W/Z-Gamma Production at NLO in Powheg Method

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Outline:

- Motivations
- Matrix element Calculations
  - Gluon radiation
  - Quark/Anti-quark radiation
- Shower in Powheg Method
  - Gluon radiation
  - Quark/Anti-quark radiation
- Outlook

Why Vector Boson-Photon Production?

Testing Standard Model and Search for New Physics:

- Anomalous WWγ coupling: CP– conserving κ,λ, and $\tilde{\kappa}$? $\tilde{\lambda}$
- Are there ZZγ or Zγγ couplings? Gauge symmetry breaking!
- Agree well with the standard model in Tevatron, and how about in LHC?

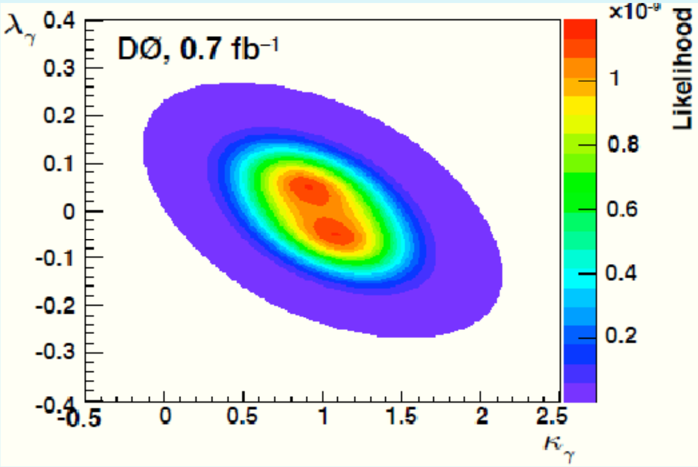
Wγ production at DØ*:

Best points:

(-0.084, -0.05),
(+0.084, +0.05)

$$|\Delta\kappa| \leq 0.51$$

$$-0.12 \leq \lambda \leq 0.13$$



*Adam Lyon, ICHEP'08

But NLO Matching ME with PS!

- LHC: high energy scale and high luminosity: we need precise NLO calculations
- NLO ME (BLO): additional parton radiation + naïve parton Shower

  ⇨ double counting and invalid in IR phase space region
- Soft gluon radiation cut δ_s and photon isolation cut for quark radiation δ_c to cancel the IR divergence

Methods to match NLO matrix element

with NLO parton shower:

Implement Powheg in Herwig++

Catani-Seymour framework

- Born and radiation phase space mapping $\Phi_{n+1} \Rightarrow \overline{\Phi}^{(\alpha)}{}_B, \Phi^{(\alpha)}{}_{rad}$

  C-S variables: (e.g. $x_{i,ab}, \tilde{v}_i, \phi$ in gluon radiation)

- Separate the real radiation into pieces $R^{(\alpha)}(\Phi^{(\alpha)}{}_{rad}, \overline{\Phi}^{(\alpha)}{}_B)$ with different singular regions in Catani-Seymour Formalism:

$$R = \sum_\alpha \mathcal{D}_\alpha(\Phi^{(\alpha)}{}_{rad}, \overline{\Phi}^{(\alpha)}{}_B) + \text{ finite term}$$

- Dipole function $\mathcal{D}_\alpha(\Phi^{(\alpha)}{}_{rad}, \overline{\Phi}^{(\alpha)}{}_B) = -\frac{1}{2 p_a \cdot p_i} \frac{1}{x_{i,ab}} \left\langle \overline{\Phi}^{(\alpha)}{}_B \middle| \frac{T_b \cdot T_{ai}}{T_{ai}^2} V_\alpha(\Phi^{(\alpha)}{}_{rad}) \middle| \overline{\Phi}^{(\alpha)}{}_B \right\rangle$

  splitting function

  color charge factor

- Then $R^{(\alpha)} = R \cdot \frac{\mathcal{D}_\alpha}{\sum_{\alpha'} \mathcal{D}_{\alpha'}}$

- Make matrix element finite separately and free of cuts, and also implement Powheg in C-S subtraction framework easily

