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Gravitational scattering/radiation in/from transplanckian-energy collisions

Gabriele Veneziano

CERN

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Introduction

For more than 30 years a small (and crazy?) group of theorists has taken Trans-Planckian-Energy (TPE) collisions of strings (and later of branes) as the thought experiment of choice for addressing some fundamental issues about the merging of General Relativity and Quantum Mechanics.

The original aim was (and to a large extent still is) to understand, within a consistent theory of quantum gravity, whether and how information is preserved in a process that leads, classically, to black hole formation and, semi-classically, to an apparent loss of information via Hawking's evaporation process.

A point to be emphasized -particularly at this conference- is that we did not start from assuming a GR metric and by then quantizing fields (or strings) around it.

We just computed (QFT or QST) amplitudes in flat space-time and tried to see phenomena usually attributed to a classical geometry emerge while keeping full control over the quantum calculation and its unitary nature.

In the back of our (or at least my) mind was the belief that the apparent loss of quantum information by black hole creation and evaporation could be due to the artificial/unphysical separation of the system into a classical geometry and quantum matter.

What if one sticked to the quantum all the way through?

The game started in 1987 with parallel work on TPE string-string collisions by:

Amati, Ciafaloni & GV (ACV) and Muzinich & Soldate

in the large-b eikonal regime and by

Gross, Mende (& later Ooguri)

in the fixed-angle regime (the two have a small overlap where they can be successfully compared)

There was parallel related work by 't Hooft in the QFT limit which does make appeal from the start to a classical background metric.

Much later (> 2010) the HE scattering of a closed string off a stack of D- branes was also considered:

D'Appollonio, Di Vecchia, R. Russo & GV +...

In the first part of this talk I will give a quick review of the main results and challenges obtained along that ambitious programme.

As we shall see, in spite of much progress, we were not able to fully answer the original question, the one about quantum coherence/unitarity in the collapse regime.

Then, around 2014, some of us turned to the calculation of gravitational radiation from TPE collisions.

This more recent activity will be the subject of the second part of this talk.

I. Results & challenges on the scattering problem: a short summary

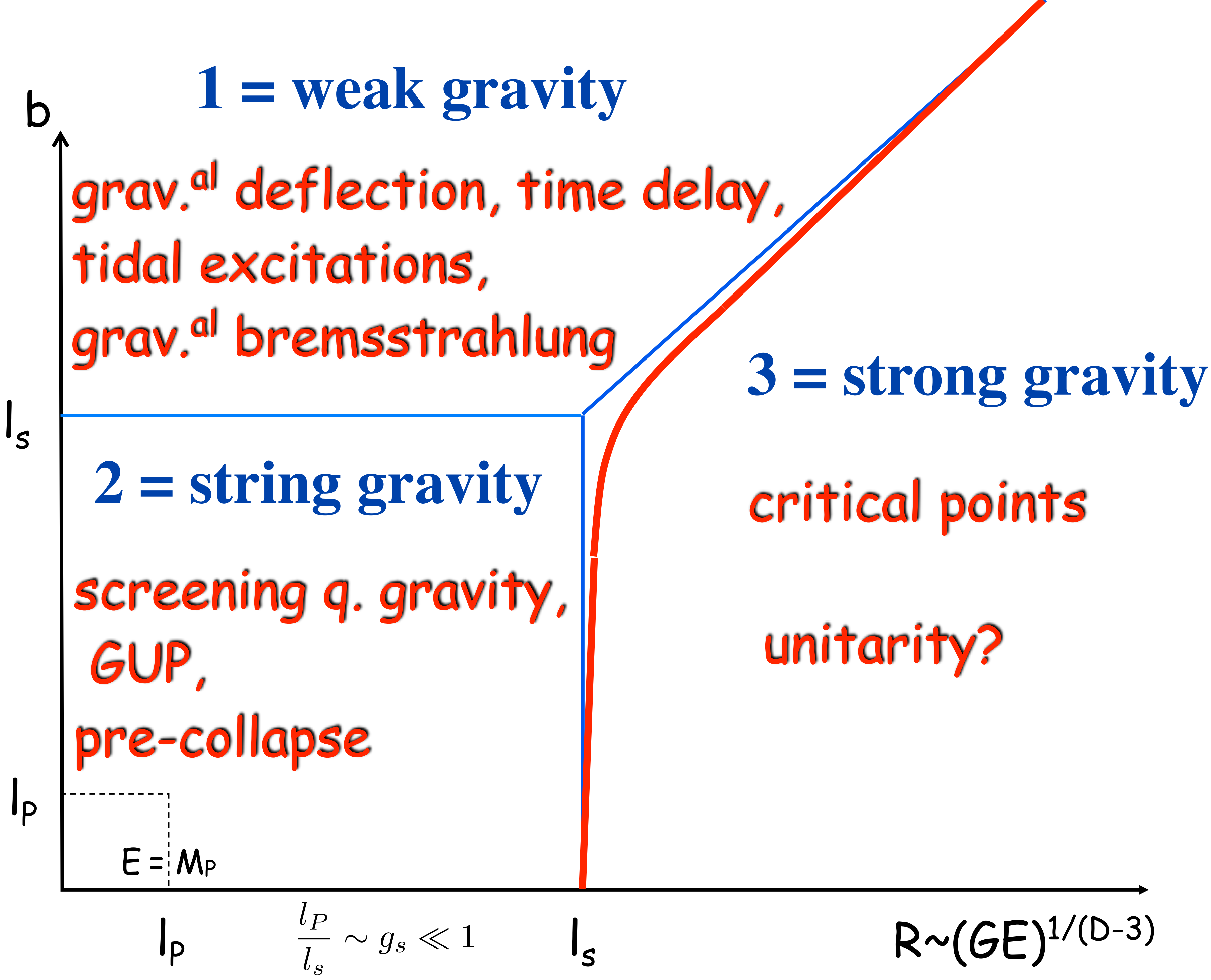
(for a longer summary see my slides at the focus week of this year's GGI workshop: "string theory from a world-sheet perspective")

Ia. String-String collisions

Parameter-space for string-string collisions @ s >> M_P^2

$$b \sim \frac{2J}{\sqrt{s}} \quad ; \quad R_D \sim \left(G\sqrt{s}\right)^{\frac{1}{D-3}} \quad ; \quad l_s \sim \sqrt{\alpha' \hbar} \quad ; \quad G\hbar = l_P^{D-2} \sim g_s^2 l_s^{D-2}$$

- 3 relevant length scales (neglecting l_P @ g_s << 1)
- Playing w/s and g_s we can make R_D/l_s arbitrary
- Several regimes emerge. Roughly just three:


1 = weak gravity
b
grav.al deflection, time delay,
tidal excitations,
grav.al bremsstrahlung
l_s
3 = strong gravity
2 = string gravity
critical points
screening q. gravity,
GUP,
unitarity?
pre-collapse
l_P
E = M_P
l_P
$\frac{l_P}{l_s} \sim g_s \ll 1$
l_s
$R \sim (GE)^{1/(D-3)}$

Results in the weak gravity regime

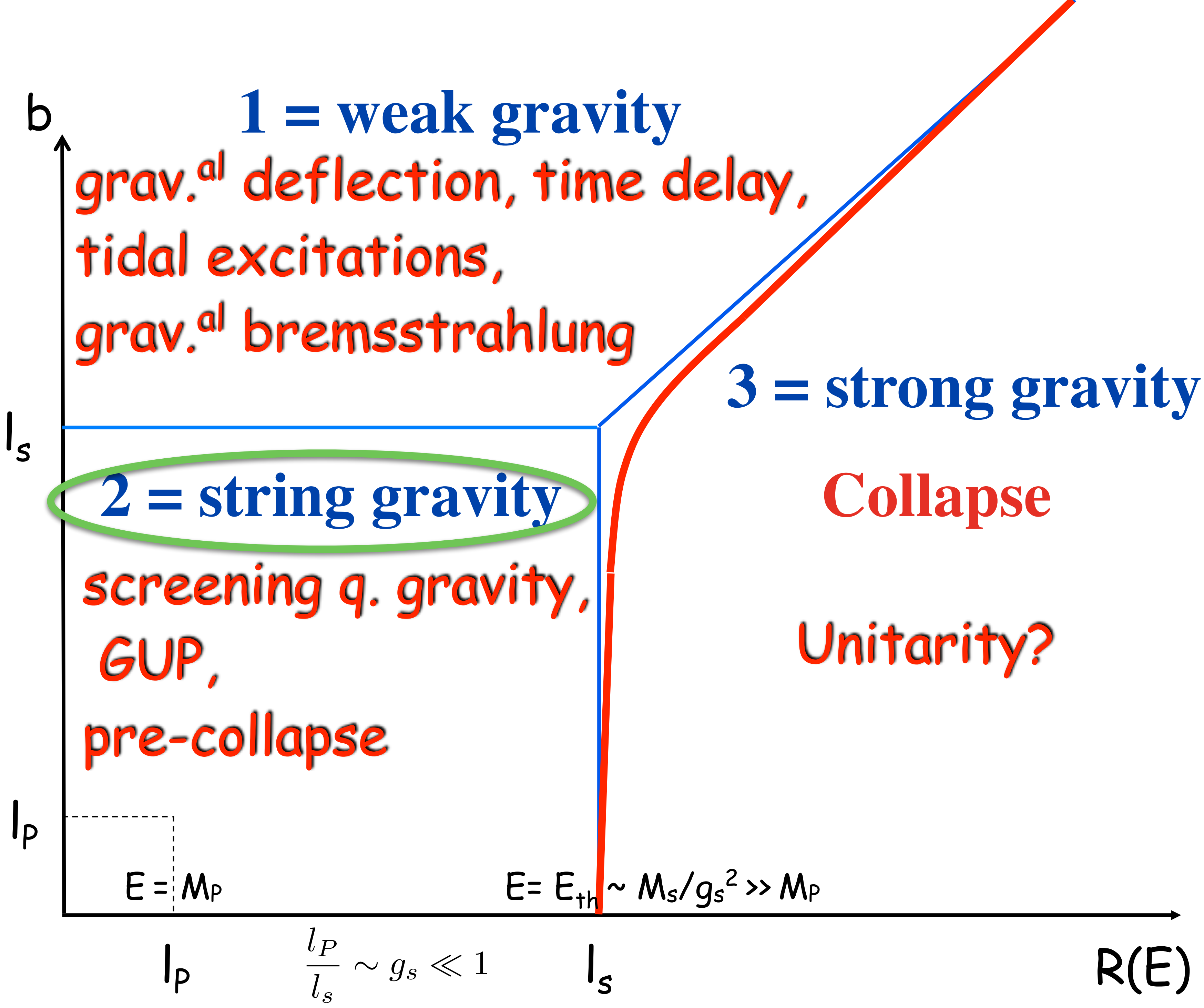
- Restoring elastic unitarity via eikonal resummation (trees violate p.w.u.)

