First Result for a Full Two-Loop Five-Gluon Amplitude

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The pentagon team

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For Many QCD Processes, Next-to-Leading Order is Insufficient

E.g. strong coupling from 3-jet/2-jet ratio:

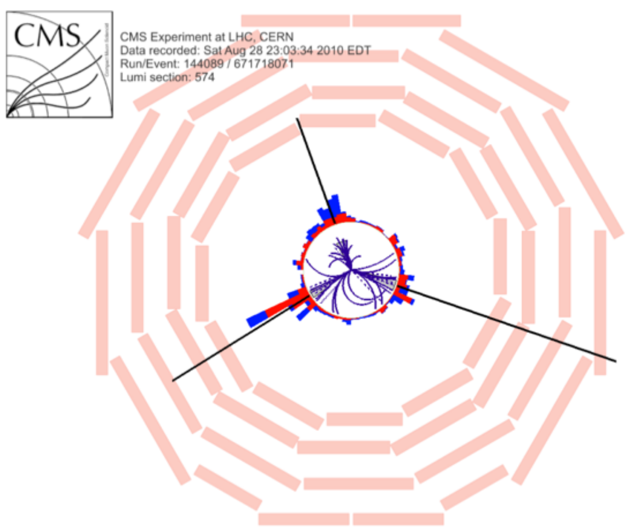
$$\alpha_S(M_Z) = 0.1148 \underset{\text{(exp)}}{\pm 0.0014} \underset{\text{(PDF)}}{\pm 0.0018} \underset{\text{(theory)}}{\pm 0.0050}$$

Large theoretical uncertainty!

Next-to-Next-to-Leading Order theory predictions needed.

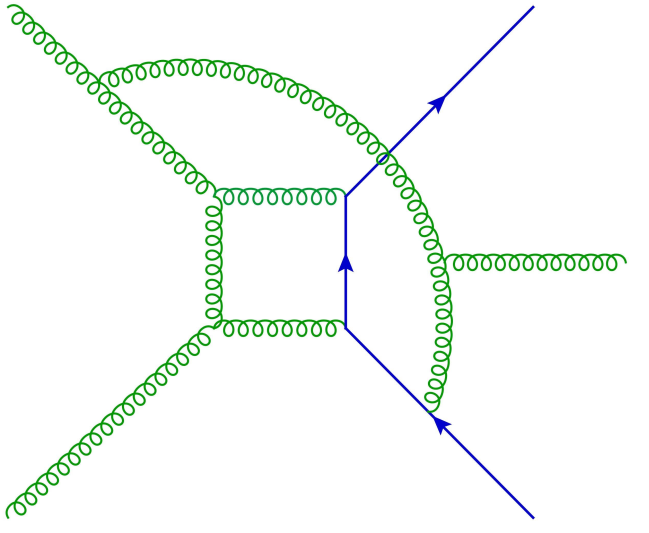
Multi-jet processes at the LHC are important for phenomenology

- Determination of strong coupling constant α_s
- tests of Standard Model
- search for new physics



Major theory bottleneck (?):

virtual two-loop amplitudes



Are virtual corrections the bottleneck?