NLO matrix element

- Subtraction formalism real radiation matrix element piece finite and can be calculated numerically
- The sum of dipoles, virtual loop and PDF & FF remnants is therefore and can be calculated together analytically:

$$[\int_{\Phi_{rad}} d\sigma^{A}_{ab}(p_a, p_b)] + d\sigma^{V}_{ab}(p_a, p_b) + d\sigma^{C}_{ab}(p_a, p_b, \mu_F^2)$$

$$\sigma^{\rm NLO}_{ab}(p_a, p_b; \mu_F^2) = \int_{m+1} \left(d\sigma^{R}_{ab}(p_a, p_b) - d\sigma^{A}_{ab}(p_a, p_b)\right) + \left[\int_{m+1} d\sigma^{A}_{ab}(p_a, p_b) + \int_{m} d\sigma^{V}_{ab}(p_a, p_b) + \int_{m} d\sigma^{C}_{ab}(p_a, p_b; \mu_F^2)\right]$$

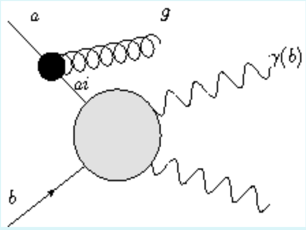
- Gluon radiation & quark radiation ← Singular Regions

soft and conlinear with incoming partons

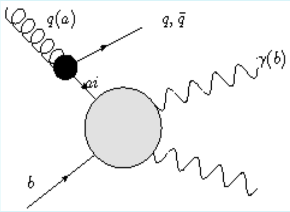
collinear with incoming partons and outgoing photon (photon fragmentation)

$\mathcal{D}^{qg,\bar{q}}$

$\mathcal{D}^{\bar{q}g,q}$



$\mathcal{D}^{gq,\bar{q}}$



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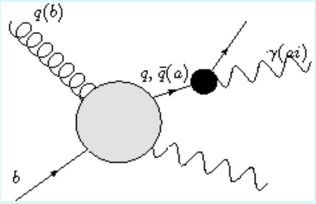
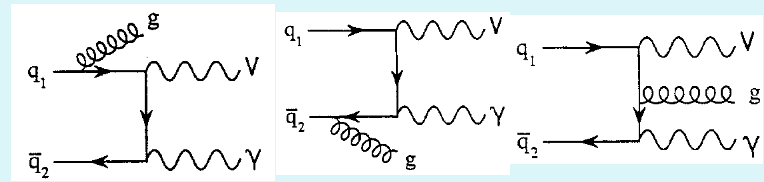


$\mathcal{D}^{b(n)}_{\gamma q}$

$\mathcal{D}_{\gamma q,W}$

Gluon Radiation for ME

- 8 diagrams for the real piece



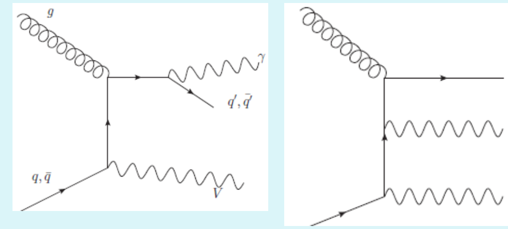
…

- Dipoles: Gluon is emitted by

one of the initial $q / \bar{q}$, with another $\bar{q}/q$ as spectator: $\mathcal{D}^{qg,\bar{q}} + \mathcal{D}^{\bar{q}g,q}$

- Splitting factor $V^{ai,b}(\Phi^{(\alpha)}{}_{rad}) = 8\pi\alpha_S C_F (\frac{1+x_{i,ab}^2}{1-x_{i,ab}})$
- The calculation of these dipoles is straight-wards
- When we integrate the C-S variables $x_{i,ab}, \tilde{v}_i, \phi$ and sum with virtual loop and PDF remnant, we will find the whole contribution is finite, can be performed Born event generation.

