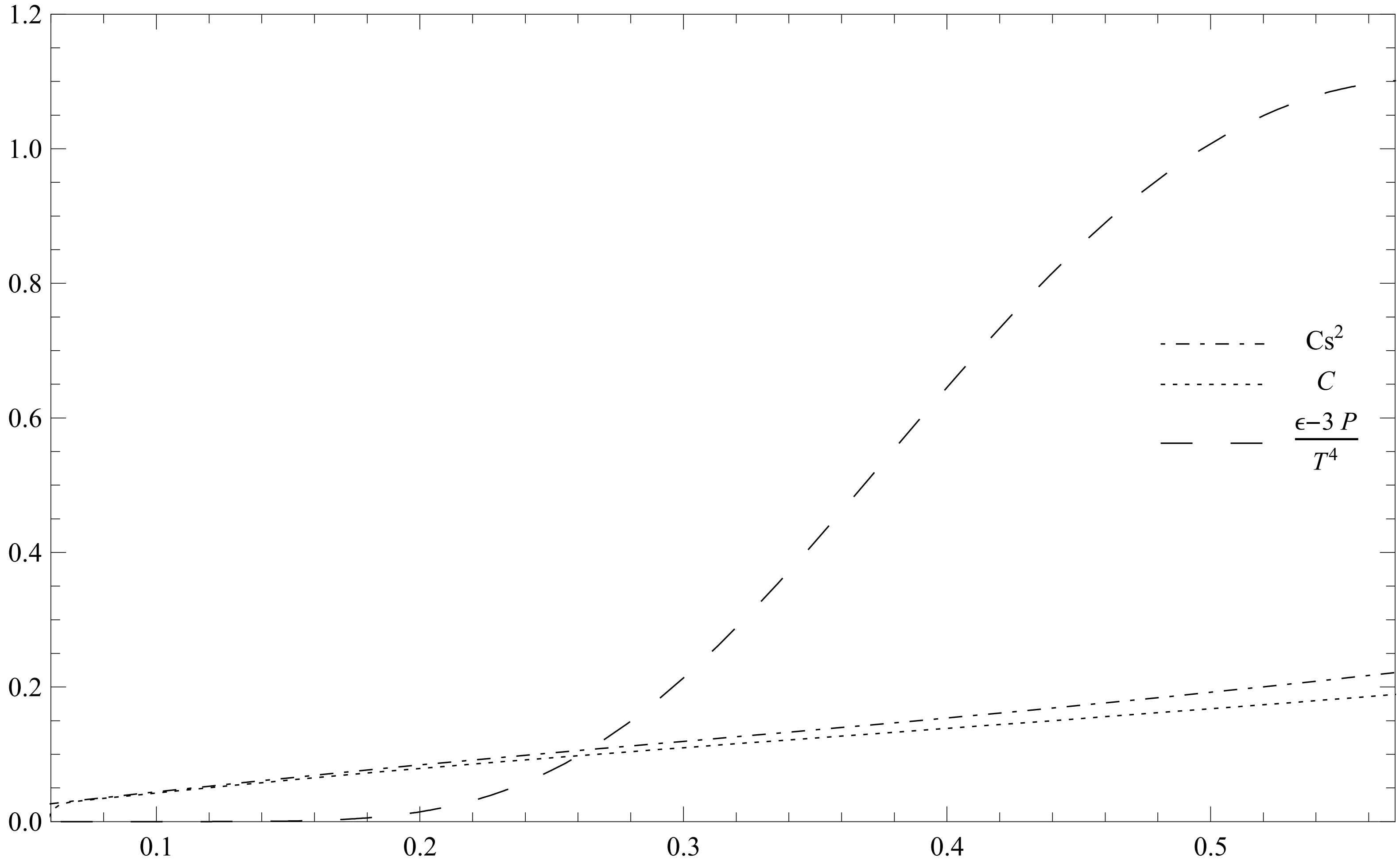


Figure 6: The NP velocity of sound, conformity and the trace anomaly relation as a functions of T/T_c. They are shown in the low-temperatures region up to $0.6T_c$.