- Gravitational deflection & time delay:an emerging Aichelburg-Sexl (AS) metric

- t-channel "fractionation" and hard scattering (large Q) from large-distance (b) physics

- Tidal excitation of colliding strings, inelastic unitarity, comparison with string in AS metric (not yet done beyond leading term in R/b = challenge # 1)

- Gravitational bremsstrahlung (see Part II)



b
1 = weak gravity
grav.al deflection, time delay,
tidal excitations,
grav.al bremsstrahlung
l_s
2 = string gravity
screening q. gravity,
GUP,
pre-collapse
3 = strong gravity
Collapse
Unitarity?
l_P
E = M_P
E= E_{th} ~ M_s/g_s^2 >> M_P
l_P
$\frac{l_P}{l_s} \sim g_s \ll 1$
l_s
R(E)

Results in the string gravity regime

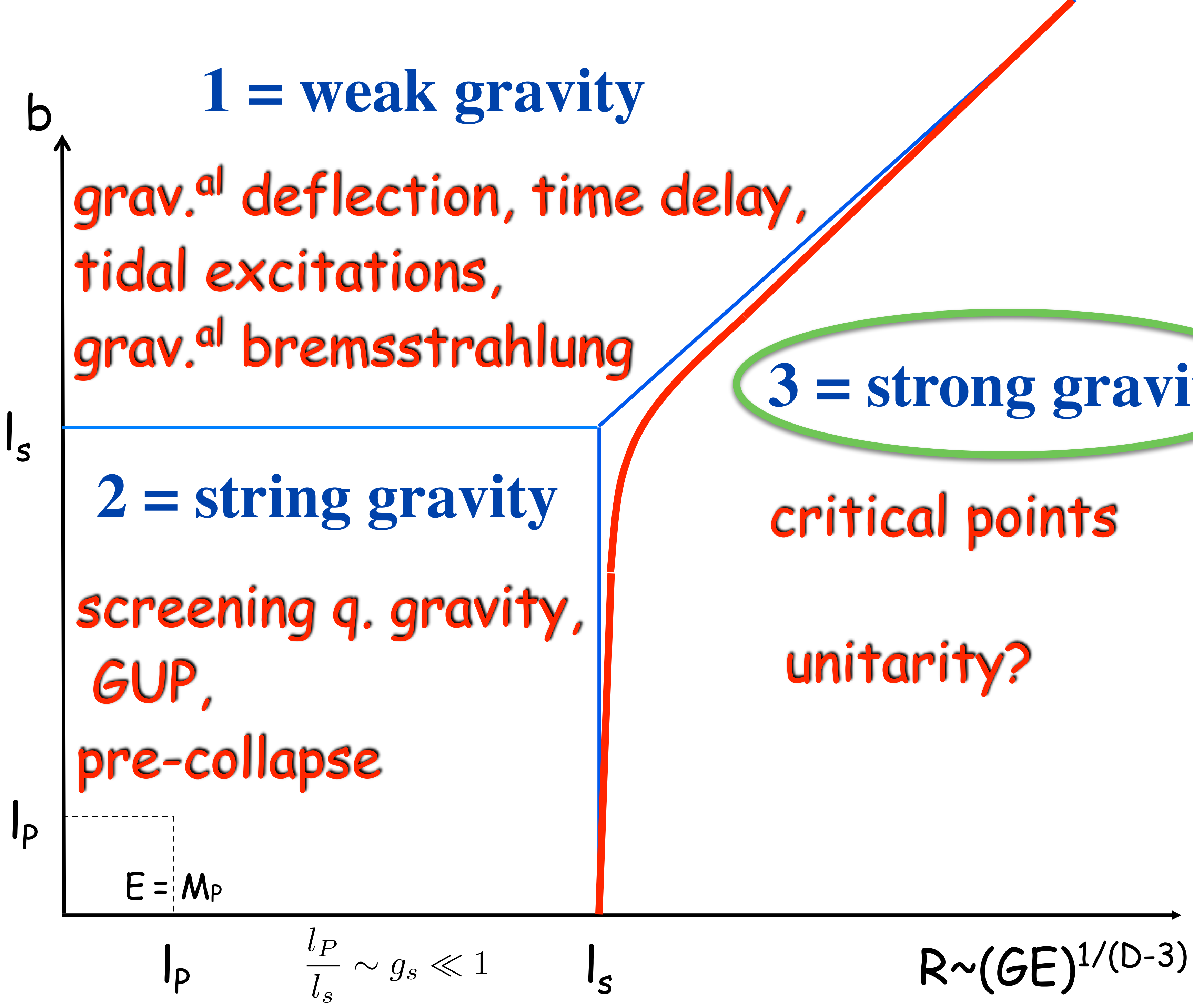
- String softening of quantum gravity @ small b: solving a causality problem (Edelstein et al)

- Maximal classical deflection and comparison/ agreement w/ Gross-Mende-Ooguri

- Generalized Uncertainty Principle

$$\Delta x \geq \frac{\hbar}{\Delta p} + \alpha' \Delta p \geq l_s$$

- s-channel "fractionation" and precocious black-hole-like behavior ($\langle E_{final} \rangle \sim M_P^2 / \langle E_{initial} \rangle$)


1 = weak gravity
b
grav.al deflection, time delay,
tidal excitations,
grav.al bremsstrahlung
3 = strong gravity
ls
2 = string gravity
critical points
screening q. gravity,
GUP,
unitarity?
pre-collapse
lP
E = MP
lP
$\frac{l_P}{l_s} \sim g_s \ll 1$
ls
R~(GE)1/(D-3)

Results in the strong gravity regime

(D=4, point-particle limit. D > 4 easier?)

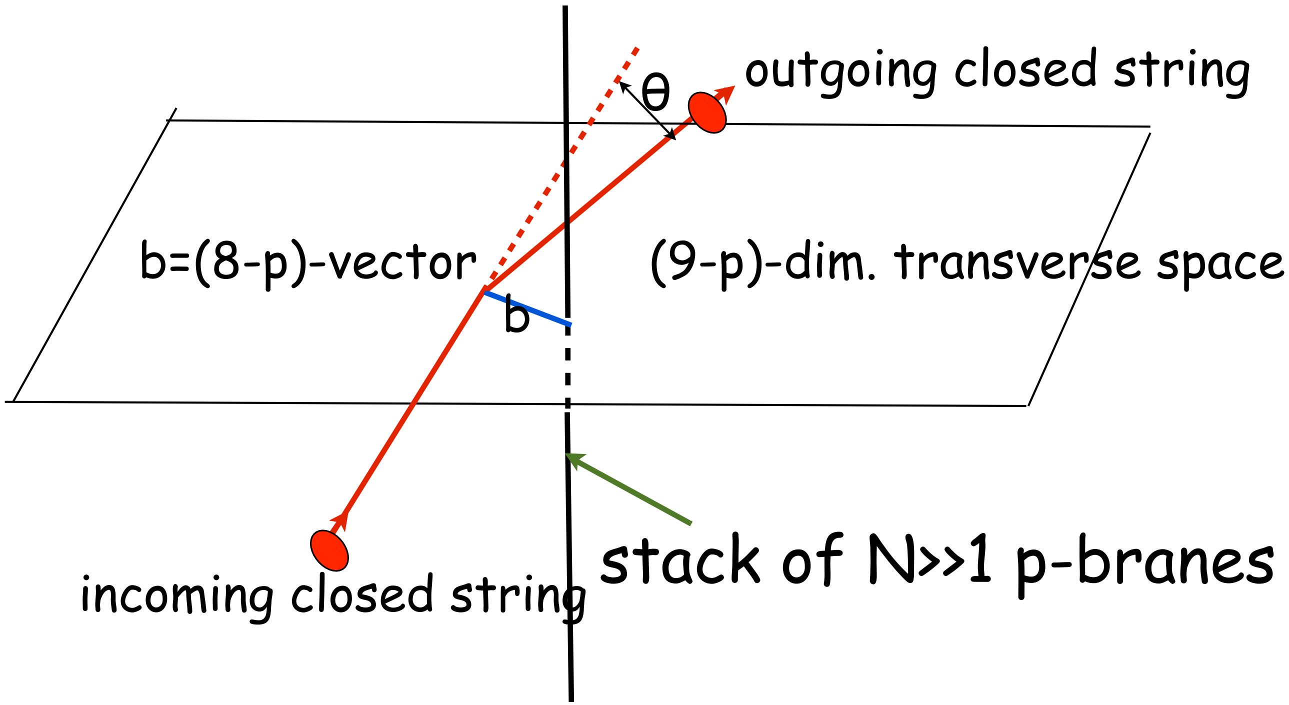
- Identifying (semi) classical contributions as effective trees. No classical correction to deflection at $O(R^2/b^2)$; correction estimated (correctly?) at $O(R^3/b^3)$. Under new scrutiny..
- An effective 2D field theory to resum trees
- Emergence of critical parameters (for real-regular solutions) in good agreement with collapse criteria based on constructing a CTS.
- Unitarity beyond cr. surf? Apparently complex-regular solns. dont work... Challenge # 2!

Ib: String-brane collisions

Another basic process in which a pure initial state evolves into a complicated (yet presumably still pure) state.

An easier problem since the string acts as a probe of a geometry determined by the heavy brane system.

