$\mathcal{N}=2$ homogeneous supergravities at one loop

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Abstract

There is a rich space of $\mathcal{N}=2$ supergravities with matter, many of which can not be constructed as truncations of $\mathcal{N}=8$ or matter-coupled $\mathcal{N}=4$ supergravity. One of the largest known family of such double-copy-constructible theories are the $\mathcal{N}=2$ homogeneous supergravities [1]. We study the one-loop divergence of these theories using the double-copy construction, and find a relation between the divergence of the supergravity amplitudes and the beta function of one of the gauge theories. Two contributions appear in the divergence, one of which is cancelled only for the four magical supergravities.

Homogeneous $\mathcal{N}=2$ supergravities

- In $5D$ with n vector multiplets; $n+1$ abelian fields A^I, n scalar fields ϕ^x and n spinors [2], Lagrangian:

$$e^{-1}\mathcal{L} = -\frac{1}{2}R - \frac{1}{4}\mathring{a}_{IJ}F^I F^J - \frac{1}{2}g_{xy}\partial_\mu\phi^x\partial^\mu\phi^y + \frac{e^{-1}}{6\sqrt{6}}C_{IJK}\epsilon^{\mu\nu\rho\lambda}F^I_{\mu\nu}F^J_{\rho\sigma}A^K_\lambda + \text{fermions} \ .$$

- The theory is completely fixed by C_{IJK}.
- Classified by de Wit Van Proeyen [3] by defining $\mathcal{V}(\xi) \equiv C_{IJK}\xi^I\xi^J\xi^K$, which was shown to take the form

$$\mathcal{V}(\xi) = \sqrt{2}(\xi^0(\xi^1)^2 - \xi^0(\xi^i)^2) + \xi^1(\xi^\alpha)^2 + \Gamma^i_{\alpha\beta}\xi^i\xi^\alpha\xi^\beta \ .$$

- Γ form a representation of a Clifford algebra $\mathcal{C}(D-5)$ for $D \geq 5$, classified by D and number of "flavours" P (and $\dot{P}$ if left/right-handed representations are independent)

The double copy construction

- Gravity amplitudes from gauge-theory by replacing color by kinematics [4];

$$\mathcal{A} \propto \sum_i \int d\tilde{l}\frac{c_i n_i}{S_i D_i} \ , \quad \mathcal{M} \propto \mathcal{A}\big|_{c_i\to\tilde{n}_i} \propto \sum_i \int d\tilde{l}\frac{n_i\tilde{n}_i}{S_i D_i} \ .$$

- The double-copy of Hom. SUGRA takes the form [1]

$$(\mathcal{N}=2 \text{ hom. sugra}) = (\mathcal{N}=2 \text{ SYM}+\frac{1}{2}\text{hyper}) \otimes (\text{YM}_D + P \text{ fermions}) \ ,$$

With fermions in a pseudo-real representation.

- D-dimensional Lagrangian for the right-hand theory is

$$\mathcal{L}_R = \frac{1}{4}F^{\hat{a}}_{\mu\nu}F^{\hat{a}\mu\nu} + \frac{i}{2}\bar{\lambda}\Gamma^\mu D_\mu\lambda \ ,$$

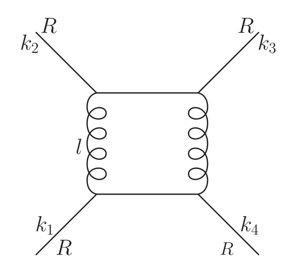
possible reality conditions on the fermions λ are given below.

D	$n_F(D,P,\dot{P})$	conditions	flavor group
5	P	R	$SO(P)$
6	$P+\dot{P}$	RW	$SO(P)\times SO(\dot{P})$
7	$2P$	R	$SO(P)$
8	$4P$	R/W	$U(P)$
9	$8P$	PR	$USp(2P)$
10	$8P+8\dot{P}$	PRW	$USp(2P)\times USp(2\dot{P})$
11	$16P$	PR	$USp(2P)$
12	$16P$	R/W	$U(P)$
$k+8$	$16\,n_F(k,P,\dot{P})$	as for k	as for k

Table 1: n_F is the number of 4 dimensional spinors. R, PR and W stand for Real, Pseudo-Real and Weyl conditions on the fermions.

Results

- One-loop amplitudes for the left-hand theory with external hypermultiplets obtained by an orbifold procedure from $\mathcal{N}=4$ SYM [5]. Color and numerator of master diagram:



$$C = (\tilde{T}^{\hat{a}}\tilde{T}^{\hat{b}})_{\hat{\alpha}}{}^{\hat{\delta}}(\tilde{T}^{\hat{a}}\tilde{T}^{\hat{b}})_{\hat{\beta}}{}^{\hat{\gamma}} \ , \quad \tilde{n} = \frac{s^2}{\langle 12\rangle\langle 34\rangle}\delta^4(Q)$$

  - Amplitudes obey color-kinematics duality.
  - Numerators do not depend on loop momenta, which simplifies double copy:

$$\mathcal{M}^{(1)} = \left(\frac{\kappa}{2}\right)^4 \sum_i \int \frac{d\tilde{l}}{S_i D_i}\tilde{n}_i n_i = \left(\frac{\kappa}{2}\right)^4 \sum_i \tilde{n}_i \int \frac{d\tilde{l}}{S_i D_i} n_i \ .$$

  These integrals give vertex and propagator corrections which are pieces of **beta functions** of $\mathcal{L}_R$.

- Substituting for $\tilde{n}$ the s channel contribution is

$$\mathcal{M}^{(1)}_{UV} \propto \frac{s\delta^4(Q)}{\langle 12\rangle\langle 34\rangle}\left\{sA^{\text{tree}}_{s,\phi}\left(\beta_{\phi,T(G)} - \frac{\beta_{\phi,T(R)}}{2}\right) + sA^{\text{tree}}_{s,A}\left(\beta_{A,T(G)} - \frac{\beta_{A,T(R)}}{2}\right)\right\}\frac{c_\Gamma}{\epsilon}$$

  - Manifest $\mathcal{N}=2$ SUSY.
  - Local (similar contributions for t and u channels).

- Feynman-rule calculation for the β function gives

$$\mathcal{M}^{(1)}_{UV} \propto \frac{s\delta^4(Q)}{\langle 12\rangle\langle 34\rangle}\left\{sA^{\text{tree}}_{s,\phi}\left(1 + \frac{n_F}{4} - \frac{n_S}{2}\right) + sA^{\text{tree}}_{s,A}\left(\frac{11}{6} + \frac{n_F}{6} - \frac{n_S}{12}\right)\right\}\frac{c_\Gamma}{\epsilon}$$

  where n_F and n_S are numbers of $4D$ fermions and scalars.

  - The scalar channel contribution can be made zero for $P=1$ and $D=7,8,10,14$, which are the four **magical supergravities**.
  - No valid choice of parameters cancels vector channel.

Conclusion

- We find no amplitude which remains finite at one loop.
- Divergence is alleviated for magical supergravities.
- Divergence of supergravity could be linked to that of the non-susy theory in the double-copy construction.
- External vectors were also computed in [1], with double copy $A^0_- = \phi \otimes A_-$, $A^a_- = A_- \otimes \phi^a$. Amplitudes agree with ours for Magical supergravities; they are **unified**.

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