The "fuzzy" bag term and the mass gap

$$P_{NP}(T) = \Delta^2 T^2 - \frac{6}{\pi^2}\Delta^2 P_1'(T) + \frac{16}{\pi^2} T[P_2(T) + P_3(T) - P_4(T)],$$

$$P_1'(T) = \int_0^{\omega_{eff}} d\omega \frac{\omega}{e^{\beta\omega} - 1},$$

$$\frac{\epsilon_{NP}(T) - 3P_{NP}(T)}{T^4} \times T^2 = \frac{\epsilon_{NP}(T) - 3P_{NP}(T)}{T^2} \longrightarrow -const., \quad T \to \infty$$

$$\frac{< G^2 >_T}{T^4} = \frac{< G^2 >_{T=0}}{T^4} - \frac{[\epsilon_{NP}(T) - 3P_{NP}(T)]}{T^2} \times \frac{1}{T^2} \sim \frac{const.}{T^2}, \quad T \to \infty.$$

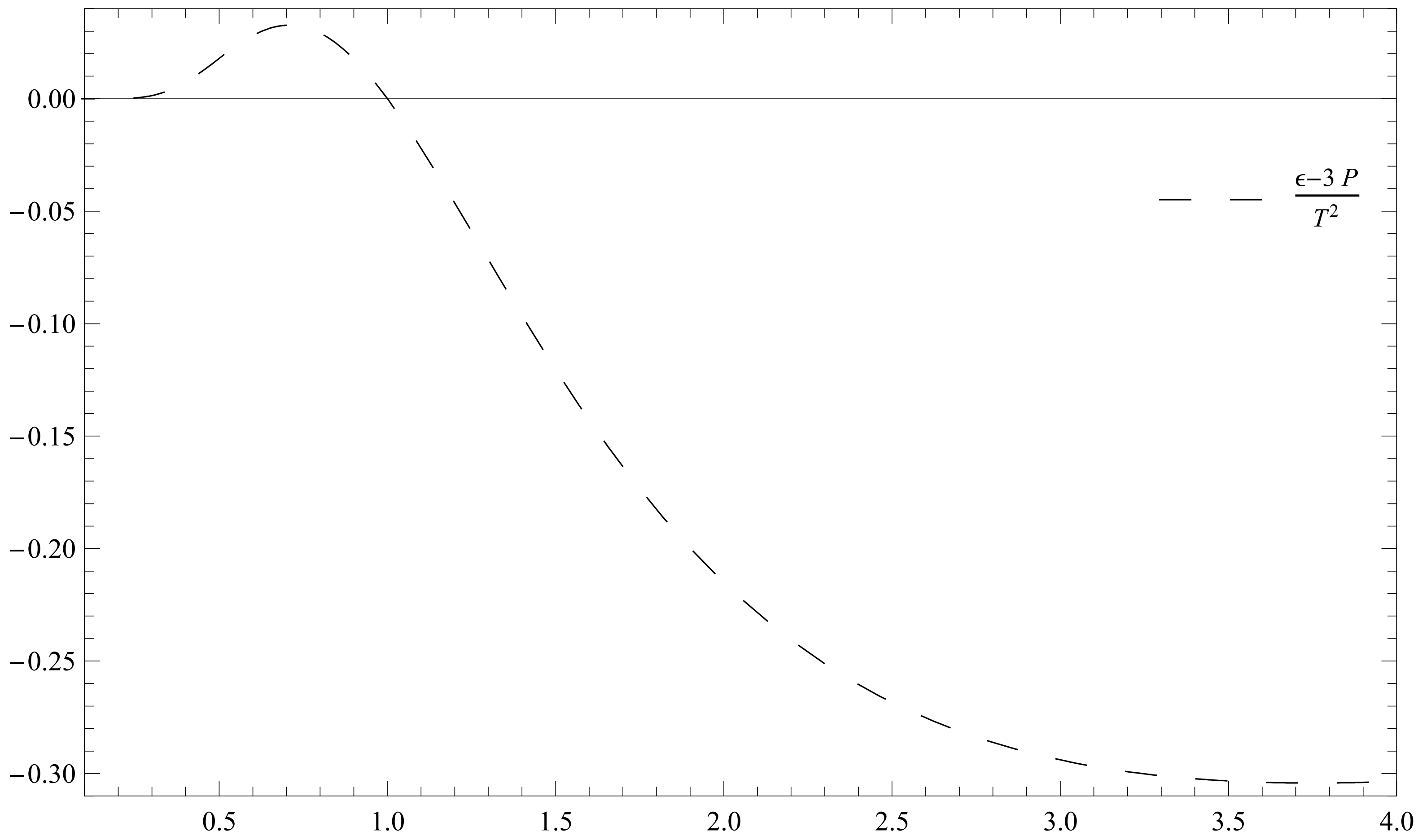


Figure 7: The NP trace anomaly relation scaled by T^2 in GeV^2 units as a function of T/T_c. It clearly goes to the finite constant in the high temperature limit