Once more: we are not assuming a metric: calculations performed in flat spacetime (D-branes introduced via boundary-state formalism)


outgoing closed string
θ
b=(8-p)-vector
(9-p)-dim. transverse space
b
stack of N>>1 p-branes
incoming closed string

Results

- Deflection angle, time delay, agreement with curved space-time calculations even at subleading order.

- Unitarity preserving tidal excitation, description of the tidally-excited microstates

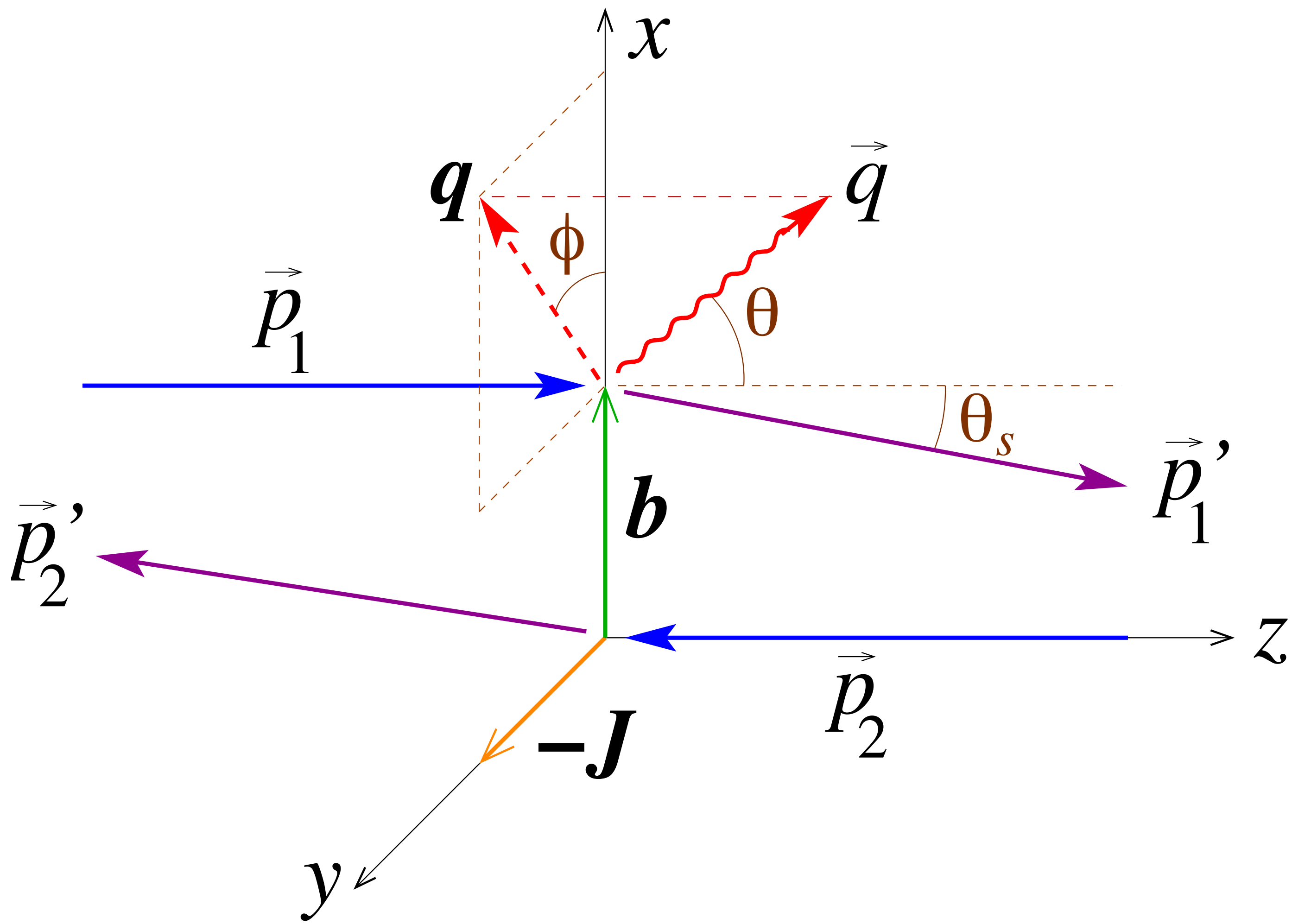
- Short-distance corrections & resolution of potential causality problems

- Absorption via closed-open transition, microscopic description of single open D-string.

- Dissipation into many open strings, thermalization? Unitarity? (Challenge # 3)

II: Gravitational radiation from ultra-relativistic collisions

The process at hand



Three methods

1. A classical GR approach
   (A. Gruzinov & GV, 1409.4555)
2. A quantum eikonal approach
   (CC&Coradeschi & GV, 1512.00281, Ciafaloni, Colferai & GV, 1812.08137)
3. A soft-theorem approach
   (Laddha & Sen, 1804.09193; Sahoo & Sen 1808.03288, Addazi, Bianchi & GV, 1901.10986)

Comments:

a. #2 goes over to #1 in the classical limit
b. They agree with #3 in the overlap of their respective domains of validity

Domains of validity

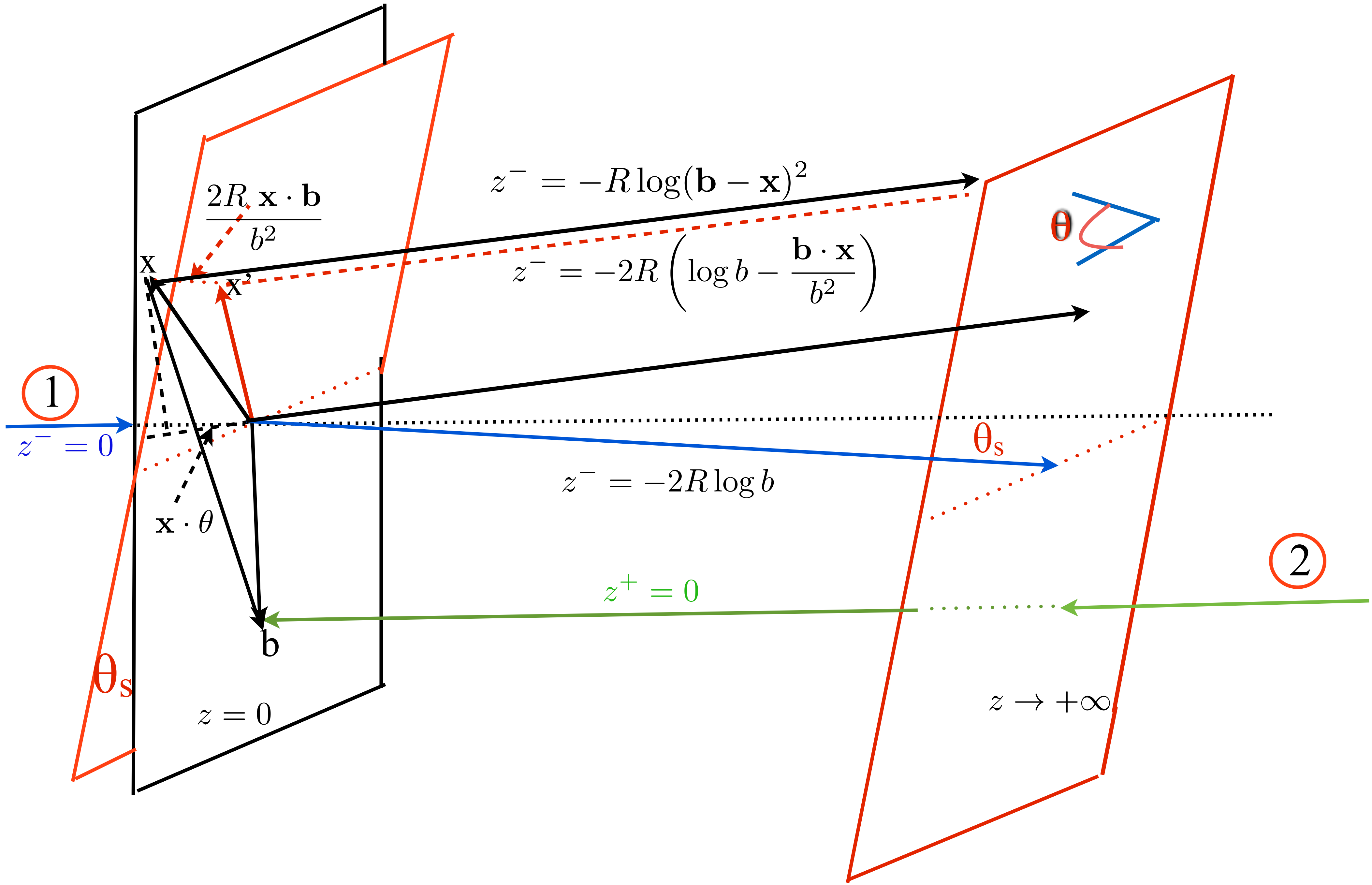
- The CGR and quantum eikonal approaches are limited to small-angle scattering but cover a wide range of GW frequencies.
- The soft-theorem approach is not limited to small deflection angles but is only valid in a much smaller frequency region.

A classical GR approach

Based on Huygens superposition principle.

For gravity this includes in an essential way the gravitational time delay in AS's shock-wave metric.

In pictures



$\frac{2R\,\mathbf{x}\cdot\mathbf{b}}{b^2}$
x
x'
$z^- = -R\log(\mathbf{b}-\mathbf{x})^2$
$z^- = -2R\left(\log b - \frac{\mathbf{b}\cdot\mathbf{x}}{b^2}\right)$
θ
1
$z^- = 0$
θ_s
$z^- = -2R\log b$
$\mathbf{x}\cdot\theta$
2
$z^+ = 0$
b
θ_s
$z = 0$
$z \to +\infty$

A quantum eikonal approach

Emission from external and internal legs throughout the whole ladder (with its suitable phase) has to be taken into account for not so soft gravitons.

One should also take into account the (finite) difference between the (infinite) Coulomb phase of the final 3-particle state and that of an elastic 2-particle state.

When this is done, the classical result of G+V is exactly recovered for $h\omega/E \to 0$!

