Amplitudes with reduced supersymmetry in gauge theory and gravity

Henrik Johansson

Uppsala U. & Nordita

July 1, 2019

Amplitudes 2019

Trinity College Dublin

Based on work with:

Claude Duhr, Gregor Kälin, Gustav Mogull, Bram Verbeek [1904.05299];

Gregor Kälin, Gustav Mogull [1706.09381];

Marco Chiodaroli, Murat Günaydin, Radu Roiban [1512.09130, 1710.08796, 1812.10434];

Alexander Ochirov [1507.00332, 1906.12292];

Outline

- Motivation: reduced SUSY amplitudes
- Construction of N=2 SQCD 4-gluon 2-loop amplitude
  - Supersymmetric decomposition & C-K duality
  - Integration of full amplitude
  - Inspection of result → transcendentality properties
- Supergravity amplitudes with reduced SUSY
  - Double copy with SQCD
  - Which theories do have we access to ?
- Conclusion

Motivation: maximal supersymmetry

Two theories frequently studied:

$$\mathcal{N} = 4\ \mathrm{SYM}: \quad \{A_\mu,\ \psi_I,\ \phi_{IJ}\}$$

At tree-level QCD effectively super-conformal

At loop-level certain terms of maximal transcendentality are the same

N=4 suggest natural integral basis, e.g. DCI integrals

$$\mathrm{QCD} \quad : \quad A_\mu + N_f\ \psi$$

We learn how to compute in QCD by studying its maximal SUSY cousin

Motivation: reduced supersymmetry

However, there are many interesting theories "inbetween"

$$\mathcal{N}=4\ \mathrm{SYM}: \quad \{A_\mu,\ \psi_I,\ \phi_{IJ}\}$$

$$\mathcal{N}=2\ \mathrm{SYM}: \quad \{A_\mu,\ \psi_i,\ \phi\} + N_f\,\{\psi_j, \phi_j\}$$

$$\mathcal{N}=1\ \mathrm{SYM}: \quad \{A_\mu,\ \psi\} + N_f\,\{\psi, \phi\}$$

$$\mathrm{QCD} \quad : \quad A_\mu + N_f\ \psi$$

→ related by SUSY decomposition at loop level

→ many unexplored oppertunities, in models closer to the real world

Motivation: supergravity w/ red. SUSY

Double copy: gravity from gauge th. → need reduced SUSY amplitudes

$$(\mathcal{N}=8\ \mathrm{SG}) = (\mathcal{N}=4\ \mathrm{SYM}) \otimes (\mathcal{N}=4\ \mathrm{SYM})$$

$$(\mathcal{N}=5,6\ \mathrm{SG}) = (\mathcal{N}=4\ \mathrm{SYM}) \otimes (\mathcal{N}=1,2\ \mathrm{SYM})$$

unique theories → no free parameters (ungauged SG)

$$\begin{aligned}(\mathcal{N}=4\ \mathrm{SG}) &= (\mathcal{N}=4\ \mathrm{SYM}) \otimes (\mathcal{N}=0\ \mathrm{YM}) \\ &= (\mathcal{N}=2\ \mathrm{SYM}) \otimes (\mathcal{N}=2\ \mathrm{SYM})\end{aligned}$$

$$(\mathcal{N}=3\ \mathrm{SG}) = (\mathcal{N}=2\ \mathrm{SYM}) \otimes (\mathcal{N}=1\ \mathrm{SYM})$$

uniquely specified by spectrum → one free parameter N_V

$$\begin{aligned}(\mathcal{N}=2\ \mathrm{SG}) &= (\mathcal{N}=2\ \mathrm{SYM}) \otimes (\mathcal{N}=0\ \mathrm{YM}) \\ &= (\mathcal{N}=1\ \mathrm{SYM}) \otimes (\mathcal{N}=1\ \mathrm{SYM})\end{aligned}$$

in 5D uniqely specified by 3pt ampl. (4D: ∞ many parameters)

Gauge theory amplitudes w/ reduced SUSY

State of the Art

- Gauge theory amplitudes at 2 loops with 4 gluons:
  - QCD [helicity avg.] Glover, Oleari, Tejeda-Yeomans (2001)
  - Pure N=1 SYM and QCD Bern, De Freitas, Dixon (2002)
  - *N*=1,2 SQED: Binoth, Glover, Marquard, van der Bij (2002)
  - Planar *N*=2 SCQCD: Leoni, Mauri, Santambrogio (2015)
    (Dixon, Kosower, Vergu, 2008)
  - *Complete N=2 SQCD: Duhr, HJ, Kälin, Mogull, Verbeek (2019)*

- Also 5-gluon 2-loop amplitude in pure YM

  Badger, Frellesvig, Zhang;

  Badger, Mogull, Ochirov, O'Connell;

  Dunbar, Jehu, Perkins;

