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Are $N_c = 3 \neq \infty$ effects important in parton showers?

- Why N_c suppressed terms may be important
- Sources of N_c suppressed terms
- A general color basis
- Ordinary parton shower
- "Color amplitude shower"
- Preliminary results
- Future plans

Work in progress, JHEP 0909:087

CERN
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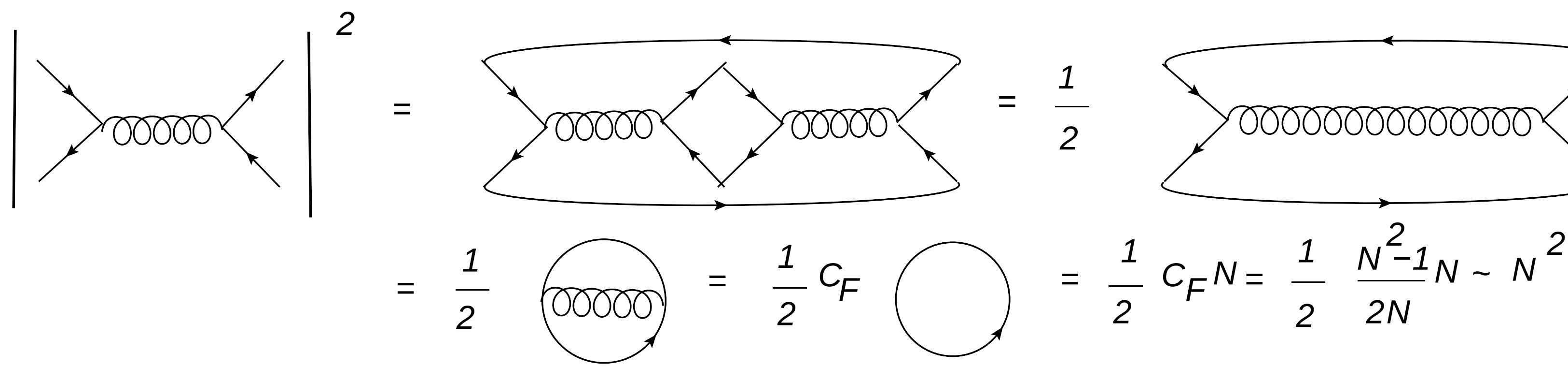
Why worry?

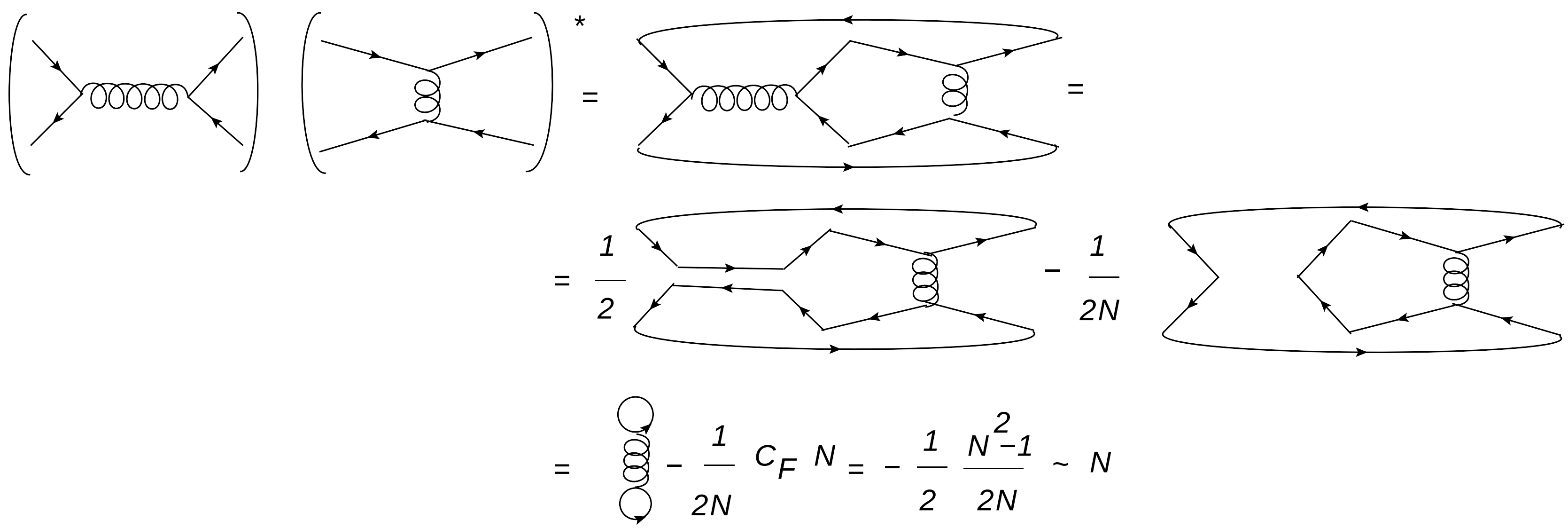
- " Non-leading color terms are suppressed by $1/N_c^2$ "
- Not always true, some simple counter examples...
- Is true for same order α diagrams with only gluons
- A parton shower is an all order (Sudakov) exponentiation

$$\Delta(t) = \exp(-\int_{t_0}^{t} \frac{dt'}{t'} \int dz \frac{\alpha}{2\pi} P(z))$$

