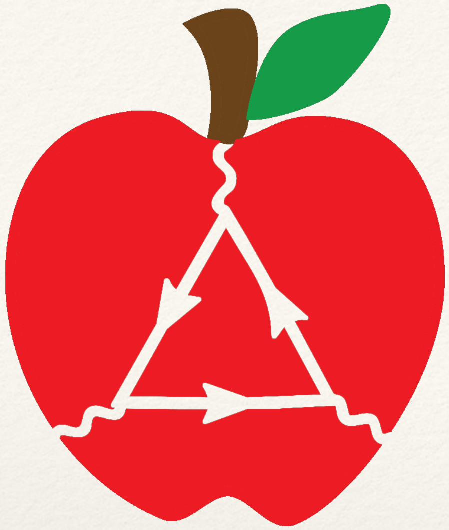

Enrico Herrmann

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In collaboration with: Jaroslav Trnka *(arXiv:1808.10446)*

+ Alex Edison, Julio Parra-Martinez *(to appear)*

Gravity amplitudes from the UV

Amplitudes 2019 - Dublin
07/05/2019

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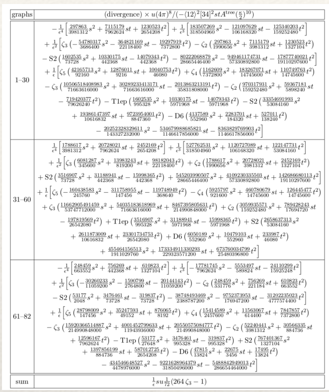
(0) Motivation

- UV behavior of (super-)gravities still mysterious



  - "enhanced" cancellations, e.g. in

    N= 4 sugra, d=5 @ L=2; [Bern,Davies,Dennen,Huang: 1209.2472]

    N=5 sugra, d=4 @ L=4 [Bern,Davies,Dennen: 1409.3089]

  - N=8 sugra: 5-loop divergence in d=24/5 [Bern,Carrasco,Chen,Edison,Johansson,Parra-Martinez,Roiban,Zeng: 1804.09311]

- Beautiful picture of N=4 SYM, where nontrivial analytic properties are understood at integrand level [talk L. Ferro]

  - Can we make similar progress in gravity?

Properties of amplitudes should be encoded before integration

(integration = tracing out = forgetting details)

Outline

- i) Gravity integrands and the UV structure:
  - off-shell nonplanar integrands?
  - unitarity cuts, maximal cuts & power counting
  - multi-particle unitarity cuts and integrand enhanced cancellations
- ii) gravity integrand construction from UV constraints
  - unique construction of 2 & 3-loop gravity integrands
- iii) Conclusions & Outlook

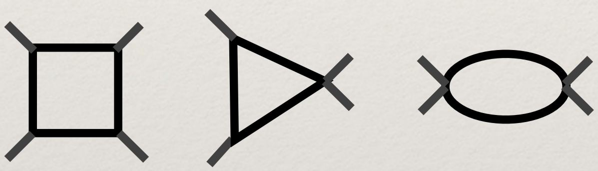
i-1) gravity integrands and the UV structure

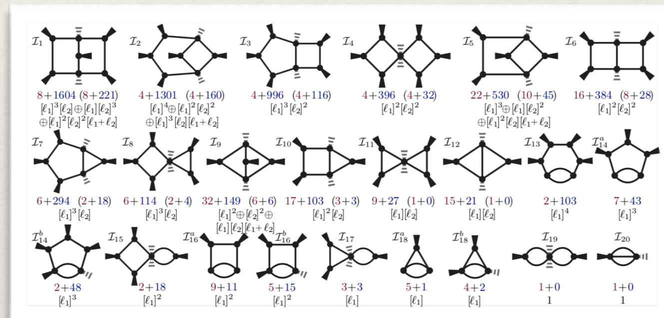
- loop-amplitudes and integrands:

$$\mathcal{M}_n^{(L)} = \sum_k c_k \int \mathcal{I}_k(p_i, \ell_j)\; d^D\ell_1 \cdots d^D\ell_L$$

kinematic coefficients

basis integrands [talk J. Bourjaily]



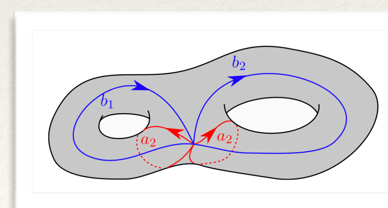


- line-1 problem: no *global* labels of an "integrand"

$$\mathcal{I}(p_i, \ell_j) \stackrel{?}{=} \sum_k c_k \mathcal{I}_k(p_i, \ell_j)$$ ✗