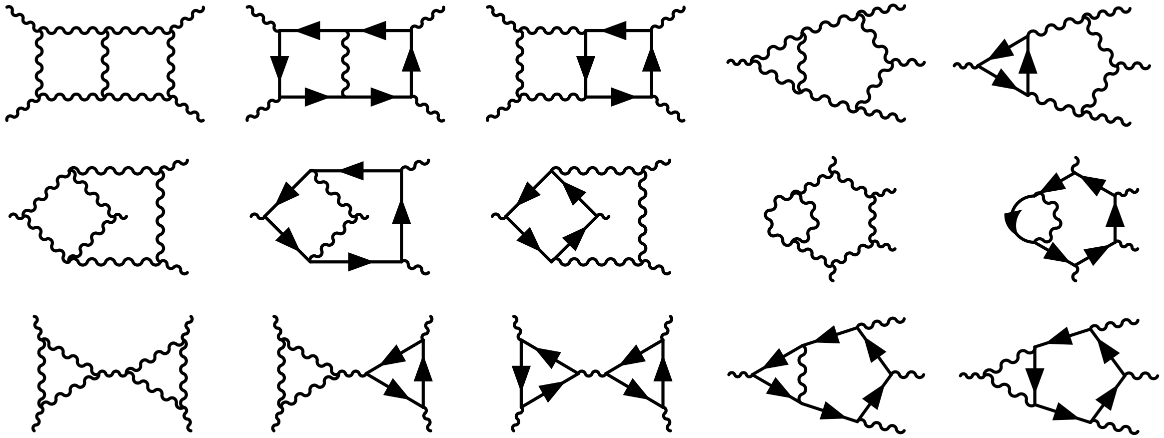
  Gehrmann, Henn, Lo Presti;

  Badger, Brønnum-Hansen, Bayu Hartanto, Peraro;

  Abreu, Febres Cordero, Ita, Page, Sotnikov

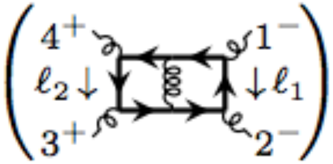
Complete $\mathcal{N}$=2 SQCD calculation

Integrand computed 2017 using color-kinematics duality and supersymmetric decomposition HJ, Kälin, Mogull

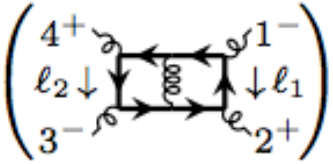


- two-loop SQCD amplitude
- color-kinematics manifest
- planar + non-planar
- N_f massless quarks
- integrand valid in $D \leq 6$

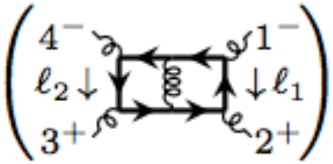
e.g. simple SQCD numerators

$n\Big($ 

$\Big) = -\kappa_{12}\mu_{12}\,,$

$n\Big($ 

$\Big) = \dfrac{\kappa_{13}}{u^2}\,\mathrm{tr}_-(1\ell_1 24\ell_2 3)$

$n\Big($ 

$\Big) = \dfrac{\kappa_{14}}{t^2}\,\mathrm{tr}_-(1\ell_1 23\ell_2 4)$

trace-rep. from 1811.09604 Kälin, Mogull, Ochirov

Color-kinematics duality

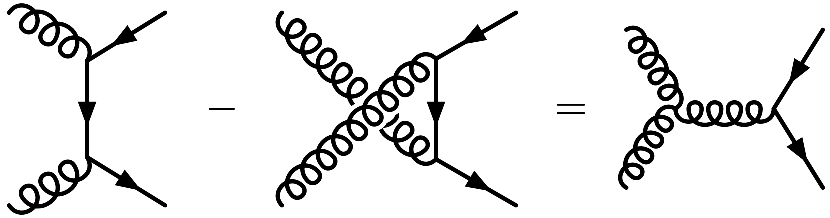
$$\mathcal{A}_m^{(L)} = \sum_{i\in\text{cubic}} \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{(N_f)^{|i|}}{S_i} \frac{n_i c_i}{D_i}$$

numerators

color factors

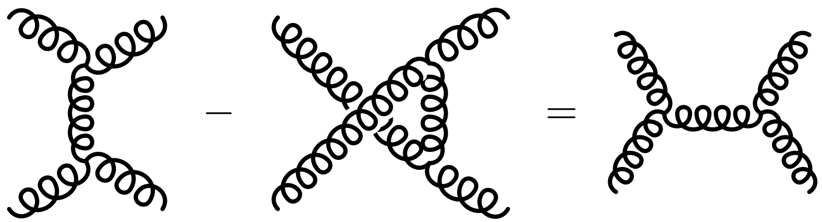
propagators

Color & kinematic numerators satisfy same relations:



commutation identity

$$T^aT^b - T^bT^a = f^{abc}T^c$$



Jacobi identity

$$f^{dac}f^{cbe} - f^{dbc}f^{cae} = f^{abc}f^{dce}$$

$$n_i - n_j = n_k$$

color $\quad c_i$

$\Updownarrow \quad \Updownarrow$

kinematics $\quad n_i$

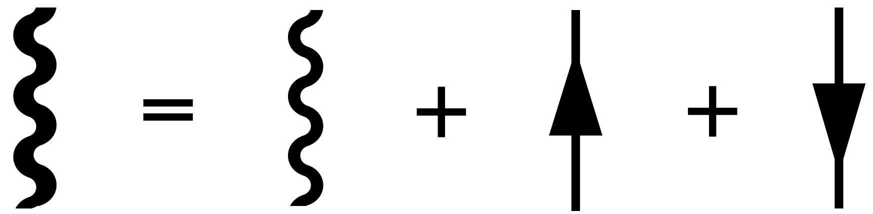
Bern, Carrasco, HJ

Supersymmetric decomposition

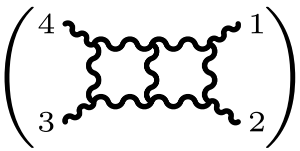
cf. BDK

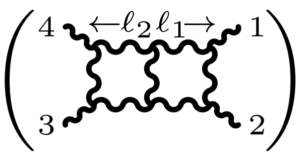
Decompose on-shell supsersymmetric states: $\mathcal{V}_{\mathcal{N}=4} = \mathcal{V}_{\mathcal{N}=2} + \Phi_{\mathcal{N}=2} + \overline{\Phi}_{\mathcal{N}=2}$

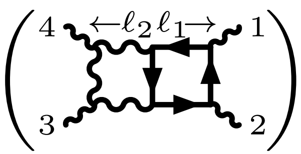
Pictorially:

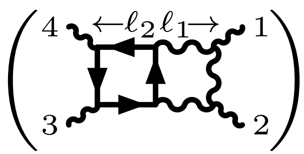


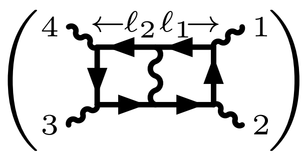
For example, the double box decomposes as:

$n^{[\mathcal{N}=4\ \mathrm{SYM}]}$ 

$= n$ 

$+ 2n$ 

$+ 2n$ 

$+ 2n$ 

→ can solve for pure *N*=2 SYM in terms of *N*=4 SYM and matter graphs

→ only matter graphs needed!

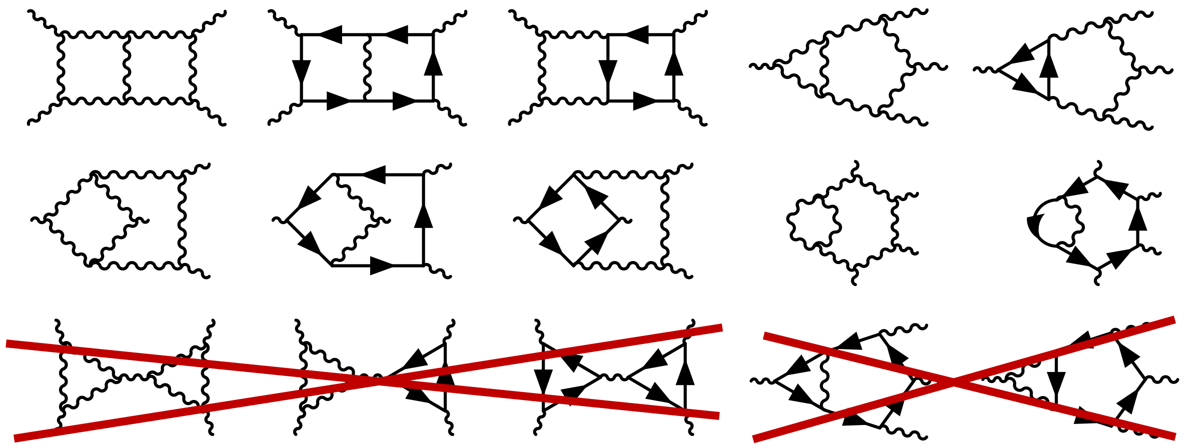
$$\mathcal{V}_{\mathcal{N}=2} = A^+ + \eta^i \psi_i^+ + \eta^1\eta^2\phi_{12} + \mathrm{h.c.}$$
$$\Phi_{\mathcal{N}=2} = (\psi_3^+ + \eta^i\phi_{i3} + \eta^1\eta^2\psi_-^4)\eta^3$$

Works similarly for any loop order and for *N*=0,1,2 SYM

Integration of complete $\mathcal{N}$=2 SQCD amp.