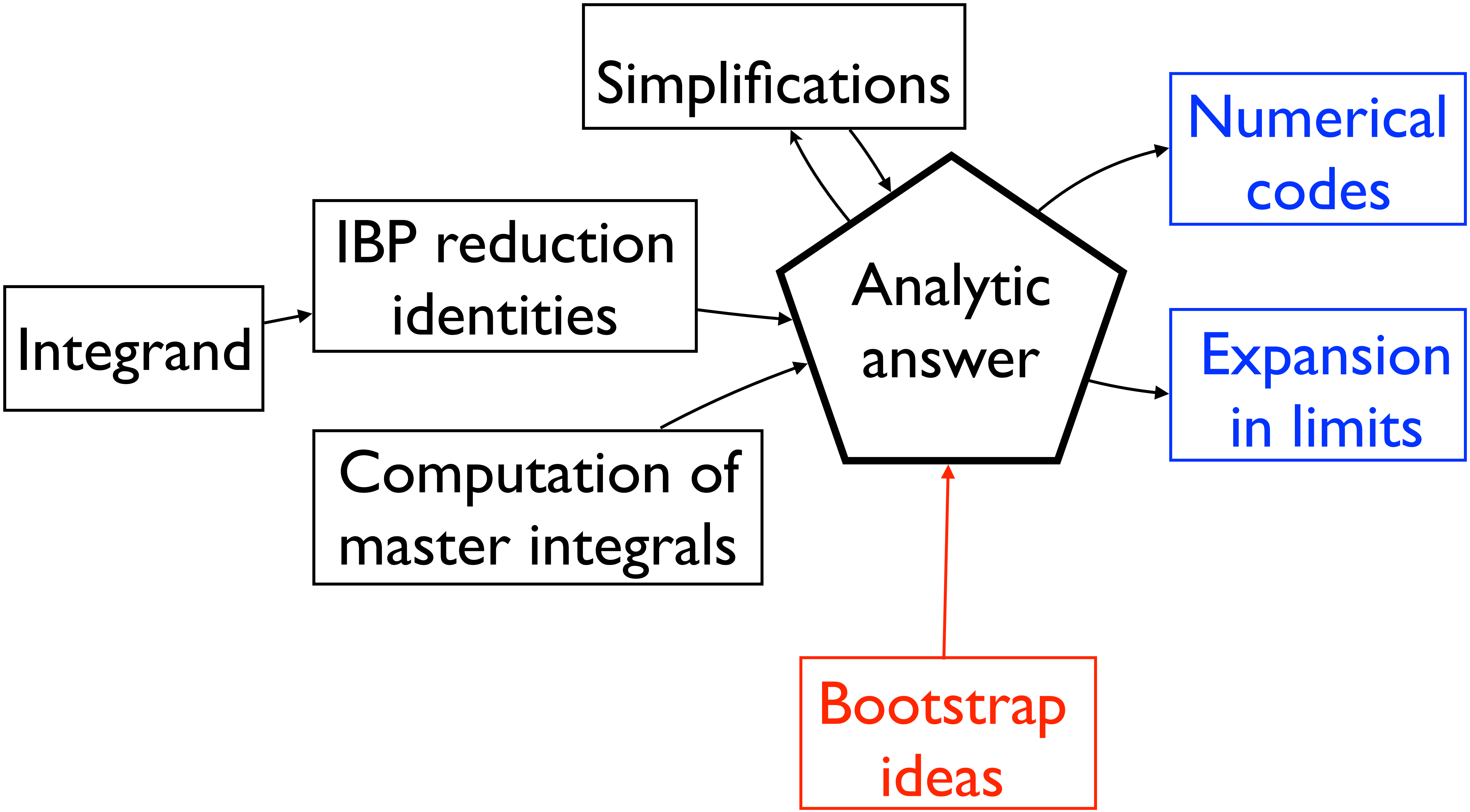
Dramatic recent progress

- All QCD amplitudes known analytically in the *planar* limit
[Gehrmann, Henn, Lo Presti '15; Dunbar, Perkins '16; Badger, Brønnum-Hansen, Hartanto, Peraro '18; Abreu, Dormans, Febres Cordero, Ita, Page '18; Abreu, Dormans, Febres Cordero, Ita, Page, Sotnikov '19] [— Ben Page's talk]

- *Symbols* of N=4 sYM and N=8 supergravity amplitudes
[Abreu, Dixon, Herrmann, Page, Zeng '18 '19]
[Chicherin, Gehrmann, Henn, Wasser, Zhang, Zoia '18 '19]

- Full-color five-gluon all-plus helicity amplitude [— this talk]

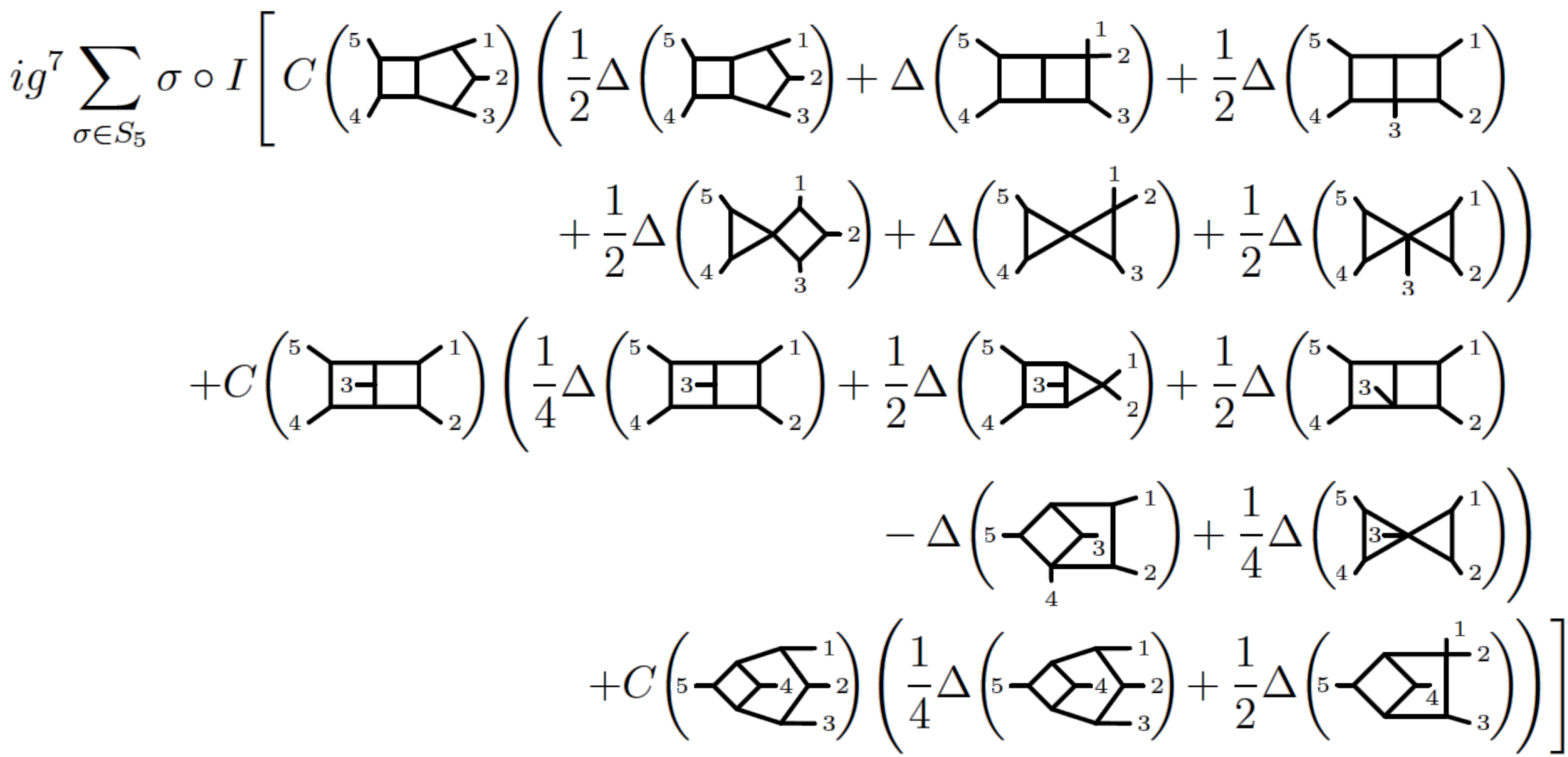
Workflow scattering amplitudes



Loop integrand for all-plus amplitude

[Badger, Mogull, Ochirov, O'Connel '15]