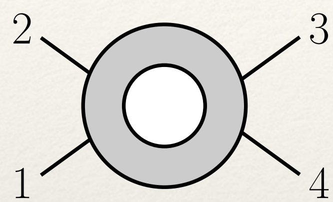
→ Study unitarity cuts of integrands!

string worldsheet?
[Tourkine 1901.02432]

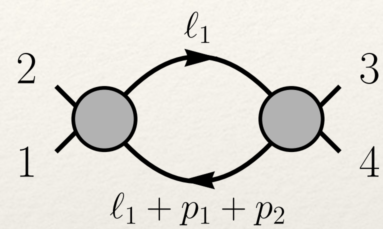


i-2) cuts of loop-integrands

- unitarity cut: [Cutkosky 1960]



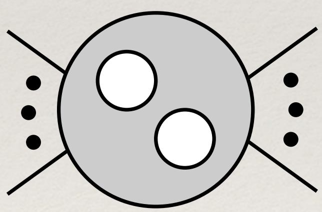
$$\ell_1^2 = (\ell_1 + p_1 + p_2)^2 = 0$$



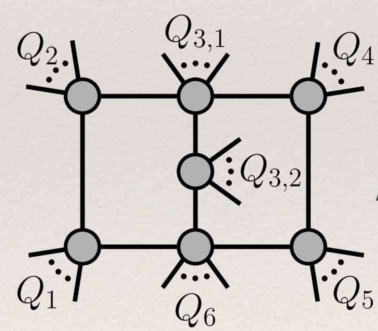
$$\mathop{\mathrm{Res}}_{\ell_1^2=0=(\ell_1+1+2)^2} \mathcal{M}_4^{(1)} = \sum_{\text{states}} \mathcal{M}_L^{(0)} \times \mathcal{M}_R^{(0)}$$

on-shell functions

- generalized unitarity: [Bern,Dixon,Kosower:9708239,0404293; Britto,Cachazo,Feng:0412103]



$$\ell_1^2 = \cdots = \ell_8^2 = 0$$

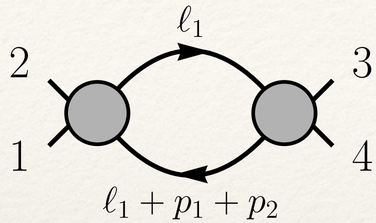


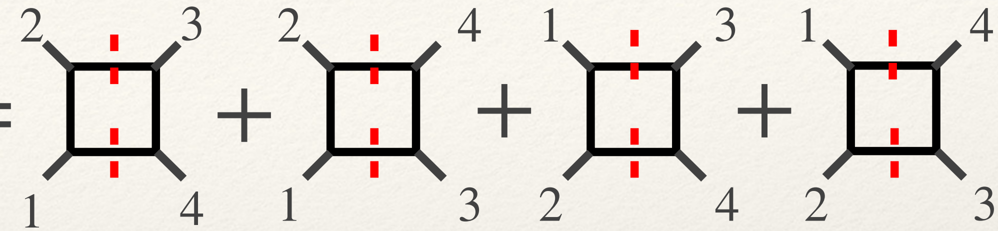
$$\mathop{\mathrm{Res}}_{\ell_i^2=0} \mathcal{M}_n^{(2)} = \sum_{\text{states}} \mathcal{M}_{[1]}^{(0)} \times \cdots \times \mathcal{M}_{[7]}^{(0)}$$

→ well defined loop-variables on the cut

i-2) cuts of loop-integrands

- unitarity cut:



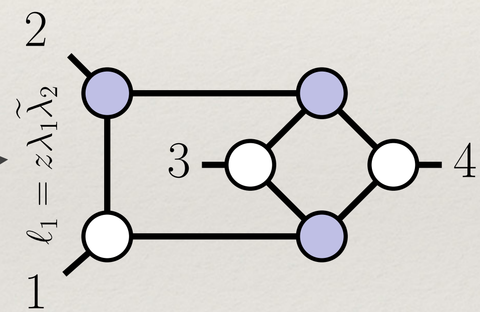


$$= \sum_{\text{states}} \mathscr{M}_L^{(0)} \times \mathscr{M}_R^{(0)}$$

- generalized unitarity:

on-shell diagram

[Arkani-Hamed,Bourjaily,Cachazo,Goncharov, Postnikov,Trnka 1212.5605]



