The Momentum Amplituhedron

Livia Ferro

Ludwig-Maximilians-Universität München

&

University of Hertfordshire

Amplitudes 2019

Trinity College Dublin, 04.07.2019

Based on: arXiv:1905.04216

with D. Damgaard, T. Lukowski, M. Parisi

Outline

* Introduction and Motivation
* Reminder: Amplituhedron
* Momentum Amplituhedron
* Examples
* Conclusions and open questions

Introduction

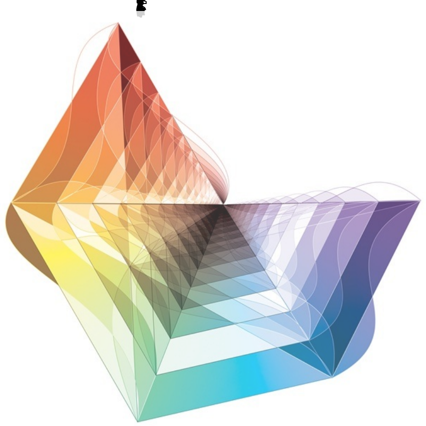
Geometrization of scattering amplitudes:

Amplitude = Volume of a geometric space
= canonical form on the space

Playground: **Maximally supersymmetric Yang-Mills theory**
Interacting 4d QFT with highest degree of symmetry

Geometrization of ampls in planar N=4 sYM: Amplituhedron

(N. Arkani-Hamed, J. Trnka)



(picture of A. Gilmore)

Introduction

Geometrization of scattering amplitudes:
Amplitude = Volume of a geometric space
= canonical form on the space

Playground: Maximally supersymmetric Yang-Mills theory
Interacting 4d QFT with highest degree of symmetry

* Positive geometries
    * amplituhedron (N. Arkani-Hamed, J. Trnka)
    * kinematic associahedron (N. Arkani-Hamed, Y. Bai, S. He, G. Yan)
    * cosmological polytope (N. Arkani-Hamed, P. Benincasa, A. Postnikov)
    * CFTs (B. Eden, P. Heslop, L.Mason; N. Arkani-Hamed, Y.-T. Huang, S.-H. Shao)
    * ...

Introduction

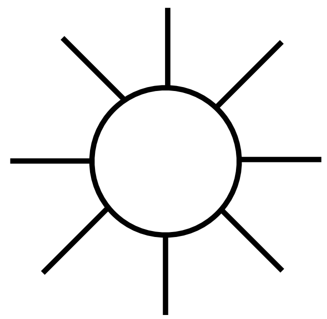
Geometrization of scattering amplitudes:
Amplitude = Volume of a geometric space
= canonical form on the space

Playground: Maximally supersymmetric Yang-Mills theory
Interacting 4d QFT with highest degree of symmetry

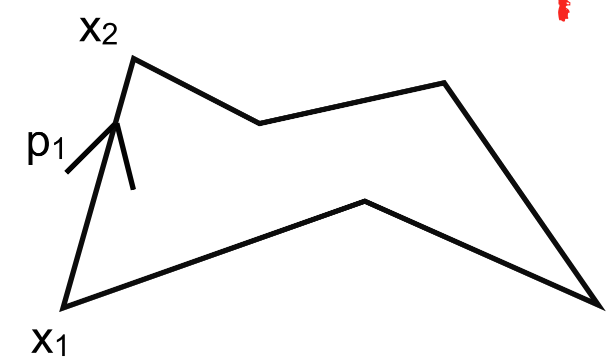
* Positive geometries
    * amplituhedron ←
    * kinematic associahedron
    * cosmological polytope
    * CFTs
    * ...

Scattering amplitudes in N=4 sYM

Amplitude duality $\longleftrightarrow$ **Wilson loop**



$$p_i^{\alpha\dot\alpha} = x_i^{\alpha\dot\alpha} - x_{i+1}^{\alpha\dot\alpha}$$
$$q_i^{\alpha A} = \theta_i^{\alpha A} - \theta_{i+1}^{\alpha A}$$



on-shell superspace $\longleftrightarrow$ dual superspace

$(\lambda_i^\alpha, \tilde\lambda_i^{\dot\alpha}, \eta_i^A)$

$(\lambda_i^\alpha, x_i^{\alpha\dot\alpha}, \theta_i^{\alpha A})$

Fourier transform on λ_i^α

Incidence relations

$$\tilde\mu_i^{\dot\alpha} := x_i^{\dot\alpha\alpha}\lambda_{i\alpha}$$
$$\chi_i^A := \theta_i^{\alpha A}\lambda_{i\alpha}$$

twistor superspace $\longleftrightarrow$ momentum-twistor superspace

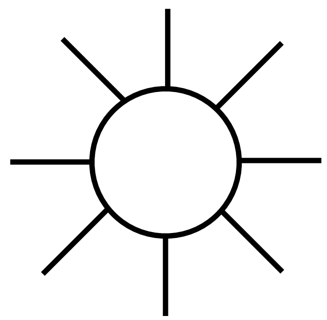
$$\mathcal{W}_i^A = (\mu_i^\alpha, \tilde\lambda_i^{\dot\alpha}, \eta_i^A)$$