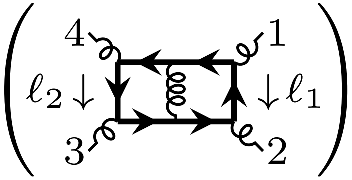
Duhr, HJ, Kälin, Mogull, Verbeek [1904.05299]

Full integration of result:
- Tensor → scalar integrals
- IBP → basis of known integr.
- Renormalized using β-fn to remove IR/UV mixing
- Catani → checked IR poles
- Alt. rep. integrated → same



Interesting features:
- only 10 diagrams contribute (6 contain matter → SUSY dec.)
- $(N_f)^2$ terms non-trivially absent
- For $N_f = 2N_c$ bubbles and triangles cancel out → conformal point
- Leading color, conformal point, only one diagram needed

$$\mathcal{A}_{\mathrm{LC}}^{\mathrm{SCQCD}} = \mathcal{A}_{\mathrm{LC}}^{\mathcal{N}=4} + \sum_{\mathrm{cyclic}} \int \frac{\mathrm{d}^{2D}\ell}{D_{\mathrm{b}}} n \left(\right)$$



Decomposing using superconformal theory

$$\mathcal{A}^{\mathcal{N}=2} = \underbrace{\mathcal{A}^{\mathcal{N}=4} + \mathcal{R}}_{\text{superconformal } N=2 \text{ QCD}} + (2N_c - N_f)\mathcal{S}$$

Duhr, HJ, Kälin, Mogull, Verbeek

Leading-color Remainder:

$$R^{(2)[2]}_{(--++)} = 12\zeta_3 + \frac{\tau}{6}\Big\{48\mathrm{Li}_4(\tau) - 24T\mathrm{Li}_3(\tau) - 24T\mathrm{Li}_3(\upsilon) + 24\mathrm{Li}_2(\tau)\left(\zeta_2 + TU\right) + 24TU\mathrm{Li}_2(\upsilon) - 24U\mathrm{Li}_3(\tau) - 24S_{2,2}(\tau) + T^4 - 4T^3U + 18T^2U^2 - 12\zeta_2T^2 + 24\zeta_2TU + 24\zeta_3U - 168\zeta_4 - 4i\pi\big[6\mathrm{Li}_3(\tau) + 6\mathrm{Li}_3(\upsilon) - 6U\mathrm{Li}_2(\tau) - 6U\mathrm{Li}_2(\upsilon) - T^3 + 3T^2U - 6TU^2 - 6\zeta_2T + 6\zeta_2U\big]\Big\} + \mathcal{O}(\epsilon)\,,$$

$$R^{(2)[2]}_{(-+-+)} = 12\zeta_3 + \frac{1}{6}\frac{\tau}{\upsilon^2}T^2(T + 2i\pi)^2 + \mathcal{O}(\epsilon)\,,$$

$$s > 0;\ t, u < 0$$
$$\tau = -t/s$$
$$\upsilon = -u/s$$
$$T = \log(\tau)$$
$$U = \log(\upsilon)$$

LC Remainder is UV and IR finite

Uniform transcendentality minimally broken!

agrees with unpublished result from 2008 by Dixon, Kosower, Vergu

cf. Leoni, Mauri, Santambrogio

Complete non-planar result

Non-planar Remainder:

Duhr, HJ, Kälin, Mogull, Verbeek

$$
\begin{aligned}
R^{(2)[1]\text{fin}}_{(--)(++)} &= \frac{2\tau}{3}\{96\text{Li}_4(\tau) - 72T\text{Li}_3(\tau) + 24T\text{Li}_3(\upsilon) \\
&+ 24T\text{Li}_2(\tau)(T-U) - 24U\text{Li}_2(\upsilon)(T-U) + 96\text{Li}_4(\upsilon) \\
&+ 24U\text{Li}_3(\tau) - 72U\text{Li}_3(\upsilon) + T^4 + 4T^3U - 18T^2U^2 \\
&+ 4TU^3 + U^4 + 24\zeta_2 TU - 12\zeta_2 T^2 - 12\zeta_2 U^2 - 654\zeta_4 \\
&- 4i\pi\big[12\text{Li}_3(\tau) + 12\text{Li}_3(\upsilon) - 12T\text{Li}_2(\tau) - 12U\text{Li}_2(\upsilon) \\
&- T^3 - 3T^2U - 3TU^2 - U^3 - 18\zeta_2 T - 18\zeta_2 U\big]\} \\
&+ \mathcal{O}(\epsilon)\,,
\end{aligned}
$$

$$
\begin{aligned}
R^{(2)[1]\text{fin}}_{(-+)(-+)} &= \frac{2\tau}{3\upsilon^2}\{48\text{Li}_4(\tau) - 24T\text{Li}_3(\tau) - 24S_{2,2}(\tau) \\
&+ 24\zeta_2\text{Li}_2(\tau) + T^4 - 84\zeta_2 T^2 - 102\zeta_4 + 24T\zeta_3 \\
&- 8i\pi\big[3T\zeta_2 - T^3\big]\} - \frac{8\tau}{3\upsilon^2}\{\underline{6\tau\text{Li}_3(\tau) - 6\tau\text{Li}_3(\upsilon)} \\
&\underline{- 6\tau T\text{Li}_2(\tau) + 6\text{Li}_3(\upsilon) - 6\upsilon U\text{Li}_2(\upsilon) + 3\tau TU^2} \\
&\underline{+ 3T\upsilon U^2 - 3TU^2 - 30\tau T\zeta_2 - 30\upsilon U\zeta_2 - 6\zeta_3} \\
&\underline{+ 3i\pi\big[2(\upsilon-\tau)\text{Li}_2(\tau) + \tau T^2 + 2T\upsilon U + \upsilon U^2 + 2\tau\zeta_2\big]}\} \\
&+ \mathcal{O}(\epsilon)\,.
\end{aligned}
$$