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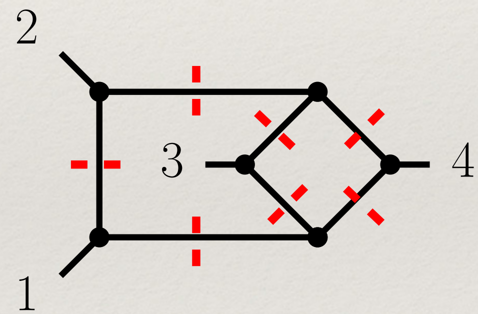
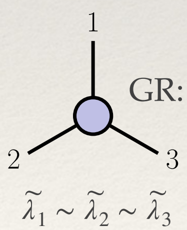
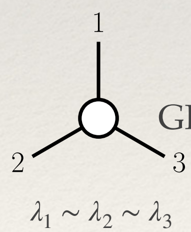




$\text{GR:} = \dfrac{\delta^{16}(\lambda_1\tilde{\eta}_1 + \text{cyc})}{(\langle 12\rangle\langle 23\rangle\langle 31\rangle)^2}$

$\tilde{\lambda}_1 \sim \tilde{\lambda}_2 \sim \tilde{\lambda}_3$



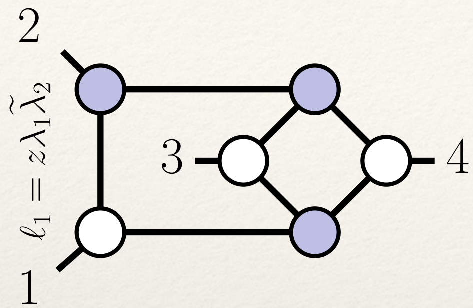
$\text{GR:} = \dfrac{\delta^{16}([23]\tilde{\eta}_1 + \text{cyc})}{([12][23][31])^2}$

$\lambda_1 \sim \lambda_2 \sim \lambda_3$

$$= \sum_{\text{states}} \mathscr{M}_{[1]}^{(0)} \times \cdots \times \mathscr{M}_{[6]}^{(0)}$$

→ maximal cut: isolates single diagram

i-2) maximal cuts and power counting



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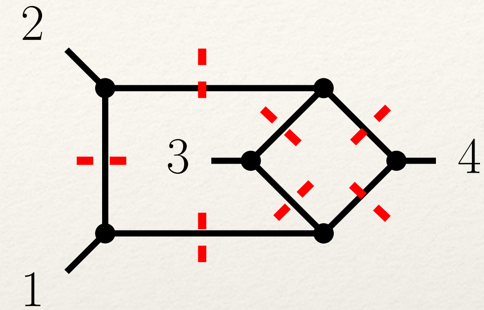
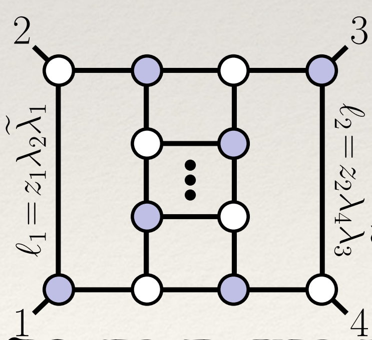


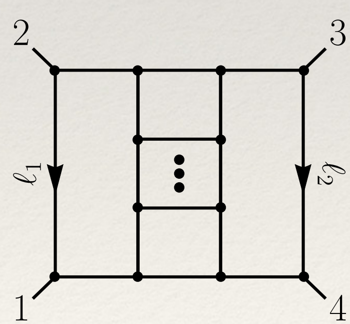
$$\sum_{\text{states}} \mathcal{M}^{(0)}_{[1]} \times \cdots \times \mathcal{M}^{(0)}_{[6]} = f(z;\, p_i) = \operatorname*{Res}_{\ell_i^2=0} \frac{\mathrm{Num}(p_i, \ell_j)}{\prod \ell_i^2} = \frac{\mathrm{Num}(p_i, \ell_j^*)}{\mathcal{J}}$$

$$f(z;\, p_i) = \frac{[12][34]^2}{z\,\langle 12\rangle\langle 13\rangle\big(z\langle 13\rangle + \langle 23\rangle\big)\big(z\langle 14\rangle + \langle 24\rangle\big)\langle 41\rangle}$$

$$\mathcal{J} = z\, s_{12}\langle 14\rangle\big(z[24] - [14]\big)\langle 13\rangle\big(z[23] - [13]\big)$$