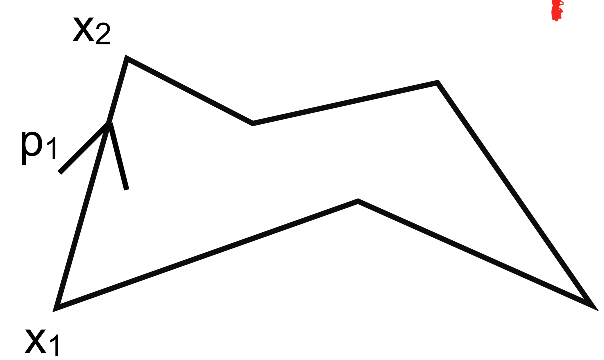
$$\mathcal{Z}_i^A = (\lambda_i^\alpha, \tilde\mu_i^{\dot\alpha}, \chi_i^A)$$

+ bosonization

Scattering amplitudes in N=4 sYM

Amplitude duality $\longleftrightarrow$ **Wilson loop**





$$p_i^{\alpha\dot\alpha} = x_i^{\alpha\dot\alpha} - x_{i+1}^{\alpha\dot\alpha}$$
$$q_i^{\alpha A} = \theta_i^{\alpha A} - \theta_{i+1}^{\alpha A}$$

on-shell superspace $\longleftrightarrow$ dual superspace

$$(\lambda_i^\alpha, \tilde\lambda_i^{\dot\alpha}, \eta_i^A) \qquad (\lambda_i^\alpha, x_i^{\alpha\dot\alpha}, \theta_i^{\alpha A})$$

Fourier transform on λ_i^α

Incidence relations

$$\tilde\mu_i^{\dot\alpha} := x_i^{\dot\alpha\alpha}\lambda_{i\alpha}$$
$$\chi_i^A := \theta_i^{\alpha A}\lambda_{i\alpha}$$

twistor superspace $\longleftrightarrow$ momentum-twistor superspace

$$\mathcal{W}_i^A = (\mu_i^\alpha, \tilde\lambda_i^{\dot\alpha}, \eta_i^A) \qquad \mathcal{Z}_i^A = (\lambda_i^\alpha, \tilde\mu_i^{\dot\alpha}, \chi_i^A)$$

+ bosonization

Amplituhedron

(picture of A. Gilmore)

Wilson loop

momentum-twistor superspace

$$\mathcal{Z}_i^A = (\lambda_i^\alpha, \tilde{\mu}_i^{\dot\alpha}, \chi_i^A)$$

+ bosonization

m

k'

$$Z = \begin{pmatrix} z_a \\ \varphi_1^A \chi_{aA} \\ \vdots \\ \varphi_k^A \chi_{aA} \end{pmatrix}$$

Generalization of polytope in Grassmannian:

$$Y_\alpha^I = \sum_{a=1}^{n} c_{\alpha a} Z_a^I$$

$I = 1,2,\ldots,k' + m$
$\alpha = 1,2,\ldots,k'$

Amplituhedron

(picture of A. Gilmore)

Amplituhedron defined as the image of the map

$$\Phi_Z : G_+(k', n) \to G(k', k' + m)$$

given by

$$Y_\alpha^I = \sum_{a=1}^{n} c_{\alpha a} Z_a^I$$

External data:
Positive matrix

= ordered maximal minors > 0

(picture of A. Gilmore)

Amplituhedron

Amplituhedron defined as the image of the map

$$\Phi_Z : G_+(k', n) \to G(k', k'+m)$$

given by

$$Y_\alpha^I = \sum_{a=1}^{n} c_{\alpha a} Z_a^I$$

$\mathbf{\Omega}_{n,k'}^{(m)}$: volume form with logarithmic singularities on all boundaries of the space

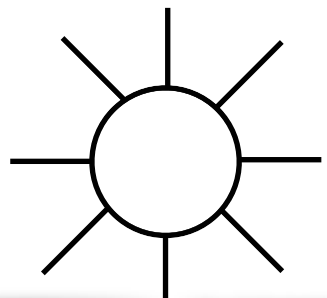
Found by push-forward of canonical forms via Φ_Z