non-U.T. weight 3

$s > 0;\ t, u < 0$

$\tau = -t/s$

$\upsilon = -u/s$

$T = \log(\tau)$

$U = \log(\upsilon)$

single trace contr. determined by double trace

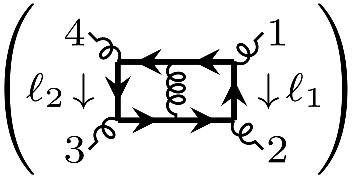
$$
R^{(2)[0]}_{(1234)} = R^{(2)[2]}_{(1234)} - R^{(2)[1]}_{(13)(24)} + \frac{1}{2}\left(R^{(2)[1]}_{(12)(34)} + R^{(2)[1]}_{(14)(23)}\right)
$$

$\mathcal{S}$ determinded by its leading color contribution

Two uniform transcendentality theories

Consider N=2 SQED, only one diagram, same as SCQCD leading color:

$$\sum_{\text{cyclic}} \int \frac{\mathrm{d}^{2D}\ell}{D_{\mathrm{b}}} n \left(\vphantom{\frac{a}{b}} \right.$$



$$\left. \vphantom{\frac{a}{b}} \right)$$

Duhr, HJ, Kälin, Mogull, Verbeek

$$\begin{aligned}\mathcal{A}^{\mathcal{N}=2\,\mathrm{QED}} &= A^{\mathcal{N}=2}_{\mathrm{LC}}(1,2,3,4) + A^{\mathcal{N}=2}_{\mathrm{LC}}(1,3,4,2) + A^{\mathcal{N}=2}_{\mathrm{LC}}(1,4,2,3) \\ &= 12\zeta_3 \times (\text{photon-decoupling identity}) + \mathrm{U.T.}\end{aligned}$$

cf. Binoth, Glover, Marquard, van der Bij

Accidental equivalence between N=4 and N=2 for SO(3)

$$(SO(3)\ N_f = 1\ \mathcal{N} = 2\ \mathrm{SQCD}) = (SO(3)\ \mathcal{N} = 4\ \mathrm{SYM})$$

Reps. have same size $3_A \sim 3_F$

generators same as structure constants

$$3V_{\mathcal{N}=2} + 3\Phi_{\mathcal{N}=2} + 3\bar{\Phi}_{\mathcal{N}=2} = 3V_{\mathcal{N}=4}$$