$\rightarrow$ $\mathrm{Num}(p_i, \ell_j^*)$ loop-independent





$$N(\ell_i, p_j) \sim \begin{cases} f_{\mathrm{YM}}(p_j) \times (\ell_1 \cdot \ell_2)^{L-3}, & \mathcal{N} = 4 \text{ SYM} \\ f_{\mathrm{GR}}(p_j) \times (\ell_1 \cdot \ell_2)^{2(L-3)}, & \mathcal{N} = 8 \text{ sugra} \end{cases}$$

e.g. in [Bern,Davies,Dennen 1409.3089]

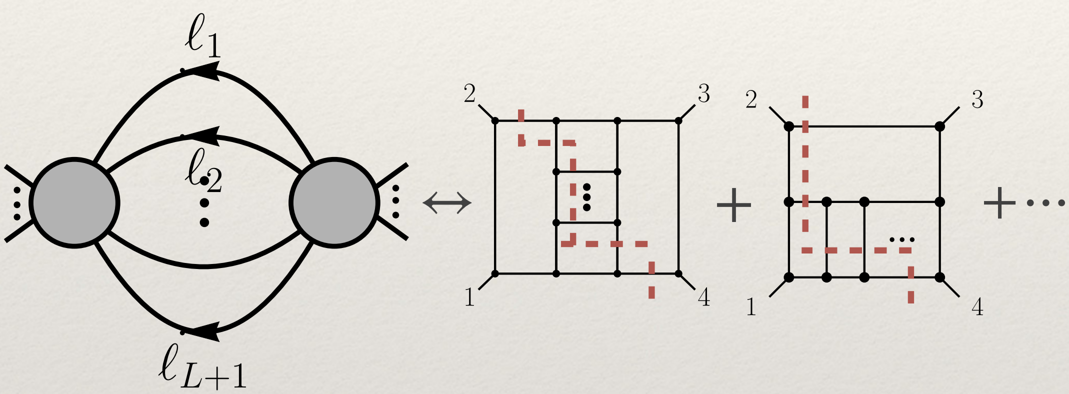
z-scaling of maximal cut dictates power counting of numerator!

i-3) multi-particle unitarity cuts and "enhanced cancellations"

[different cuts analyzed in Bern,Dixon,Roiban: 0611086 based on no-triangle hypothesize for 1-loop amplitudes in N=8 sugra]

maximal cut isolates single diagram → cuts with minimum # of on-shell props

No cancellations — cancellations between diagrams possible



- (L+1) legs cut
- almost all diagrams contribute
- flexibility to "approach" infinity

D-dimensional parametrization of on-shell loop momenta:

$$\ell_i \to \ell_i + z q_i \qquad i = 1 \ldots L$$

$$q_i \cdot q_j = 0 \qquad \ell_i \cdot q_i = 0$$

$$\Rightarrow \ell_i \cdot x \to \ell_i \cdot x + z q_i \cdot x$$

$D = 4$ limit →

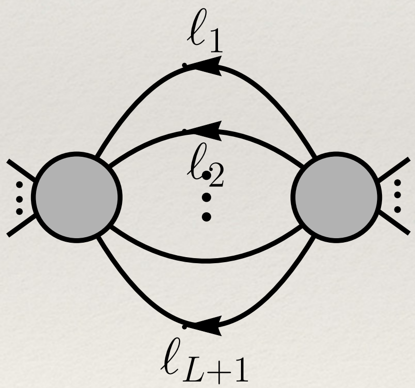
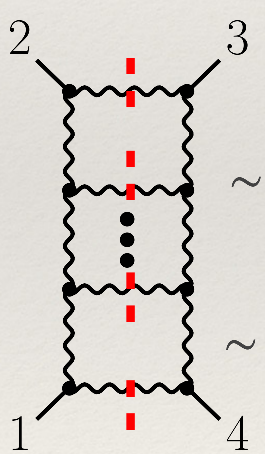
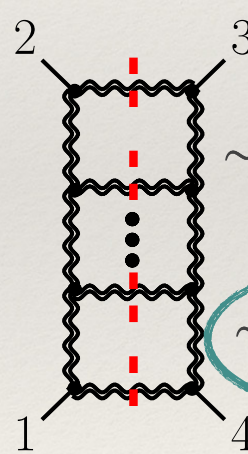
$$\ell_i = \lambda_{\ell_i}\left(\tilde{\lambda}_{\ell_i} + z c_i \tilde{\chi}\right)$$

gravity cut scaling compared to a) SYM b) individual diagrams ?