The classical result/limit

Frequency + angular spectrum ($s = 4E^2$, $R = 4GE$)

$$\frac{dE^{GW}}{d\omega\ d^2\tilde{\theta}} = \frac{GE^2}{\pi^4}|c|^2 \ ;\ \tilde{\boldsymbol{\theta}} = \boldsymbol{\theta} - \boldsymbol{\theta}_s \ ;\ \boldsymbol{\theta}_s = 2R\frac{\boldsymbol{b}}{b^2}$$

$$c(\omega, \tilde{\boldsymbol{\theta}}) = \int \frac{d^2x\ \zeta^2}{|\zeta|^4}\ e^{-i\omega \mathbf{x}\cdot\tilde{\boldsymbol{\theta}}} \left[e^{-2iR\omega\Phi(\mathbf{x})} - 1\right]$$

$$\zeta = x + iy \qquad \Phi(\mathbf{x}) = \frac{1}{2}\ln\frac{(\mathbf{x}-\mathbf{b})^2}{b^2} + \frac{\mathbf{b}\cdot\mathbf{x}}{b^2}$$

$$c(\omega, \boldsymbol{\theta}) = \int \frac{d^2x\ \zeta^2}{|\zeta|^4} e^{-i\omega\mathbf{x}\cdot\boldsymbol{\theta}} \left[e^{-iR\omega\ln\frac{(\mathbf{x}-\mathbf{b})^2}{b^2}} - e^{+2iR\omega\frac{\mathbf{b}\cdot\mathbf{x}}{b^2}}\right]$$

Re ζ^2 and Im ζ^2 correspond to usual (+,x) GW polarizations, ζ^2, ζ^{*2} to the two circular ones (not each other's cc!).

Subtracting the deflected shock wave is crucial!

Analytic results: a Hawking knee* & an unexpected bump

*yet another precocious BH behavior?

For $b^{-1} < \omega < R^{-1}$ the GW-spectrum is almost flat in ω

$$\frac{dE^{GW}}{d\omega} \sim \frac{4G}{\pi} \theta_s^2 E^2 \log(\omega R)^{-2}$$

Below $\omega = b^{-1}$ it "freezes" reproducing the zero-f-limit

$$\frac{dE^{GW}}{d\omega} \to \frac{4G}{\pi} \theta_s^2 E^2 \log(\theta_s^{-2})$$

Above $\omega = R^{-1}$ drops, takes a "scale-invariant" form:

Hawking knee!

$$\frac{dE^{GW}}{d\omega} \sim \theta_s^2 \frac{E}{\omega}$$

This gives a log ω^* in the "efficiency" for a cutoff at ω^*

At $\omega \sim R^{-1}\theta_s^{-2}$ the above spectrum becomes $O(Gs\,\theta_s^4)$ i.e. of the same order as terms we neglected.
Also, if continued above $R^{-1}\theta_s^{-2}$, the so-called "Dyson bound" ($dE/dt < 1/G$) would be violated. Using $\omega^* \sim R^{-1}\theta_s^{-2}$ we find (to leading-log accuracy):

$$\frac{E^{GW}}{\sqrt{s}} = \frac{1}{2\pi}\,\theta_s^2\,\log(\theta_s^{-2})$$

For $\omega > \omega^*$ G+V argued for a $G^{-1}\omega^{-2}$ spectrum which (extrapolated to $\theta_s \sim 1$) turns out to be that of a time-integrated BH evaporation!

Challenge #4: ω^* & spectrum above

The fine spectrum below 1/b

$$\frac{dE^{GW}}{d\omega} \to \frac{4G}{\pi} \theta_s^2 E^2 \log(\theta_s^{-2}) \qquad (\omega b \ll 1)$$

$$\frac{dE^{GW}}{d\omega} \sim \frac{4G}{\pi} \theta_s^2 E^2 \log(\omega R)^{-2} \quad (\omega b \gg 1 \gg \omega R)$$

suggest naive (monotonic) interpolation around

ωb ~ 1, e.g.

$$\frac{dE^{GW}}{d\omega} \sim \frac{4G}{\pi} \theta_s^2 E^2 \log\left(\frac{b^2}{R^2(1+\omega^2 b^2)}\right) \sim \frac{4G}{\pi} \theta_s^2 E^2 \left[\log\left(\frac{b^2}{R^2}\right) - O(\omega^2 b^2)\right]$$

This appears not to be the case...

A careful study of the region $\omega R \ll 1$, but with ωb generic, shows that:

- At $\omega b < (\ll) 1$ there are corrections of order $(\omega b)\log(\omega b)$, $(\omega b)^2\log^2(\omega b)$ (higher logs suppressed).

- First noticed by Sen et al. in the context of soft thrms in D=4. Here they come from the mismatch between the two- and three-body Coulomb phase.

- These logarithmically enhanced sub and sub-sub leading corrections disappear at $\omega b > 1$ so that the previously found $\log(1/\omega R)$ behavior (for $\omega b > 1 > \omega R$), as well as the Hawking knee, remain valid.

- The ωb (both w/ and w/out $\log(\omega b)$) correction only appears for circularly polarized (definite helicity) GWs but disappear either for the (more standard) + and x polarizations, or after summing over them*), or, finally, after integration over the azimuthal angle.

- The $(\omega b)\log(\omega b)$ terms are in complete agreement with what had been previously found by A. Sen and collaborators using soft-graviton theorems to sub-leading order (see below).

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*)Indeed we found: $A(\lambda=-2,\omega) = A^*(\lambda=+2,-\omega)$ implying that the $O(\omega)$ correction to unpol.^{ed} flux vanishes.

- The leading $(\omega b)^2 \log^2(\omega b)$ correction to the total flux is positive and produces a bump at $\omega b \sim 0.5$.

- Could not be compared to Sen et al. who only considered $\omega b \log(\omega b)$ corrections.

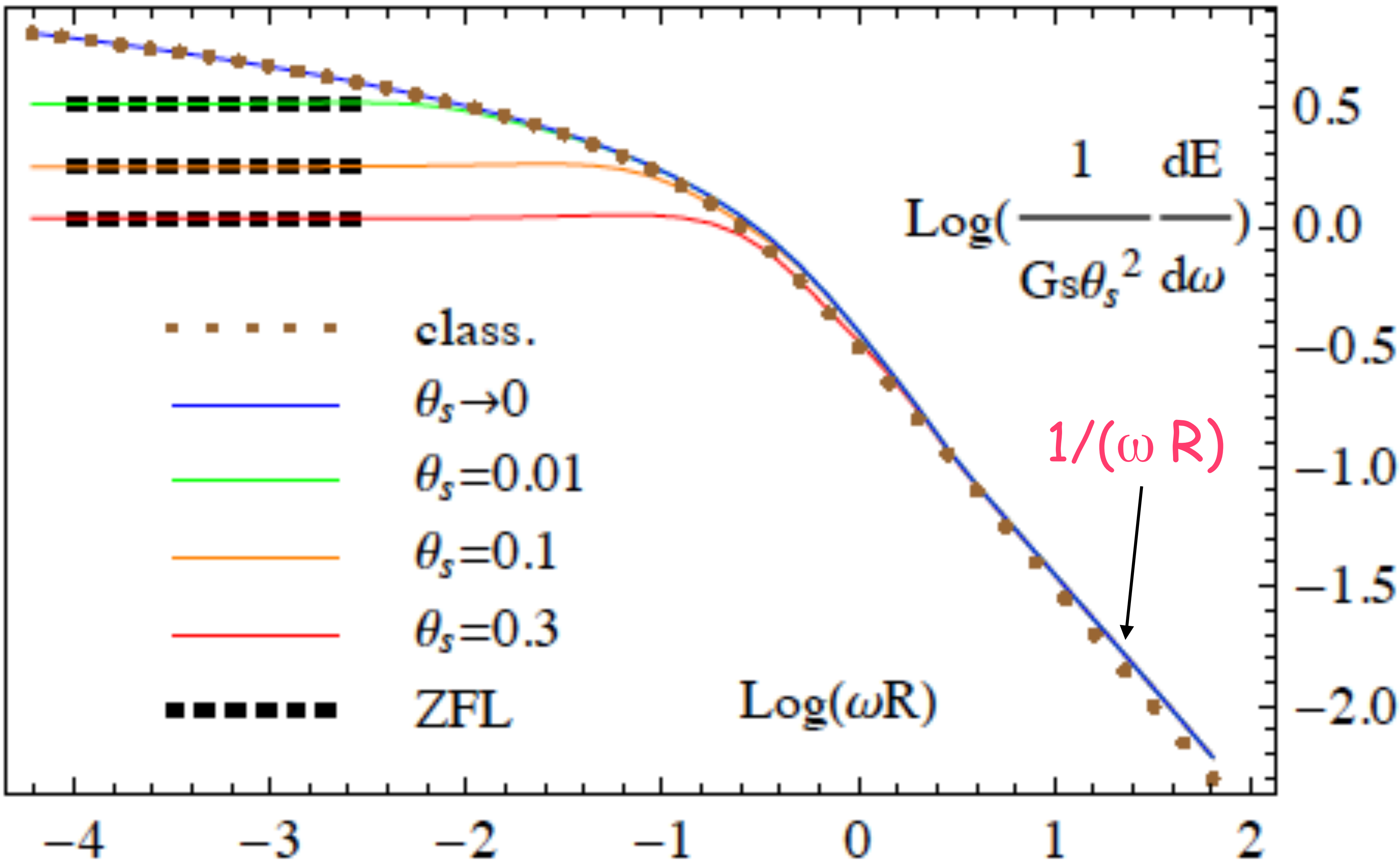
- Now confirmed by Sahoo (private comm. by Sen) but there are still questions about $O(\omega b)$.

- Can be compared successfully with soft-graviton approach if Sen et al.'s recipe is adopted at $O(\omega^2)$, see below.