$$T^a_{ij} \sim \epsilon_{aij}$$

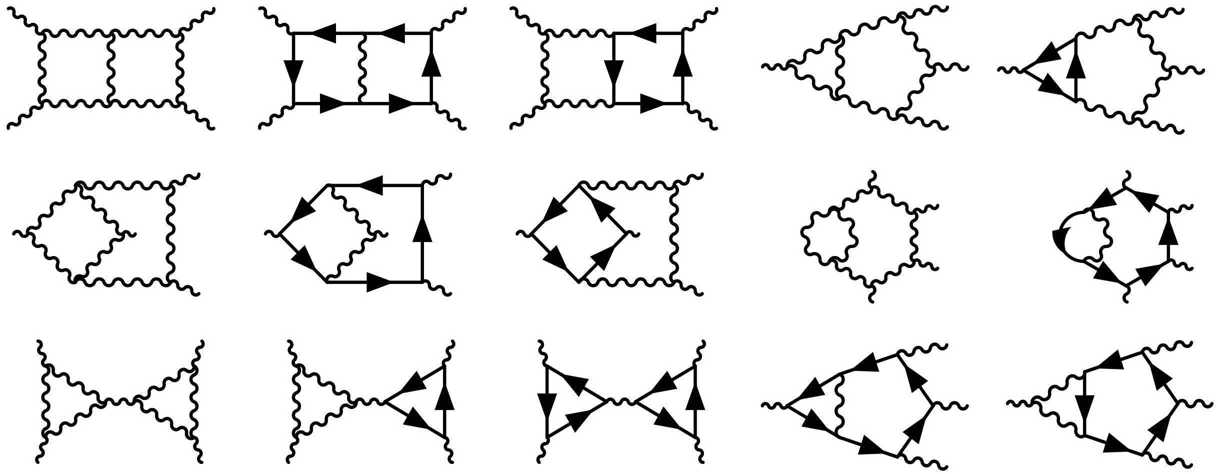
$$f^{abc} \sim \epsilon_{abc}$$

Supergravity amplitudes w/ reduced SUSY

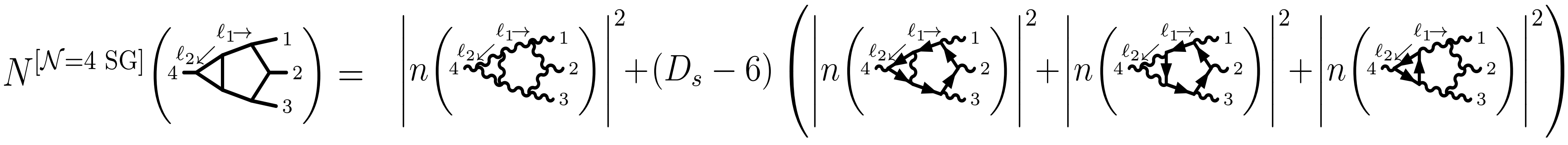
Double copy of N=2 super-QCD

HJ, Kälin, Mogull

- two-loop SQCD amplitude
- color-kinematics manifest
- N_f massless quarks
- integrand valid in $D \leq 6$



Double copy gives half-maximal (N=4) supergravity numerator:



Pure supergravities given by $D_s = D = 4, 5, 6$ cf. HJ, Ochirov ('14)

Extracted 5D UV divergence → vanishes → enhanced cancellation

cf. Bern, Davies, Dennen, Huang $(\mathcal{N}=4)\otimes(\mathcal{N}=0)$

$\mathcal{N}$=(1,1) vs $\mathcal{N}$=(2,0) supergravity

HJ, Kälin, Mogull

In 6D there are two half-maximal supergravities:

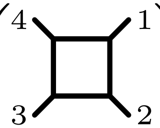
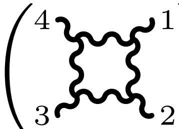
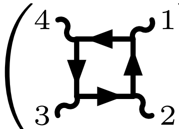
$$\Big(\mathcal{N}=(1,1)\ \mathrm{SG}\Big) = \Big(\mathcal{N}=(1,0)\ \mathrm{SQCD}\Big) \otimes \Big(\mathcal{N}=(0,1)\ \mathrm{SQCD}\Big)$$

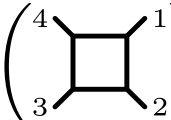
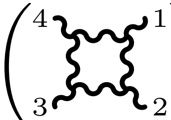
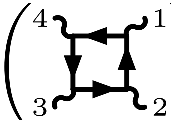
(graviton + vector multiplets)

$$\Big(\mathcal{N}=(2,0)\ \mathrm{SG}\Big) = \Big(\mathcal{N}=(1,0)\ \mathrm{SQCD}\Big) \otimes \Big(\mathcal{N}=(1,0)\ \mathrm{SQCD}\Big)$$

(graviton + tensor multiplets)

In terms of gravity numerators:

(1,1) → $N^{[\mathcal{N}=(1,1)\ \mathrm{SG}]}\Bigg($  $\Bigg) = \Bigg|\, n\Bigg($  $\Bigg)\Bigg|^2 + (D_s-6)\,\Bigg|\, n\Bigg($  $\Bigg)\Bigg|^2$

(2,0) → $N^{[\mathcal{N}=(2,0)\ \mathrm{SG}]}\Bigg($  $\Bigg) = \Bigg(n\Bigg($  $\Bigg)\Bigg)^2 + (N_T-1)\,\Bigg(n\Bigg($  $\Bigg)\Bigg)^2$

same simple relation at two loops!

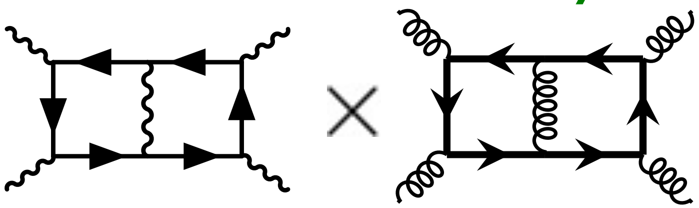
$\mathcal{N}$=2 supergravity double copies

Recall that double copy works
if one side obeys C-K duality

HJ, Ochirov;
Chiodaroli, Gunaydin, HJ, Roiban;
Ben-Shahar, Chiodaroli;
Mogull, Kälin, HJ (in progress)

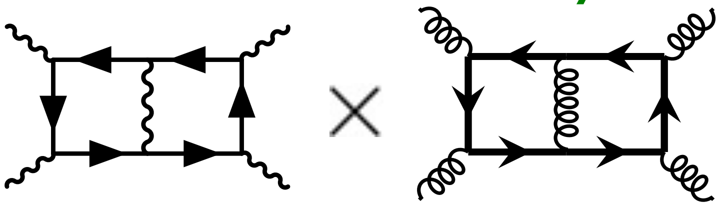
$$(\text{2-loop } \mathcal{N}=2 \text{ SQCD}) \otimes (D\text{-dim. QCD Feynman rules})$$

$\mathcal{N}$=2 SQCD × 4D Feynman



→ $\mathcal{N}$=2 Luciani Model (1978)

