Scattering amplitudes as cluster-adjacent polylogarithms

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in collaboration with

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Alphabets from clusters

$\mathcal{N} = 4$ headstart:

A priori knowledge of singularities $\sim$ **the alphabet** unlocks the bootstap approach for $\mathcal{N} = 4$ amplitudes

Results up to $(n, L) = (6, 7)$ or $(7, 4)$ **w/o loop integration**

[Caron-Huot, Dixon, Drummond, Dulat, Foster, Harrington, Henn, McLeod, ÖCG, Papathanasiou, Pennington, Spradlin, von Hippel]

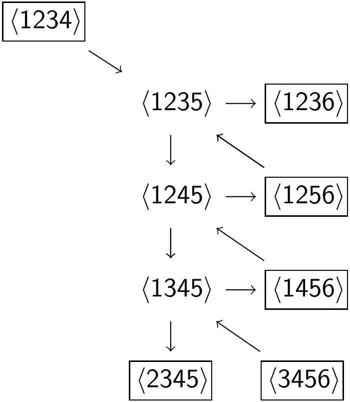
Symbol letters of $n = 6, 7$ amplitudes in $\mathcal{N} = 4$ sYM are "$\mathcal{A}$-coordinates" of $\mathrm{Gr}(4, n)$ cluster algebras [Golden, Goncharov, Spradlin, Vergu, Volovich]

- $n = 6, 7$: all loops & helicities
- $n \geq 8$: Captures known rational letters of (N)MHV @ 2-loops but $\mathrm{Gr}(4, \geq 8)$ cluster algebra is **infinite**

Branch points from $\mathrm{Gr}(4, n)$ cluster algebras

[Fomin, Zelevinsky; Golden, Goncharov, Spradlin, Vergu, Volovich]

$\mathrm{Gr}(4,6)$ **example:** Six-particle scattering



Cluster: Quiver diagram with nodes in $\mathcal{A}$ and adjacency matrix b_{ij}.

Nodes: All-adjacent Plückers: **frozen** , others **active**

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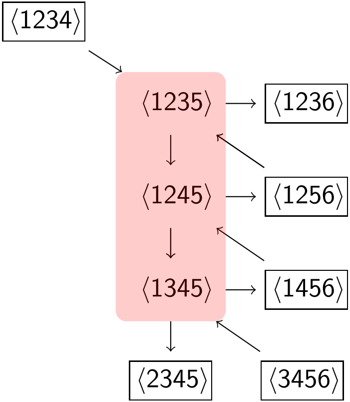

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A_3-type: The active nodes are connected in an A_3 Dynkin diagram

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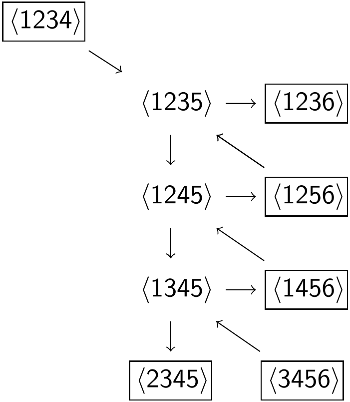

Mutations "mutation on an active node" transform the cluster to a new one:

$$a'_k = \frac{1}{a_k}\left[\prod_{i\,|\,b_{ik}>0} a_i^{b_{ik}} + \prod_{i\,|\,b_{ik}<0} a_i^{-b_{ik}}\right]$$

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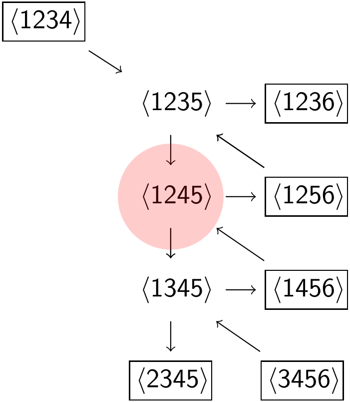

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Example mutate the initial cluster on $\langle 1245\rangle$:

$$\langle 1245\rangle \mapsto \frac{\langle 1235\rangle\langle 1456\rangle + \langle 1256\rangle\langle 1345\rangle}{\langle 1245\rangle} = \langle 1356\rangle$$