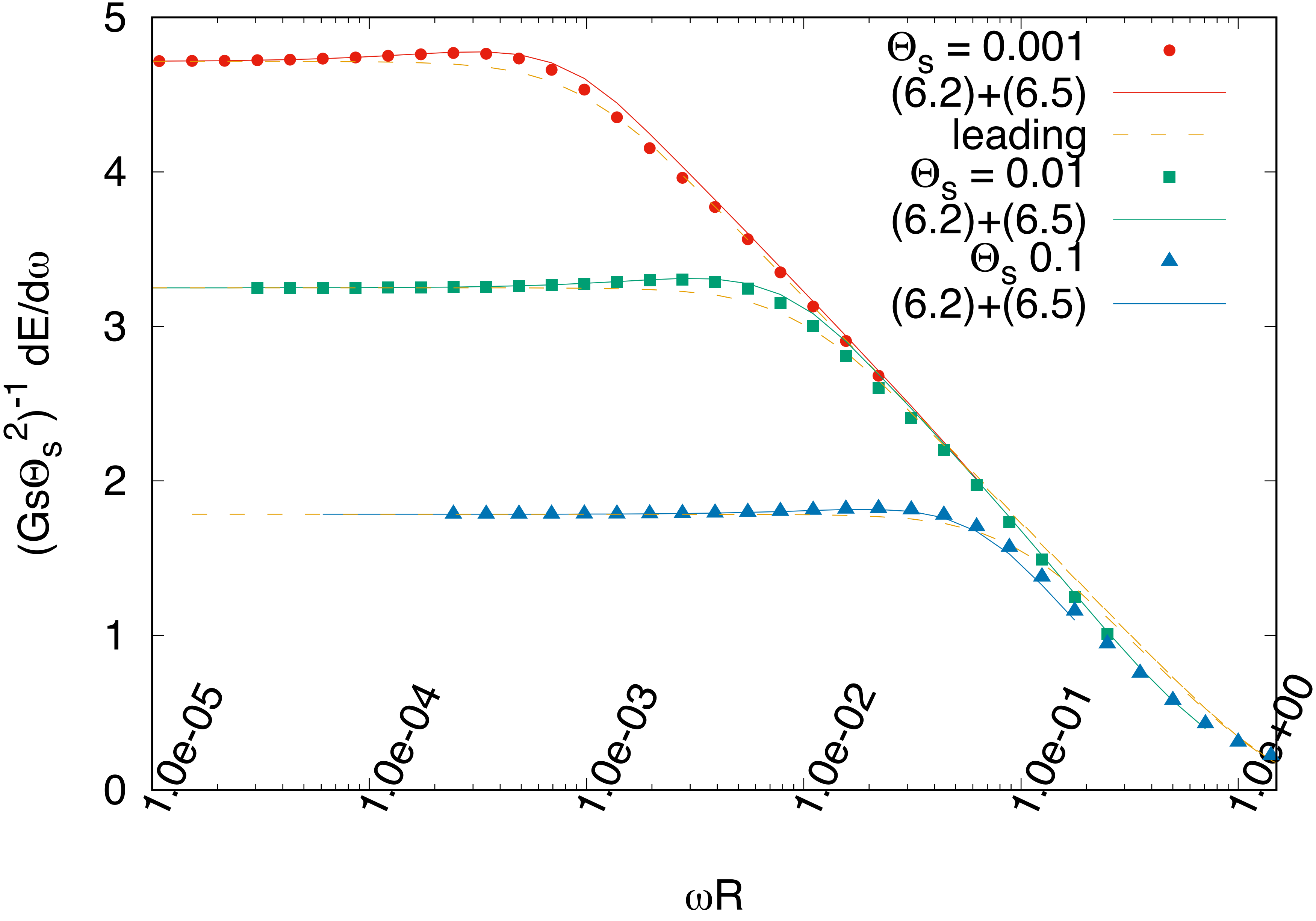
Numerical results
(skip 8 slides?)

Ciafaloni, Colferai, Coradeschi & GV-1512.00281
Ciafaloni, Colferai & GV-1812.08137

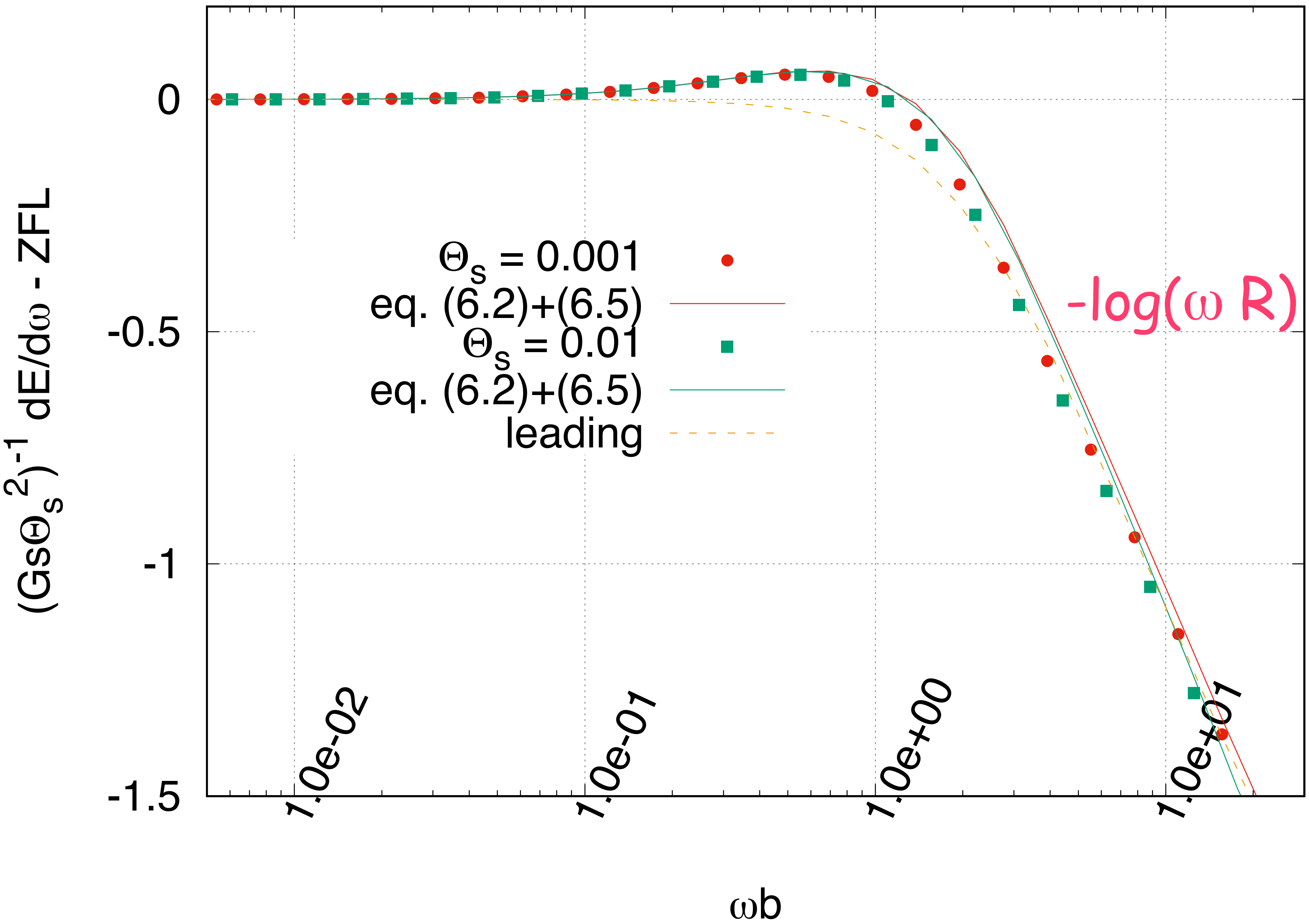
(CCCV 1512.00281)



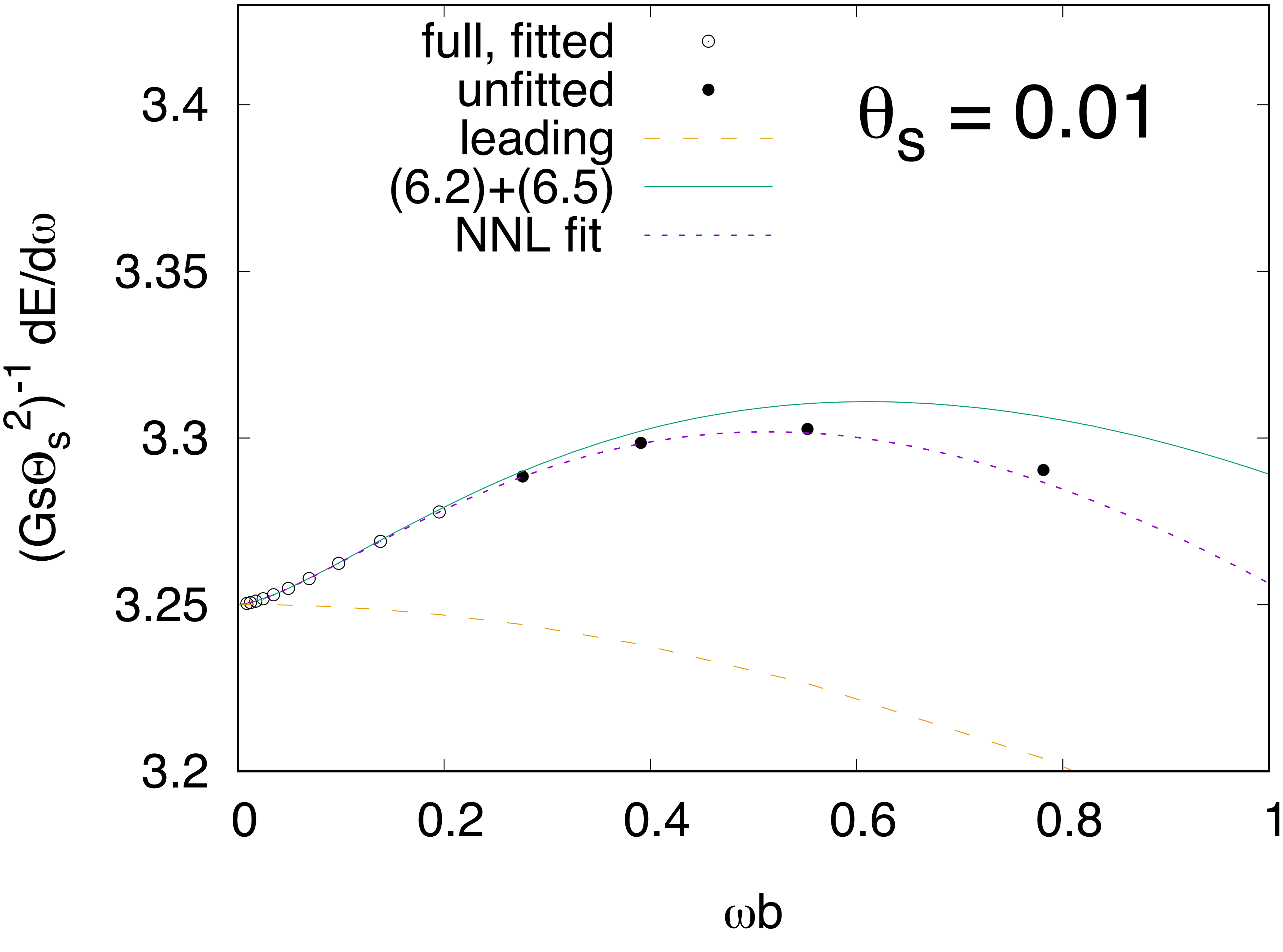
(CCV 1812.08137)