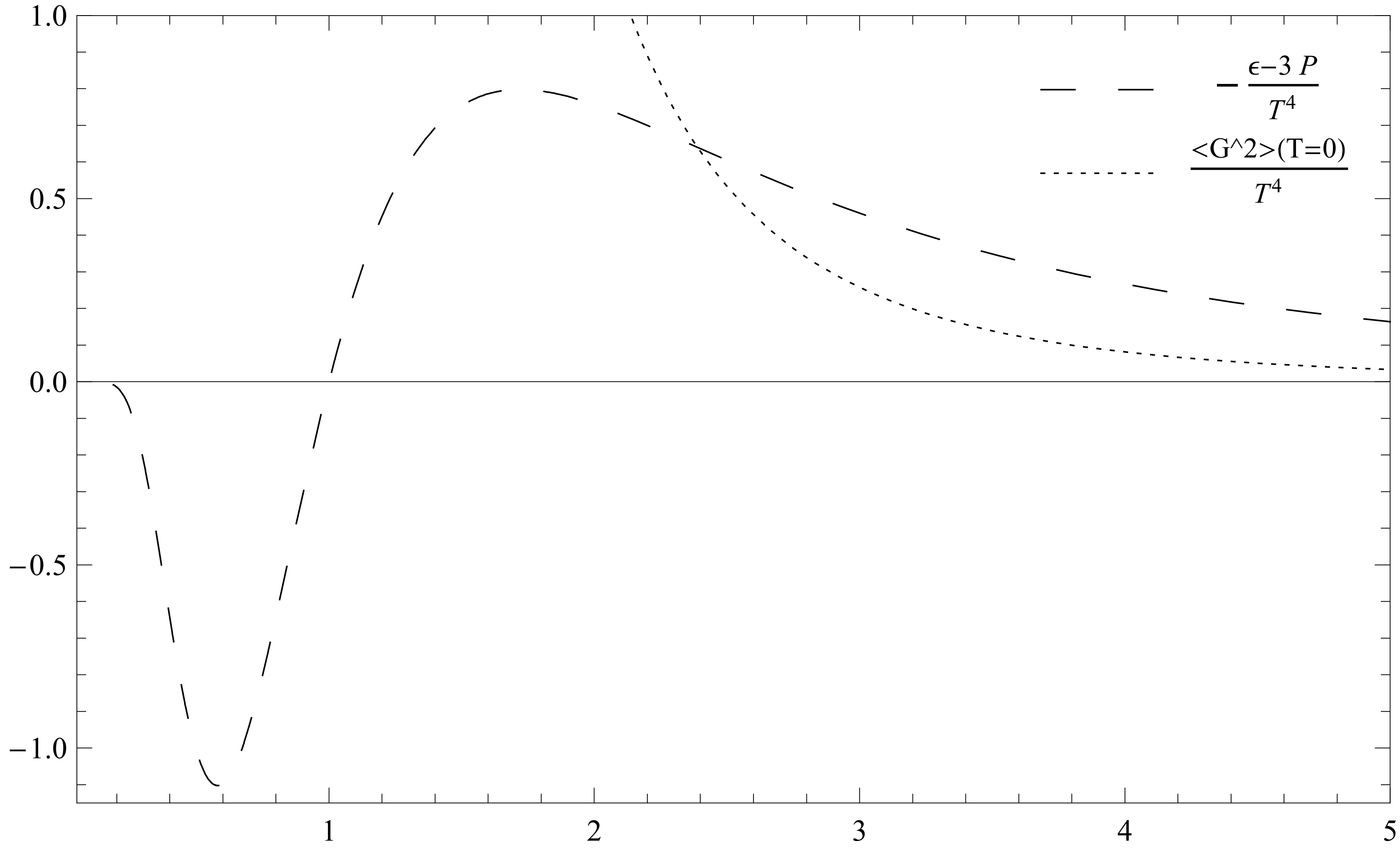


Figure 8: The NP zero temperature gluon condensate and the NP trace anomaly relation with minus sign. Both scaled by T^4 and are shown as a functions of T/T_c.