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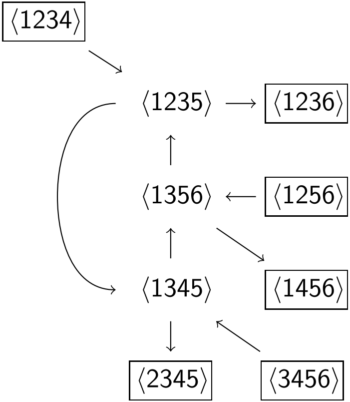
[Fomin, Zelevinsky; Golden, Goncharov, Spradlin, Vergu, Volovich]

$\mathrm{Gr}(4, 6)$ **example:** Six-particle scattering



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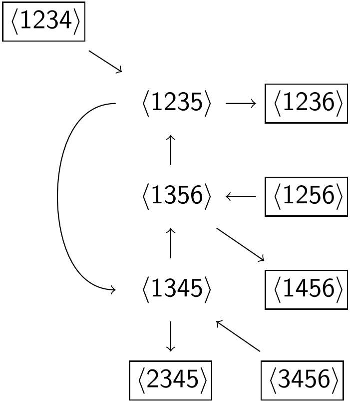
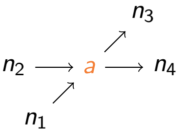


Cluster $\mathcal{X}$ coordinates



$$X(a) = \frac{n_1 n_2}{n_3 n_4}$$

NB: $\mathcal{X}(a)$ depend on the cluster

The Gr(4, 6) $\cong A_3$ / Stasheff polytope

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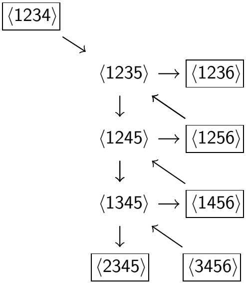

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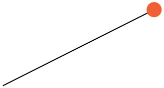

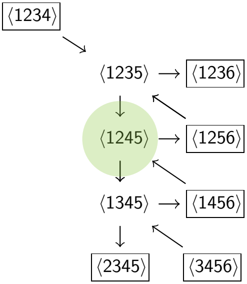

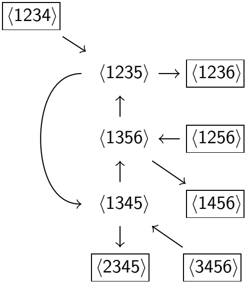

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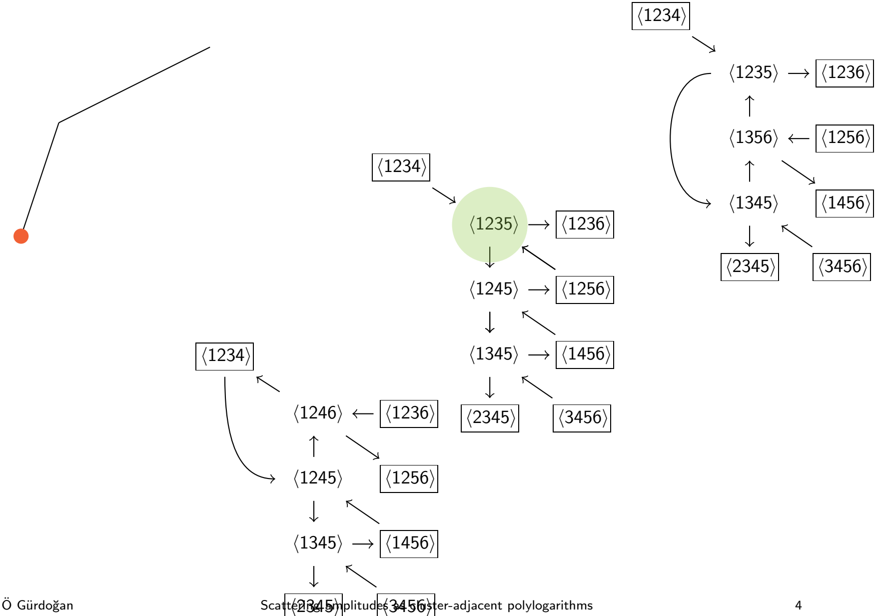