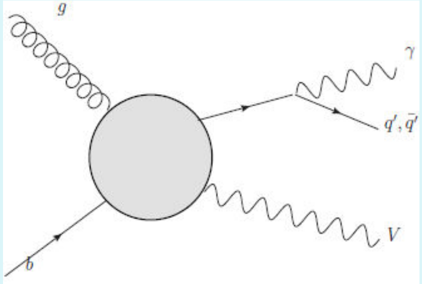
Quark/Anti-quark Radiation for ME

- Also 8 diagrams for real piece similarly  ...
- 2 singular regions → 2 kinds of Dipoles:
  - $q/\bar{q}$ is emitted by the initial gluon, with another $\bar{q}/q$ as spectator: QCD type, just similar to the previous one $\mathcal{D}^{gq,\bar{q}}$
  - $q/\bar{q}$ is emitted by the final photon, with initial $q'/\bar{q}'$ and W boson as spectators: QED type dipoles $\mathcal{D}_{\gamma q}^{b(n)} + \mathcal{D}_{\gamma q,W}$
- No $q/\bar{q}$ virtual loop, but gluon PDF and photon fragmentation function remnants
- Such QED type dipoles are not standard, how to construct them?
  - color charges T_c ⟹ electric charges Q_c
  - In soft limit and collinear limit, back to splitting function $\hat{P}_{\gamma q}(z,\varepsilon)$
- Electric charges aren't conversed when only include $\mathcal{D}_{\gamma q}^{b(n)} + \mathcal{D}_{\gamma q,W}$ then electric charges aren't well defined! ☹

Quark/Anti-quark Radiation for ME

- Study the electric conserved dipoles for photon radiation instead
- 6 such dipoles $\mathcal{D}_c^{(n)a\gamma} + \mathcal{D}_W^{(n)b\gamma} + \mathcal{D}_{c\gamma}^{(n)b} + \mathcal{D}_{c\gamma,W}^{(n)} + \mathcal{D}_{W\gamma}^{(n)b} + \mathcal{D}_{W\gamma,c}^{(n)}$

  the soft & collinear limits are just the same as $q' / \bar{q}'$ radiation since the same real matrix element



- Electric charges are well defined, dipole structures:

  just like QCD final state gluon radiation; photon & p_T cuts: safe
- Initial gluon and Z: electric neutral, thus not contribute
- Then $\mathcal{D}_{\gamma c}^{(n)b}(p_\gamma, p_V, p_c, p_a, p_b) = -\frac{1}{2p_c \cdot p_\gamma} \quad {}_{m,ab}\langle \tilde{p}_{\gamma c}, \tilde{p}_V; \tilde{p}_a, \tilde{p}_b | \frac{Q_b \cdot Q_{c\gamma}}{Q_{c\gamma}^2} V_{c\gamma}^{(n)b} | \tilde{p}_{\gamma c}, \tilde{p}_V; \tilde{p}_a, \tilde{p}_b \rangle_{m,ab}$

  dipoles $\mathcal{D}_{\gamma c,W}^{(n)}(p_\gamma, p_V, p_c, p_a, p_b) = -\frac{1}{2p_c \cdot p_\gamma} \quad {}_{m,ab}\langle \tilde{p}_{\gamma c}, \tilde{p}_V; \tilde{p}_a, \tilde{p}_b | \frac{Q_W \cdot Q_{c\gamma}}{Q_{c\gamma}^2} V_{c\gamma,W}^{(n)} | \tilde{p}_{\gamma c}, \tilde{p}_V; \tilde{p}_a, \tilde{p}_b \rangle_{m,ab}$
- Phase space mapping: gluon remains in z-axis & notice constraints on angle θ': C-S mapping $z_{\gamma q n}, u_q, \phi'$ is totally identical
- We find $\mathcal{D}_{\gamma q}^{b(n)} + \mathcal{D}_{\gamma q,W} = \frac{(Q_W + Q_b) \cdot Q_{c\gamma}}{Q_{c\gamma}^2} \frac{\langle V_{\gamma c}^{(n)b} \rangle}{2p_\gamma \cdot p_c} = \frac{\langle V_{\gamma c}^{(n)b} \rangle}{2p_\gamma \cdot p_c} = \frac{\sum_{b'} Q_{b'} \cdot Q_{c\gamma}}{Q_b \cdot Q_{c\gamma}} \mathcal{D}_{\gamma c}^{(n)b}$

  where $\langle s | V_{\gamma c}^{(n)b}(z_{\gamma cn}) | s' \rangle = 8\pi\mu^{2\epsilon}\alpha Q_c^2 \delta_{s,s'} [\frac{1 + (1 - z_{\gamma cn})^2}{z_{\gamma cn}} - \epsilon z_{\gamma cn}]$