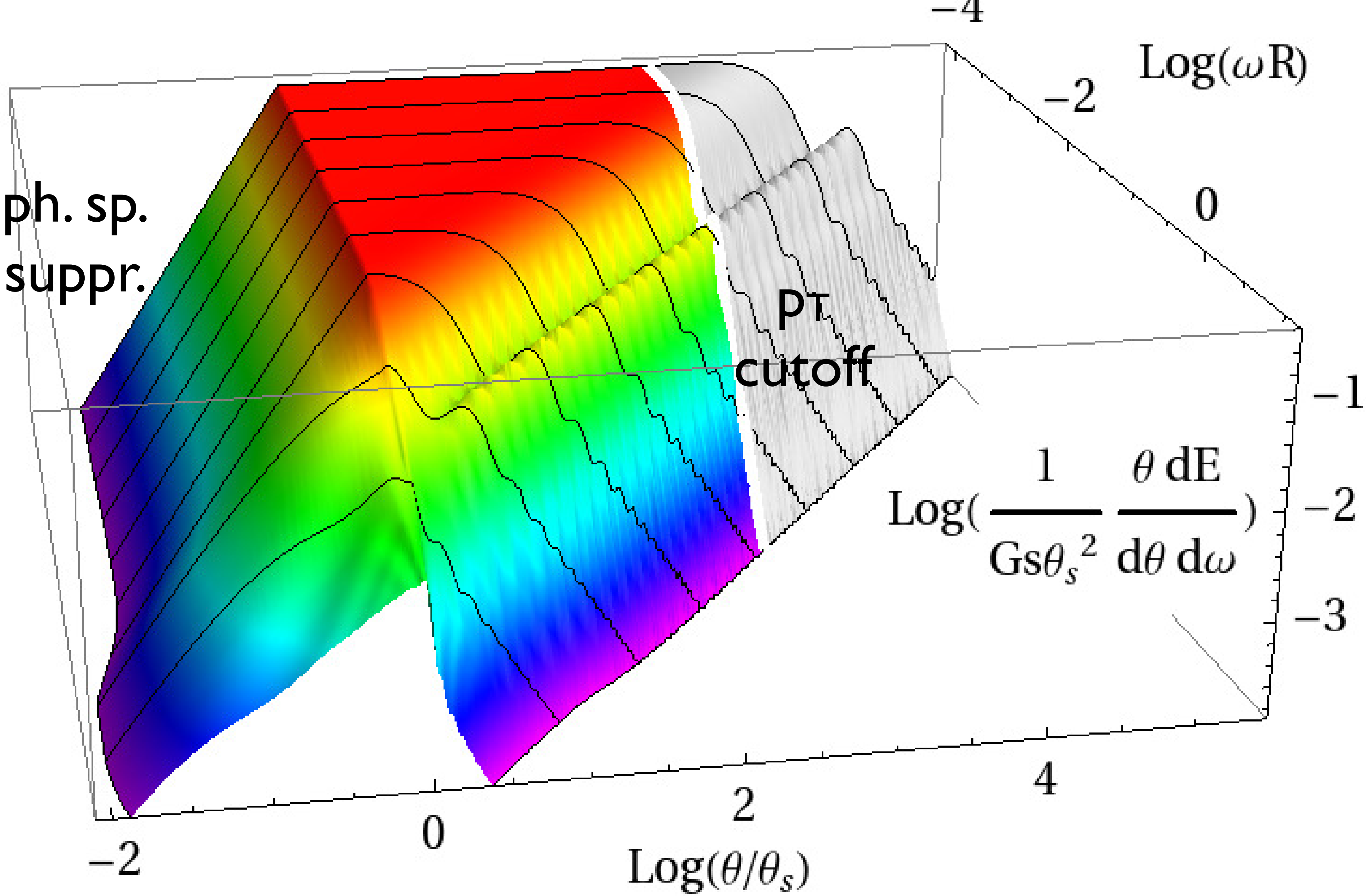
(CCV 1812.08137)



(CCV 1812.08137)



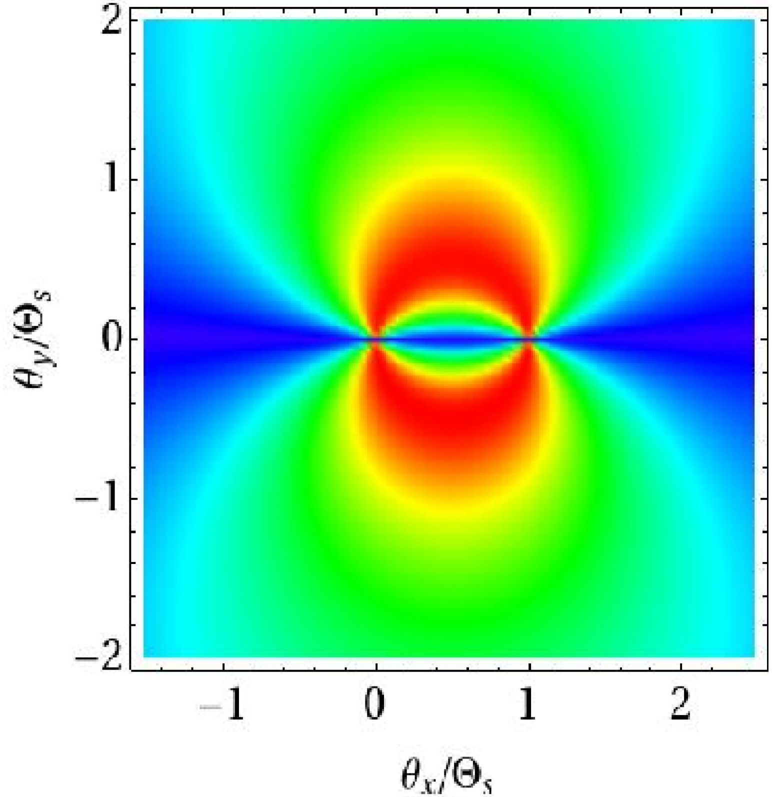
$\theta_s = 10^{-3}$



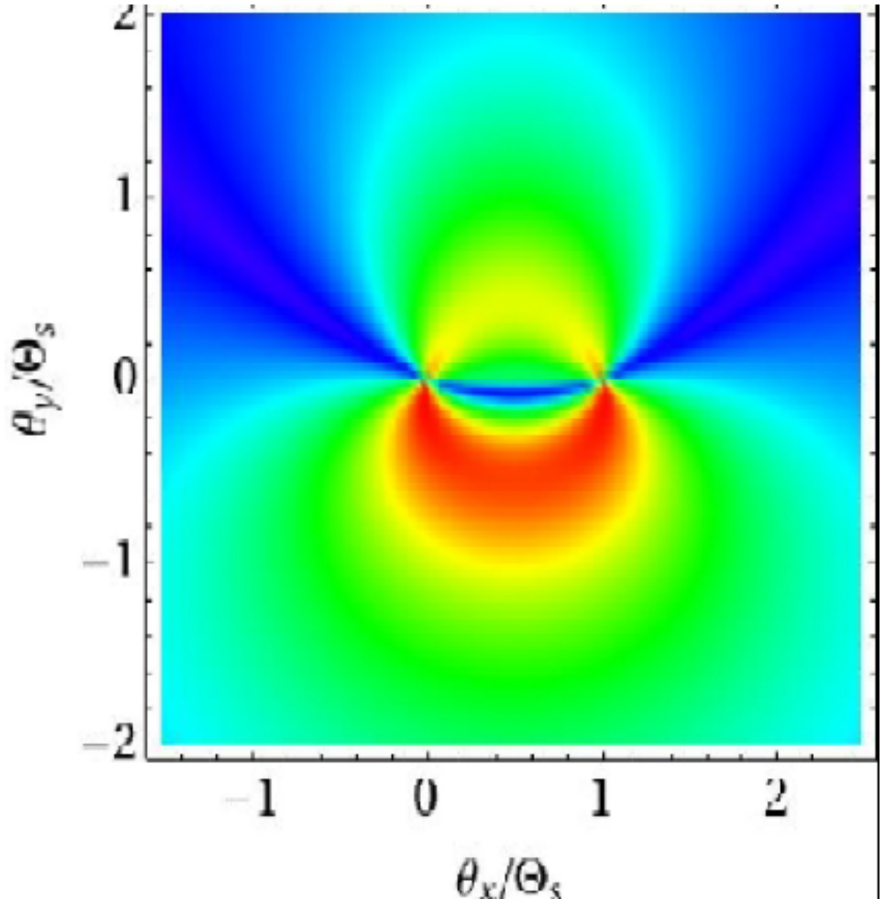
M. Ciafaloni, D. Colferai & GV, 1505.06619