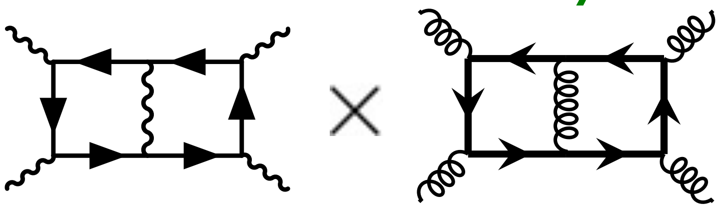
$\mathcal{N}$=2 SQCD × 5D Feynman



→ $\mathcal{N}$=2 Generic non-Jordan family

Günaydin, Sierra, Townsend (1986)

$\mathcal{N}$=2 SQCD × 6D Feynman



→ $\mathcal{N}$=2 Generic Jordan family

Günaydin, Sierra, Townsend (1984)

Magical and homogeneous SUGRAs

Maxwell-Einstein 5d supergravity theories — Günaydin, Sierra, Townsend

$$e^{-1}\mathcal{L} = -\frac{R}{2} - \frac{1}{4}\mathring{a}_{IJ}F^{I}_{\mu\nu}F^{J\mu\nu} - \frac{1}{2}g_{xy}\partial_\mu\varphi^x\partial^\mu\varphi^y + \frac{e^{-1}}{6\sqrt{6}}C_{IJK}\epsilon^{\mu\nu\rho\sigma\lambda}F^{I}_{\mu\nu}F^{J}_{\rho\sigma}A^{K}_{\lambda}$$

Lagrangian is entirely determined by constants C_{IJK} → 3pt amplitudes

---

- Homogenous scalar manifold

$$C_{IJK} \sim \Gamma^{a}_{\alpha\beta}$$

de Wit, Van Proeyen

- Double copy:

$$(\mathcal{N}=2\ \mathrm{SQCD}) \otimes (D, N_f\ \mathrm{QCD})$$

	$\#f_D = 0$	$\#f_D = 1$	$\#f_D = 2$	$\#f_D = 3$	$\cdots$
$D = 4$	— Luciani model —				
$D = 5$		— Generic non-Jordan Family —			
$D = 6$		— Generic Jordan Family —			
$D = 7$		$\mathbb{R}$			
$D = 8$		$\mathbb{C}$			
$D = 9$					
$D = 10$		$\mathbb{H}$			
$\cdots$					
$D = 14$		$\mathbb{O}$			
$\cdots$					

- Magical theories

$$(\mathcal{N}=2\ \mathrm{SQCD}) \otimes (D=7,8,10,14\ \mathrm{QCD})$$
$$= \mathrm{Magical}\ \mathcal{N}=2\ \mathrm{Supergravity}$$
$$(\mathbb{R},\mathbb{C},\mathbb{H},\mathbb{O}\ \mathrm{type})$$

Chiodaroli, Günaydin, HJ, Roiban ('15)

→ see talk by Borsten

What about gauged supergravity ?

Need more parameters even in *N*=4 SYM !

$$(\mathcal{N}=8\ \mathrm{SG}) = (\mathcal{N}=4\ \mathrm{SYM}) \otimes (\mathcal{N}=4\ \mathrm{SYM})$$

$$(\mathcal{N}=5,6\ \mathrm{SG}) = (\mathcal{N}=4\ \mathrm{SYM}) \otimes (\mathcal{N}=1,2\ \mathrm{SYM})$$

unique theories → no free parameters (ungauged SG)

$$(\mathcal{N}=4\ \mathrm{SG}) = (\mathcal{N}=4\ \mathrm{SYM}) \otimes (\mathcal{N}=0\ \mathrm{YM})$$
$$= (\mathcal{N}=2\ \mathrm{SYM}) \otimes (\mathcal{N}=2\ \mathrm{SYM})$$

$$(\mathcal{N}=3\ \mathrm{SG}) = (\mathcal{N}=2\ \mathrm{SYM}) \otimes (\mathcal{N}=1\ \mathrm{SYM})$$

uniquely specified by spectrum → one free parameter N_V

$$(\mathcal{N}=2\ \mathrm{SG}) = (\mathcal{N}=2\ \mathrm{SYM}) \otimes (\mathcal{N}=0\ \mathrm{YM})$$
$$= (\mathcal{N}=1\ \mathrm{SYM}) \otimes (\mathcal{N}=1\ \mathrm{SYM})$$

in 5D uniqely specified by 3pt ampl. (4D: ∞ many parameters)

Gauged supergravity – U(1) gauging

Make gravitini charged under some local U(1) -- subgroup of R-symmetry
→ Covariantize derivatives

$$\mathcal{D}_\mu \Psi^i_\nu = \nabla_\mu \Psi^i_\nu + g V_I A^I_\mu \Psi^i_\nu$$

→ Add terms to Lagrangian to not explicitly break SUSY
→ SUSY spontaneously broken in flat space → massive gravitini