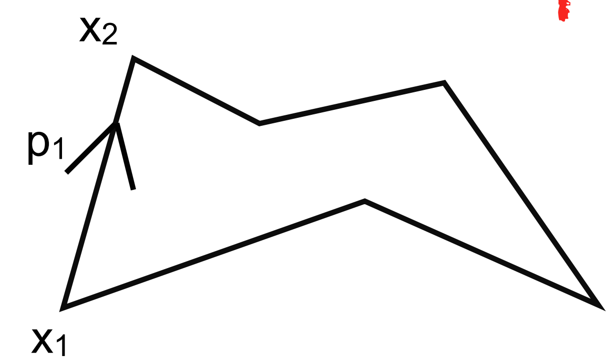
$$\mathbf{\Omega}_{n,k'}^{(m)} = \sum_{\sigma} \mathrm{dlog}\, \alpha_1^\sigma \wedge \mathrm{dlog}\, \alpha_2^\sigma \wedge \ldots \wedge \mathrm{dlog}\, \alpha_{k'm}^\sigma$$

Logarithmic differential form

Scattering amplitudes in N=4 sYM

Amplitude duality $\longleftrightarrow$ **Wilson loop**





$$p_i^{\alpha\dot\alpha} = x_i^{\alpha\dot\alpha} - x_{i+1}^{\alpha\dot\alpha}$$
$$q_i^{\alpha A} = \theta_i^{\alpha A} - \theta_{i+1}^{\alpha A}$$

on-shell superspace $(\lambda_i^\alpha, \tilde\lambda_i^{\dot\alpha}, \eta_i^A)$ $\longleftrightarrow$ **dual superspace** $(\lambda_i^\alpha, x_i^{\alpha\dot\alpha}, \theta_i^{\alpha A})$

Fourier transform on λ_i^α (on-shell superspace $\longleftrightarrow$ twistor superspace)

Incidence relations (dual superspace $\longleftrightarrow$ momentum-twistor superspace)

$$\tilde\mu_i^{\dot\alpha} := x_i^{\dot\alpha\alpha}\lambda_{i\alpha}$$
$$\chi_i^A := \theta_i^{\alpha A}\lambda_{i\alpha}$$

twistor superspace $\longleftrightarrow$ **momentum-twistor superspace**

$$\mathcal{W}_i^A = (\mu_i^\alpha, \tilde\lambda_i^{\dot\alpha}, \eta_i^A)$$

$$\mathcal{Z}_i^A = (\lambda_i^\alpha, \tilde\mu_i^{\dot\alpha}, \chi_i^A)$$

+ bosonization

Scattering amplitudes in N=4 sYM

Amplitude

on-shell superspace

$$(\lambda_i^{\alpha}, \tilde{\lambda}_i^{\dot{\alpha}}, \eta_i^A)$$

Non-chiral superspace

$$(\lambda_i^a, \eta_i^r \,|\, \tilde{\lambda}_i^{\dot{a}}, \tilde{\eta}_i^{\dot{r}}),\ a, \dot{a}, r, \dot{r} = 1,2$$

$$\begin{cases} \tilde{q}^{\dot{a}r} = \sum_{i=1}^{n} \tilde{\lambda}_i^{\dot{a}} \eta_i^r \\ q^{ar} = \sum_{i=1}^{n} \lambda_i^a \tilde{\eta}_i^{\dot{r}} \end{cases}$$

Associate a,ȧ with SU(2)xSU(2) R-symmetry indices:

$$\eta^a \to d\lambda^a, \quad \tilde{\eta}^{\dot{a}} \to d\tilde{\lambda}^{\dot{a}}$$

n-point super-amplitude in non-chiral space <-> 2n form

(S. He, C. Zhang)

Amplitudes as forms

$(\lambda_i^a, \eta_i^r \,|\, \tilde{\lambda}_i^{\dot{a}}, \tilde{\eta}_i^{\dot{r}}),\ a, \dot{a}, r, \dot{r} = 1{,}2$

n-point super-amplitude in non-chiral space <-> 2n form

$$\mathscr{A}_{n,k} := (dq)^4 \wedge \mathbf{\Omega_{n,k}}$$

2n-4 form

Example: 4-point MHV

$$\mathbf{\Omega}_{4,2} = \frac{(d\tilde{q})^4}{st} = \mathrm{dlog}\frac{\langle 12\rangle}{\langle 13\rangle} \wedge \mathrm{dlog}\frac{\langle 23\rangle}{\langle 13\rangle} \wedge \mathrm{dlog}\frac{\langle 34\rangle}{\langle 13\rangle} \wedge \mathrm{dlog}\frac{\langle 41\rangle}{\langle 13\rangle}$$

Geometry whose canonical form gives the amplitude form?

Momentum amplituhedron!