i-3) multi-unitarity cut UV scaling

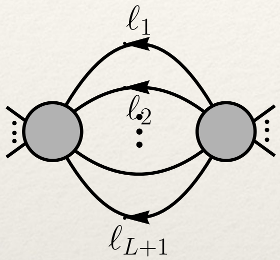
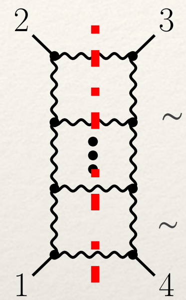
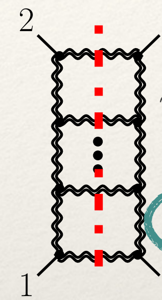
- Naive scaling of Feynman diagrams $(\ell_i \cdot \ell_j) \sim z, \quad \ell^2 \sim z, \quad \epsilon \cdot \ell \sim z$

2L+2 vertices 3L+1 props - (L+1) cuts = 2L uncut props

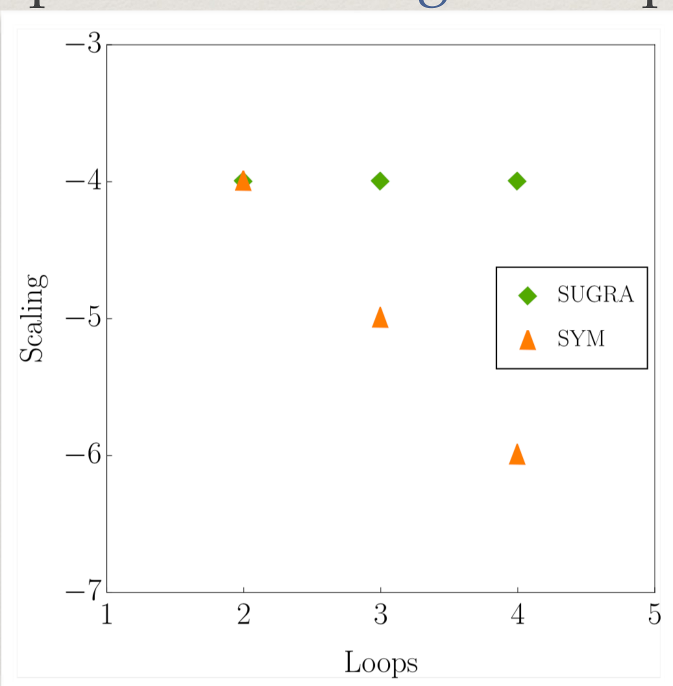
	(super-) Yang-Mills	(super-) gravity
	 $\sim \frac{(\epsilon\cdot\ell)^4\,[\ell_i\cdot\ell_j]^{\frac{2L+2-4}{2}}}{[\ell^2]^{2L}}$ $\sim \frac{z^{L+3}}{z^{2L}} = z^{-L+3} \xrightarrow{\text{susy}} z^{-L-1}$	 $\sim \frac{(\epsilon\cdot\ell)^8\,[\ell_i\cdot\ell_j]^{2L+2-4}}{[\ell^2]^{2L}}$ $\sim z^6 \xrightarrow{\text{susy}} z^{-2}$

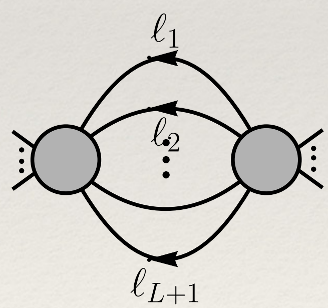
Compare diagram scaling to full cut?

i-3) cut vs. diagram UV scaling

$(\ell_i\cdot\ell_j)\sim z,\quad \ell^2\sim z,\quad \epsilon\cdot\ell\sim z$	sYM	suGRa
	 $\sim \frac{(\epsilon\cdot\ell)^4[\ell_i\cdot\ell_j]^{\frac{2L+2-4}{2}}}{[\ell^2]^{2L}}$ $\sim \frac{z^{L+3}}{z^{2L}} = z^{-L+3} \overset{\text{susy}}{\longrightarrow} z^{-L-1}$	 $\sim \frac{(\epsilon\cdot\ell)^8[\ell_i\cdot\ell_j]^{2L+2-4}}{[\ell^2]^{2L}}$ $\sim z^6 \overset{\text{susy}}{\longrightarrow} z^{-2}$