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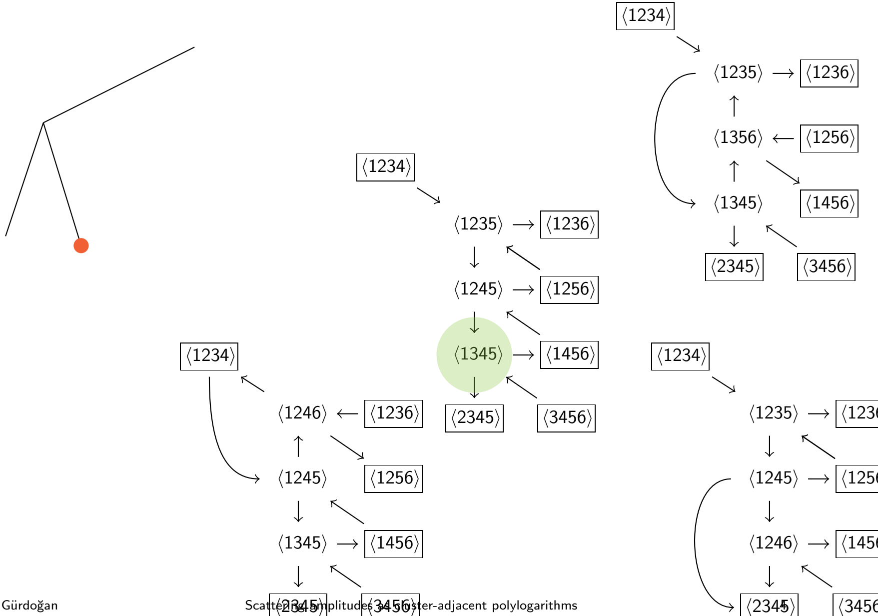

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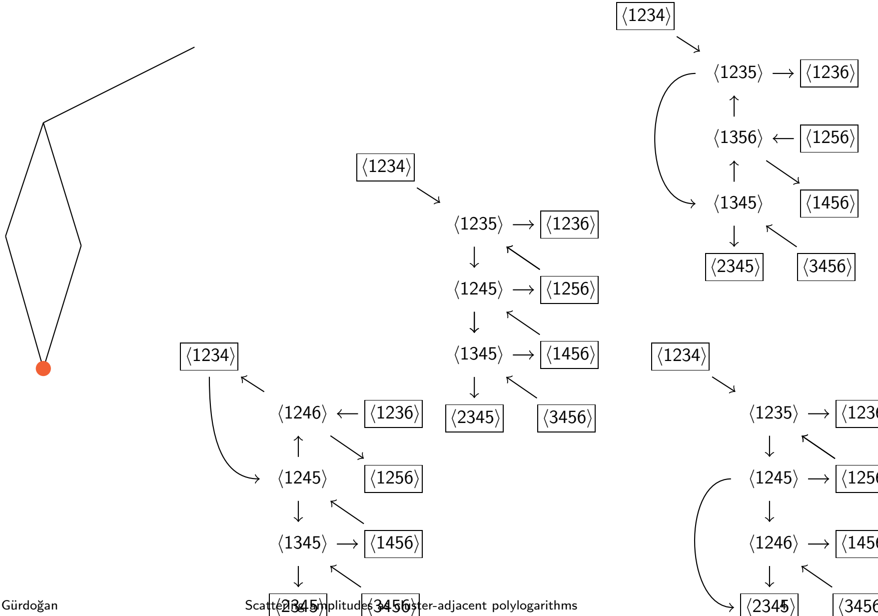

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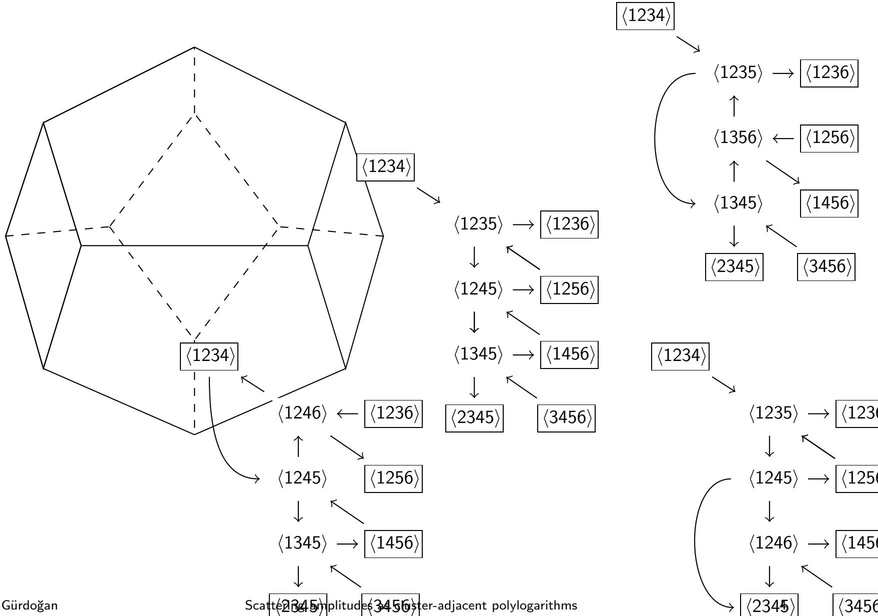