Momentum Amplituhedron

Bosonized spinor helicity variables:

$$\tilde{\Lambda}_i^{\dot{A}} = \begin{pmatrix} \tilde{\lambda}_i^{\dot{a}} \\ \tilde{\phi}_{\dot{a}}^{\dot{\alpha}} \cdot \tilde{\eta}_i^{\dot{a}} \end{pmatrix}, \qquad \dot{A} = (\dot{a}, \dot{\alpha}) = 1, \ldots, k+2 \qquad \Lambda_i^{A} = \begin{pmatrix} \lambda_i^{a} \\ \phi_{a}^{\alpha} \cdot \eta_i^{a} \end{pmatrix}, \qquad A = (a, \alpha) = 1, \ldots, n-k+2$$

$$\left\{ \text{matrix } \tilde{\Lambda} \text{ positive} \quad \text{and} \quad \text{matrix } \Lambda^{\perp} \text{ positive} \right\}$$

Positive region

Momentum amplituhedron: Image of the positive Grassmannian $G_+(k,n)$ through the map

$$\Phi_{(\Lambda,\tilde{\Lambda})} : G_+(k,n) \to G(k,k+2) \times G(n-k,n-k+2)$$

defined as:

$$\tilde{Y}_{\dot{\alpha}}^{\dot{A}} = \sum_{i=1}^{n} c_{\dot{\alpha}i} \tilde{\Lambda}_i^{\dot{A}} \qquad Y_{\alpha}^{A} = \sum_{i=1}^{n} c_{\alpha i}^{\perp} \Lambda_i^{A}$$

Momentum Amplituhedron

Check 1.

Dimension

$$\Phi_{(\Lambda,\tilde{\Lambda})} : G_+(k,n) \to G(k,k+2) \times G(n-k,n-k+2)$$

dimension 2n

Dimension of G+(k,n) through the map is lower dimensional:

Momentum amplituhedron lives in the co-dimension 4 surface

$$P^{a\dot{a}} = \sum_{i=1}^{n} \left(Y^{\perp} \cdot \Lambda\right)_i^a \left(\tilde{Y}^{\perp} \cdot \tilde{\Lambda}\right)_i^{\dot{a}} = 0$$

Equivalent to momentum conservation

Dimension 2n-4 ☑

Momentum Amplituhedron

Check 2.

Sign flips

Conjecture from Scatt. Eqs (S. He, C. Zhang)

$$\tilde{Y}^{\dot{A}}_{\dot{\alpha}} = \sum_{i=1}^{n} c_{\dot{\alpha}i}\,\tilde{\Lambda}^{\dot{A}}_i \qquad Y^A_\alpha = \sum_{i=1}^{n} c^{\perp}_{\alpha i}\,\Lambda^A_i$$

$\left\{\text{matrix } \tilde{\Lambda} \text{ positive} \quad \text{and} \quad \text{matrix } \Lambda^{\perp} \text{ positive}\right\}$

$\{[\tilde{Y}12], [\tilde{Y}13], \ldots, [\tilde{Y}1n]\}$ k sign flips ?

$\{\langle Y12\rangle, \langle Y13\rangle, \ldots, \langle Y1n\rangle\}$ k-2 sign flips ?

Momentum Amplituhedron

Check 2.

Sign flips

(S. He, C. Zhang)

$$\tilde{Y}^{\dot{A}}_{\dot{\alpha}} = \sum_{i=1}^{n} c_{\dot{\alpha}i}\,\tilde{\Lambda}^{\dot{A}}_i \qquad Y^{A}_{\alpha} = \sum_{i=1}^{n} c^{\perp}_{\alpha i}\,\Lambda^{A}_i$$

$\{$matrix $\tilde{\Lambda}$ positive and matrix $\Lambda^{\perp}$ positive$\}$

= ordinary amplituhedron
with m=2, k=k'

$\{[\tilde{Y}12], [\tilde{Y}13], \ldots, [\tilde{Y}1n]\}$ k sign flips ☑

Momentum Amplituhedron

Check 2.

Sign flips

(S. He, C. Zhang)

$$\tilde{Y}^{\dot{A}}_{\dot{\alpha}} = \sum_{i=1}^{n} c_{\dot{\alpha}i}\,\tilde{\Lambda}^{\dot{A}}_i \qquad Y^{A}_{\alpha} = \sum_{i=1}^{n} c^{\perp}_{\alpha i}\,\Lambda^{A}_i$$

{matrix $\tilde{\Lambda}$ positive and matrix $\Lambda^{\perp}$ positive}

$\{\langle Y12\rangle, \langle Y13\rangle, \ldots, \langle Y1n\rangle\}$?

Define $X^{\bar{A}}_{\dot{\alpha}} = (\Lambda^{\perp})^{\bar{A}}_i\, c_{\dot{\alpha}i}\,,\quad \bar{A} = 1,\ldots,k-2\,,\dot{\alpha} = 1,\ldots,k \qquad X \in G(k-2,k)$

Ordinary amplituhedron with m=2, k->k-2

$\{(X12), (X13), \ldots, (X1n)\}$ k-2 sign flips

$$(Xij) = \epsilon_{\dot{\alpha}_1\ldots\dot{\alpha}_k} X^{1}_{\dot{\alpha}_1}\ldots . X^{k-2}_{\dot{\alpha}_{k-2}} c_{\dot{\alpha}_{k-1},i}\, c_{\dot{\alpha}_k,j}$$

But $(Xij) = \langle Yij\rangle$

$\{\langle Y12\rangle, \langle Y13\rangle, \ldots, \langle Y1n\rangle\}$ k-2 sign flips ☑

Momentum Amplituhedron

Check 2.