- explicit local integrand representations and check d-dim large z-scaling





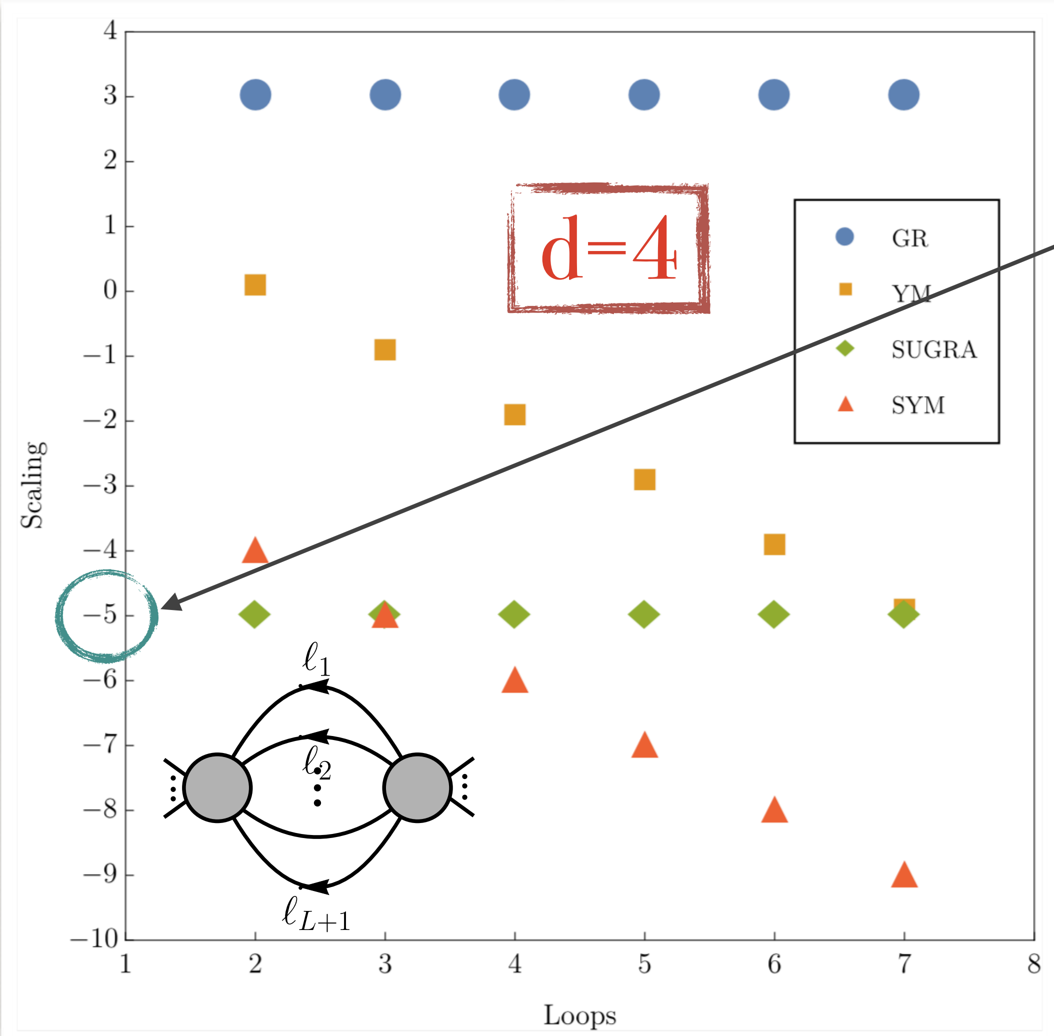
SYM : z^{-L-2} vs. naive z^{-L-1}

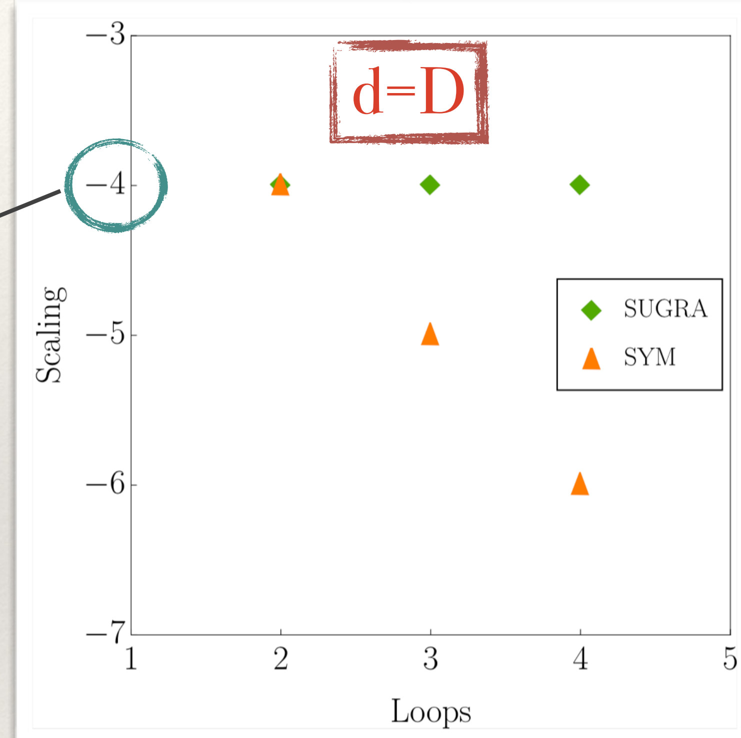
sugra : z^{-4} vs. naive z^{-2}

Is there anything special about d=4 ?

i-3) d=4 UV scaling of cut

- use KLT and BCFW for N=4 SYM amps to check 4-dim large z-scaling





In d=4, SUGRA drops 1 power in large z scaling!

@ L=2: drop due to Gram determinant that vanishes in d=4