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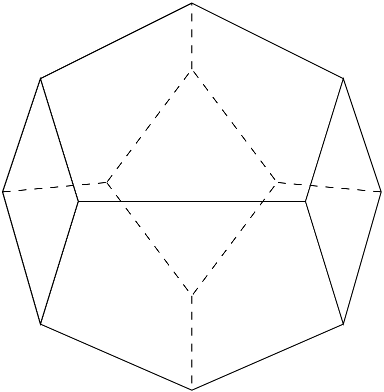

The associahedron

The mutations of clusters close on a polytope.

Cluster-adjacent polylogarithms

Cluster Adjacency

Consecutive symbol letters of scattering amplitudes must appear in the same cluster.

[Drummond, Foster, ÖCG]

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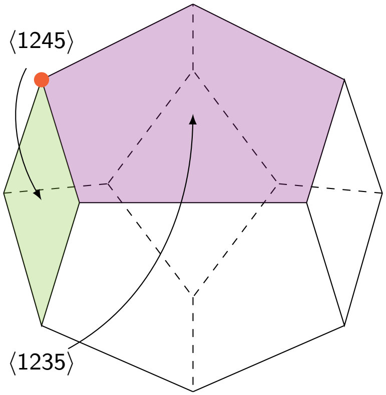

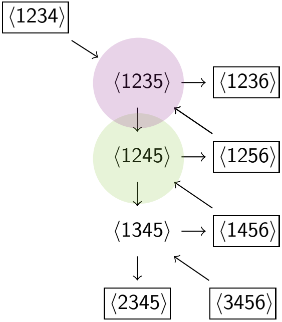

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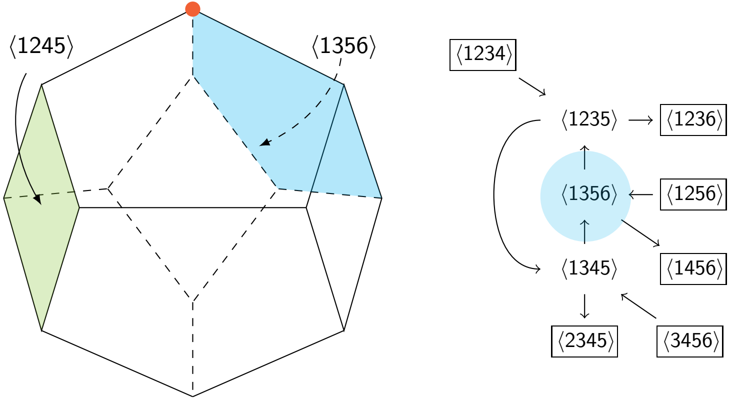

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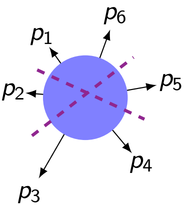

Brackets in overlapping invariants mutate to each other:

$$\langle 1245 \rangle \sim (p_2+p_3+p_4)^2 \mapsto \langle 2356 \rangle \sim (p_2+p_6+p_1)^2$$

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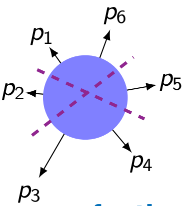

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- Various **further conditions**

shorthand:		
$(12) = \langle 3456\rangle$	(12) :	Frozen, adjacent to anything
etc	(13) :	$\{(13),(14),(15),(35),(36),(46)$ & frozen$\}$
	(14) :	$\{(13),(14),(15),(24),(46)$ & frozen$\}$

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Physical branch points + integrability + **extended Steinmann**

[Caron-Huot, Dixon et al]

$$\Leftrightarrow$$

Cluster adjacency

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Probed by the **Sklyanin bracket** [Golden, McLeod, Spradlin, Volovich] .

see Anastasia's talk!