Angular (polar and azimuthal) distribution

ωR = 10^{-3}



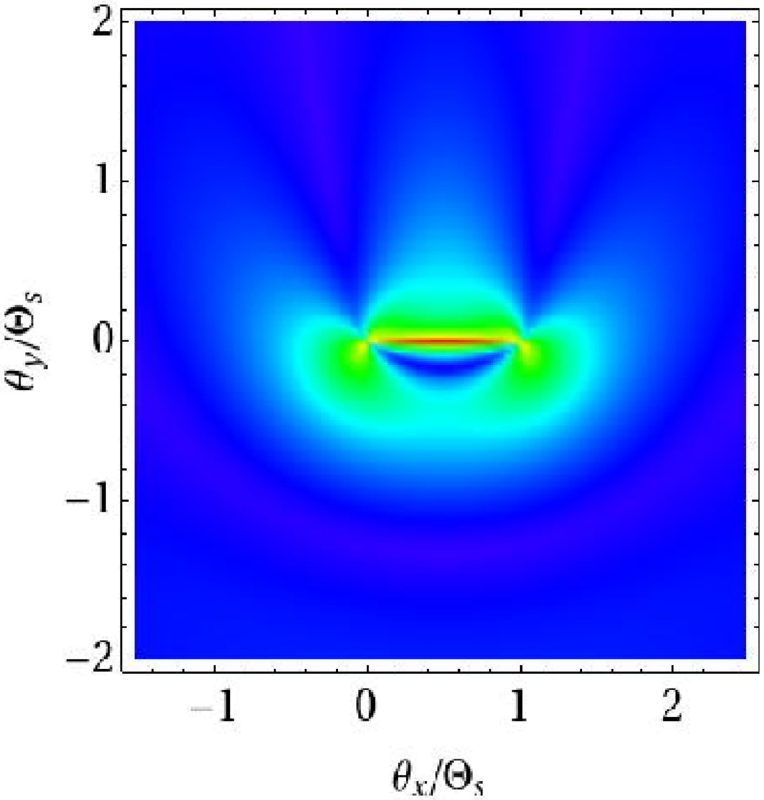
ωR = 0.125



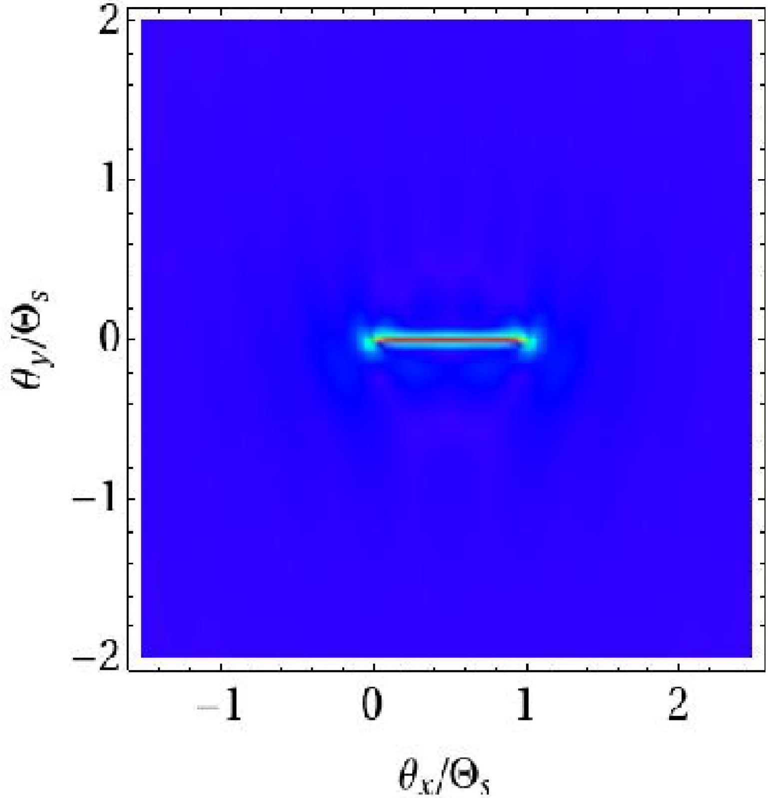
M. Ciafaloni, D. Colferai, F. Coraldeschi & GV, 1512.00281

Angular (polar and azimuthal) distribution

$\omega R = 1.0$



$\omega R = 8.0$



Selected for PRD's picture gallery…

A soft-theorem approach

Beyond the ZFL via soft theorems

(Laddha & Sen, 1804.09193;

Sahoo & Sen, 1808.03288,

Addazi, Bianchi & GV, 1901.10986)

Low-energy (soft) theorems for photons and gravitons (Low, Weinberg, … sixties) had a revival recently (Strominger, Cachazo, Bern, Di Vecchia, Bianchi…). In the case of a soft graviton of momentum q we have (for spinless hard particles)

$$\mathcal{M}_{N+1}(p_i;q) \approx \sum_{i=1}^{N} \left[\frac{p_i h p_i}{q p_i} + \frac{p_i h J_i q}{q p_i} - \frac{q J_i h J_i q}{2 q p_i} \right] \mathcal{M}_N(p_i) \equiv S_0 + S_1 + S_2$$

$$J_i^{\mu\nu} = p_i^{\mu} \partial / \partial p^i_{\nu} - p_i^{\nu} \partial / \partial p^i_{\mu}$$

NB: sub and sub-sub leading terms may need corrections at loop level & from IR sing.s @ D=4.

Recovering the ZFL (m=0 case)

Keeping just the leading term in the x-section:

$$\int \frac{d^3q}{2|q|(2\pi)^3} \sum_{s=\pm 2} \left| \sum_{i=1}^{N} \frac{p_i h_s p_i}{q p_i} \right|^2 |\mathcal{M}_N(p_i)|^2$$

sum over polarizations gives the integrand

$$\sum_{i,j} \frac{p_i^\mu p_i^\nu}{q p_i} \Pi_{\mu\nu,\rho\sigma} \frac{p_j^\rho p_j^\sigma}{q p_j} = \sum_{i,j} \frac{(p_i p_j)^2}{q p_i q p_j} .$$

$$B_0 = \frac{8\pi G}{\hbar} \int \frac{d^3q}{2|q|(2\pi)^3} \sum_{i,j} \frac{(p_i p_j)^2}{q p_i q p_j} = -\frac{2G}{\pi\hbar} \log\frac{\Lambda}{\lambda} \sum_{i,j} (p_i p_j) \log\frac{|p_i p_j|}{\mu^2}$$

$$\frac{dE_0^{GW}}{d\omega} = \hbar\omega \frac{dN_0}{d\omega} = -\frac{2G}{\pi} \sum_{i,j} (p_i p_j) \log\frac{|p_i p_j|}{\mu^2}$$

$$B_0 = \frac{8\pi G}{\hbar} \int \frac{d^3q}{2|q|(2\pi)^3} \sum_{i,j} \frac{(p_i p_j)^2}{q p_i q p_j} = -\frac{2G}{\pi\hbar} \log\frac{\Lambda}{\lambda} \sum_{i,j} (p_i p_j) \log\frac{|p_i p_j|}{\mu^2}$$

$$\frac{dE_0^{GW}}{d\omega} = \hbar\omega \frac{dN_0}{d\omega} = -\frac{2G}{\pi} \sum_{i,j} (p_i p_j) \log\frac{|p_i p_j|}{\mu^2}$$

Result does not depend on μ and is free of mass (collinear) divergences. For 2->2 scattering:

$$\frac{dE^{GW}}{d\omega}(\omega = 0) = \frac{4G}{\pi} \left(s \log s + t \log(-t) + u \log(-u) \right)$$

At small deflection angle (|t| << s):

$$\frac{dE^{GW}}{d\omega} \to \frac{Gs}{\pi} \theta_E^2 \log(4e\theta_E^{-2}) \quad ; \quad \omega \to 0$$

Next-to-Leading ($O(\omega)$) correction

$$B_1 = 8\pi G \int \frac{d^3q}{2|q|(2\pi)^3} \sum_{i,j} \sum_{s=\pm 2} \left[\frac{(p_i h^s p_i)(p_j h^{(-s)} J_j q)}{qp_i qp_j} + (i \leftrightarrow j) \right]$$

summing over polarizations

$$B_1 = 8\pi G \int \frac{d^3q}{2|q|(2\pi)^3} \sum_{i,j} \frac{p_i p_j}{qp_i qp_j} [p_i \overrightarrow{J}_j + p_j \overleftarrow{J}_i] q$$

For given i,j the relevant integral

$$I^\mu_{ij} = \int \frac{d^3q}{2|q|(2\pi)^3} \frac{p_i p_j q^\mu}{qp_i qp_j} = \int \frac{d^4q}{(2\pi)^3} \delta_+(q^2) \frac{p_i p_j q^\mu}{qp_i qp_j} \ ; \ \delta_+(q^2) = \delta(q^2)\Theta(-q_0)$$

has collinear divergences. These are nicely avoided through a little trick (additional terms vanish after sum)

$$I^\mu_{ij} \to \tilde{I}^\mu_{ij} = \int \frac{d^4q}{(2\pi)^3} \delta_+(q^2) \frac{[(p_i p_j) q^\mu - (qp_j) p^\mu_i - (qp_i) p^\mu_j]}{(p_i q)(p_j q)}$$

We also add a $\delta(qP + 2E\omega_0)$ (w/ P the c.o.m. momentum) to fix the c.o.m. $\omega = \omega_0$ in a covariant way. Quantity in sq. brackets orthogonal to p_i, p_j. Then we get

$$\frac{dE_1}{d\omega} = -2\frac{G\sqrt{s}\hbar\omega}{\pi}\sum_{ij}\frac{\log\left[\frac{-s(p_ip_j)}{2(Pp_i)(Pp_j)}\right]}{\tilde{s}_{ij}}\left[(p_ip_j)P - (Pp_j)p_i - (Pp_i)p_j\right]^\mu\left(\frac{\overleftarrow{\partial}}{\partial p_i} + \frac{\overrightarrow{\partial}}{\partial p_j}\right)_\mu$$

$$\tilde{s}_{ij} = -\Pi^2 = s + \frac{2(Pp_i)(Pp_j)}{p_ip_j}$$

(note absence of singularities when latter vanishes)

To be sandwiched (divided) between (by) $S_{if}^{+}S_{fi}$

Vanishing of O(ω) correction for 2->2

$$\frac{dE_1}{d\omega} = -2\frac{G\sqrt{s}\hbar\omega}{\pi}\sum_{ij}\frac{\log\left[\frac{-s(p_ip_j)}{2(Pp_i)(Pp_j)}\right]}{\tilde{s}_{ij}}\left[(p_ip_j)P - (Pp_j)p_i - (Pp_i)p_j\right]^\mu\left(\frac{\overleftarrow{\partial}}{\partial p_i} + \frac{\overrightarrow{\partial}}{\partial p_j}\right)_\mu$$

Terms with i = j do not contribute. Terms with (i, j = 1, 2 and 3,4) vanish because projector = 0. For (i,j = 1,3) the derivatives only contribute when acting on (p_1p_3): this produces a p_1 or p_3 which get killed by the contraction. The result (recall that we summed over pol.^{s}!) agrees with those obtained in the eikonal approach and also with Sen et al. for the log-enhanced term.