ii) gravity integrand construction from UV constraints

- Turn things upside down

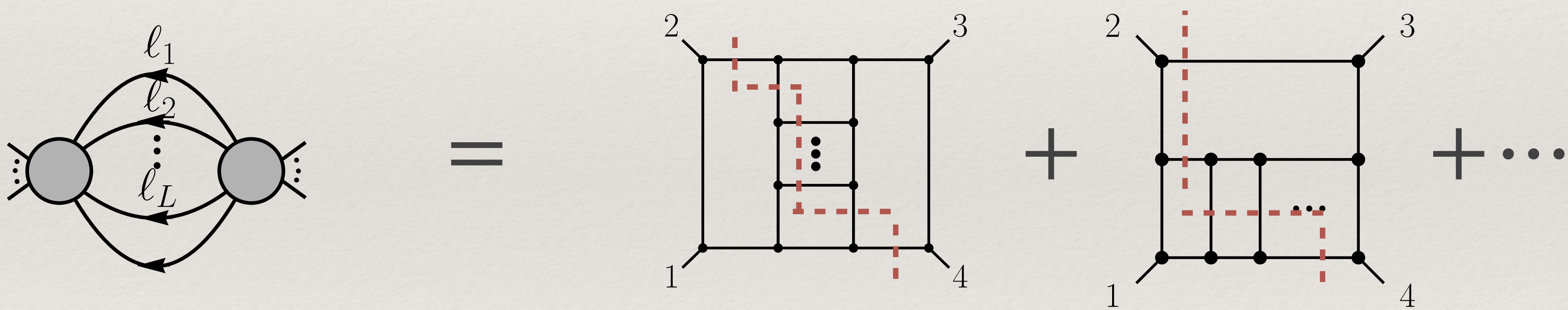
Impose the good large-z scaling on multi-particle unitarity cuts as constraints on an ansatz?

Is there a unique object that satisfies the large-z scaling?

ii) gravity integrand construction from UV constraints

- write numerator ansatz for each local diagram
  - triangle-power counting
  - numerator respects diagram symmetries