$$\mathcal{A}^{(2)}(1^+,2^+,3^+,4^+,5^+) =$$



Naively numerators with up to degree five/six

Color decomposition

- The amplitudes are vectors in color space

$$\mathcal{A}_5^{(1)} = \sum_{\lambda=1}^{12} N_c A_\lambda^{(1,0)} T_\lambda + \sum_{\lambda=13}^{22} A_\lambda^{(1,1)} T_\lambda$$

$$\mathcal{A}_5^{(2)} = \sum_{\lambda=1}^{12} \left(N_c^2 A_\lambda^{(2,0)} + A_\lambda^{(2,2)} \right) T_\lambda + \boxed{\sum_{\lambda=13}^{22} A_\lambda^{(2,1)} T_\lambda}$$

$A_\lambda^{(2,1)}$ truly new piece (due to color relations)

- Basis of single and double traces

$$T_1 = \mathrm{Tr}(12345) - \mathrm{Tr}(15432)$$

$$T_{13} = \mathrm{Tr}(12)\,[\mathrm{Tr}(345) - \mathrm{Tr}(543)]$$

(and permutations thereof)

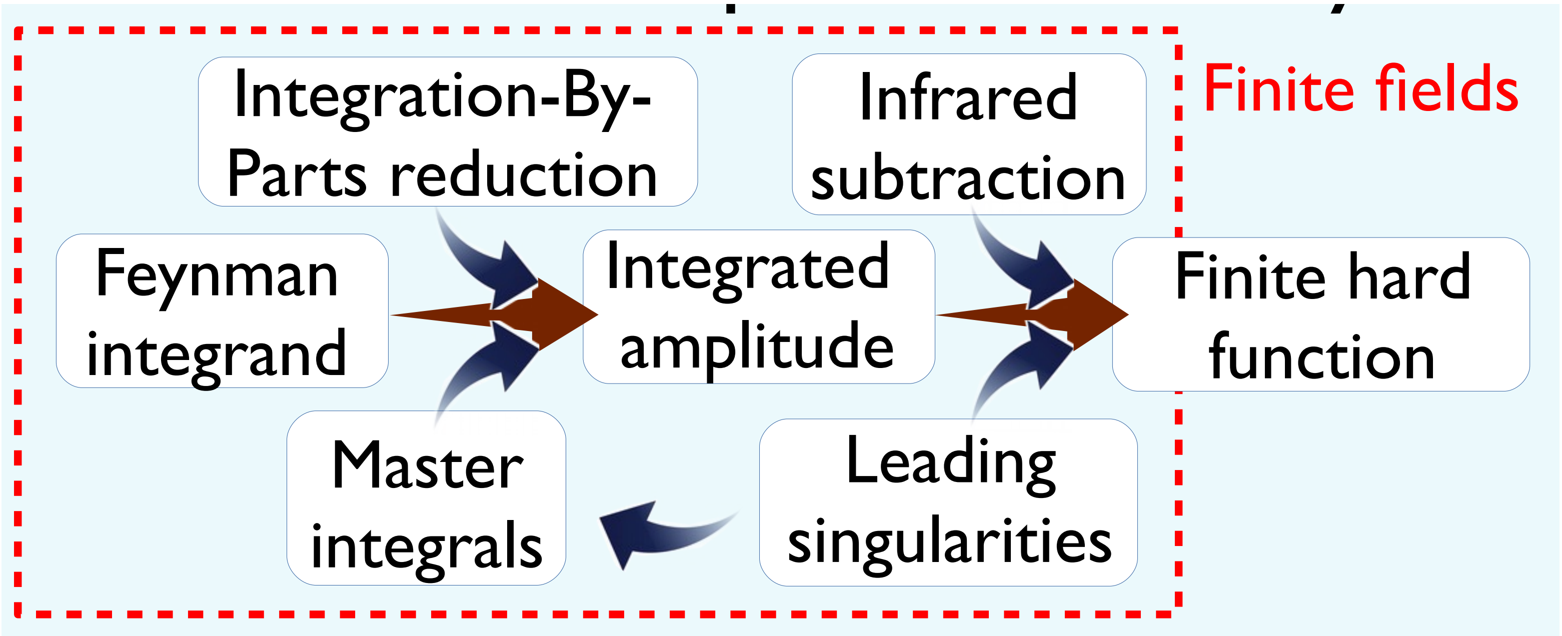
Finite field methods significantly improve integration-by-parts (IBP) reduction algorithms

- Finite field and rational reconstruction

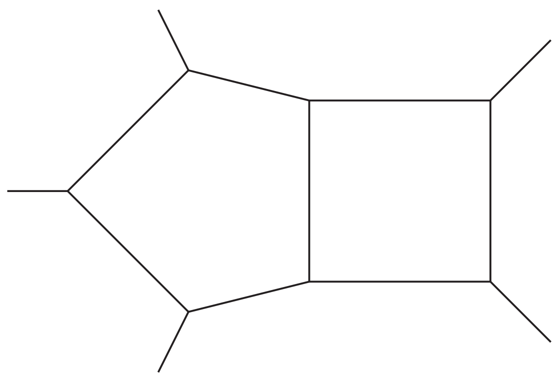
  [Schabinger, von Manteuffel, '15] [Peraro, '16, '19]
  [Maierhöfer, Usovitch, '18] [Smirnov, Chukharev, '19]

