Asymptotic symmetries and coherent states in QCD

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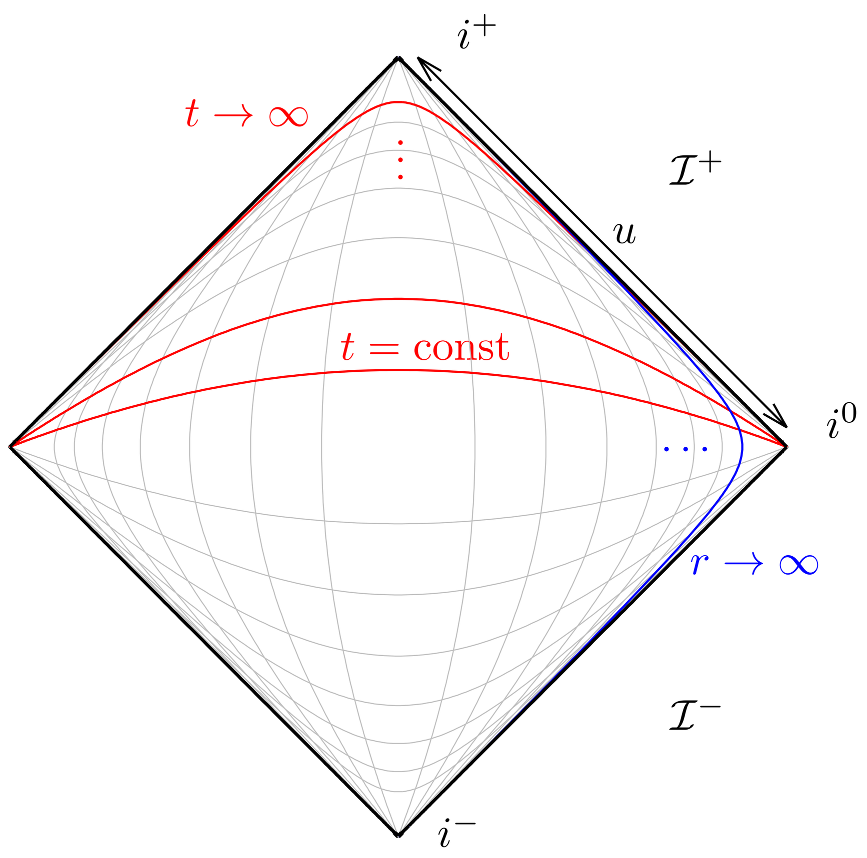
Summary

- Framework: Perturbative QCD, assuming factorization of hard and soft scales
- The charge is derived from QCD asymptotic symmetries at leading order in the large radius expansion
- The states are dressed according to the Faddeev-Kulish construction: the coherent states were studied by S.Catani,G.Marchesini and M.Ciafaloni in the 90'
- Infrared-finite S-matrix defined from the soft Möller operators at leading logarithmic order in the soft divergences
- The insertion of the charge with an ordering prescription gives rise to a Ward identity at one-loop leading logarithmic order
- Ordering ambiguities arise in the order of soft limits which give rise to different interpretation of the same result

A new exciting one-loop Ward identity in QCD with FK dressed states at leading logarithmic order!

Charges and coherent states definition



Using retarded radial coordinates for $\mathcal{I}^+$

$$ds^2 = -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z} \qquad \text{with} \qquad \gamma_{z\bar{z}} = \frac{2}{(1+z\bar{z})^2}$$

the leading YM soft charge in the Lorenz gauge $\nabla_\mu \mathcal{A}^{a,\mu}(x) = 0$ is

$$Q^{\rm lin}_\epsilon = \int_{\mathcal{I}^+} d^2z du \epsilon^a(z,\bar{z})\,[\partial_u(\partial_z A^a_{\bar{z}} + \partial_{\bar{z}} A^a_z)]$$

We split the QCD Hamiltonian into soft and hard parts

$$H^I(t) = H^E_h(t) + H^E_s(t)$$

where in the eikonal regime at late times the soft hamiltonian receives contribution also from non-linear self-interactions of the gluons. The soft Möller operator reads

$$\Omega_E = \mathcal{P}_\omega \exp\Big[\int_\lambda^E \widetilde{dq}\, \mathcal{J}_q \cdot \Pi_q\Big]$$

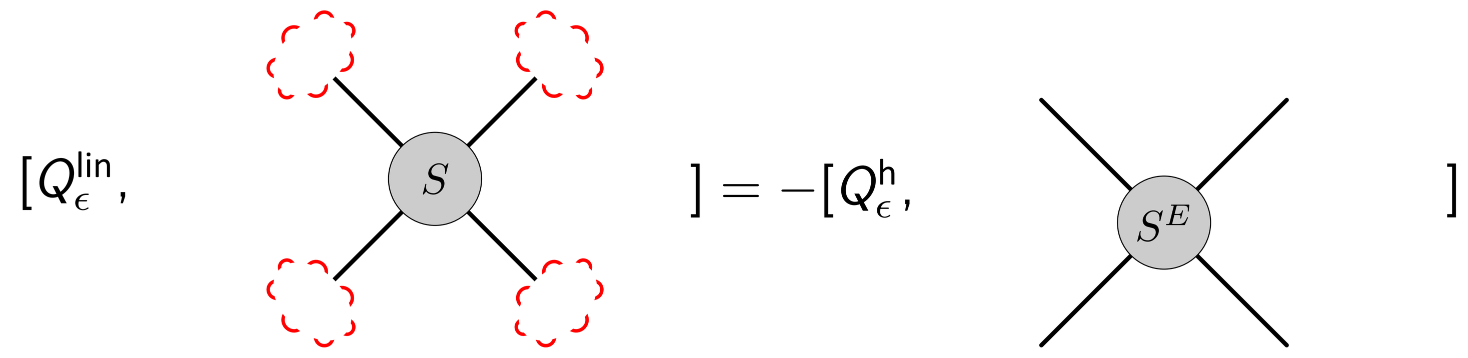
where $\Pi^a_\mu(q) = a^a_\mu(q) - a^{a\dagger}_\mu(q)$ is the famous displacement operator and

$$\mathcal{J}^a_{q\,\mu} = g_{\rm YM} \int_{\omega_q} \widetilde{dp}\; [\rho^a_{\rm f}(\mathbf{p}) + \rho^a_{\rm g}(\mathbf{p})]\, \frac{p_\mu}{p\cdot q}$$