- Certainly not only one power in α is needed
- Also, even if non-leading terms always were N_c^2 suppressed, the number of suppressed terms grow $\sim (N_{\text{partons}}!)^2$
  $\rightarrow$ Importance naively grows like $\sim (N_{\text{partons}}!)^2/N_c^2$

Why worry?





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Some rescuing mechanisms?

- In the collinear regions, the emitted parton can be seen as coming from only one parton and the color structure is trivial
  $\rightarrow$ no need for N_c suppressed terms
- Random averaging:
  The suppressed terms sometimes contributes positively to the cross section, and sometimes negatively, perhaps they tend to cancel
- α_s suppression: $1/N_c^1$ suppressed terms tend to also be associated with powers of α_s
- Current parton showers actually do work quite well, this is a reason for believing that there is a suppression mechanism

Different sources of N_c-suppressed terms

- In a tree level parton shower (no virtual gluon exchange), N_c-suppressed terms are dropped as interferences are ignored
- Although an ordinary (tree level) parton shower resum the most important terms, there are suppressed terms from virtual gluon exchanges which rearrange the color structure, in

  $$\exp(\text{large} + \text{moderate})$$

  the moderate number is not irrelevant!
  $\longrightarrow$ different source of N_c-suppressed terms
- To treat the later source a basis for describing the color space is needed

A basis for the color space

The color space is a finite dimensional vector space equipped with a scalar product

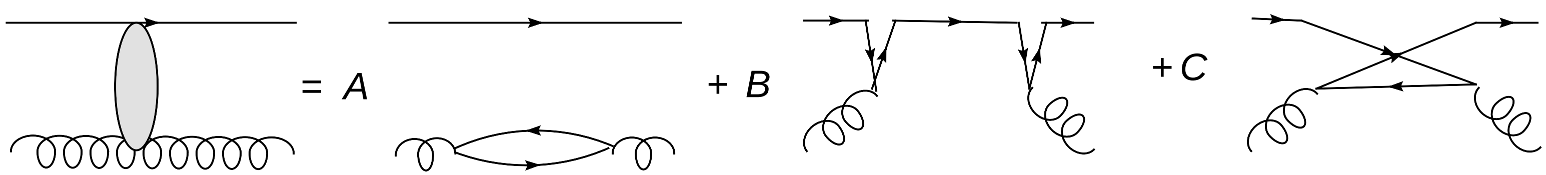
$$< A, B >= \sum_{a,b,c,\ldots} A_{a,b,c,\ldots} (B_{a,b,c,\ldots})^*$$

Individual colors are not observed so we always sum/average over them. One way of constructing a complete basis (for any fixed number of external colored) particles is to:

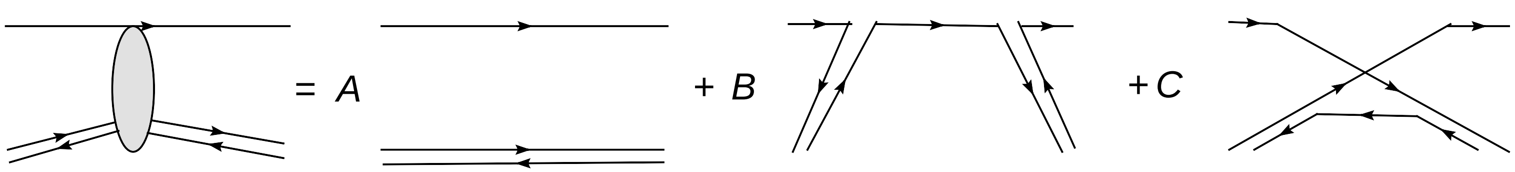
- Decompose all gluons into $\mathrm{q\bar{q}}$-pairs
- Connect quarks and anti-quarks in all possible ways, such that the $\mathrm{q\bar{q}}$ pairs corresponding the the same gluon are not connected
- (If only gluons, make sure the internal quarks and anti-quarks enter on equal footing)