  - faster IBP's
  - better scaling for multi scale problems

- We further optimise by reconstructing only the physical answer

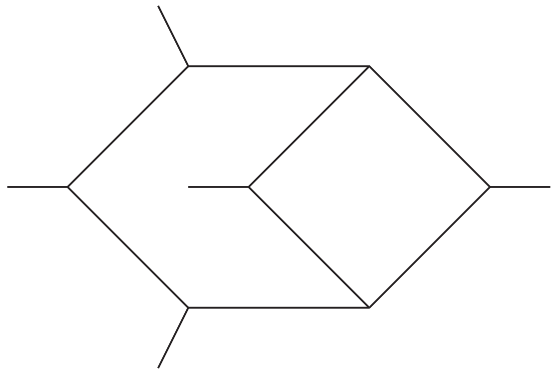


All master integrals known for massless two-loop five-particle scattering



[Gehrmann, Henn, Lo Presti ’15, ’18]

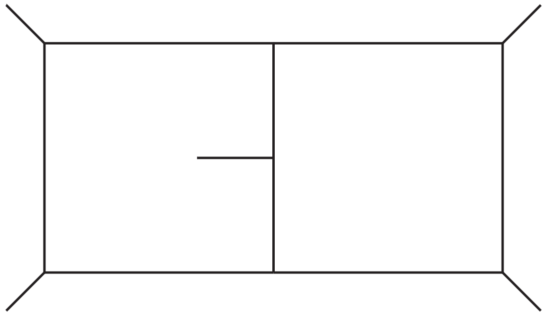
[Papadopoulos, Tommasini, Wever ’15]



[Böhm, Georgoudis, Larsen, Schönemann, Zhang, ’18]

[Abreu, Page, Zeng, ’18]

[Chicherin, Gehrmann, Henn, Lo Presti, Mitev, Wasser, ’18]



[Abreu, Dixon, Herrmann, Page, Zeng, ’18]

[Chicherin, Gehrmann, Henn, Wasser, Zhang, Zoia, ’18]

⟶ Result described by pentagon functions

Algorithmic construction of canonical basis

- Main idea: analyse leading singularities

  [Arkani-Hamed, Bourjaily, Cachazo, Trnka '10] [Henn '13]

  Closely related to dlog integrands

  [Arkani-Hamed, Bourjaily, Cachazo, Trnka '14]
  [Bern, Herrmann, Litsey, Stankowicz, Trnka '14]

- Implementation as algorithm to find all D=4 dlog integrands [Wasser '16]

Subtlety: integrands that vanish in D=4 can be important

⟶ D-dimensional leading singularity analysis based on Baikov representation

[Chicherin, Gehrmann, Henn, Wasser, Zhang, Zoia '18]

Pentagon functions

- Proposed in [Chicherin, Henn, Mitev '17]
- Confirmed in [Abreu, Dixon, Herrmann, Page, Zeng, '18]
  [Chicherin, Gehrmann, Henn, Lo Presti, Mitev, Wasser, '18]

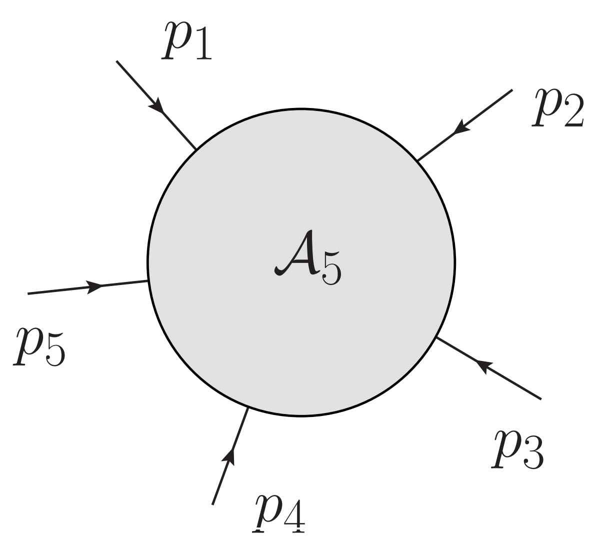
Iterated integrals along path γ

$$\int_{\gamma} d \log W_{i_1} \cdots d \log W_{i_n}$$

31 integration kernels W related to branch cuts

- Planar pentagon functions: fast numerical implementations [Gehrmann, Henn, Lo Presti, '18]
  [https://pentagonfunctions.hepforge.org/]

Kinematics of five-particle scattering



Massless particles: $p_i^2 = 0$

Scalar invariants: $s_{ij} = (p_i + p_j)^2 = 2p_i \cdot p_j$

Five independent: $(\; s_{12}, \; s_{23}, \; s_{34}, \; s_{45}, \; s_{51} \;)$

One pseudo-scalar $\epsilon_5 \equiv \epsilon_{\mu\nu\rho\sigma} p_1^\mu p_2^\nu p_3^\rho p_4^\sigma$

$$\Delta \sim (\epsilon_5)^2$$

Integration kernels encode all possible physical and spurious singularities of amplitudes