The heptagon: $\mathrm{Gr}(4,7) \cong E_6$

odd n in $\mathrm{Gr}(4,n)$:
homogenise active nodes with frozen-adjacent Plückers and ignore latter

$$a_{11} = \frac{\langle 1234\rangle\langle 1567\rangle\langle 2367\rangle}{\langle 1237\rangle\langle 1267\rangle\langle 3456\rangle} \quad a_{51} = \frac{\langle 1(23)(45)(67)\rangle}{\langle 1234\rangle\langle 1567\rangle} \quad a_{41} = \frac{\langle 2457\rangle\langle 3456\rangle}{\langle 2345\rangle\langle 4567\rangle}$$

$$a_{31} = \frac{\langle 1567\rangle\langle 2347\rangle}{\langle 1237\rangle\langle 4567\rangle} \quad a_{21} = \frac{\langle 1234\rangle\langle 2567\rangle}{\langle 1267\rangle\langle 2345\rangle} \quad a_{61} = \frac{\langle 1(34)(56)(72)\rangle}{\langle 1234\rangle\langle 1567\rangle}$$

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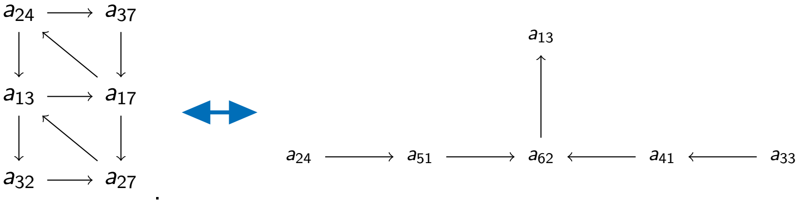

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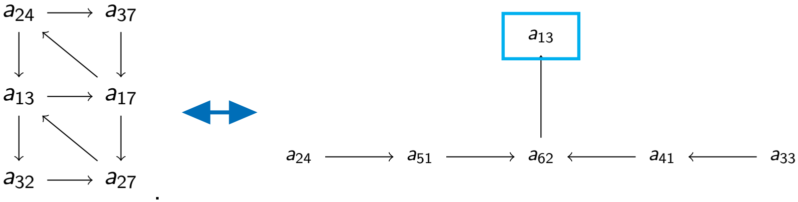

A_5 subalgebra: $20 + 1$ neighbours

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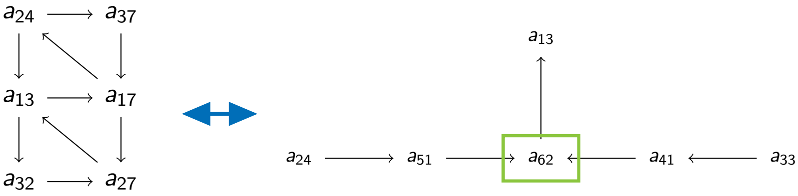

$A_2 \times A_2 \times A_1$ subalgebra: (5+5+2)+1 neighbours

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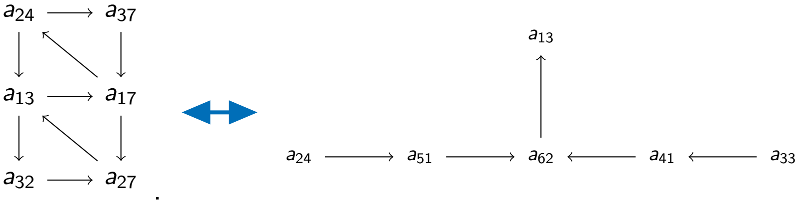