The sub-sub leading ($O(\omega^2)$) correction

The calculation is much more involved, but the final result takes a (relatively) simple, elegant form

$$
\begin{aligned}
B_2|\mathcal{S}_{if}|^2 &= \mathcal{S}^\dagger_{if}\frac{G\omega^2}{\pi}(C_1+C_2+C_3)\,\mathcal{S}_{fi} \\
C_1 &= -3\sum_i \overleftarrow{D}_i \sum_j \overrightarrow{D}_j + 4\sum_i(\overleftarrow{D}_i+\overrightarrow{D}_i)^2 \\
C_2 &= \sum_{i\neq j}\frac{P^2}{\tilde{s}_{ij}}\log\frac{P^2 p_i p_j}{2Pp_iPp_j}[p_ip_j(\overleftrightarrow{\partial}_{ij})^2 - 2p_i(\overleftrightarrow{\partial}_{ij})p_j(\overleftrightarrow{\partial}_{ij})] \\
C_3 &= \sum_{i\neq j}\frac{2}{p_ip_j\tilde{s}_{ij}}\left[1+\frac{P^2}{\tilde{s}_{ij}}\log\frac{P^2p_ip_j}{2Pp_iPp_j}\right](p_ip_j)^2\left(Q^\mu_{ij}(\overleftrightarrow{\partial}_{ij})_\mu\right)^2
\end{aligned}
$$

$$D_i = p_i\partial_i \quad \text{(no sum)}$$

$$\overleftrightarrow{\partial}_{ij\,\nu} \equiv \overleftarrow{\partial}_{i\nu} + \overrightarrow{\partial}_{j\nu} \qquad Q^\mu_{ij} \equiv \left(P^\mu - \frac{Pp_j}{p_ip_j}p^\mu_i - \frac{Pp_i}{p_ip_j}p^\mu_j\right)$$

Specializing to a 2->2 process

$$B_2|\mathcal{S}_{if}|^2 = \frac{dE_2^{GW}}{d(\hbar\omega)}|\mathcal{S}_{if}|^2 = 2\frac{G\hbar\omega^2}{\pi} \times$$
$$\mathcal{S}_{if}^\dagger \left\{ \overleftarrow{D}^2 + \overrightarrow{D}^2 + [st + us \log\left(-\frac{u}{s}\right)] \overleftrightarrow{\Delta}_{st}^2 + [su + ts \log\left(-\frac{t}{s}\right)] \overleftrightarrow{\Delta}_{su}^2 \right\} \mathcal{S}_{fi}$$

$$D \equiv s\partial_s + t\partial_t + u\partial_u \ : \ \overleftrightarrow{\Delta}_{st} \equiv \left(\overleftarrow{\partial}_s - \overleftarrow{\partial}_t - \overrightarrow{\partial}_s + \overrightarrow{\partial}_t\right) \ldots$$

The above combinations of derivatives are unambiguous. They act on either A(s,t) or on A'(s,u) or on A''(t,u) yielding the same result for the same physical amplitude.

Example I

A tree-level 2->2 amplitude, e.g. single graviton exchange in a+b->a+b (w/ a ≠ b)

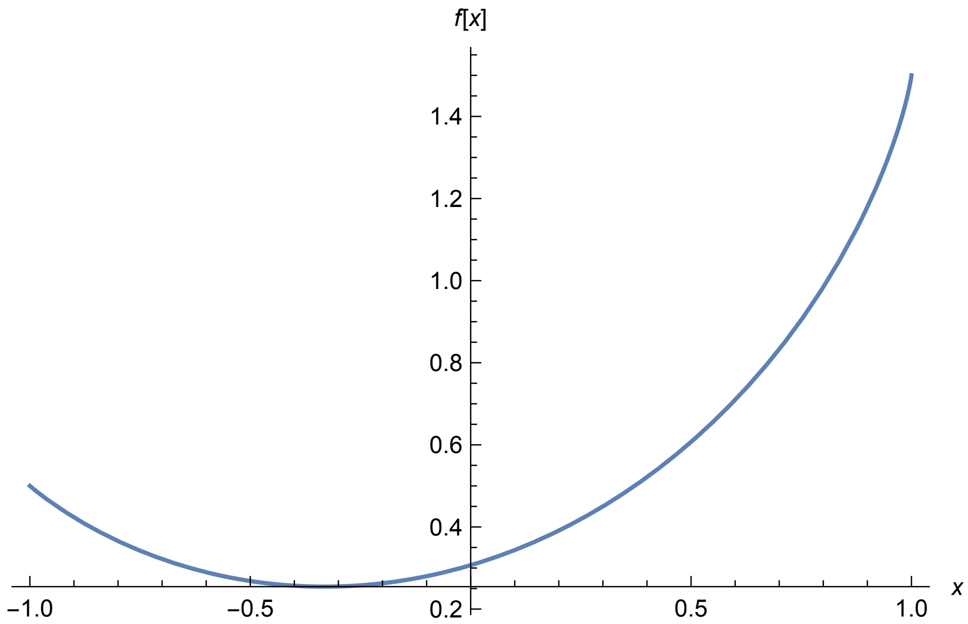
$$A(s,t) = -\frac{su}{t} = \frac{s^2}{t} + s = \frac{u^2}{t} + u$$

we find after a little algebra:

$$\frac{dE_2^{GW}}{d\omega} = \frac{4G(\hbar\omega)^2}{\pi} f(x) \quad ; \quad x = cos\theta_s$$

Corrections to ZFL look quantum and $O(h^2\ \omega^2/Q^2)$
But if we use $Q = h/b$ they become $O(\ \omega^2 b^2)$ (i.e. classical?)

$$f(x) = 1 - \frac{2}{1-x} + \frac{2}{1+x} - \frac{1+x}{1-x}\left(\frac{2}{1-x} - \frac{1-x}{2}\right)\log\left(\frac{1+x}{2}\right)$$
$$+\frac{1-x}{1+x}\left(\frac{2}{1+x} + \frac{1+x}{2}\right)\log\left(\frac{1-x}{2}\right).$$



This result has been checked via a long, explicit calculation in N=8 SUGRA.

Example II : Resummed eikonal a la ACV.

Because of phase O(action/h) derivatives act, to leading order, on the exponent (Cf. WKB). The powers of h cancel and we get a classical contribution. Unfortunately, the infinite Coulomb phase does NOT drop out.

The reason is quite clear: the derivative operators in J_i feel the change of the Coulomb phase due to the change of the hard momenta. Such a change is itself IR divergent. However, also the final soft graviton contributes an IR div. Coulomb phase which is exactly as needed for the cancellation (Cf. CCV18).

The standard soft-graviton recipe misses it and should be amended.

If we follow Sen et al's recipe for dealing with the Coulomb IR logs we can match the result with the one obtained in CCV-18 (for the unpolarized, angle-integrated flux).

We get, like CCV18, a positive correction of order $(\omega b)^2 \log^2(\omega b)$ (but, unlike in CCV18, with a precise coefficient in front) confirming the already mentioned bump in the spectrum around $\omega b = 0.5$.

Complementarity w/ other calculations

- Grav.^{al} bremss. from a gravit^{al} collision occurs @ $O(G^3)$; same as a recent calculation of the 3PM conservative potential/deflection angle (Bern et al. 1901.04424, applied to EOB by Buonanno et. al. 1901.07102).
- Even more recently, a paper by Henn & Mistlberger (1902. 07221) has computed massless 2->2 scattering to three loops ($O(G^4)$) in N=8 SUGRA. Comparison under way…
- A complete answer including radiation @ 3PM level within reach? Important for improving EOB (Damour, 1710.10599).

Summarizing part II

- GW's from ultra-relativistic collisions is an interesting (though probably academic) theoretical problem.

- It is challenging both analytically and numerically, both classically and quantum mechanically.

- The ZFL (for $dE^{GW}/d\omega$) is classical & well understood. In order to go beyond the ZFL two approaches have been followed (besides the CGR one of G+V):

- The first follows the eikonal ACV approach, is limited (so far) to small deflection angles, but extends to frequencies somewhat beyond $1/R \gg 1/b$

- It is free from IR infinities which, however, bring about logarithmic enhancements at $\omega < 1/b$ and are responsible for a peak in the flux around $\omega b = 0.5$.

- The second goes via the soft-graviton theorems. It is not limited to small-angle scattering but is restricted to the $\omega b < 1$ regime.

- Because of IR divergences in 4D, the non-leading soft terms are ill defined and need modifications.

- At sub-(and now sub-sub?)-leading level a recipe due to Sen and collaborators looks to be confirmed by the eikonal-approach results.
- At sub-sub-leading level that same recipe confirms the CCV-18 prediction of a bump in the flux @ $\omega b \sim 0.5$
- Eventually, one would like to extend these results to arbitrary masses and kinematics and to combine them with recent ones on the conservative gravitational potential at 3PM level, leading hopefully to a full understanding of gravitational scattering <u>and</u> radiation at that level.
- With such a motivation in mind I'm pleased to announce:

Workshop on

Gravitational scattering, inspiral, and radiation

(GGI, May 18-July 5, 2020, not incompatible with Amplitudes 2020)

Thank you!