Integration kernels $d\log W_i \quad i = 1\ldots 31$

Letter	s notation	momentum notation	cylic
W_1	s_{12}	$2p_1 \cdot p_2$	$+\ (4)$
W_6	$s_{34}+s_{45}$	$2p_4 \cdot (p_3+p_5)$	$+\ (4)$
W_{11}	$s_{12}-s_{45}$	$2p_3 \cdot (p_4+p_5)$	$+\ (4)$
W_{16}	$s_{45}-s_{12}-s_{23}$	$2p_1 \cdot p_3$	$+\ (4)$
W_{21}	$s_{34}+s_{45}-s_{12}-s_{23}$	$2p_3 \cdot (p_1+p_4)$	$+\ (4)$
W_{26}	$\frac{s_{12}s_{23}-s_{23}s_{34}+s_{34}s_{45}-s_{12}s_{15}-s_{45}s_{15}-\sqrt{\Delta}}{s_{12}s_{23}-s_{23}s_{34}+s_{34}s_{45}-s_{12}s_{15}-s_{45}s_{15}+\sqrt{\Delta}}$	$\frac{\mathrm{tr}[(1-\gamma_5)\not{p}_4\not{p}_5\not{p}_1\not{p}_2]}{\mathrm{tr}[(1+\gamma_5)\not{p}_4\not{p}_5\not{p}_1\not{p}_2]}$	$+\ (4)$
W_{31}	$\sqrt{\Delta}$	$\mathrm{tr}[\gamma_5\not{p}_1\not{p}_2\not{p}_3\not{p}_4]$	

Table 2. Interpretation of pentagon alphabet in terms of particle momenta.

adapted from [Gehrmann, Henn, Lo Presti, '18]

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W_6	$s_{34} + s_{45}$	$2p_4 \cdot (p_3 + p_5)$	
W_{11}	$s_{12} - s_{45}$	$2p_3 \cdot (p_4 + p_5)$	
W_{16}	$s_{45} - s_{12} - s_{23}$	$2p_1 \cdot p_3$	
[illegible]	[illegible] $+ s_{45} - s_{12} - s_{23}$	$2p_3 \cdot (p_1 + p_4)$	$+ (4)$
[illegible]	$\frac{[illegible]s_{34}+s_{34}s_{45}-s_{12}s_{15}-s_{45}s_{15}-\sqrt{\Delta}}{[illegible]s_{34}+s_{34}s_{45}-s_{12}s_{15}-s_{45}s_{15}+\sqrt{\Delta}}$	$\frac{\mathrm{tr}[(1-\gamma_5)\not{p}_4\not{p}_5\not{p}_1\not{p}_2]}{\mathrm{tr}[(1+\gamma_5)\not{p}_4\not{p}_5\not{p}_1\not{p}_2]}$	
[illegible]	$\sqrt{\Delta}$	$\mathrm{tr}[\gamma_5\not{p}_1\not{p}_2\not{p}_3\not{p}_4]$	

Soft/ collinear limits

Vanishing Gram determinant

Phases

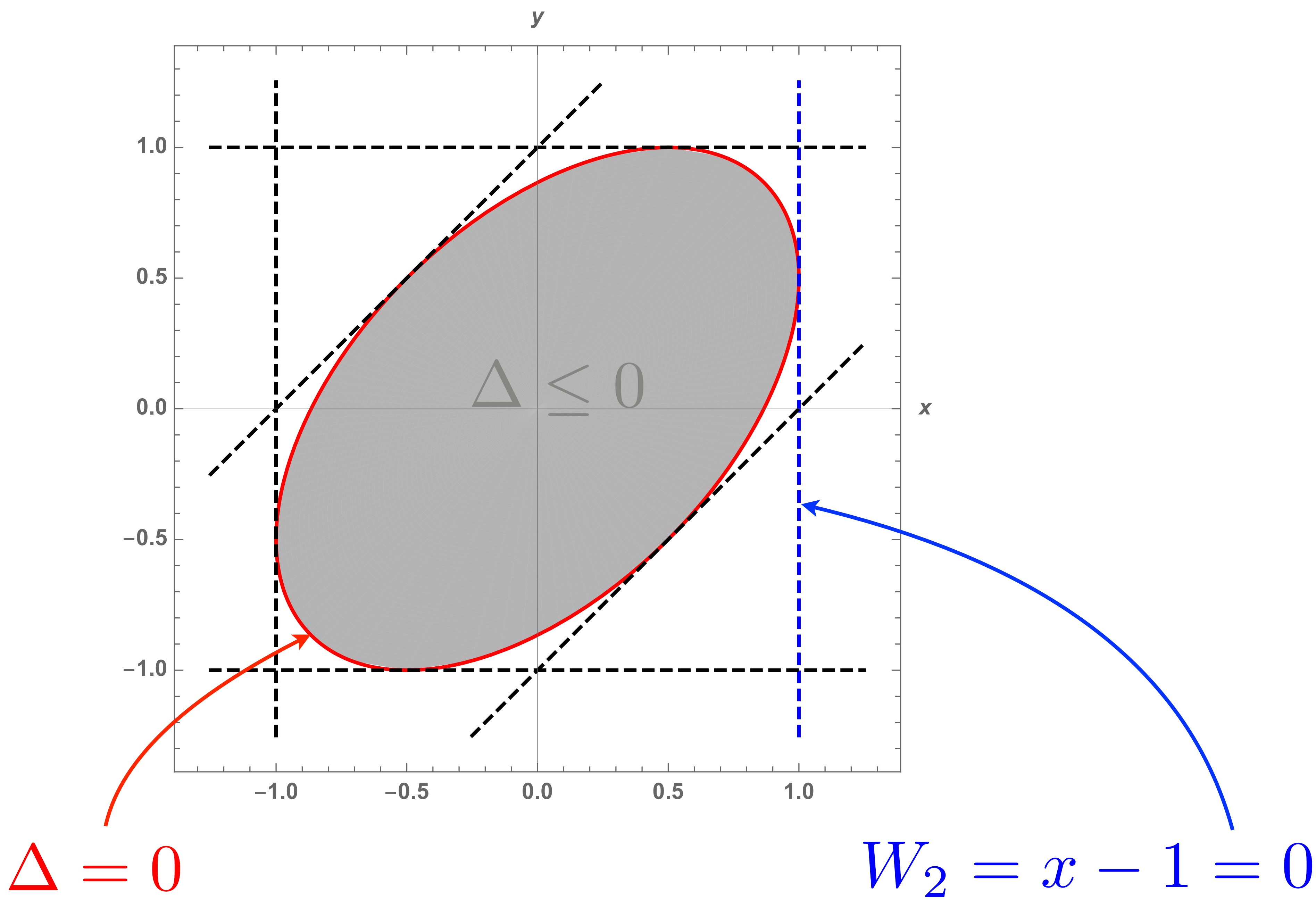
Table 2. Interpretation of pentagon alphabet in terms of particle momenta.