Sign flips

(S. He, C. Zhang)

$$\tilde{Y}^{\dot{A}}_{\dot{\alpha}} = \sum_{i=1}^{n} c_{\dot{\alpha}i}\,\tilde{\Lambda}^{\dot{A}}_i \qquad Y^A_\alpha = \sum_{i=1}^{n} c^{\perp}_{\alpha i}\,\Lambda^A_i$$

$\{$matrix $\tilde{\Lambda}$ positive and matrix $\Lambda^{\perp}$ positive$\}$

$\{[\tilde{Y}12], [\tilde{Y}13], \ldots, [\tilde{Y}1n]\}$ k sign flips ☑

$\{\langle Y12\rangle, \langle Y13\rangle, \ldots, \langle Y1n\rangle\}$ k-2 sign flips ☑

Momentum Amplituhedron

$$\tilde{Y}^{\dot{A}}_{\dot{\alpha}} = \sum_{i=1}^{n} c_{\dot{\alpha} i}\, \tilde{\Lambda}^{\dot{A}}_i \qquad Y^{A}_{\alpha} = \sum_{i=1}^{n} c^{\perp}_{\alpha i}\, \Lambda^{A}_i$$

Boundaries

$\langle Y\, i\, i+1 \rangle = 0\,,$	$[\tilde{Y}\, i\, i+1] = 0$	Collinear limits
$S_{i,i+1\ldots,i+p} = 0\,,$	$p = 2,\ldots,n-4$	Factorizations

Uplift of planar Mandelstam variables

$$S_{i,i+1\ldots,i+p} = \sum_{i \le j_1 < j_2 \le i+p} \langle Y j_1 j_2 \rangle [\tilde{Y} j_1 j_2]$$

Momentum Amplituhedron

$$\tilde{Y}^{\dot{A}}_{\dot{\alpha}} = \sum_{i=1}^{n} c_{\dot{\alpha} i}\, \tilde{\Lambda}^{\dot{A}}_i \qquad\qquad Y^{A}_{\alpha} = \sum_{i=1}^{n} c^{\perp}_{\alpha i}\, \Lambda^{A}_i$$

Boundaries

$\langle Y\, i\, i+1 \rangle = 0\,,$	$[\tilde{Y}\, i\, i+1] = 0$	Collinear limits
$S_{i,i+1\ldots,i+p} = 0\,,$	$p = 2,\ldots,n-4$	Factorizations

Volume form
=
Differential form with log singularities on all boundaries
=
Sum over cells of push-forwards of canonical diff-form

! Positive geometry -> no singularities inside momentum amplituhedron. Positivity of $\Lambda^{\perp}$ and $\tilde{\Lambda}$ is not enough to guarantee Mandelstams $S>0$. But: space of allowed kinematics large.

Momentum Amplituhedron

$$\tilde{Y}^{\dot{A}}_{\dot{\alpha}} = \sum_{i=1}^{n} c_{\dot{\alpha}i}\,\tilde{\Lambda}^{\dot{A}}_{i} \qquad Y^{A}_{\alpha} = \sum_{i=1}^{n} c^{\perp}_{\alpha i}\,\Lambda^{A}_{i}$$

➡ Volume function

$$\mathbf{\Omega}_{n,k} \wedge d^4P\,\delta^4(P) = \prod_{\alpha=1}^{n-k} \langle Y_1 \ldots Y_{n-k} d^2 Y_\alpha \rangle \prod_{\dot{\alpha}=1}^{k} [\tilde{Y}_1 \ldots \tilde{Y}_k d^2 \tilde{Y}_{\dot{\alpha}}]\,\delta^4(P)\,\mathbf{\Omega}_{n,k}$$

➡ Amplitude

Reference subspaces

$$\mathscr{A}^{\text{tree}}_{n,k} = \delta^4(p) \int d\phi^1_a \ldots d\phi^{n-k}_a \int d\tilde{\phi}^1_{\dot{a}} \ldots d\tilde{\phi}^k_{\dot{a}}\;\Omega_{n,k}(Y^*, \tilde{Y}^*, \Lambda, \tilde{\Lambda})$$

➡ Integral representation

$$\delta^4(P)\,\Omega_{n,k} = \int \frac{d^{(n-k)\cdot(n-k)}g}{(\det g)^{n-k}} \int \omega_{n,k} \prod_{\alpha=1}^{n-k} \delta^{(n-k+2)}(Y^A_\alpha - g^\beta_\alpha (c^\perp)_{\beta i}\,\Lambda^A_i) \prod_{\dot{\alpha}=1}^{k} \delta^{(k+2)}(\tilde{Y}^{\dot{A}}_{\dot{\alpha}} - c_{\dot{\alpha}i}\,\tilde{\Lambda}^{\dot{A}}_i)$$