The infrared finite S-matrix is defined as

$$S^E = \Omega^E_- S \Omega^{E\dagger}_+$$

One loop Ward identity calculation



At leading log order the dressing factorizes in colour space so that

$$\|\{p_i,\alpha_i\}\rangle\!\rangle \equiv \Omega^{E\dagger}|\{p_i,\alpha_i\}\rangle = \prod_{i\in{\rm in}} \mathcal{U}^{p_i E}_{\alpha_i\beta_i}(\Pi)\, b^\dagger_{\beta_i}(p_i)|0\rangle$$

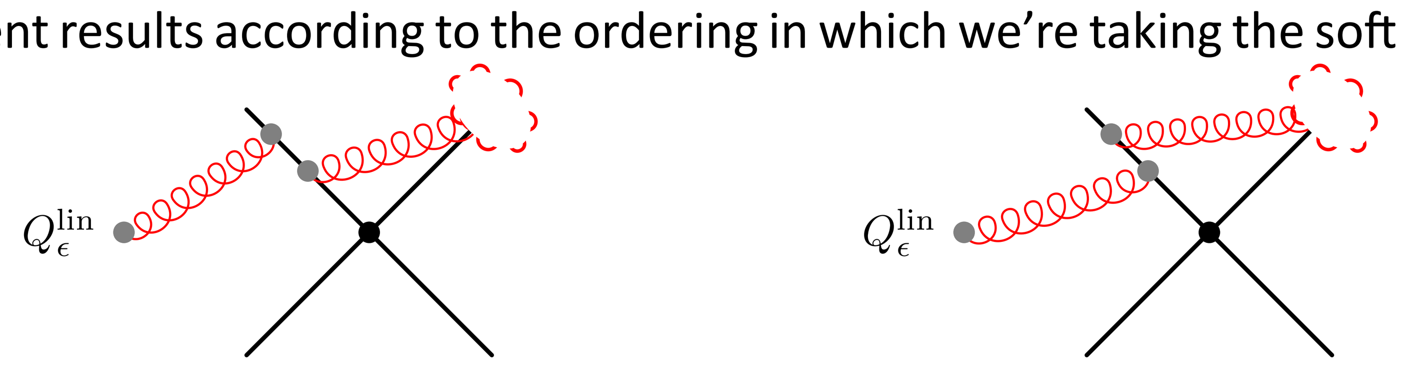
and at tree level we have

$$\langle\!\langle\{p_f,\alpha_f\}\|\,[Q^{\rm lin}_\epsilon, S]\,\|\{p_i,\alpha_i\}\rangle\!\rangle = -\Big[\sum_{\ell\in{\rm out}} Q^{\rm h}_\epsilon(p_\ell) - \sum_{\ell\in{\rm in}} Q^{\rm h}_\epsilon(p_\ell)\Big]\,\mathcal{M}^{(0)}_n$$

where $\mathcal{M}^{(0)}_n$ is the n-particle tree level scattering amplitude and the hard charge is

$$Q^{h,a}_\epsilon(p) = -8\pi^2 \int \widetilde{dq}\,\frac{\delta(\omega)}{\sqrt{\gamma_{z\bar{z}}}}\Big[\partial_z \epsilon^a(\widehat{q})\frac{\varepsilon^- \cdot p}{q\cdot p} + \partial_{\bar{z}} \epsilon^a(\widehat{q})\frac{\varepsilon^+ \cdot p}{q\cdot p}\Big]$$

At one-loop leading log there are many more contributions and we have found different results according to the ordering in which we're taking the soft limits:



Ordering O_1 Ordering O_2

$$\langle\!\langle 0\|\,Q^{\rm lin}_\epsilon S\|{\rm in}\rangle\!\rangle\big|_{\mathcal{O}(g^3_{\rm YM})} \overset{O_1}{=} -\frac{g^2 C_A}{16\pi^2\widehat{\epsilon}^2}\sum_{\ell\in{\rm in}} Q^{\rm h}_\epsilon(p_\ell)\langle\!\langle 0\|S\|{\rm in}\rangle\!\rangle$$

$$\langle\!\langle 0\|\,Q^{\rm lin}_\epsilon S\|{\rm in}\rangle\!\rangle\big|_{\mathcal{O}(g^3_{\rm YM})} \overset{O_2}{=} 0 + \mathcal{O}(\tfrac{1}{\widehat{\epsilon}})$$

While in the first case the Ward identity receives pure one-loop corrections (which may be included in the hard charge), in the second case we have found that the Ward identity is unaltered by loop effects:

$$\langle\!\langle {\rm out}\|[Q^{\rm lin}_\epsilon, S]\|{\rm in}\rangle\!\rangle\big|_{\mathcal{O}(g^3_{\rm YM})} \overset{O_2}{=} 0 + \mathcal{O}(\tfrac{1}{\widehat{\epsilon}})\,.$$

Acknowledgments

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