adapted from [Gehrmann, Henn, Lo Presti, '18]

Physical s_{12} channel

Positive s-channel energies, real particle momenta: $\Delta \leq 0$

Sketch for kinematics: $s_{i,i+1} = (3, -1+x, 1, 1, -1+y)$



Pentagon functions in terms of familiar functions (to mathematicians and particle theorists)

- Integration kernels $d\log W_i \quad i = 1\ldots 31$
- At NNLO, up to four iterations (weight) needed:

$$[W_a, W_b, W_c, W_d]_{X_0} = \int d\log W_a \int d\log W_b \int d\log W_c \int d\log W_d$$

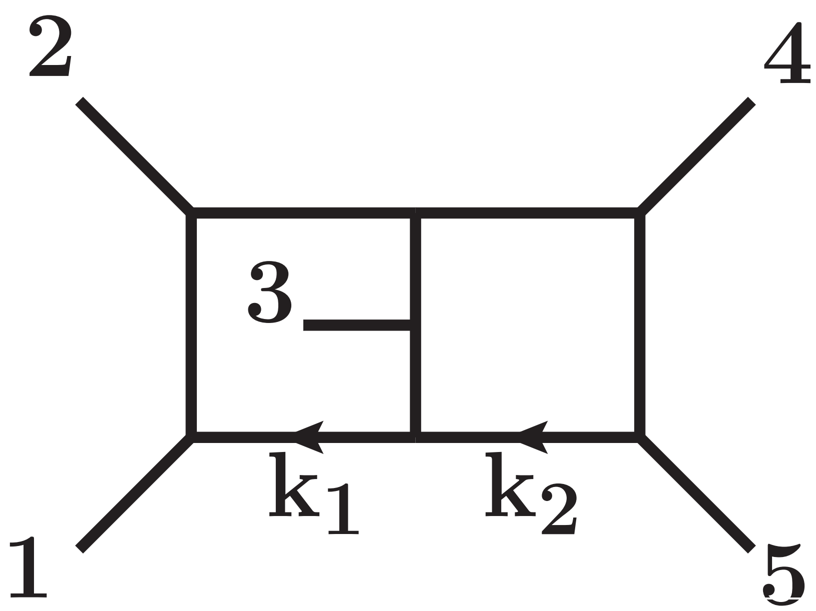
Boundary point: $X_0 = (3, -1, 1, 1, -1)$

Up to weight 2, logarithms and dilogarithms:

- Weight 1: $[W_1]_{X_0} = \log(s_{12}/3) \quad [W_2]_{X_0} = \log(-s_{23})$
- Weight 2: $[W_5/W_2, W_{12}/W_2]_{X_0} = -\mathrm{Li}_2\left(1 - \frac{s_{15}}{s_{23}}\right)$

- In general: Goncharov polylogarithms

Numerical evaluation in physical region



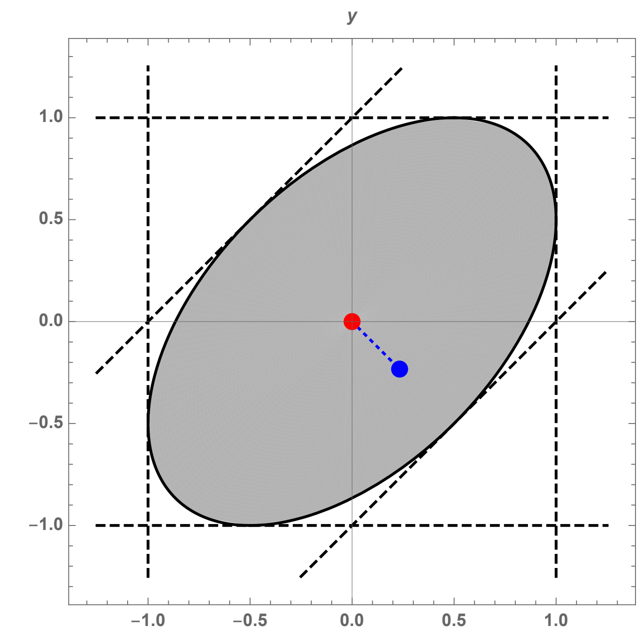
- Expressed in terms of pentagon functions
- Boundary values from physical consistency conditions
- Result in terms of Goncharov polylogarithms
- Numerical evaluation: GINAC
- Checks using SecDec