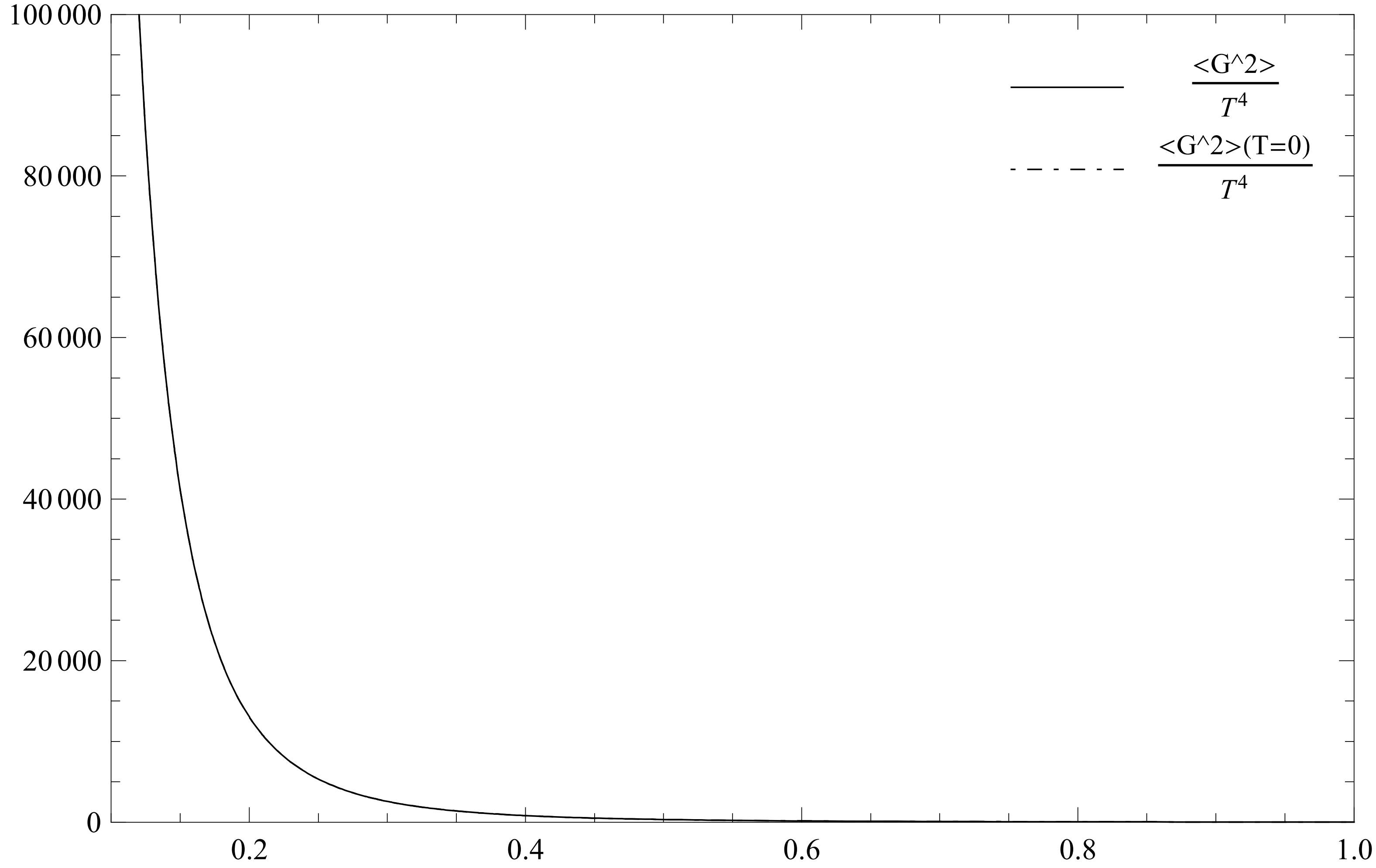


Figure 9: The NP gluon condensate scaled by T^4 as a function of T/T_c. The NP zero temperature gluon condensate scaled by T^4 is also shown. At low temperatures below T_c they coincides, as it should be, since the NP trace anomaly goes to zero in the $T \to 0$ limit.