Powheg Method in Catani-Seymour framework

- NLO accuracy matching parton shower with matrix element
- Smooth IR region to high p_T region, no phase-space slicing
- Generate shower in single singular region defined in C-S framework every time as we did in ME: C-S variable to $p_T^{\min}$ cut
  - The hardest radiation is generated by Sudakov form factor:

$$\Delta^{f_b}(\Phi_n, p_T) = \exp\left\{ - \sum_{\alpha_r \in \{\alpha_r | f_b\}} \int \frac{\left[d\Phi_{\text{rad}} R(\Phi_{n+1}) \, \theta(k_T(\Phi_{n+1}) - p_T) \right]_{\alpha_r}^{\bar{\Phi}_n^{\alpha_r} = \Phi_n}}{B^{f_b}(\Phi_n)} \right\}$$

  - The cross-section of Powheg:

$$d\sigma = \sum_{f_b} \bar{B}^{f_b}(\Phi_n) \, d\Phi_n \left\{ \Delta^{f_b}(\Phi_n, p_T^{\min}) + \sum_{\alpha_r \in \{\alpha_r | f_b\}} \frac{\left[d\Phi_{\text{rad}} \, \theta(k_T - p_T^{\min}) \, \Delta^{f_b}(\Phi_n, k_T) \, R(\Phi_{n+1}) \right]_{\alpha_r}^{\bar{\Phi}_n^{\alpha_r} = \Phi_n}}{B^{f_b}(\Phi_n)} \right\}$$

Generate Radiation Events: Highest-pT-bid Method

- Real-to-Born ratio: different flavor structures f_b: gluon radiation $q\bar{q}' \to \gamma V \to \gamma V g$ vs. $q'/\bar{q}'$ radiation $qg \to \bar{q}'V \to \bar{q}'V\gamma$; and 2 different singular regions α_r each
- Generate radiation events with the probability:

$$\left[\frac{R(\Phi_{n+1})}{B^{f_b}(\Phi_n)}\Delta^{f_b}(\Phi_n, k_T(\Phi_{n+1}))\right]_{\alpha_r}^{\bar{\Phi}_n^{\alpha_r}=\Phi_n} d\Phi_{rad}^{\alpha_r}$$ with $$\Delta^{f_b}(\Phi_n, p_T) = \prod_{\alpha_r \in \{\alpha_r | f_b\}} \Delta_{\alpha_r}^{f_b}(\Phi_n, p_T)$$

- Hardest shower events $[R^{\alpha_r}(\Phi_{n+1})]^{\bar{\Phi}_n^{\alpha_r}=\Phi_n}$ differ for different singular region α_r when mapping to unique underlying Born $\overline{\Phi}^{(\alpha)}{}_n$
- We can use highest-pT-bid method to generate shower events for each α_r by their own probability

$$\left[\frac{R^{\alpha_r}(\Phi_{n+1})}{B^{f_b}(\Phi_n)}\Delta_{\alpha_r}^{f_b}(\Phi_n, k_T(\Phi_{n+1}))\right]^{\bar{\Phi}_n^{\alpha_r}=\Phi_n} d\Phi_{rad}^{\alpha_r}$$

and choose the one with highest pT in each iteration.

Gluon Radiation for Powheg

- In Sudakov we should do p_T integration, thus map C-S variables to p_T: $p_T = \bar{v}_i (1 - x_{iab} - \bar{v}_i)\hat{\hat{s}} / x_{iab}$
- Since $x_{iab} \in [\bar{x}, 1]$, we can integrate out x_{iab} in Sudakov to get $dk_T^2 d\bar{v}_i d\phi$: $dx_{iab}\delta(k_T^2 - (1 - x_{iab} - \bar{v}_i)\hat{\hat{s}} / x_{iab})\cdots$ and new constraint on $\bar{v}_i$ and $k^2_{T,\max}$
- The R-B ratio is too complicated to integrate. However, we can estimate the upper bounding function of R/B with C-S dipole and then use veto technique: solve k_T for uniform number $r = \Delta^{\alpha_r}_{F,U}(k_T) / \Delta^{\alpha_r}_{F,U}(k_{T,0})$ to generate event according to probability $R/B \cdot \Delta_{\mathrm{F}}(k_T)$ by taking care the Jacobian, approximate upper bound and integrate out $\bar{v}_i$:

$$\Delta^{ag,b}_{F,U}(k_T) = \exp[-\frac{N^{ag,b}_U \cdot 4\pi}{b_0}(\ln q^2 \frac{\hat{\hat{s}}}{\Lambda^2} \cdot \ln\frac{\ln k^2_{T,\max} / \Lambda^2}{\ln k_T^2 / \Lambda^2} - \ln\frac{k^2_{T,\max}}{k_T^2})]$$ where $$q^2 = \frac{(2 - \bar{x})^2}{4\bar{x}}$$

- The scale of QCD coupling is taken care $\alpha_S(k_T^2)$ rather than constant
- Use Lambert-W function to solve the equation of k_T to generate it, then generate $\bar{v}_i$ and uniform ϕ

Quark/Anti-quark Radiation for Powheg

- α_r : $q'/\bar{q}'$ collinear with initial gluon is similar to gluon radiation case, has no difficulty.
- α_r : $q'/\bar{q}'$ collinear with photon: when using the dipole to estimate R-B ratio, 2 Born MEs are of different flavor structures:

$$[\frac{R^{\gamma q}(\Phi_{n+1}^{\alpha_r})}{B(\Phi_n)}]^{\bar{\Phi}_n=\Phi_n} \leq \propto [\frac{1+(1+z)^2}{z}] \frac{|M^{qV}|^2}{|M^B|^2} \cdot \frac{L_g}{L_q} \cdot \tilde{F}$$

However, the last 3 factor can be estimated into upper bound constant $N_U^{\gamma q} \cdot \frac{\alpha_S(k_T)}{\alpha}$

- Mapping C-S variables to p_T, it's too complicated in lab frame, and it's impossible to succeeding integration: p_T should defined according to the direction of $k_q + k_\gamma$, so try to approximate by that in centre-of-mass frame!
- A simpler try: according to the direction of $\tilde{k}_{\gamma q}$: $k_T^2 = \frac{4uA^2}{(\hat{s}-m_V^2)^2}[(1-z)(1-u)(\hat{s}-m_V^2)-um_V^2]$

  and integrate out $z(k_T^2, u)$, problem: z=0 singularity (soft photon)

Quark/Anti-quark Radiation for Powheg

- This case is just approximation $A=(k_a+k_b)\cdot\tilde{k}_{\gamma q}=(\hat{s}-m_V^2)/2<<\hat{s}$, making $z\to 0$ k_T^2 is finite, which isn't physical
- It's still possible according to the direction of k_q+k_γ approximate by reasonable $m_V^2<<\hat{s}$: $k_T^2=uz^2[(1-z)(1-u)(\hat{s}-m_V^2)-um_V^2]/(z-u)^2$
- There is cube of z: integrate out $u(k_T^2,z)$
- When $z\to 0$, $u_+(k_T^2,z)$ can be finite: constraint on u $u\in(0,u_{\lim}(z)]$ makes $z\in(z_-,1]$, ($z_\pm=\frac{2\hat{s}-m_V^2\pm\sqrt{m_V^2[4(\hat{s}-m_V^2)+m_V^2]}}{2(\hat{s}-m_V^2)}$), solve the spurious soft photon problem.
- But we have no upper limit on k_T^2 now. $u(k_T^2,z)\to 0$ is regularized by $k_T^{\min}$ in the integration of k_T
- So the upper bound of $\Delta_{F,U}^{\gamma q}(k_T)$ should be finite and simple when we do further approximation.

Outlook

- When we finish quark/qnti-quark radiation for Powheg, the whole NLO calculations is finally completed
- Complete the codes soon and then we have numerical results to compare with the previous WGamma MC tools and expect to be used in experimental data in near future
- Anomalous WWγ couplings and beyond SM
- W/Z decay: ask for additional contributions of photon radiates from decayed lepton