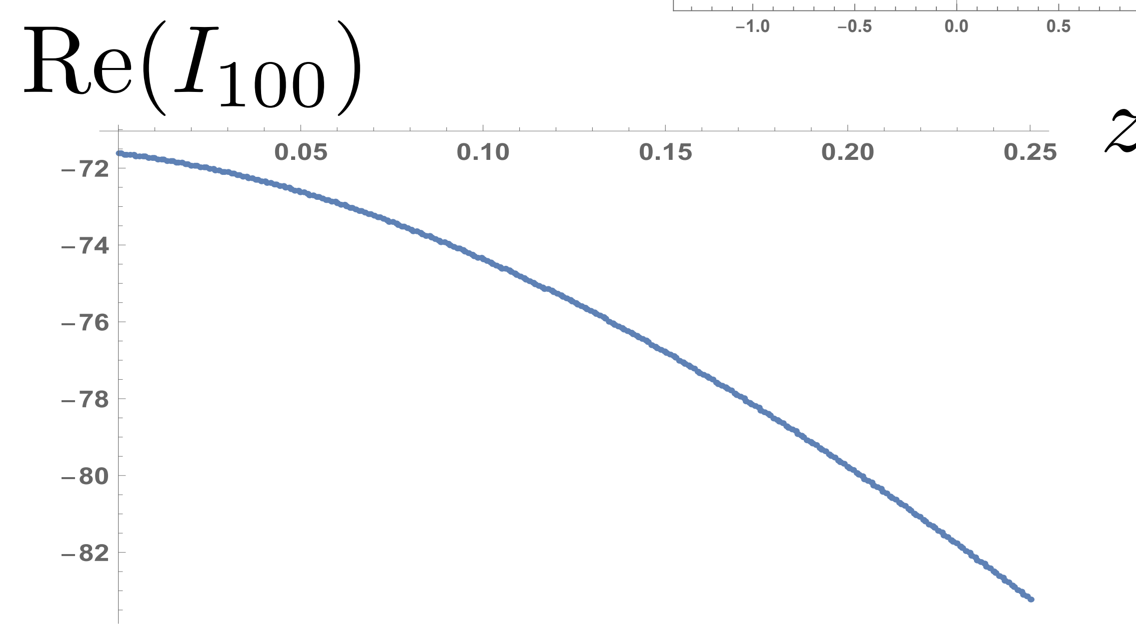
Boundary point:

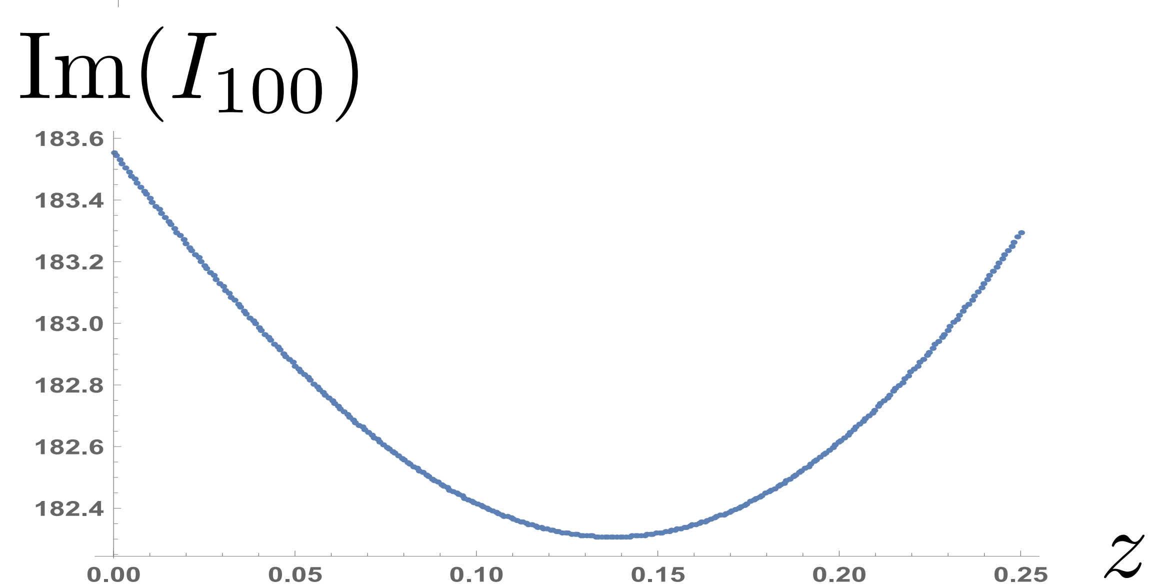
$$x = y = 0$$

Integration

$$x = -y = \frac{z}{z^2+1}$$







Infrared singularities factorize

$$\mathcal{A} = \mathcal{Z} \cdot \mathcal{A}^f$$

Captures all IR singularities (poles in ε)
Matrix in color space

Finite

- We define an infrared finite hard function

$$\mathcal{H} = \lim_{\varepsilon \longrightarrow 0} \mathcal{A}^f$$

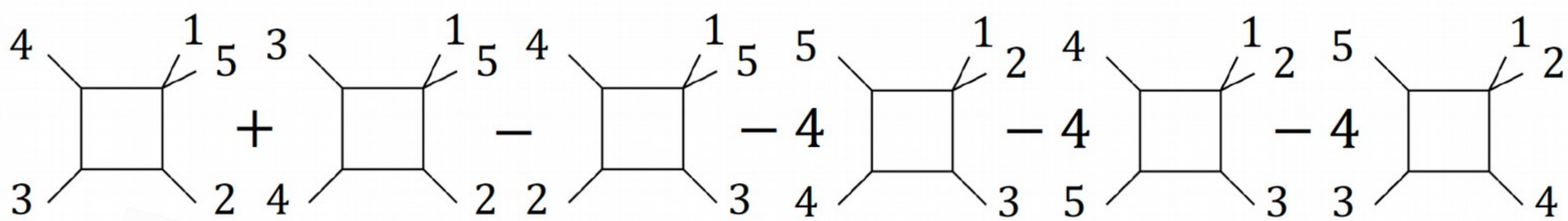
- Truly new piece of information
- Much simpler than finite part of amplitude