Momentum Amplituhedron

$$\tilde{Y}^{\dot{A}}_{\dot{\alpha}} = \sum_{i=1}^{n} c_{\dot{\alpha} i}\, \tilde{\Lambda}^{\dot{A}}_{i} \qquad Y^{A}_{\alpha} = \sum_{i=1}^{n} c^{\perp}_{\alpha i}\, \Lambda^{A}_{i}$$

Examples

* MHV$_4$ amplitudes:

$$\alpha_1 = \frac{\langle Y12\rangle}{\langle Y13\rangle}, \alpha_2 = \frac{\langle Y23\rangle}{\langle Y13\rangle}, \alpha_3 = \frac{\langle Y34\rangle}{\langle Y13\rangle}, \alpha_4 = \frac{\langle Y14\rangle}{\langle Y13\rangle}$$

$$\mathbf{\Omega}_{4,2} = \bigwedge_{j=1}^{4} \mathrm{dlog}\alpha_j = \mathrm{dlog}\frac{\langle Y12\rangle}{\langle Y13\rangle} \wedge \mathrm{dlog}\frac{\langle Y23\rangle}{\langle Y13\rangle} \wedge \mathrm{dlog}\frac{\langle Y34\rangle}{\langle Y13\rangle} \wedge \mathrm{dlog}\frac{\langle Y14\rangle}{\langle Y13\rangle}$$

$$= \frac{\langle 1234\rangle^2}{\langle Y12\rangle\langle Y23\rangle\langle Y34\rangle\langle Y41\rangle}\langle Yd^2Y_1\rangle\langle Yd^2Y_2\rangle$$

Momentum Amplituhedron

$$\tilde{Y}^{\dot{A}}_{\dot{\alpha}} = \sum_{i=1}^{n} c_{\dot{\alpha} i}\, \tilde{\Lambda}^{\dot{A}}_i \qquad Y^{A}_{\alpha} = \sum_{i=1}^{n} c^{\perp}_{\alpha i}\, \Lambda^{A}_i$$

Examples

* MHV_4 amplitudes:

$$\boldsymbol{\Omega}_{4,2} = \frac{[1234]^2}{[\tilde{Y}12][\tilde{Y}23][\tilde{Y}34][\tilde{Y}41]}[\tilde{Y}d^2\tilde{Y}_1][\tilde{Y}d^2\tilde{Y}_2]$$

Momentum Amplituhedron

$$\tilde{Y}^{\dot{A}}_{\dot{\alpha}} = \sum_{i=1}^{n} c_{\dot{\alpha}i}\, \tilde{\Lambda}^{\dot{A}}_i \qquad Y^{A}_{\alpha} = \sum_{i=1}^{n} c^{\perp}_{\alpha i}\, \Lambda^{A}_i$$

Examples

* MHV$_4$ amplitudes:

$$\Omega_{4,2} = \frac{[1234]^2}{[\tilde{Y}12][\tilde{Y}23][\tilde{Y}34][\tilde{Y}41]}[\tilde{Y}d^2\tilde{Y}_1][\tilde{Y}d^2\tilde{Y}_2]$$

$$\Omega_{4,2} = \frac{\langle 1234\rangle^2}{\langle Y12\rangle\langle Y23\rangle\langle Y34\rangle\langle Y41\rangle}\langle Yd^2Y_1\rangle\langle Yd^2Y_2\rangle$$

Related by momentum conservation

$$\Omega_{4,2} = \frac{\langle 1234\rangle^2[1234]^2}{\langle Y12\rangle\langle Y23\rangle[\tilde{Y}12][\tilde{Y}23]}$$

$$\mathscr{A}^{\text{tree}}_{4,2} = \delta^4(p)\frac{\delta^4(q)\delta^4(\tilde{q})}{\langle 12\rangle_\lambda\langle 23\rangle_\lambda[12]_{\tilde{\lambda}}[23]_{\tilde{\lambda}}}$$

Momentum Amplituhedron

$$\tilde{Y}^{\dot{A}}_{\dot{\alpha}} = \sum_{i=1}^{n} c_{\dot{\alpha}i}\,\tilde{\Lambda}^{\dot{A}}_i \qquad Y^{A}_{\alpha} = \sum_{i=1}^{n} c^{\perp}_{\alpha i}\,\Lambda^{A}_i$$