- multi-unitarity cut! L+1 props on-shell



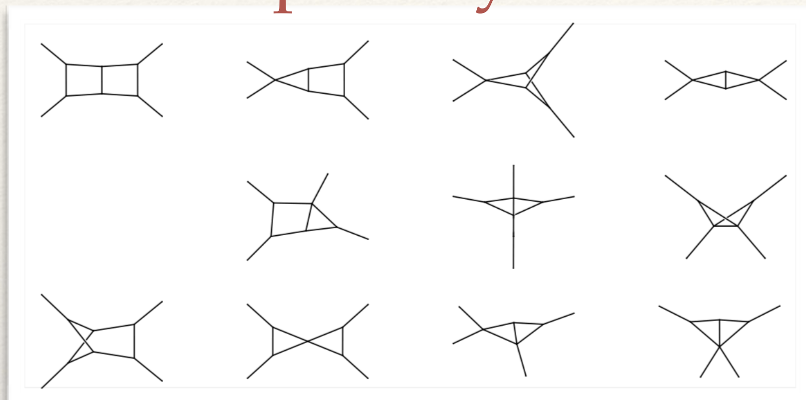
$$\mathscr{M}(z)\,|_{\text{cut}} \sim z^{a} = z^{b_1} + z^{b_2} + z^{b_3} + \cdots$$

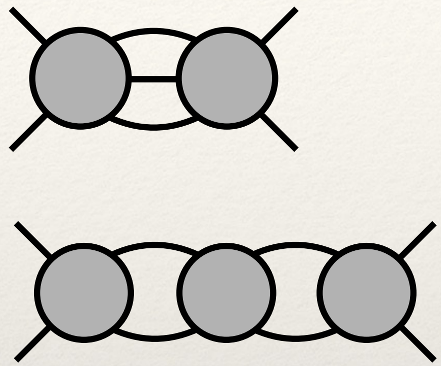
- demand: $\forall b_i > a$, set coefficients to zero as $\ell_i(z) \overset{z\to\infty}{\to} \infty$

Can impose the scaling in all directions at infinity!

ii) gravity integrand construction from UV constraints

- 2-loop analysis:

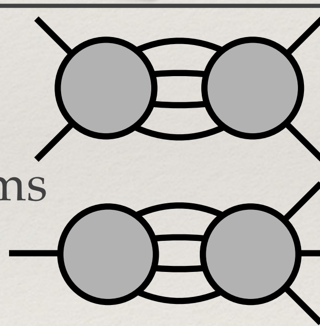


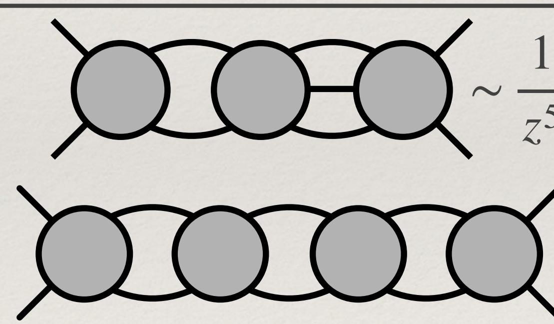


$\sim \frac{1}{z^5}$, $\sim \frac{1}{z^4}$ } $N_{db} = N_{npdb} = a\, s^2$, $N_{\text{other}} = 0$

- 3-loop analysis:

$\sim$ 2700 parameters, 83 diagrams

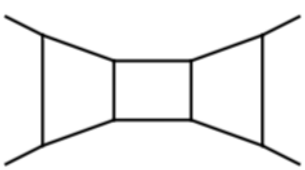




$\sim \frac{1}{z^5}$ $\sim \frac{1}{z^5}$ $\sim \frac{1}{z^5}$ $\sim \frac{1}{z^4}$

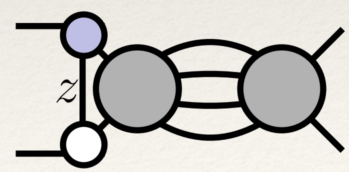
50 unfixed dof

problem:



$= a_1 s^4 + a_2 s^3 t + a_2 s^2 t^2 \sim \frac{1}{z^6}$

distinguish via BCFW-shift of external momenta →



We find a unique solution after shifting external kinematics as well

iii) Conclusions & Outlook

- analyzed cuts of (super-)gravity integrands through 7-loops!
  - improvement of d-dimensional cut scaling
  - further drop in UV scaling in d=4
    - status of 7-loop divergence?
  - how much of the UV scaling can be understood from trees?
    - analyze more general deformations of gravity tree-amplitudes
- imposing UV constraints on ansatz fixed N=8 sugra amplitude uniquely to L=3, n=4
  - How does this extend to higher loops, higher points? [Bourjaily,EH,Trnka 1812.11185]
  - Including deformation of external momenta, do we have the full list of homogeneous constraints that "define" gravity?
  - Can we "geometrize" these properties?

THANK YOU FOR YOUR ATTENTION