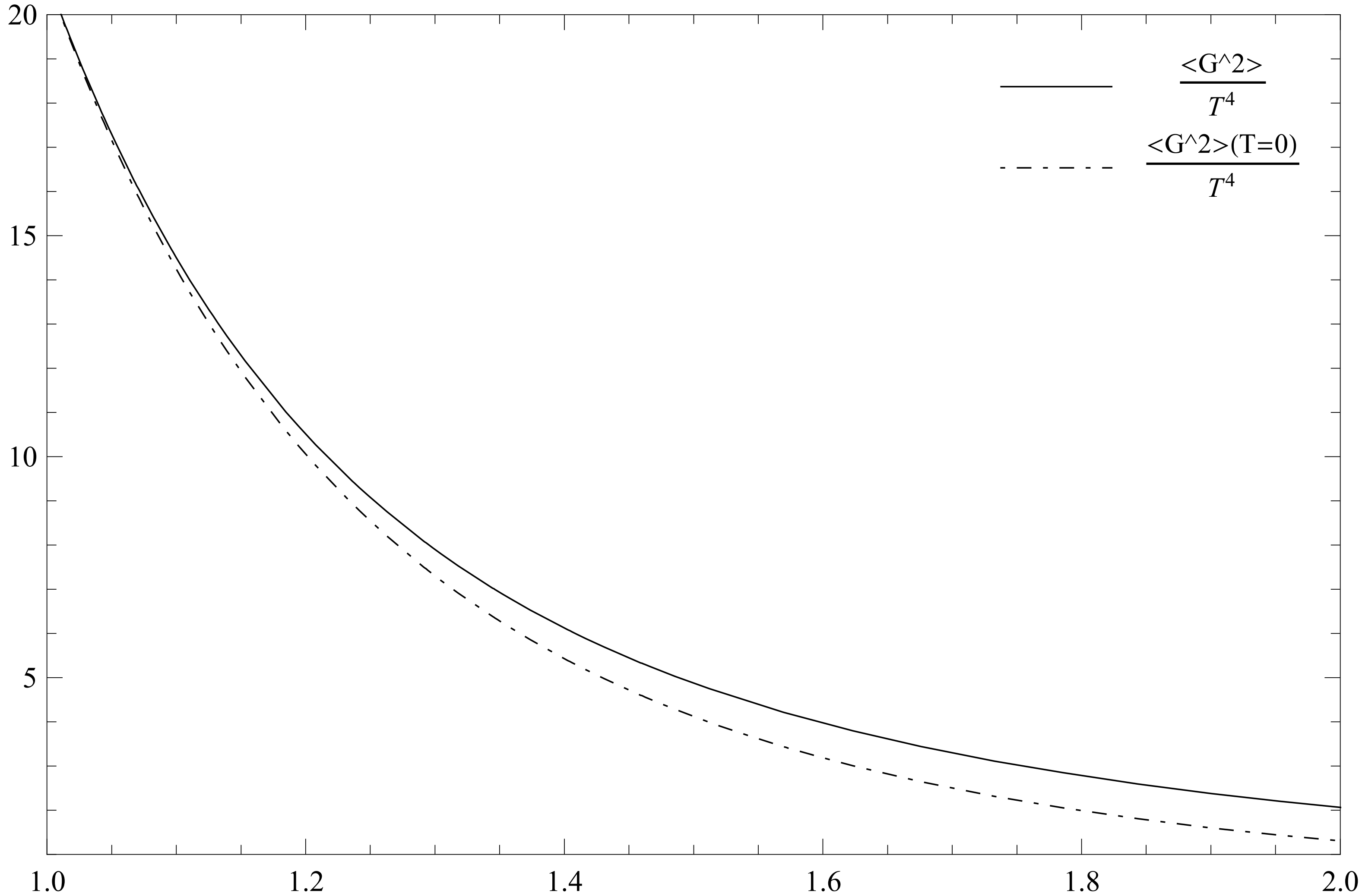


Figure 10: The NP gluon condensate scaled by T^4 as a function of T/T_c. The NP zero temperature gluon condensate scaled by T^4 is also shown. The difference between them is already seen in the moderate temperature region $(1-2)T_c$.