Examples

* NMHV$_6$ amplitudes:

$$\Omega_{6,3} = \Omega^{(612)}_{6,3} + \Omega^{(234)}_{6,3} + \Omega^{(456)}_{6,3} = \Omega^{(123)}_{6,3} + \Omega^{(345)}_{6,3} + \Omega^{(561)}_{6,3}$$

$$\alpha_1 = \frac{\langle Y12\rangle}{\langle Y13\rangle}, \alpha_2 = \frac{\langle Y23\rangle}{\langle Y13\rangle}, \alpha_3 = \frac{[\tilde{Y}\hat{3}4]}{[\tilde{Y}\hat{1}\hat{3}]}, \alpha_4 = \frac{[\tilde{Y}64]}{[\tilde{Y}\hat{1}\hat{3}]}, \alpha_5 = \frac{[\tilde{Y}6\hat{1}]}{[\tilde{Y}\hat{1}\hat{3}]}, \alpha_6 = \frac{[\tilde{Y}4\hat{1}]}{[\tilde{Y}\hat{1}\hat{3}]}, \alpha_7 = \frac{[\tilde{Y}45]}{[\tilde{Y}64]}, \alpha_8 = \frac{[\tilde{Y}56]}{[\tilde{Y}64]}$$

$$\hat{\tilde{\Lambda}}_1 = \tilde{\Lambda}_1 + \frac{\langle Y23\rangle}{\langle Y13\rangle}\tilde{\Lambda}_2, \hat{\tilde{\Lambda}}_3 = \tilde{\Lambda}_3 + \frac{\langle Y12\rangle}{\langle Y13\rangle}\tilde{\Lambda}_2$$

$$\Omega^{(123)}_{6,3} = \frac{\big(\langle Y12\rangle[12456] + \langle Y13\rangle[13456] + \langle Y23\rangle[23456]\big)^2([\tilde{Y}45]\langle 12345\rangle + [\tilde{Y}46]\langle 12346\rangle + [\tilde{Y}56]\langle 12356\rangle)^2}{S_{123}\langle Y12\rangle\langle Y23\rangle[\tilde{Y}45][\tilde{Y}56]\langle Y1|5+6|4\tilde{Y}]\langle Y3|4+5|6\tilde{Y}]}$$

Momentum Amplituhedron

$$\tilde{Y}^{\dot{A}}_{\dot{\alpha}} = \sum_{i=1}^{n} c_{\dot{\alpha} i}\, \tilde{\Lambda}^{\dot{A}}_i \qquad Y^{A}_{\alpha} = \sum_{i=1}^{n} c^{\perp}_{\alpha i}\, \Lambda^{A}_i$$

Examples

* NMHV$_6$ amplitudes:

$$\Omega_{6,3} = \Omega^{(612)}_{6,3} + \Omega^{(234)}_{6,3} + \Omega^{(456)}_{6,3} = \Omega^{(123)}_{6,3} + \Omega^{(345)}_{6,3} + \Omega^{(561)}_{6,3}$$

$$\Omega^{(123)}_{6,3} = \frac{\big(\langle Y12\rangle[12456] + \langle Y13\rangle[13456] + \langle Y23\rangle[23456]\big)^2 \big([\tilde{Y}45]\langle 12345\rangle + [\tilde{Y}46]\langle 12346\rangle + [\tilde{Y}56]\langle 12356\rangle\big)^2}{S_{123}\langle Y12\rangle\langle Y23\rangle[\tilde{Y}45][\tilde{Y}56]\langle Y1|5+6|4\tilde{Y}]\langle Y3|4+5|6\tilde{Y}]}$$

Momentum Amplituhedron

$$\tilde{Y}^{\dot{A}}_{\dot{\alpha}} = \sum_{i=1}^{n} c_{\dot{\alpha} i}\, \tilde{\Lambda}^{\dot{A}}_i \qquad Y^{A}_{\alpha} = \sum_{i=1}^{n} c^{\perp}_{\alpha i}\, \Lambda^{A}_i$$

Examples

* NMHV$_6$ amplitudes:

$$\Omega_{6,3} = \Omega^{(612)}_{6,3} + \Omega^{(234)}_{6,3} + \Omega^{(456)}_{6,3} = \Omega^{(123)}_{6,3} + \Omega^{(345)}_{6,3} + \Omega^{(561)}_{6,3}$$

$$\Omega^{(123)}_{6,3} = \frac{\left(\langle Y12\rangle[12456] + \langle Y13\rangle[13456] + \langle Y23\rangle[23456]\right)^2 \left([\tilde{Y}45]\langle 12345\rangle + [\tilde{Y}46]\langle 12346\rangle + [\tilde{Y}56]\langle 12356\rangle\right)^2}{S_{123}\langle Y12\rangle\langle Y23\rangle[\tilde{Y}45][\tilde{Y}56]\langle Y1|5+6|4\tilde{Y}]\langle Y3|4+5|6\tilde{Y}]}$$