From double copy:

Günaydin, Chiodaroli, HJ, Roiban

$$\Big(\text{gauged SUGRA}\Big) = \Big(\text{Higgsed YM}\Big) \otimes \Big(\cancel{\text{super}}\ \text{YM}\Big)$$

For example:

$$\Big(\mathcal{N}=8\ \mathrm{SG}_{\mathrm{U}(1)}\Big) = \Big(\mathcal{N}=4\ \mathrm{SYM}_{\mathrm{Coulomb}}\Big) \otimes \Big(\mathcal{N}=2\ \mathrm{SQCD}_{\mathrm{massive}}\Big)$$

Several other known DC examples, including non-abelian gaugings

Non-SUSY massive (QCD)x(QCD)

What is the square of QCD ?

HJ, Ochirov [1906.12292]

Gluon: $$A_\mu \otimes A_\nu \sim h_{\mu\nu} \oplus Z \oplus \bar{Z}$$

Massive quarks: $$q_i \otimes \bar{q}_i \sim V_i^\mu \oplus \varphi_i$$

Lagrangian determined by matching to double copy of QCD ampl.

$$\begin{aligned}
\mathcal{L}_{(\mathrm{QCD})^2} = & -\frac{2}{\kappa^2}R + \frac{\partial_\mu \bar{Z}\, \partial^\mu Z}{(1-\frac{\kappa^2}{4}\bar{Z}Z)^2} + \sum_{\imath=1}^{N_f}\bigg\{-\frac{1}{2}V^*_{\imath\mu\nu}V_\imath^{\mu\nu} + m_\imath^2 V^*_{\imath\mu}V_\imath^\mu\Big[1-\frac{\kappa}{2}(Z+\bar{Z})+\frac{\kappa^2}{2}\bar{Z}Z\Big] \\
& + \partial_\mu\varphi_\imath^*\,\partial^\mu\varphi_\imath - m_\imath^2\varphi_\imath^*\varphi_\imath\Big[1-\frac{\kappa}{2}(Z+\bar{Z})+\frac{\kappa^2}{16}(Z^2+\bar{Z}^2+8\bar{Z}Z)\Big] \\
& + \frac{i\kappa}{4}m_\imath\Big[(\varphi_\imath^* V_{\imath\mu}+V^*_{\imath\mu}\varphi_\imath)\partial^\mu(Z-\bar{Z}) - (Z-\bar{Z})(\partial^\mu\varphi_\imath^*\,V_{\imath\mu}+V^*_{\imath\mu}\partial^\mu\varphi_\imath)\Big] \\
& - \frac{i\kappa^2}{4}m_\imath(\varphi_\imath^* V_{\imath\mu}+V^*_{\imath\mu}\varphi_\imath)(\bar{Z}\partial^\mu Z - Z\partial^\mu\bar{Z}) + \frac{\kappa^2}{8}\Big[\varphi_\imath^*\varphi_\imath\partial_\mu\bar{Z}\,\partial^\mu Z + \bar{Z}Z\partial_\mu\varphi_\imath^*\,\partial^\mu\varphi_\imath\Big]\bigg\} \\
& + \sum_{\imath,\jmath=1}^{N_f}\bigg\{\frac{\kappa^2}{8}\varphi_\imath^*\varphi_\imath\Big[\partial_\mu\varphi_\jmath^*\,\partial^\mu\varphi_\jmath - 3m_\jmath^2\varphi_\jmath^*\varphi_\jmath + 2m_\jmath^2 V^*_{\jmath\mu}V_\jmath^\mu\Big] \\
& + \frac{\kappa^2}{4}m_\imath m_\jmath\Big[\varphi_\imath^*\varphi_\jmath^* V_{\imath\mu}V_\jmath^\mu + \varphi_\imath\varphi_\jmath V^*_{\imath\mu}V_\jmath^{*\mu} + 2\varphi_\imath^*\varphi_\jmath V^*_{\jmath\mu}V_\imath^\mu\Big]\bigg\} + \mathcal{O}(\kappa^3).
\end{aligned}$$

Summary

- Argued that amplitudes with reduced supersymmetry in SYM and SG are interesting and rich objects to study

- Computed the first complete N=2 SQCD 2-loop amplitude
  → general N_f and N_c
  → superconformal decomposition: compact result for SCQC sector
  → violates U.T. mildly – only ζ_3 in planar sector
  → identified two U.T. theories - U(1) SQED and SO(3) SQCD

- Considered supergravity amplitudes with reduced SUSY
  → SQCD amplitudes critical for double copy
  → Infinite families of N=2 Maxwell-Einstein homogeneous SGs

- Gauged supergravities → R-symmetry is gauged
  - (spontaneously broken gauge sym.) $\otimes$ (explicitly broken SUSY)
  - Need to generalize to AdS vacua → preserves SUSY

- Looking forward to future "data mining" of reduced SUSY ampl. Simple-to-obtain gauge ampl. → landscape of SG theories