	a_{1i}	a_{2i}	a_{3i}	a_{4i}	a_{5i}	a_{6i}
a_{11}	● ○ ○ ◆ ◆ ○ ○	◆ ◆ ○ ● ◆ ● ○	◆ ○ ● ◆ ● ○ ◆	● ○ ◆ ○ ○ ◆ ○	● ○ ◆ ○ ○ ◆ ○	◇ ◆ ○ ○ ○ ○ ◆
a_{21}	◆ ○ ● ◆ ● ○ ◆	● ○ ● ◆ ◆ ● ○	◆ ○ ◆ ◆ ○ ◆ ●	◆ ○ ◆ ○ ● ◆ ○	○ ◆ ● ○ ◆ ○ ◆	○ ◆ ○ ● ○ ◆ ○
a_{31}	◆ ◆ ○ ● ◆ ● ○	◆ ● ◆ ○ ◆ ◆ ○	● ○ ● ◆ ◆ ● ○	◆ ○ ◆ ● ○ ◆ ○	○ ◆ ○ ◆ ○ ● ◆	○ ○ ◆ ○ ● ○ ◆
a_{41}	● ○ ◆ ○ ○ ◆ ○	◆ ○ ◆ ● ○ ◆ ○	◆ ○ ◆ ○ ● ◆ ○	● ◇ ◆ ○ ○ ◆ ◇	● ○ ○ ○ ○ ○ ○	◇ ◆ ◇ ○ ○ ◇ ◆
a_{51}	● ○ ◆ ○ ○ ◆ ○	○ ◆ ○ ◆ ○ ● ◆	○ ◆ ● ○ ◆ ○ ◆	● ○ ○ ○ ○ ○ ○	● ◇ ◆ ○ ○ ◆ ◇	◇ ◆ ◇ ○ ○ ◇ ◆
a_{61}	◇ ◆ ○ ○ ○ ○ ◆	○ ○ ◆ ○ ● ○ ◆	○ ◆ ○ ● ○ ◆ ○	◇ ◆ ◇ ○ ○ ◇ ◆	◇ ◆ ◇ ○ ○ ◇ ◆	● ◇ ○ ◇ ◇ ○ ◇

The adjacency table for $\mathrm{Gr}(4,7)$ branch points

◇: non-neighbour, ○: mutation non-neighbour,

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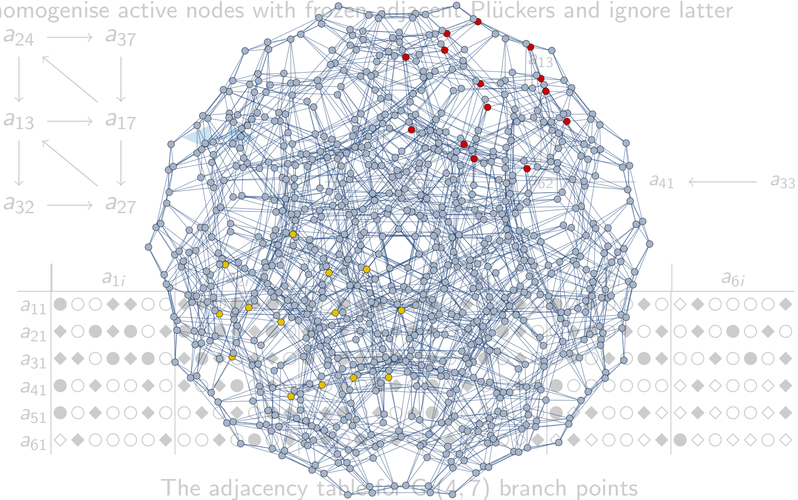

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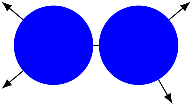
Tree amplitudes



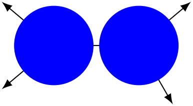
BCFW recursion of tree amplitudes via factorisation on physical poles.

NMHV: $\mathcal{A}^{\text{NMHV}} = \sum_{2<i<j\leq n} \left[1\, i-1\, i\, j-1\, j\right]$, where

$$[abcde] = \frac{\delta(\langle abcd\rangle \chi_e + \text{cyclic})}{\langle abcd\rangle\langle abce\rangle\langle abde\rangle\langle acde\rangle\langle bcde\rangle} \ni \text{physical and spurious poles}.$$

N$^{>1}$MHV: More complicated terms e.g. $[12345][56781] \in \mathcal{A}_{8,2}$

Tree amplitudes



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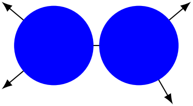
Poles of BCFW terms are cluster neighbours

[Drummond, Foster, ÖCG]

Tree amplitudes



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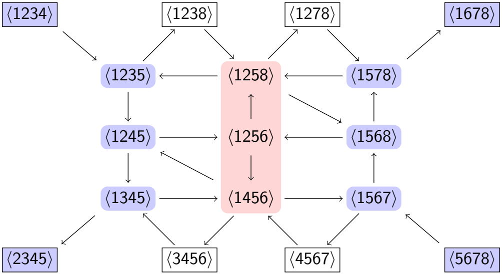

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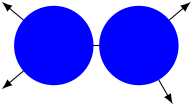

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Also Yangian invariants... **see Anastasia's talk!**

NMHV loop amplitudes

- Expansion in R-invariants: $E = \sum [abcde] \quad f_{abcde}$

Rational R-invariants CA poles in Plückers

Polylog coefficient functions CA branch cuts

Dual superconformal symmetry:
$\bar{Q}$ equation $\Rightarrow$ final entries of $f \leftrightarrow [\cdots]$

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$\bar{Q}$ final entries of NMHV coefficient functions are cluster compatible with the R-invariants.