A basis for the color space
Example: qg → qg

- Split gluons into $q\bar{q}$ pairs and connect lines in all possible ways



- In the $N_c \to \infty$ limit

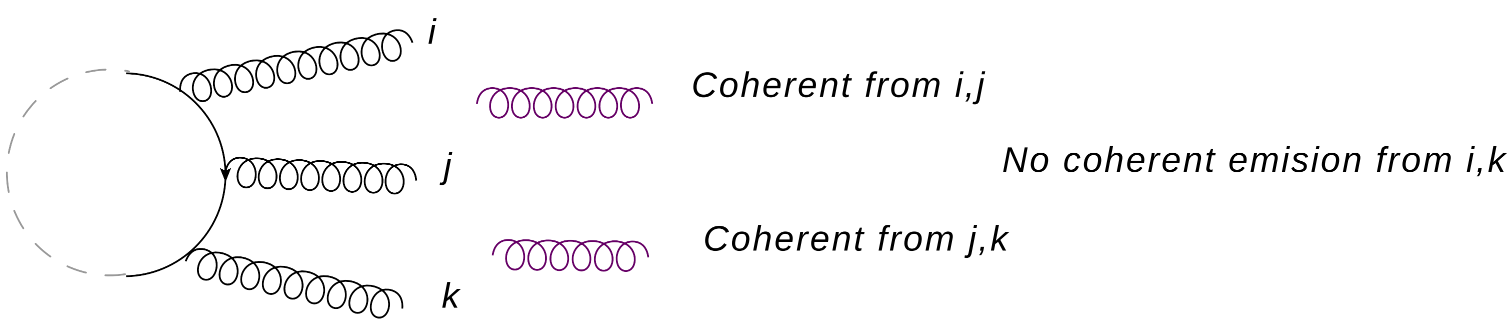


A basis for the color space

- The number of basis tensors grows roughly factorially
- For $N_{\mathrm{q}} = N_{\bar{\mathrm{q}}}$ and $N_g = 0$, there are precisely $N_{\mathrm{q}}!$ basis states
- For states with gluons even more
- Hence the naive importance of suppressed terms $\sim (N_{\text{partons}}!)^2/N_c^2$
- The basis constructed in this way is complete
- Overcomplete for $N_c \neq \infty$ (and many partons)
- Virtual gluon exchange directly gives back a linear combination of the basis tensors $\rightarrow$ can easily be treated in this basis
- By virtual gluon exchange leading N_c terms are diagonal

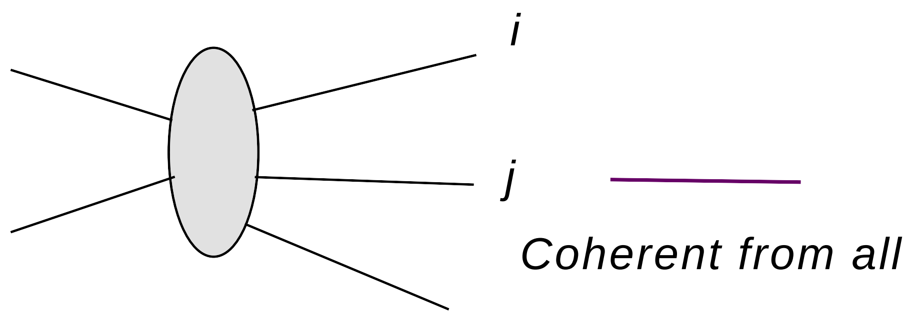
An ordinary parton shower

- Works at the cross section level
- Can be thought of in the language of the $N_c \to \infty$ limit of the above basis (apart from C_F...).
- Also, it is easy to prove that in this limit only "color neighbors" radiate, i.e. only neighboring partons on the quark-lines in the basis $\to$ above basis superior for comparing to parton showers



A (toy) amplitude color shower

- Treat: $N_c = 3$ color $\otimes$ random number
- Emit one parton at the time (imagine an evolution time)
- Keep *all* contributions to the emission



- "shower $<$ amplitude shower $<$ all Feynman diagrams calculation"
- Will the ratio

$$\frac{|A(N_{\text{partons}})|^2|_{\text{Leding terms}}}{|A(N_{\text{partons}})|^2|_{\text{All terms}}} \neq 1?$$

Preliminary results

- Starting with $q\bar{q}$ and radiate N_g gluons
- $\left\langle \frac{|A(N_{\text{partons}})|^2|_{\text{Leding terms}}}{|A(N_{\text{partons}})|^2|_{\text{All terms}}} \right\rangle$

N_g	$C_F = 4/3$	$C_F = 3/2$
1	1	9/8
2	0.97	1.22
3	0.92	1.31
4	0.85	1.36
5	0.77	1.39

- Importance of suppressed terms does grow, but not like $N_{\text{partons}}!^2$. Random averaging?
- Treatment of C_F is very important

Future plans

- Continue checking
- Further investigate simple results
- Add virtual gluon exchange (rearrange the color without emission)
- Speed up program by saving intermediate results
- Incorporate sensible momentum space