Spurious singularities

Divergences on the 15 boundaries of the momentum amplituhedron:

$$\langle Yi\,i+1\rangle = 0\,,\ i = 1,\ldots,6\,, \quad [\tilde{Y}i\,i+1] = 0\,,\ i = 1,\ldots,6\,, \quad S_{i,i+1,i+2} = 0\,,\ i = 1,2,3$$

Momentum Amplituhedron

$$\tilde{Y}^{\dot{A}}_{\dot{\alpha}} = \sum_{i=1}^{n} c_{\dot{\alpha} i}\,\tilde{\Lambda}^{\dot{A}}_i \qquad Y^{A}_{\alpha} = \sum_{i=1}^{n} c^{\perp}_{\alpha i}\,\Lambda^{A}_i$$

Examples

* NMHV$_6$ amplitudes:

$$\Omega_{6,3} = \Omega^{(612)}_{6,3} + \Omega^{(234)}_{6,3} + \Omega^{(456)}_{6,3} = \Omega^{(123)}_{6,3} + \Omega^{(345)}_{6,3} + \Omega^{(561)}_{6,3}$$

$$\Omega^{(123)}_{6,3} = \frac{\left(\langle Y12\rangle[12456] + \langle Y13\rangle[13456] + \langle Y23\rangle[23456]\right)^2 ([\tilde{Y}45]\langle 12345\rangle + [\tilde{Y}46]\langle 12346\rangle + [\tilde{Y}56]\langle 12356\rangle)^2}{S_{123}\langle Y12\rangle\langle Y23\rangle[\tilde{Y}45][\tilde{Y}56]\langle Y1|5+6|4\tilde{Y}]\langle Y3|4+5|6\tilde{Y}]}$$

Reduces to

$$\delta^4(q)(\tilde{\eta}_4[56]_{\tilde{\lambda}} + \tilde{\eta}_5[64]_{\tilde{\lambda}} + \tilde{\eta}_6[45]_{\tilde{\lambda}})^2$$

Momentum Amplituhedron

$$\tilde{Y}^{\dot{A}}_{\dot{\alpha}} = \sum_{i=1}^{n} c_{\dot{\alpha} i}\, \tilde{\Lambda}^{\dot{A}}_i \qquad Y^{A}_{\alpha} = \sum_{i=1}^{n} c^{\perp}_{\alpha i}\, \Lambda^{A}_i$$

Examples

* NMHV_6 amplitudes:

$$\Omega_{6,3} = \Omega^{(612)}_{6,3} + \Omega^{(234)}_{6,3} + \Omega^{(456)}_{6,3} = \Omega^{(123)}_{6,3} + \Omega^{(345)}_{6,3} + \Omega^{(561)}_{6,3}$$

$$\Omega^{(123)}_{6,3} = \frac{(\langle Y12\rangle[12456] + \langle Y13\rangle[13456] + \langle Y23\rangle[23456])^2 ([\tilde{Y}45]\langle 12345\rangle + [\tilde{Y}46]\langle 12346\rangle + [\tilde{Y}56]\langle 12356\rangle)^2}{S_{123}\langle Y12\rangle\langle Y23\rangle[\tilde{Y}45][\tilde{Y}56]\langle Y1|5+6|4Y]\langle Y3|4+5|6Y]}$$

Reduces to

$$\delta^4(q)(\tilde{\eta}_4[56]_{\tilde{\lambda}} + \tilde{\eta}_5[64]_{\tilde{\lambda}} + \tilde{\eta}_6[45]_{\tilde{\lambda}})^2$$

Reduces to

$$\delta^4(\tilde{q})\big(\eta_1\langle 23\rangle_{\lambda} + \eta_2\langle 31\rangle_{\lambda} + \eta_3\langle 12\rangle_{\lambda}\big)^2$$

Agrees with the one found in (S. He, C. Zhang):

$$\Omega^{(123)}_{6,3} = \frac{\delta^4(q)\delta^4(\tilde{q})(\tilde{\eta}_4[56]_{\tilde{\lambda}} + \tilde{\eta}_5[64]_{\tilde{\lambda}} + \tilde{\eta}_6[45]_{\tilde{\lambda}})^2\big(\eta_1\langle 23\rangle_{\lambda} + \eta_2\langle 31\rangle_{\lambda} + \eta_3\langle 12\rangle_{\lambda}\big)^2}{s_{123}\langle 12\rangle\langle 23\rangle[45][56]\langle 1|5+6|4]\langle 3|4+5|6]}$$

Conclusions

New positive geometry for tree-level amplitudes in N=4 sYM in spinor helicity space

- ☑ Bosonized spinor helicity variables + positivity constraints
- ☑ Sign-flip conditions + Mandelstam with additional constrs
- ☑ Amplitude from the canonical form with log singularities on the boundaries

- ☐ Loop amplitudes?
- ☐ Extension to other theories: e.g. non-planar, less-, non-supersymmetric?

Thank you!