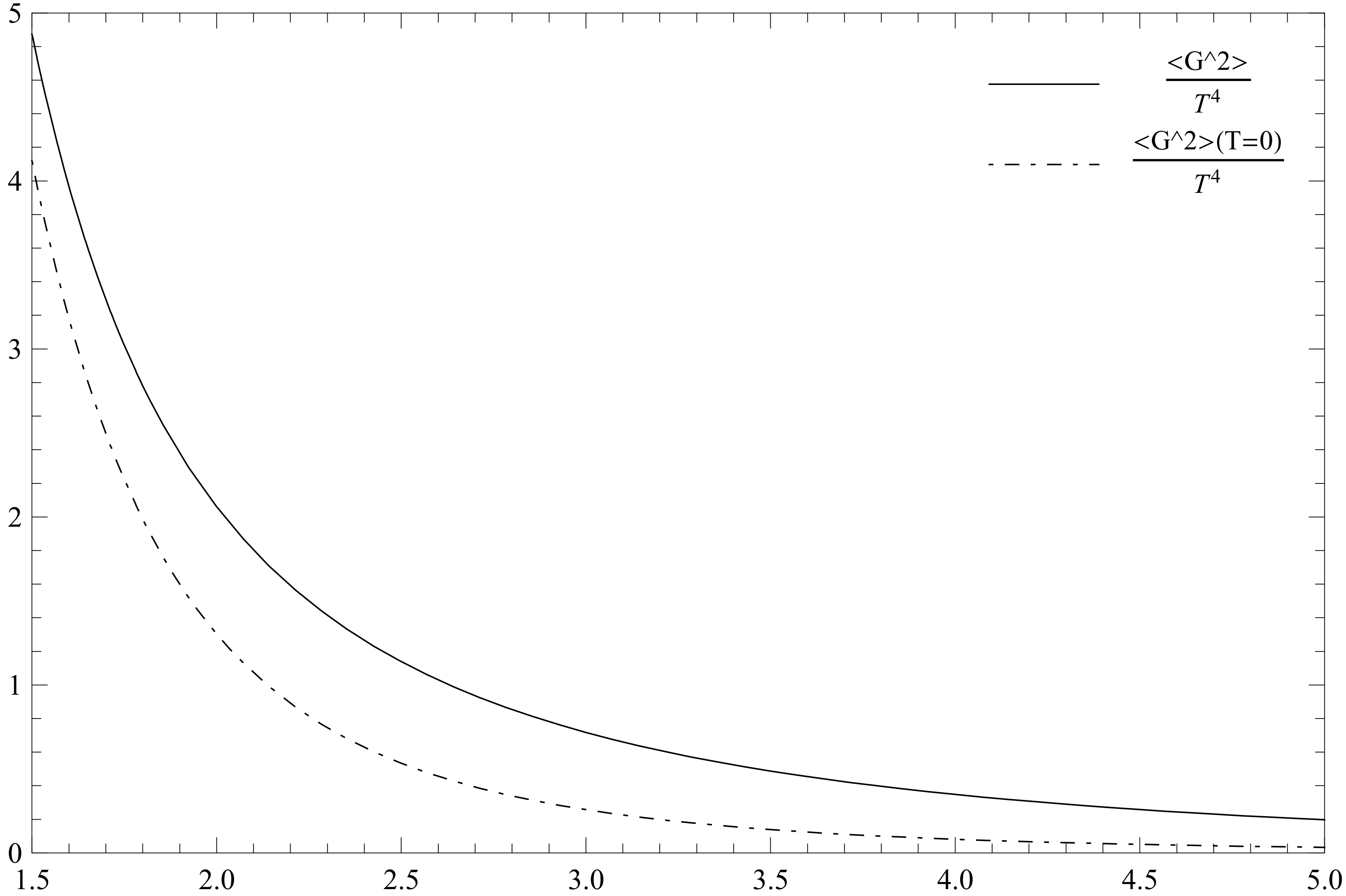


Figure 11: The NP gluon condensate scaled by T^4 as a function of T/T_c. The NP zero temperature gluon condensate scaled by T^4 is also shown.

The dynamical structure of the NP GM

1. $\omega' = \sqrt{\omega^2 + 3\Delta^2} = \sqrt{\omega^2 + m'^2_{eff}}, \quad m'_{eff} = 1.17\ GeV$

2. $\bar{\omega} = \sqrt{\omega^2 + (3/4)\Delta^2} = \sqrt{\omega^2 + \bar{m}^2_{eff}}, \quad \bar{m}_{eff} = 0.585\ GeV$

3. The NP massless gluons associated directly with Δ^2

4. The NP massless gluons not directly associated with Δ^2

5. The two new forms of the GM below and above $3.75T_c$

6. A possible "mixed" phase around T_c

Conclusions

(i). The confining dynamics at non-zero temperatures is taken into account through the T-dependent Bag constant and the NP YM part of the full GM EoS. The both terms essentially depend on the mass gap Δ^2.

(ii). The value of the characteristic temperature $T_c = 266.5\ MeV$ is uniquely and exactly fixed.

(iii). The low-temperature region up to $0.6T_c$ is under our control.

(iv). At T_c all the main full thermodynamic quantities will have a discontinuities, while the full pressure will remain continuous.

(v). The existence of the $\Delta^2 T^2$ term in the NP pressure, which will remain in the full pressure as well.

(vi). Because of the $\Delta^2 T^2$ term the full trace anomaly and the gluon condensate will go down as $1/T^2$ at high temperatures, and not as $1/T^4$.

(vii). The presence of the four different types of massive and massless gluonic excitations of the NP origin.

(viii). In the moderate temperature region up to $3.75T_c$ the NP vacuum effects are still significant. This points out on the existence of a mixed phase in the GM around T_c from its both sides.

(ix). All the full thermodynamic quantities, therefore, will approach their SB limits rather slowly.

(x). A possible existence of a new forms of the GM below and above $3.75T_c$ has been also pointed out.

Evaluation of the $P_{PT}(T)$ and inclusion of quarks in order to derive the NP analytical QGP EoS.

Quark Confinement

I. Necessary condition $S(p) \neq Z_2/(\hat{p} - m_{phys})$,

i.e., quarks are always <u>off mass-shell</u> objects

II. Sufficient condition
Discrete spectrum (no continuum) in bound states

Quark Confinement is absolute and permanent
Deconfinement phase transition does not exist
What is known as
the Deconfinement phase transition is, in fact,
the Dehadronization phase transition