Simple final result for two-loop hard function

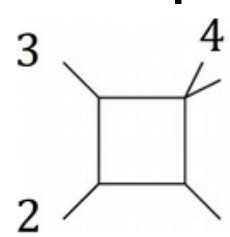
Permutations of the external legs — Color SU(N_c) — Gluon spin dimension — Spinor-helicity variables

$$\mathcal{H}^{(2)}_{\substack{\text{double}\\ \text{trace}}} = \sum_{S_5/\Sigma} \mathrm{Tr}(12)[\mathrm{Tr}(345) - \mathrm{Tr}(543)] \sum_{\Sigma} \Bigg\{ 6\,\kappa^2 \left[\frac{\langle 24\rangle[14][23]}{\langle 12\rangle\langle 23\rangle\langle 45\rangle^2} + 9\,\frac{\langle 24\rangle[12][23]}{\langle 12\rangle\langle 34\rangle\langle 45\rangle^2} \right]$$

$$+ \kappa\,\frac{[15]^2}{\langle 23\rangle\langle 34\rangle\langle 42\rangle} \Big[\quad + \quad - \quad - 4 \quad - 4 \quad - 4 \quad \Big] \Bigg\}$$



Finite part of one-mass box function:

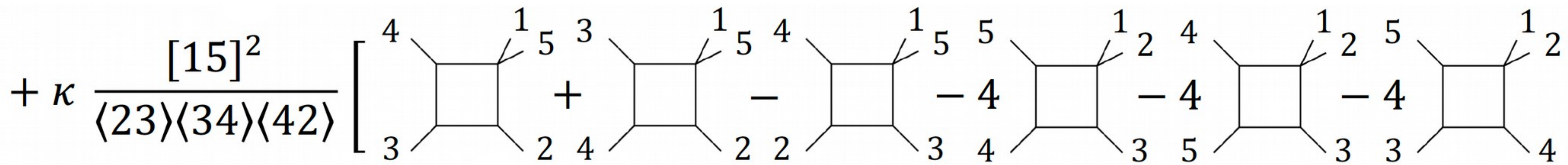


$$= \mathrm{Li}_2\left(1 - \frac{s_{12}}{s_{45}}\right) + \mathrm{Li}_2\left(1 - \frac{s_{23}}{s_{45}}\right) + \log^2\left(\frac{s_{12}}{s_{23}}\right) + \frac{\pi^2}{6}$$

Gluon spin dimension: $\kappa = \dfrac{g_\mu{}^\mu - 2}{6}$

- Formula is valid in all physical regions $s_{ij} \to s_{ij} + i0$
- Correct factorisation in collinear limit

Hints of conformal symmetry in highest transcendental weight part



- Coefficients of box functions are conformally invariant!

$$k_{\alpha\dot{\alpha}} \frac{[45]^2}{\langle 12\rangle\langle 23\rangle\langle 31\rangle} = 0$$

Generator of conformal boosts $k_{\alpha\dot{\alpha}} = \sum_{i=1}^{5} \frac{\partial^2}{\partial\lambda_i^{\alpha}\, \partial\tilde{\lambda}_i^{\dot{\alpha}}}$

[Witten '03]

- Box functions satisfy anomalous conformal Ward identities [Chicherin, Sokatchev '17; Chicherin, Henn, Sokatchev '18]

New, manifestly conformal form of one-loop amplitude

$$A_1^{(1,0)} = \frac{\kappa}{2}\,\frac{s_{12}s_{23} + s_{23}s_{34} + s_{34}s_{45} + s_{45}s_{51} + s_{51}s_{12} + \mathrm{tr}(\gamma_5\, p_4 p_5 p_1 p_2)}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle} =$$

$$= \frac{\kappa}{5}\sum_{\text{cyclic}}\left[\frac{[24]^2}{\langle 13\rangle\langle 35\rangle\langle 51\rangle} + 2\,\frac{[23]^2}{\langle 14\rangle\langle 45\rangle\langle 51\rangle}\right]$$

One-loop formula is conformally invariant:

$$k_{\alpha\dot{\alpha}} = \sum_{I=1}^{5}\frac{\partial^2}{\partial\lambda_i^{\alpha}\partial\tilde{\lambda}_i^{\dot{\alpha}}} \qquad\qquad k_{\alpha\dot{\alpha}}\,A_1^{1,0} = 0$$

Summary

- Very first full five-gluon two-amplitude

Including non-planar part and at function level

- Result fits on only two lines,

and has intriguing conformal symmetry properties!

- All master integrals for generic five-particle QCD

amplitudes are known in the physical region

Full analytical and numerical control

MIAPP program on Scattering Amplitudes
July 13 - August 7, 2020

www.munich-iapp.de

Program on LHC physics: August 10 - Sept 4, 2020