NMHV loop amplitudes

NMHV 7-particle amplitude @ 4 loops ✓

[Drummond, Foster, ÖCG, Papathanasiou]

$$E_7 = (12)\ f_{(12)} + (13)\ f_{(13)} + (14)\ f_{(14)} + \text{cyclic}, \qquad (12) = [34567] \text{ etc}$$

- computed with the help of cluster adjacency
- R invariants satisfy linear relations
- Integrability of E_7 satisfied on the support of said identities

$\bar{Q}$ final entries of NMHV coefficient functions are cluster compatible with the R-invariants.

Scattering equations, tropical geometry and cluster algebras

Scattering equations and tropical geometry

- **Biadjoint amplitudes** on $\mathrm{Gr}(3, n)$.

  Define a "potential" $S = \sum_{i,j,k} s_{ijk}[ijk]$

$$\mathcal{A}(\mathbb{I}|\alpha) \sim \int \mathrm{d}x_i \mathrm{d}y_i \frac{\delta(\partial_x S)\,\delta(\partial_y S)}{([123][234]\cdots[n12])\big|_\alpha ([123][234]\cdots[n12])},$$

  where α is a permutation of the labels.

- Recently discovered link to **tropical geometry** [Cachazo et al 19] [2]:

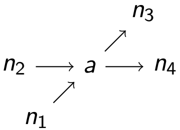
$$\mathcal{A}(\mathbb{I}|\alpha) = \sum \text{over facets of } \mathrm{Trop}\,\mathrm{Gr}(3, n) \text{ compatible with the ordering } \alpha$$

- Canonically-ordered $\mathcal{A}(\mathbb{I}|\mathbb{I}) \leftrightarrow$ **positive part** : $\mathrm{Trop}\,\mathrm{Gr}^+(3, n)$

The $\mathrm{Trop}\,\mathrm{Gr}^{+}(k,n)$ **fan**

Evaluate Plücker coordinates $(i_1, i_2, \ldots, i_k)$ in terms of the $\mathcal{X}$ coordinates of the initial cluster

$$X(a) = \frac{n_1 n_2}{n_3 n_4}$$



and **tropicalise** all of them, ie replace

$$\begin{aligned} \text{multiplication:} \quad & x_1 \cdot x_2 \longrightarrow x_1 + x_2 \\ \text{addition:} \quad & x_1 + x_2 \longrightarrow \min(x_1, x_2) \end{aligned}$$

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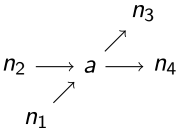

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$\mathrm{Gr}(2,5)$:

$$(1i) = (23) \mapsto 0, \quad (24) \mapsto \min(0, x_1), \quad (25) \mapsto \min(0, x_1, x_1 + x_2),$$
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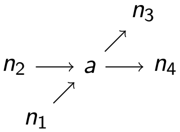

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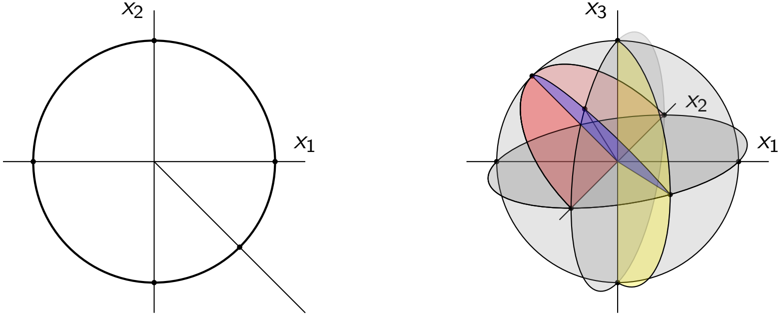
Collection of **domains of collective linearity**

$\rightsquigarrow$ **Speyer-Williams fan** $F_{k,n}$

Simplicial fans in $\mathrm{Gr}(2, n)$

Obtain polyhedra by intersecting the fan with the unit sphere:



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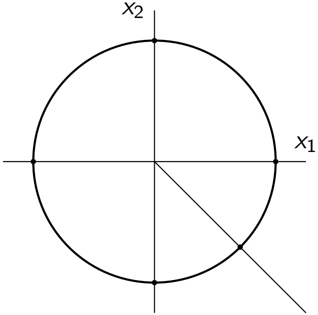

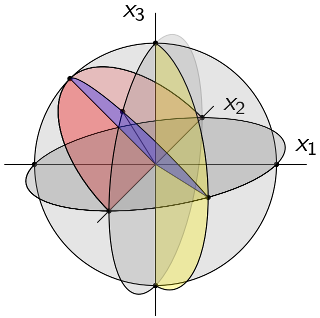

- simplicial fans • Duals of Gr$(2, n)$ associahedra
- explicit correspondence: **Rays $\leftrightarrow$ cluster $\mathcal{A}$ coordinates**
- Initial cluster: unit vectors $\{\mathbf{e}_1, \ldots, \mathbf{e}_m\}$ • Mutations translated to rays:

Explicit mutation formula for the rays of the fan!

Non-simpliciality in $\mathrm{Gr}(3, n)$

...or constellations on a 3D night sky

- $\mathrm{Gr}(3,6)$**:** The fans $F_{3,n\geq 6}$ are **not simplicial**

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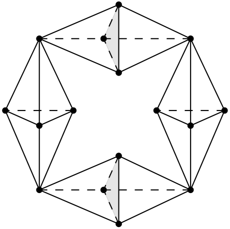

  - The cluster algebra **triangulates** $F_{3,6}$
  - From the volume of one simplex:

$$\mathrm{Vol}^{-1}\left(\begin{array}{c}\text{initial}\\ \text{simplex}\end{array}\right) = \prod_{r\,\in\,\text{in. cluster}} \sum_{i,j,k} s_{ijk}\,\mathrm{Trop}\,(ijk)\Big|_r$$

  - obtain the amplitude $\mathcal{A} = \sum_{\text{clusters}} \mathrm{Vol}(\text{simplex})$

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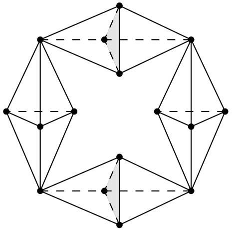

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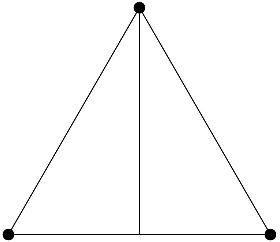
- Gr(3, 8):

Finite cluster algebra, triangulates $F_{3,8}$ into finite number of simplices

but

generates **redundant triangulations** of simplices into simplices



Finiteness in $\mathrm{Gr}(4, n)$

- First genuine case is $\mathrm{Gr}(4, 8)$ $\mathrm{Gr}(4, 7) \cong \mathrm{Gr}(3, 7)$, $\mathrm{Gr}(4, 6) \cong \mathrm{Gr}(2, 6)$
- Cluster algebra of **infinite** type
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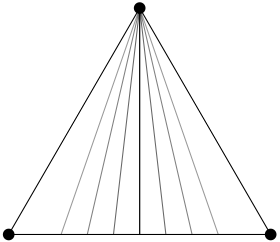
What about **the fan** $F_{4,8}$?

- **356** rays generated by mutations.
Cfr $\mathrm{Gr}(3,7/)T^7$ of [Arkani-Hamed, Lam, Spradlin]
- 169,192 clusters contain a cyclically-complete set of (up to **quintic**) $\mathcal{A}$ coordinates
- $\supset$ 8-point 2-loop NMHV letters of [Prlina et al 2017]
- **infinite redundant** triangulations



(cartoon)

Summary & further aspects

- A **geometric principle** governing the analytic structure of scattering amplitudes
- **Steinmann relations** interpreted in terms cluster algebras
- Cluster algebras to "bootstrap" solutions to **scattering equtions**
- (Candidate) **finite** alphabet for **8-particle** scattering
- Implications of adjacency **beyond pairs** . Eg triplet rules

$$\cdots a \otimes X(a) \otimes a' \cdots \qquad \text{or} \qquad \cdots a \otimes b \otimes c \cdots$$

- Construction of cluster-adjacent A_n **spaces** with various initial entries
- Adjacency for non-dual-conformal processes?

Thank you!